TMD densities from the Parton branching method

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Parton Branching method

1) Introduction to the Parton Branching solution of DGLAP

2) Parton showers with virtuality or angle as an ordering variable, discussion about kT distribution of the TMDs

3) NNLO DGLAP evolution performed by Parton Branching method

See also the DIS talk by O. Lelek:
Collinear and TMD densities from Parton Branching method

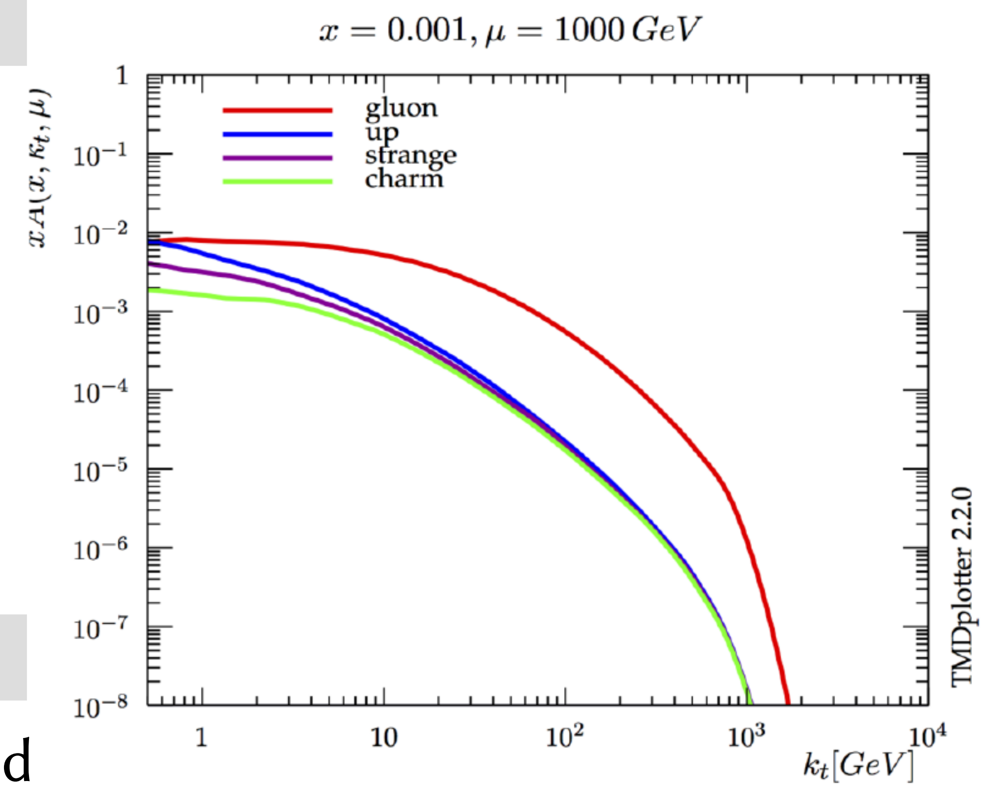
Motivation

Parton branching method is:

- An analogy to the MC parton showers but is used to solve evolution equation
- In case of DGLAP equation the collinear part exactly reproduce semi-analytical solution

And allows:

- Trace the k_T of each emissions and determine the k_T part of PDFs
- Study different kinds of branching branching dynamics (ordering conditions, resolution condition) and determine their effect on PDFs



Different flavors have different shapes of kT distribution.

Similar approaches in:
S. Jadach et al., Comput.Phys.Commun. 181 (2010) 393.
H. Tanaka, Prog. Theor. Phys., 110:963, 2003.

DGLAP splittings decomposition

- The evolution employs momentum weighted densities

$$f_a(x,\mu^2) \to \tilde{f}_a(x,\mu^2) = x f_a(x,\mu^2)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\tilde{f}_a(x,\mu^2) = \sum_b \int_x^1 \frac{\mathrm{d}z}{z} z P_{ab}\left(\alpha_s(\mu^2), z\right) \tilde{f}_b(x/z,\mu^2)$$

Parton Branching solution relays on:

1) Decomposition of the splitting kernels
(valid at least to NNLO)

$$zP_{ab}\left(\alpha_s, z\right) = D_{ab}(\alpha_s)\delta(1-z) + K_{ab}(\alpha_s, z)\frac{1}{(1-z)_+} + R_{ab}(\alpha_s, z)$$

Where K and R do not contain any power-like singularities like $1/z$ or $1/(1-z)$

2) Sum rules $\sum_b \int_0^1 \mathrm{d}z\, zP_{ba}\left(\alpha_s(\mu^2), z\right) = 0, \quad \text{for every flavor } a$

Sudakov Formalism

- With momentum sum rules and Sudakov, the evolution can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\frac{\tilde{f}_a(x,\mu^2)}{\Delta_a(\mu^2)} = \sum_b \int_x^{z_m} \frac{\mathrm{d}z}{z} z P_{ab}\left(\alpha_s(\mu^2), z\right) \frac{\tilde{f}_b(x/z,\mu^2)}{\Delta_a(\mu^2)}$$

- Where the Sudakov is:

$$\Delta_a(\mu^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu^2}{\mu^2} \sum_b \int_0^{z_m} \mathrm{d}z\, z P_{ba}(\alpha_s(\mu^2), z)\right)$$

- The cut-off $z_m < 1$ determine what is still resolvable branching
- The delta part and +prescription of splittings is outside of the integration range (soft emissions resumed by Sudakov)
- This solution is identical to DGLAP as soon as z_m is large enough

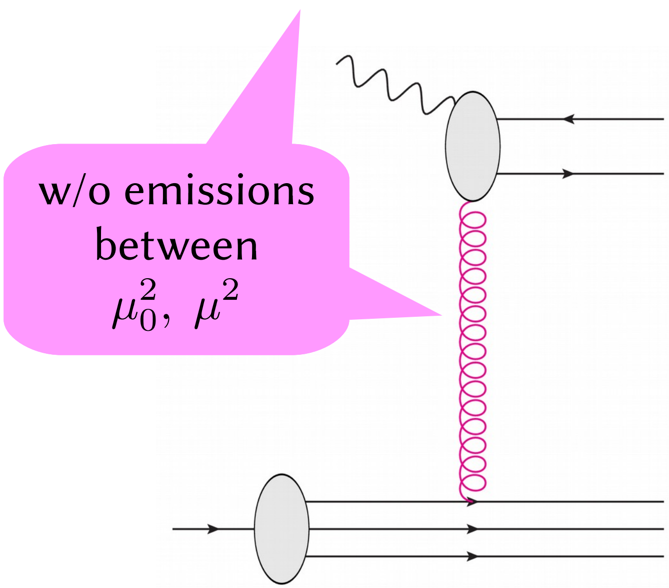
Iterative solution

- Integral form of the evolution equation:

$$\tilde{f}_a(x,\mu^2) = \Delta_a(\mu^2)\tilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_m} \frac{\mathrm{d}z}{z} z P_{ab}\left(\alpha_s(\mu'^2), z\right) \tilde{f}_b(x/z, \mu'^2)$$

- Iterative solution:

$$\tilde{f}_a^{(1)}(x,\mu^2) = \Delta_a(\mu^2)\tilde{f}_a^{(0)}(x,\mu_0^2)$$



Iterative solution

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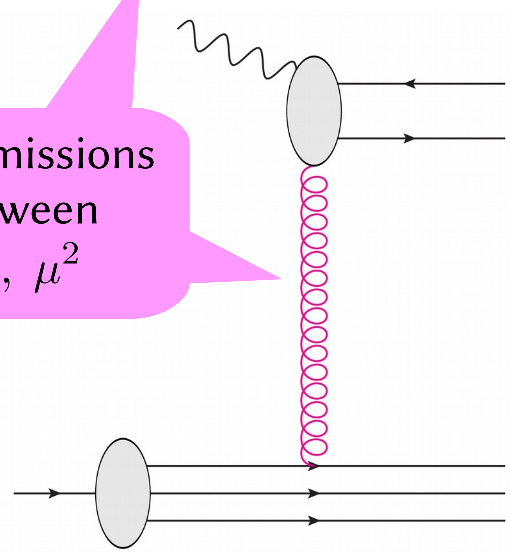
Splitting probability

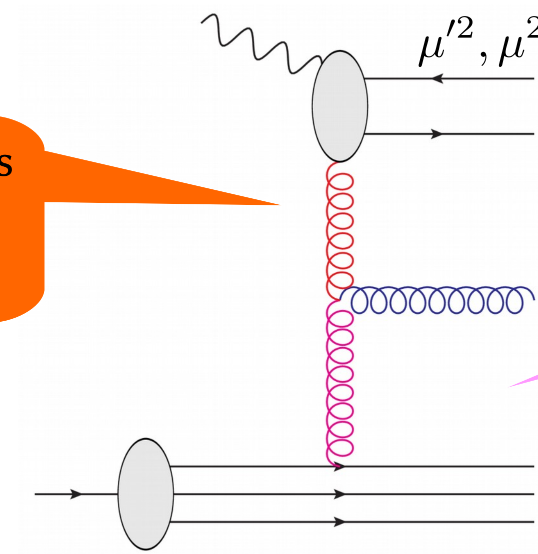
- Iterative solution:

$$\tilde{f}_a^{(2)}(x,\mu^2) = \Delta_a(\mu^2)\tilde{f}_a^{(0)}(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_m} \frac{\mathrm{d}z}{z} z P_{ab}\left(\alpha_s(\mu'^2), z\right) \Delta_b(\mu'^2) \tilde{f}_b^{(0)}(x/z, \mu_0^2)$$

w/o emissions between $\mu_0^2,\ \mu^2$



w/o emissions between $\mu'^2,\ \mu^2$

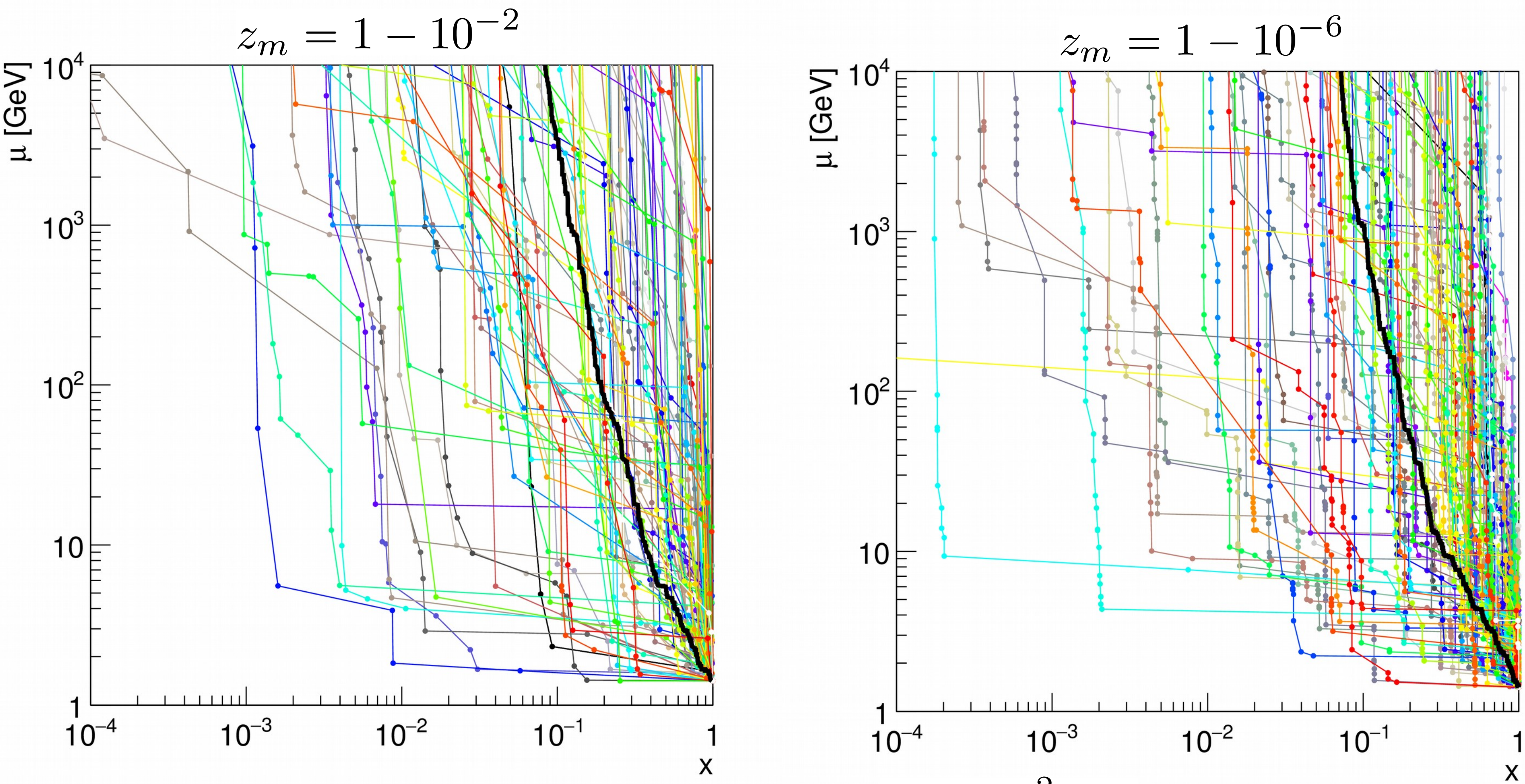


w/o emissions between $\mu_0^2,\ \mu'^2$

Monte Carlo solution

1) Starting with $\tilde{g}(x, \mu_0^2) = \tilde{g}(x, 2) = \delta(1-x)$ (for these plots)
2) The position of every next branching (dot) depends only on the previous one and is randomly generated using Sudakov and splitting kernels

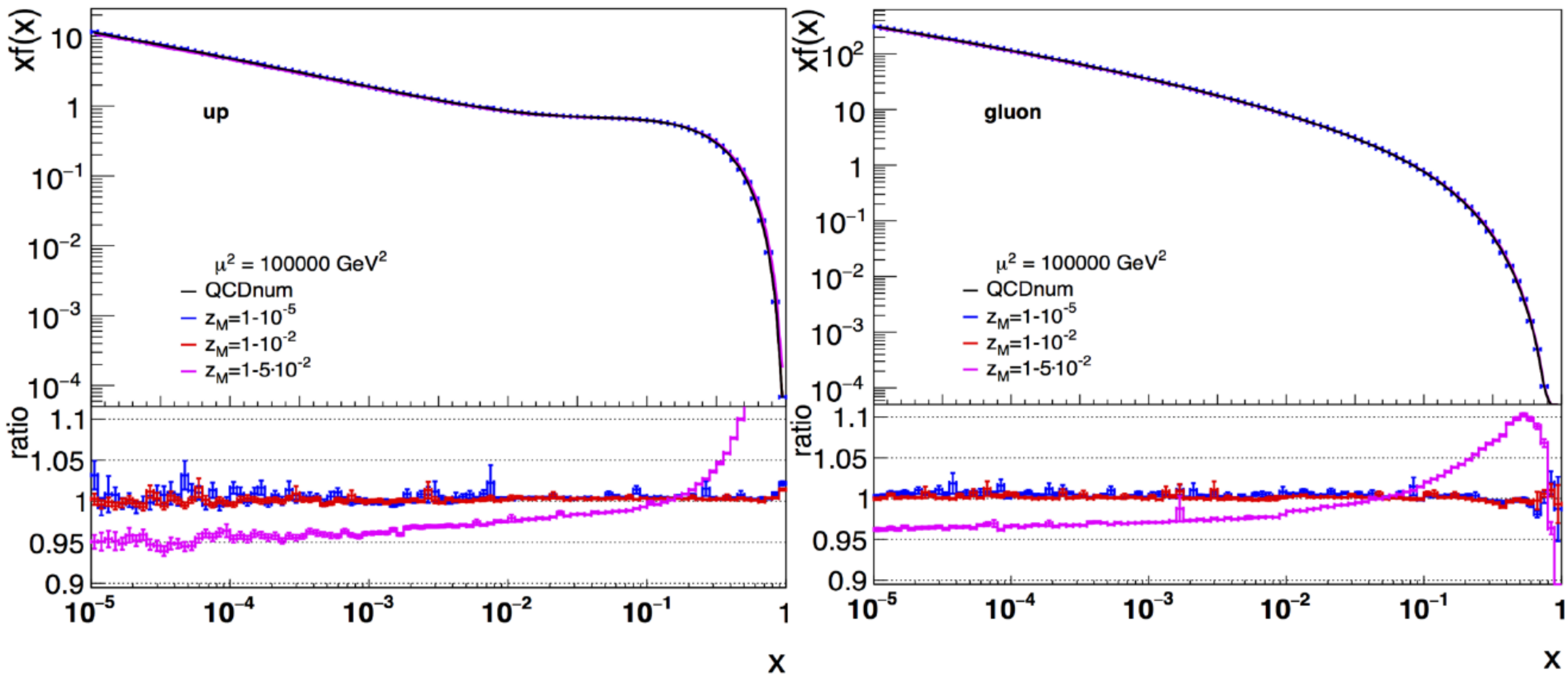
Higher z_m cut-off cause more soft emissions (dots with similar x)



100 LO MC evolution paths from the point $x = 1, \mu^2 = 2$ plotted

Resolvable branching dependence

- The parameter z_m separate resolvable branchings from non-resolvable and virtual one



- The z_m affects high-x region, no difference if $z_m > 0.99$
- Momentum sum rules still holds irrespectively on z_m
- Possibility to use $z_m(\mu^2)$ like in showers of MC generators.

Virtuality and angular ordering

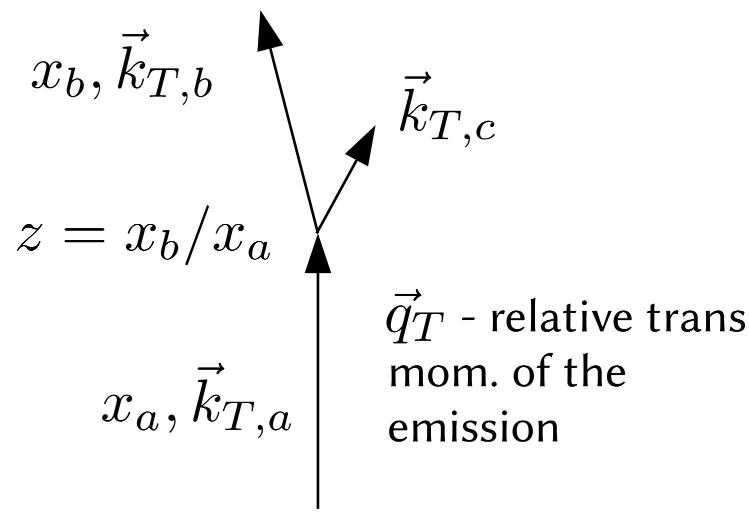
- The Parton Branching method allows to study different parton shower ordering conditions
  $\longrightarrow$ the bridge between MC parton showers and PDF fits from analytic DGLAP solution

- **Virtuality ordering** ($\mu^2 \stackrel{\text{def}}{=\!=} Q^2$)

$$q_T^2 = (1-z)\,Q^2 \stackrel{\text{def}}{=\!=} (1-z)\;\mu^2$$

- **Angular ordering** ($\mu^2 \stackrel{\text{def}}{=\!=} \theta^2$)

$$q_T^2 = (1-z)^2\theta^2 \stackrel{\text{def}}{=\!=} (1-z)^2\mu^2$$



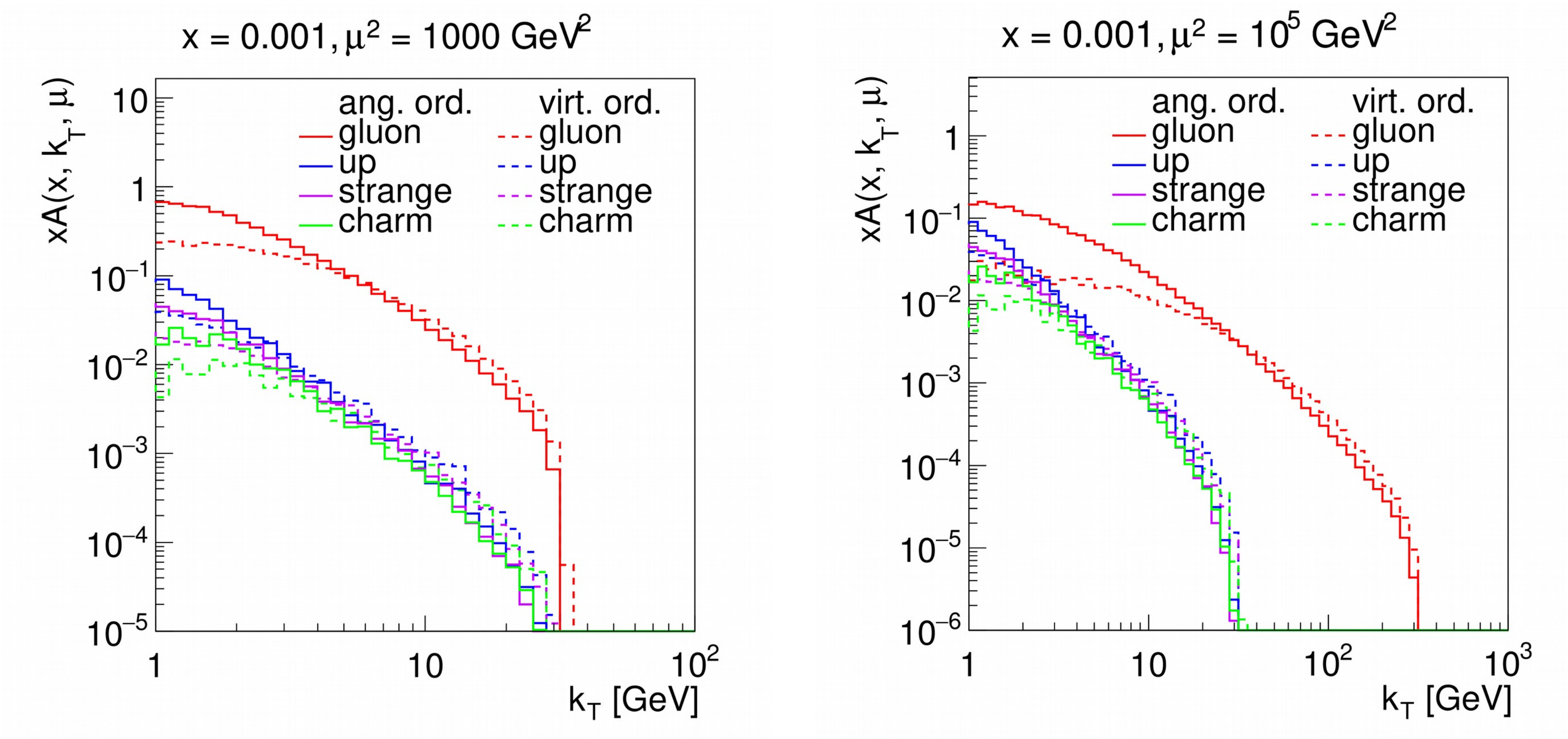
- k_T distribution as a probe of the parton shower coherence effects presented in case of angular ordered shower (e.g. in Drell-Yan process)

$$\vec{k}_{T,b} = z\vec{k}_{T,a} + \vec{q}_T$$

$$\vec{k}_{T,c} = (1-z)\vec{k}_{T,a} - \vec{q}_T$$

TMD distributions for various flavors

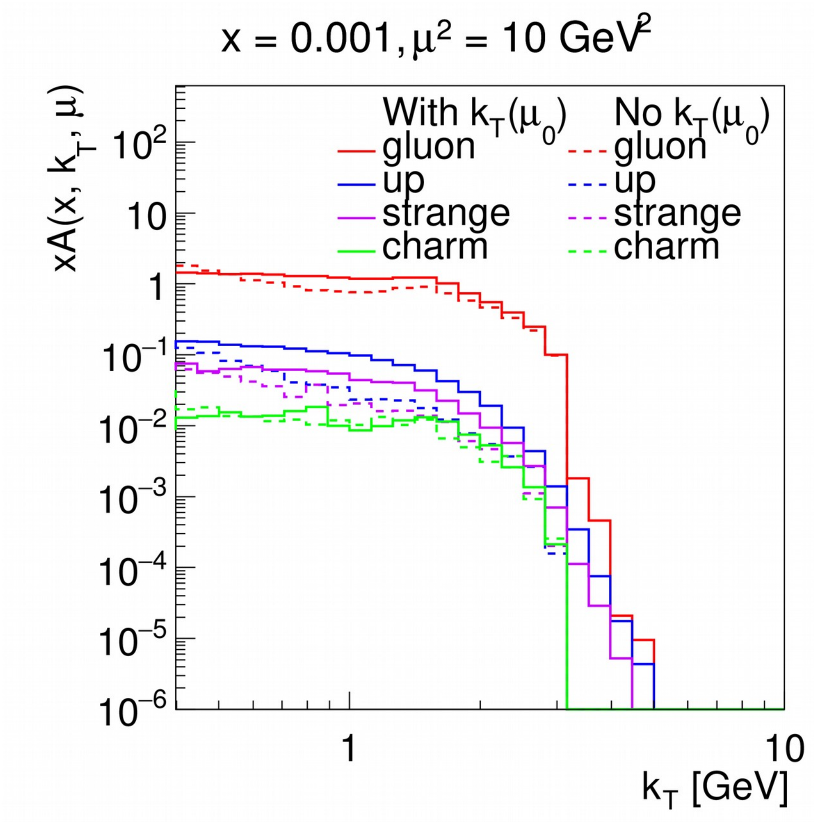
- At higher scales the quark k_T significantly smaller than the gluon one (quarks radiate less)
- Angular ordering leads to smaller k_T virtuality ordering

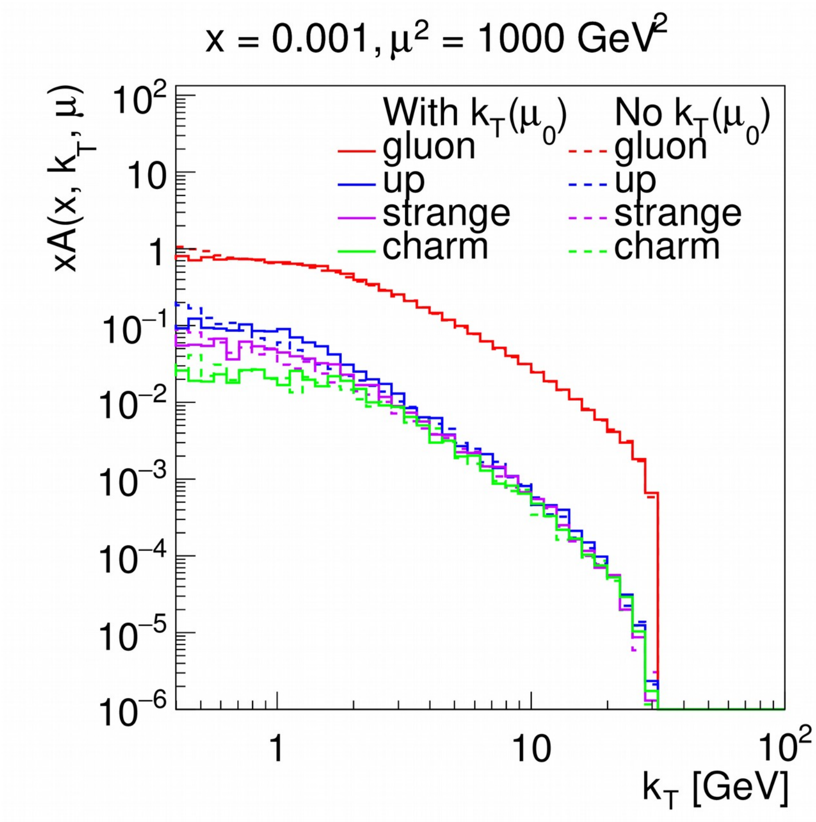


At the starting scale $\mu^2 = 2$ all flavors has the same Gaussian distribution of k_T with variance 1 GeV², correct assumption?

Effect of the intrinsic momentum

- Shown for **angular ordering**
- At higher scales or higher k_T the effect of intrinsic momentum negligible





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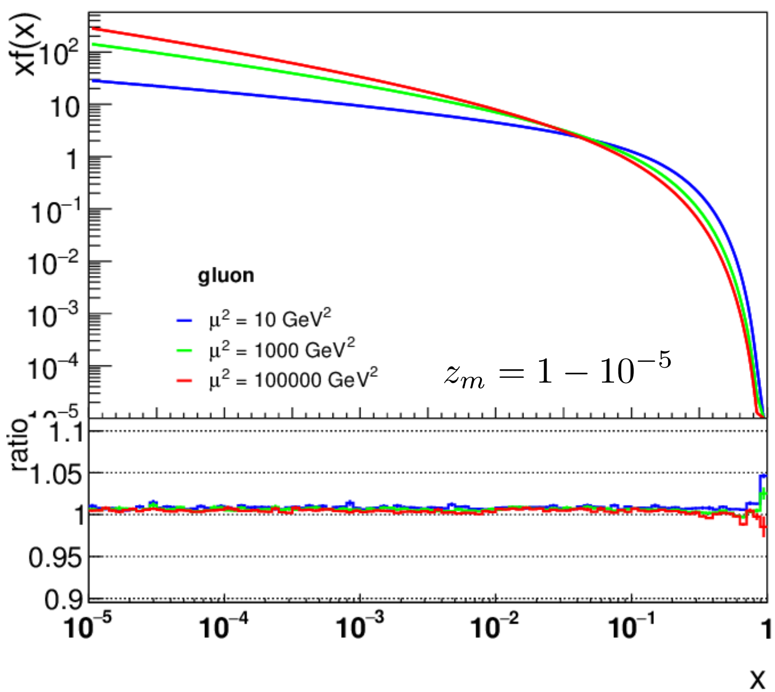
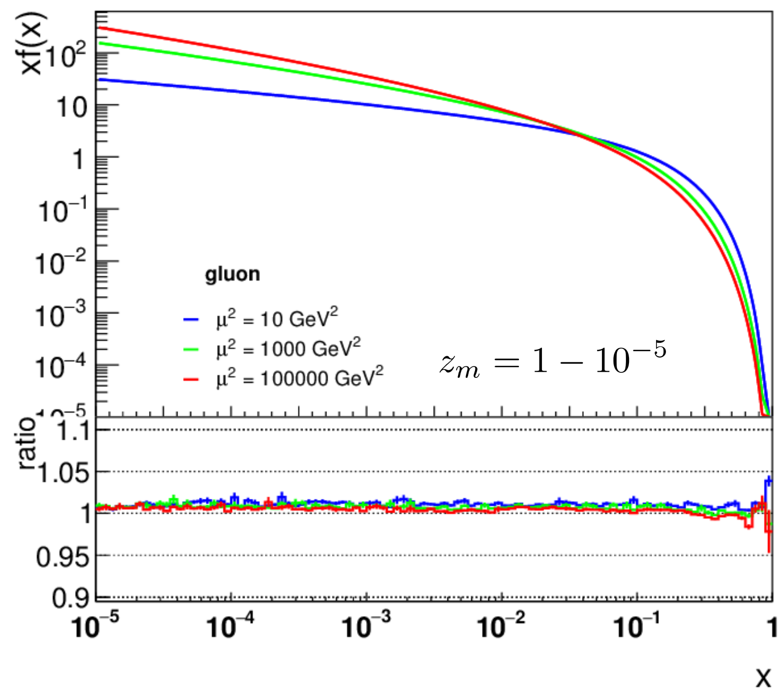
Validation of the method against QCDnum

- The approach successfully validated at LO and NLO precision by comparison to the semi-analytical DGLAP solution

LO evolution



NLO evolution

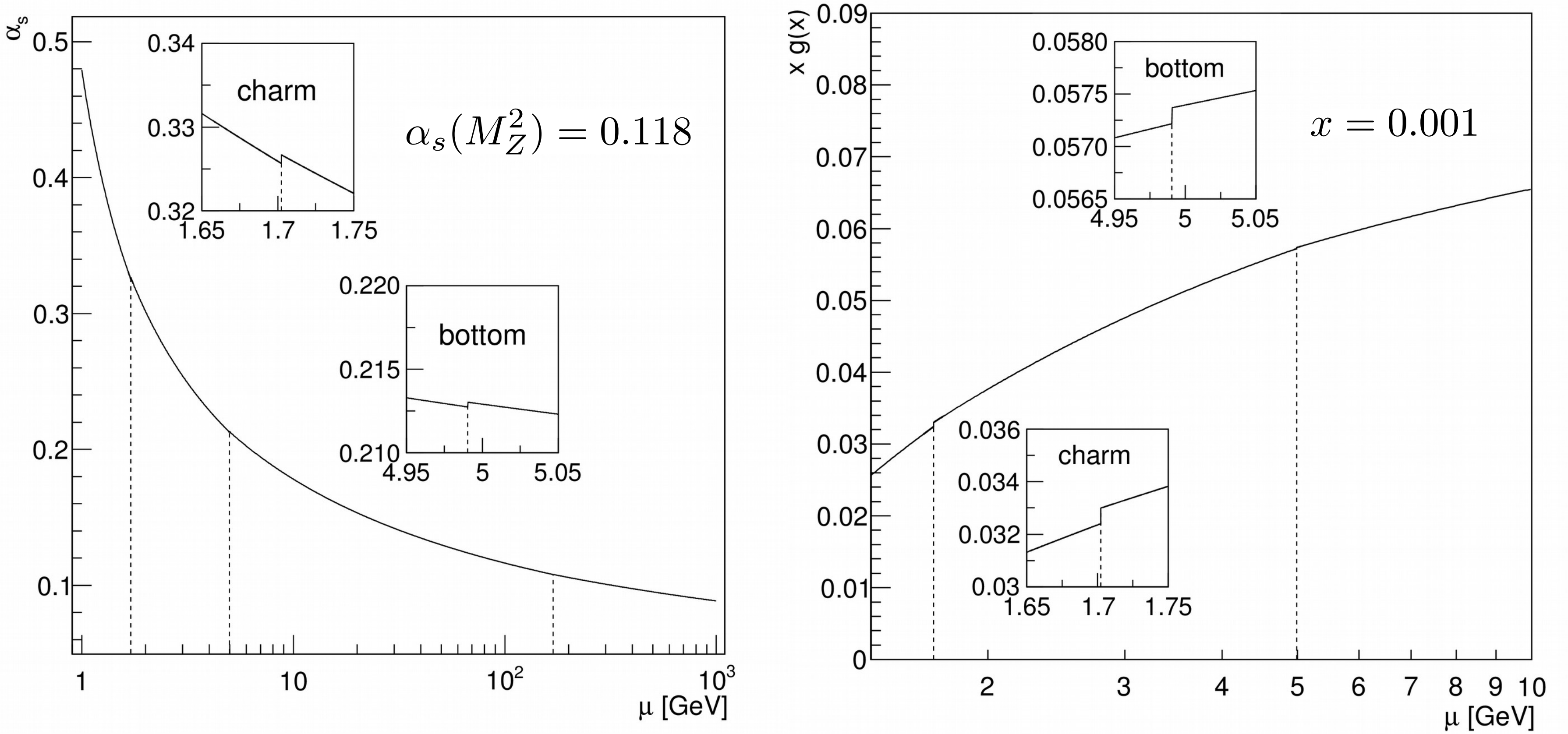


- The uncertainties are the statistical, depending on number of "events"

QCDnum: M. Botje, Comput.Phys.Commun. 182 (2011) 490

Extension of the method to NNLO evolution

- In NNLO VFNS discontinuities both in α_S and PDFs
- These discontinuities ensure continuity of observables, e.g. F_2



M. Buza et al., Eur. Phys. J. C1, 301 (1998), hep-ph/9612398

Extension of the method to NNLO evolution

- Sizes of discontinuities $\Delta f_i(x, m_c^2) = f_i(x, m_c^{+2}) - f_i(x, m_c^{-2})$ evaluated by convolution-like formulas (here for charm):

$$\Delta g(x, m_c^2) = \alpha_s^2(m_c^{+2}) \left[A_{gq} * q_s + A_{gg} * g\right]$$
$$\Delta q_i(x, m_c^2) = \alpha_s^2(m_c^{+2}) \;\; A_{qq} * q_i$$
$$\Delta c(x, m_c^2) = \alpha_s^2(m_c^{+2}) \left[A_{hq} * q_s + A_{hg} * g\right]$$

1) Having form of: $zA_{ab}(z) = D_{ab}\delta(1-z) + K_{ab}(z)\dfrac{1}{(1-z)_+} + R_{ab}(z)$

2) Preserving sum rules: $\displaystyle\sum_b \int_0^1 \mathrm{d}z\, zA_{ba}(z) = 0, \quad \text{for every flavor } a$

⇩

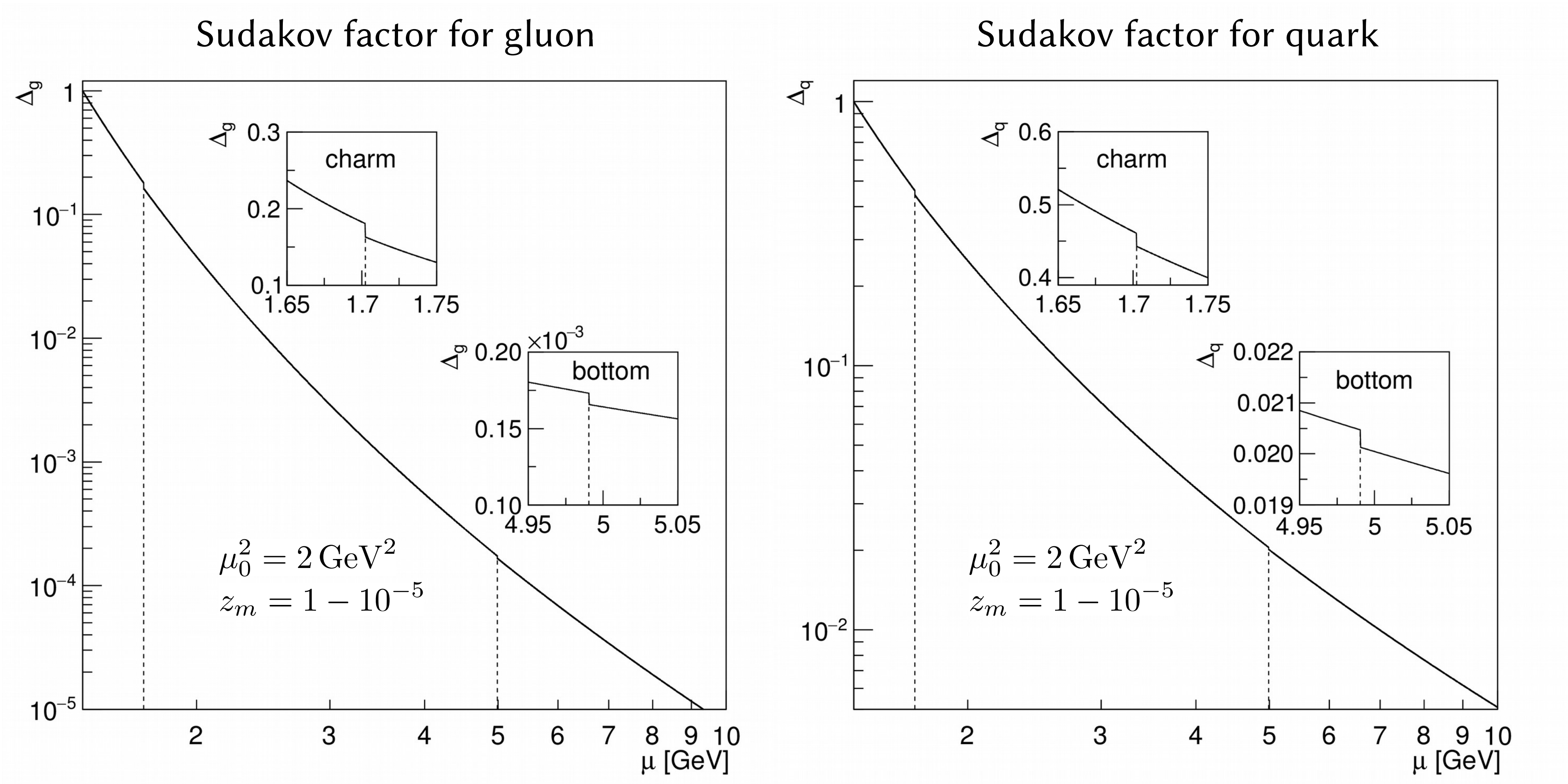
1) Incorporate discontinuity kernels into splitting:

$$zP_{ab}(\alpha_s, z) = zP_{ab}^{\text{reg}}(\alpha_s, z) + zA_{ab}^{\text{charm}}\alpha_s^2(m_c^{+2})\,\delta(\log\mu^2 - \log m_c^2) + \{\text{bottom}, \text{top}\}$$

2) Use Parton Branching method work-flow

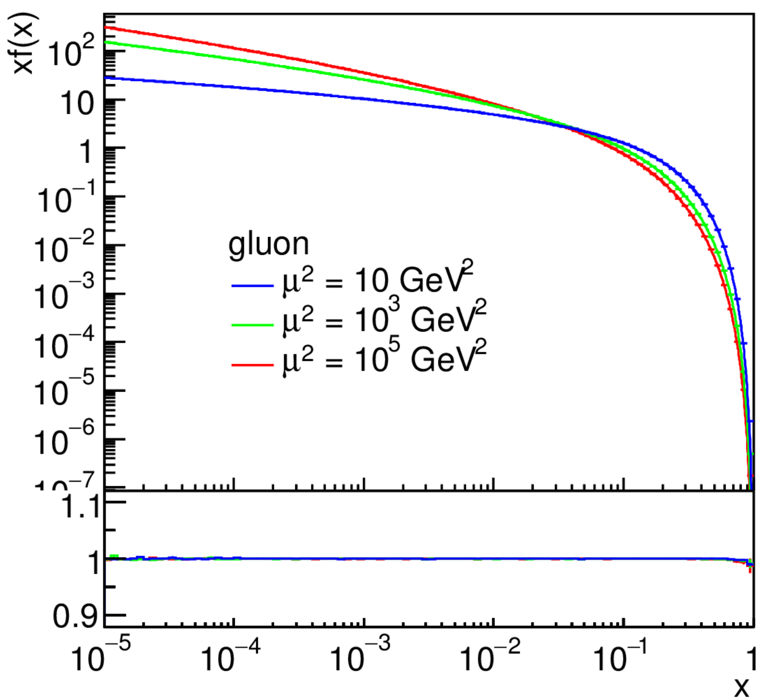
Extension of the method to NNLO evolution

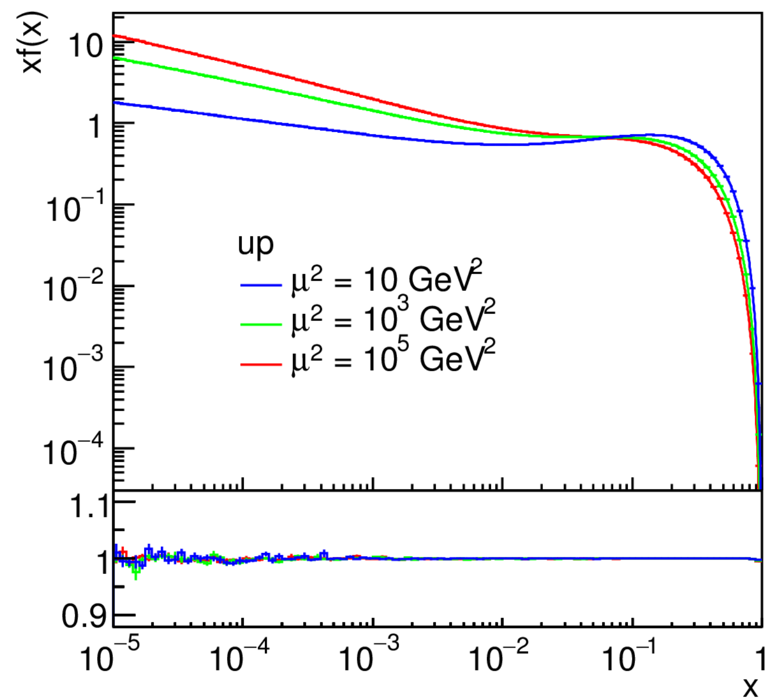
- **Consequence:** Sudakov factor with steps
- At discontinuities the branchings happen according to A_{ab}, elsewhere by standard P_{ab}^{reg}



Parton branching evolution method at NNLO

- NNLO calculations verified against semi-analytical DGLAP evolution (QCDnum), checked with level ~1% accuracy

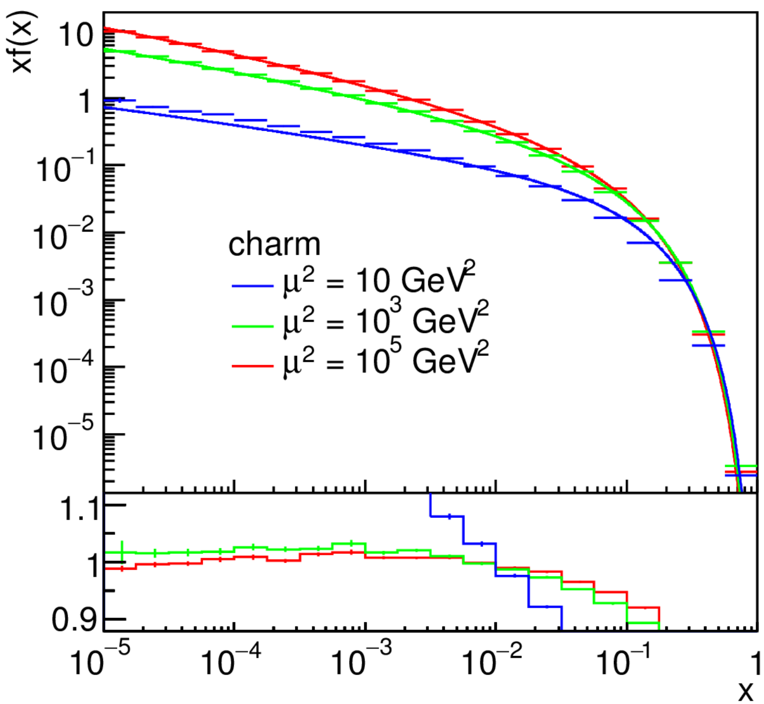




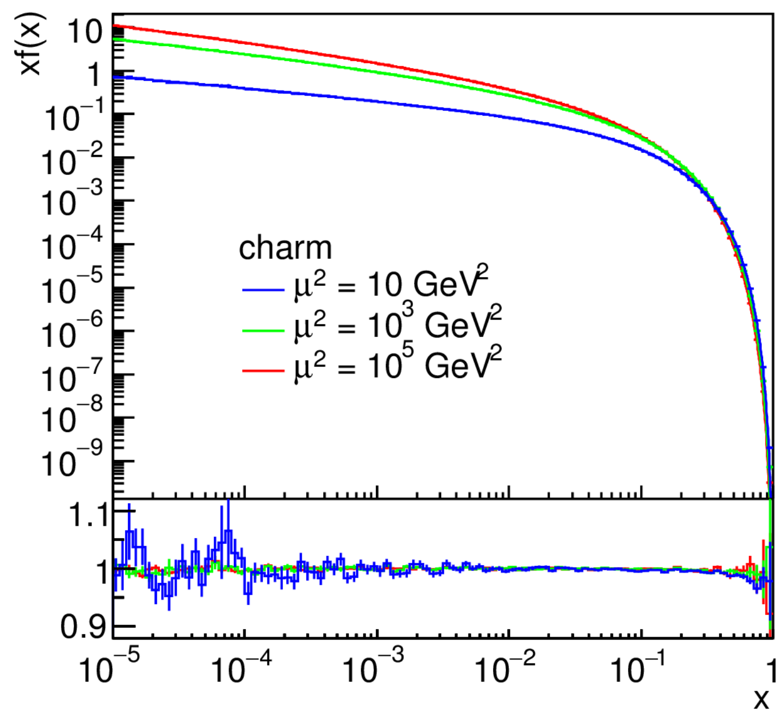
Parton branching evolution method at NNLO

- The parton branching method with discontinuous Sudakov correctly describes all discontinuities emerging with NNLO

Without step-like Sudakov



With step-like Sudakov



- Effect of discontinuities most prominent for charm distribution at lower scales $\longrightarrow$ discontinuities matter

Conclusion

- The developed Parton Branching method solves DGLAP equation at LO, NLO and **NNLO** "collinear" accuracy
- Possibility to study effects of different ordering conditions and resolution criteria in the shower

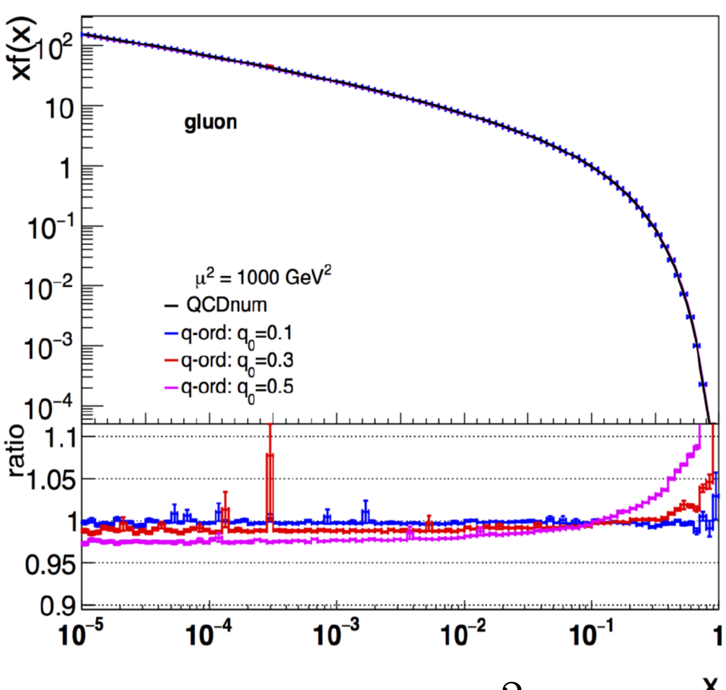
- The Parton Branching evolution implemented within **xFitter**, $\longrightarrow$ first TMDs at LO and NLO obtained form HERA inclusive DIS data
- Applications for LHC processes like DY, jets...

More results will follow

Extension of the method by scale-dependent resolution parameter

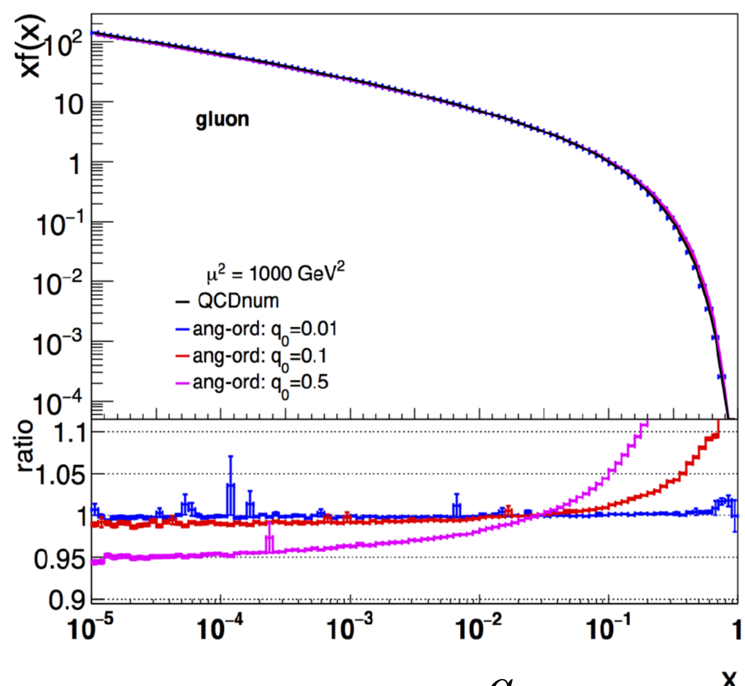
- Automatic sum rules conservation allows to study various definition of the resolvable branching $z_m(\mu^2)$
- In case of $z_m(\mu^2) \not\to 1$ the evolution in general differs from DGLAP

q-ordering condition



$$z_m = 1 - \frac{q_0^2}{\mu^2}$$

Angular ordering condition



$$z_m = 1 - \frac{q_0}{\mu}$$