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Wigner Distributions of Quarks

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In collaboration with Jai More and Sreeraj Nair, arXiv 1701.00339 [hep-ph]

Plan of the talk

- Wigner distributions for quarks
- Wigner distributions for different polarization of target and quark
- Calculation for a dressed quark at one loop
- Numerical results
- Summary and Conclusions

Wigner Distribution for Quarks

Partonic picture of nucleons in terms of quarks and gluons : joint position and momentum space information. In classical physics phase space distributions

Quantum mechanics : because of uncertainty principle position and momentum cannot be determined simultaneously. One cannot have density interpretation of such phase space variables. They are positive definite only in the classical limit

For a one-dimensional quantum system with wave function ψ(x) the Wigner function is defined as

$$W(x,p) = \int dy e^{ip\cdot y}\psi^*(x-y/2)\psi(x+y/2)$$

Matrix element of the Wigner operator for a nucleon state can be interpreted as distribution of partons in 6 D (3 position and 3 momentum)

X. Ji, PRL (2003); Belitsky, Ji, Yuan , PRD (2004)

Wigner Distributions

5 D wigner distribution in infinite momentum frame : boost invariant description

Lorce, Pasquini, PRD 84, 014015 (2011)

Integrating over transverse momentum Wigner distributions reduce to GPDs in impact parameter space; integrating over transverse position, they become TMDs

“Mother Distributions” : contain information coded in GPDs and TMDs and even more

Related to GTMDs ; give information on orbital angular momentum of quarks as well as spin-orbit correlation

Meissner, Metz, Schlegel, JHEP 08 (2009) 056

Gluon Wigner distributions and GTMDs discussed in

Lorce and Pasquini, JHEP 1309 (2013) 138; Ji, Xiong, Yuan, PRD 88, 014041 (2013)

Wigner distributions for quarks

$$\rho^{[\Gamma]}(b_\perp, k_\perp, x, \sigma) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} W^{[\Gamma]}(\Delta_\perp, k_\perp, x, \sigma);$$

$\Delta_\perp$ Momentum transfer in the transverse direction

$b_\perp$ Impact parameter conjugate to $\Delta_\perp$

$$W^{[\Gamma]}(\Delta_\perp, k_\perp, x, \sigma) = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(xp^+ z^-/2 - k_\perp . z_\perp)}$$
$$\left\langle p^+, \frac{\Delta_\perp}{2}, \sigma \middle| \overline{\psi}(-\frac{z}{2}) \Omega \Gamma \psi(\frac{z}{2}) \middle| p^+, -\frac{\Delta_\perp}{2}, \sigma \right\rangle \Big|_{z^+=0}.$$

$k_\perp$ Average transverse momentum of quark conjugate to $z_\perp$

Ω : gauge link, Γ : Dirac matrix   We take gauge link to unity in light-front gauge

Dressed Quark Target

Instead of a proton, we take the target to be a quark dressed with a gluon

State is expanded in Fock space in terms of multi-particle light-front wave functions

$$\left|p^+, p_\perp, \sigma\right\rangle = \Phi^\sigma(p) b^\dagger_\sigma(p)|0\rangle + \sum_{\sigma_1\sigma_2}\int [dp_1]\int [dp_2]\sqrt{16\pi^3 p^+}\delta^3(p - p_1 - p_2)$$

$$\Phi^\sigma_{\sigma_1\sigma_2}(p; p_1, p_2) b^\dagger_{\sigma_1}(p_1) a^\dagger_{\sigma_2}(p_2)|0\rangle;$$

Two-component formalism , light-front gauge

$\Phi^\sigma_{\sigma_1\sigma_2}(p; p_1, p_2)$ Two-particle LFWF; Zhang, Harindranath, PRD (1993)

$\Phi^\sigma(p)$ Gives normalization of the state Related to boost invariant LFWF

Composite spin ½ state with a gluonic degree of freedom, two-particle LFWF calculated analytically

Wigner Distribution for Quarks

Unpolarized target and different quark polarizations

$$\rho_{UU}(b_\perp, k_\perp, x) = \frac{1}{2}\left[\rho^{[\gamma^+]}(b_\perp, k_\perp, x, \hat{e}_z) + \rho^{[\gamma^+]}(b_\perp, k_\perp, x, -\hat{e}_z)\right]$$

$$\rho_{UL}(b_\perp, k_\perp, x) = \frac{1}{2}\left[\rho^{[\gamma^+\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_z) + \rho^{[\gamma^+\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_z)\right]$$

$$\rho^j_{UT}(b_\perp, k_\perp, x) = \frac{1}{2}\left[\rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_z) + \rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_z)\right]$$

ρ_{UL} and ρ_{LU} are equal in this model .

e_z : Polarization of the target state

ρ_{UL} : no TMD or GPD limit. Represents quark spin-orbit correlation

ρ_{UT} : related to Boer-Mulders function in TMD limit, to $\tilde{H}_T$ in GPD limit

Wigner Distribution for Quarks

Longitudinally polarized target and different quark polarization

$$\rho_{LU}(b_\perp, k_\perp, x) = \frac{1}{2}\Big[\rho^{[\gamma^+]}(b_\perp, k_\perp, x, \hat{e}_z) - \rho^{[\gamma^+]}(b_\perp, k_\perp, x, -\hat{e}_z)\Big]$$

$$\rho_{LL}(b_\perp, k_\perp, x) = \frac{1}{2}\Big[\rho^{[\gamma^+\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_z) - \rho^{[\gamma^+\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_z)\Big]$$

$$\rho^j_{LT}(b_\perp, k_\perp, x) = \frac{1}{2}\Big[\rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_z) - \rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_z)\Big]$$

TMD limit : ρ_{LT} related to the worm-gear function $h_{1L}^{\perp}$

Related to the GPDs H_T and $\tilde{H}_T$

ρ_{LU} : related to orbital angular momentum of the quark

Wigner Distribution for Quarks

Transversely polarized target and different quark polarizations

$$\rho^{i}_{TU}(b_\perp, k_\perp, x) = \frac{1}{2}\Big[\rho^{[\gamma^+]}(b_\perp, k_\perp, x, \hat{e}_i) - \rho^{[\gamma^+]}(b_\perp, k_\perp, x, -\hat{e}_i)\Big]$$

$$\rho^{i}_{TL}(b_\perp, k_\perp, x) = \frac{1}{2}\Big[\rho^{[\gamma^+\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_i) - \rho^{[\gamma^+\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_i)\Big]$$

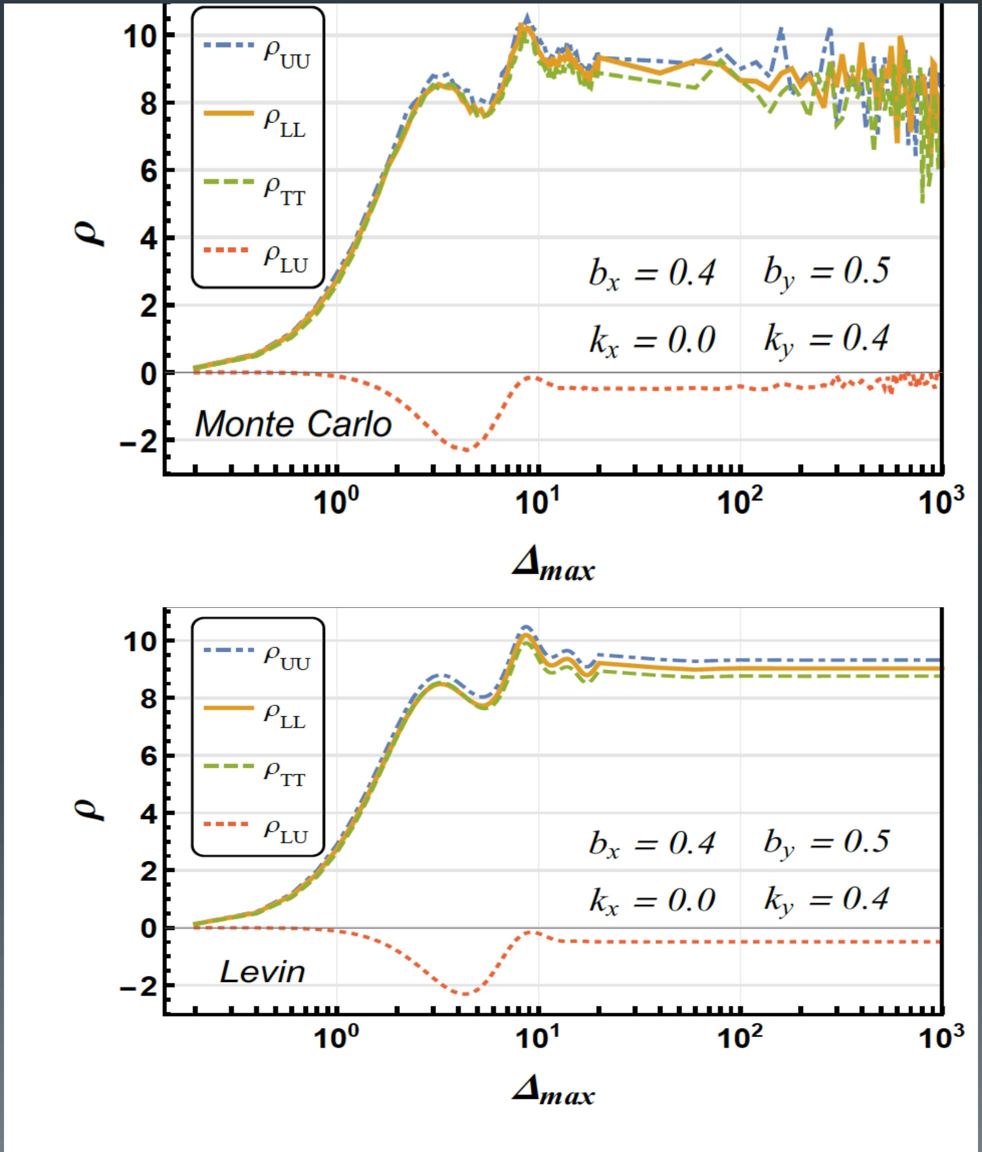
$$\rho_{TT}(b_\perp, k_\perp, x) = \frac{1}{2}\delta_{ij}\Big[\rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_i) - \rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_i)\Big]$$

$$\rho^{\perp}_{TT}(b_\perp, k_\perp, x) = \frac{1}{2}\epsilon_{ij}\Big[\rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, \hat{e}_i) - \rho^{[i\sigma^{+j}\gamma^5]}(b_\perp, k_\perp, x, -\hat{e}_i)\Big]$$

Pretzelous Wigner distribution : quark and target transversely polarized in orthogonal directions : zero in our model

ρ_{TL} : TMD limit is related to the other worm-gear function g_{1T} ;

Integration Technique



Earlier study : MC integration method. Low value of upper integration limit for convergence : Δ_{max} dependence

AM, Nair, Ojha, PRD (2015)

Levin method : for oscillatory integrand. Better convergence. Results agree for smaller values of cutoff

Δ_{max} = 20 GeV, m=0.33 GeV

Results are independent of cutoff

Contribution from single particle sector

3 D plots of ‘transverse’ Wigner distributions in b and k space

x integrated in b space from 0 to 1 and in k space from 0 to 0.9

To get the correct result at x=1, contribution from single particle sector needs to be taken into account. This contributes to ρ_{UU}, ρ_{LL}, and ρ_{TT}

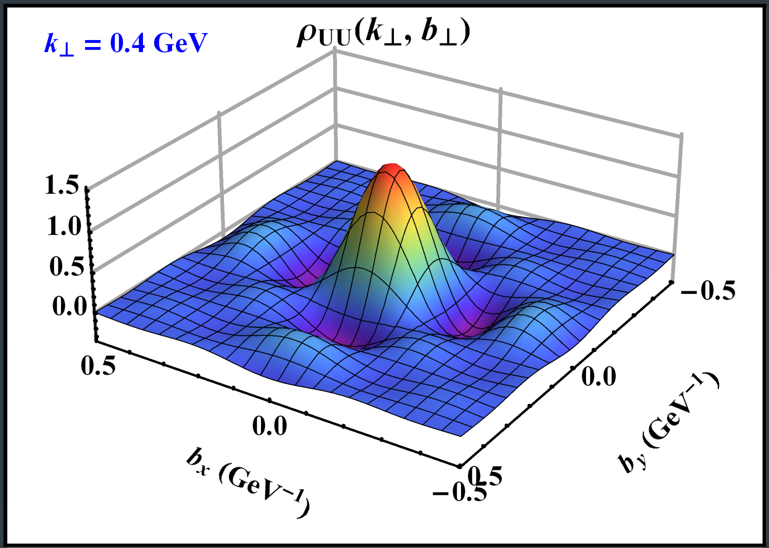
This is of the form $N\delta(1-x)\delta^2(b_\perp)\delta^2(k_\perp)$

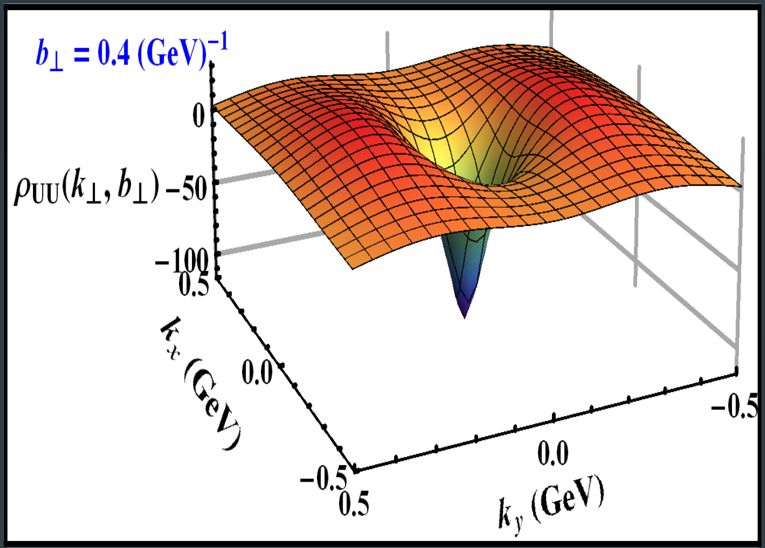
There is also a contribution due to the normalization of the state

Contribution from the normalization of the state combines with the contribution from the two particle sector to give the familiar plus distribution in the pdf for a dressed quark

Harindranath, Kundu, Zhang, PRD 59, 094013 (1999)

Numerical Results : ρ_{UU}



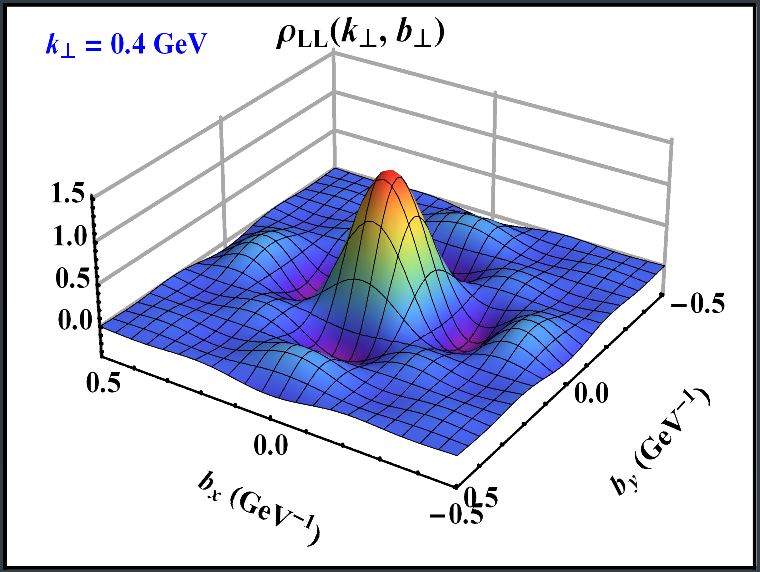


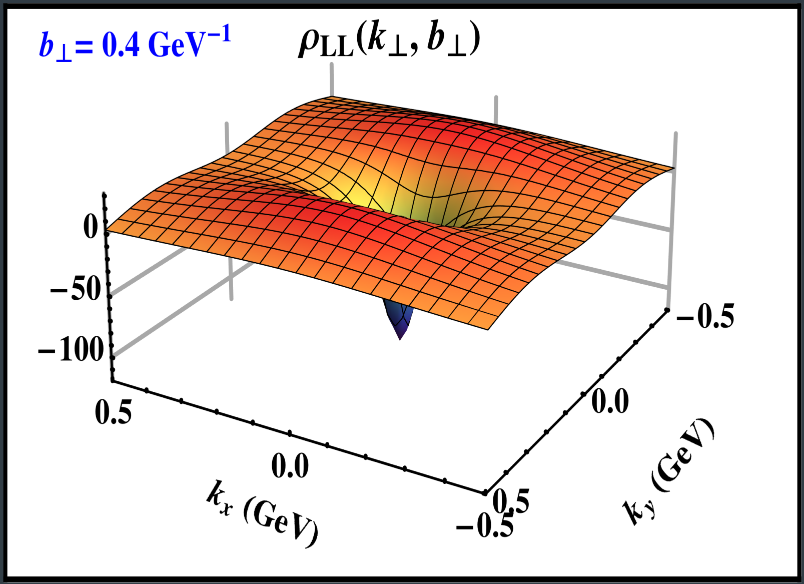
b(k) space plot : for fixed value of $k_\perp (b_\perp)$

Single particle contribution does not affect this plot

Positive peak in b space similar to LFCQM (Lorce and Pasquini , PRD 93, 034040 (2016)) ; spectator model (Liu and Ma, PRD 91, 034019 (2015))

Numerical Results : ρ_{LL}

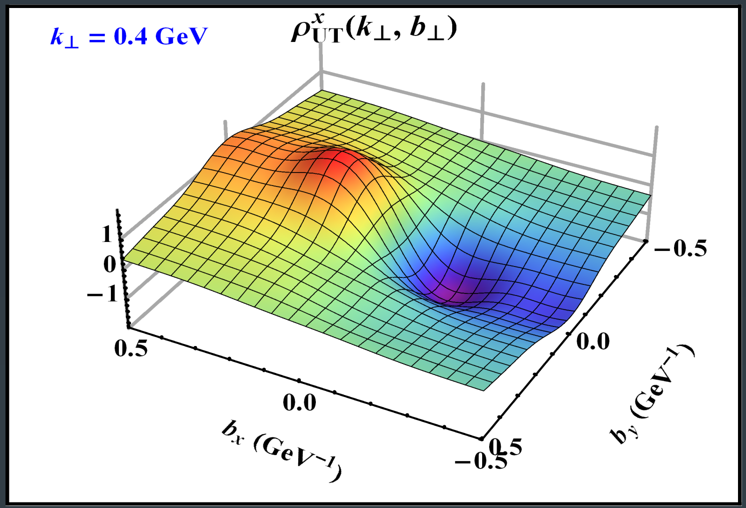


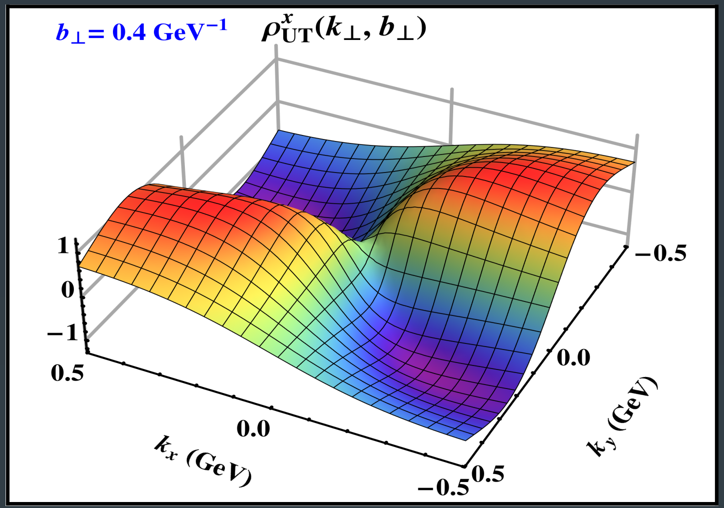


Analytic expression similar to ρ_{UU}, difference in mass term

Positive peak in b space similar to other model calculations

Numerical Results : ρ^{x}_{UT}



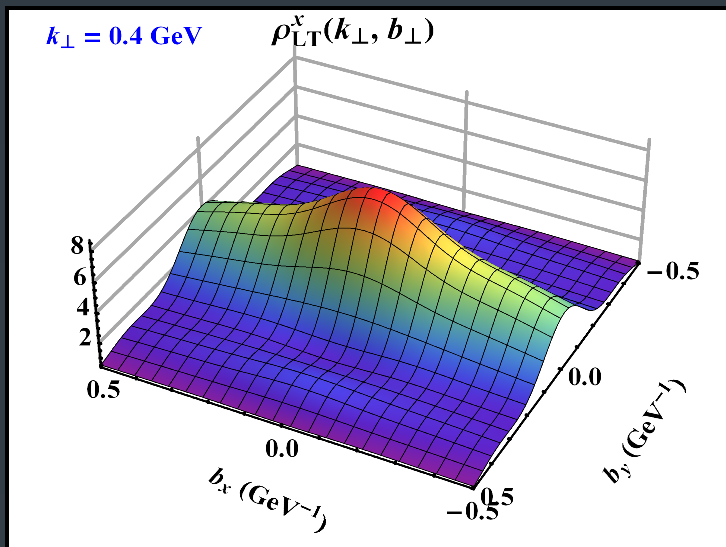


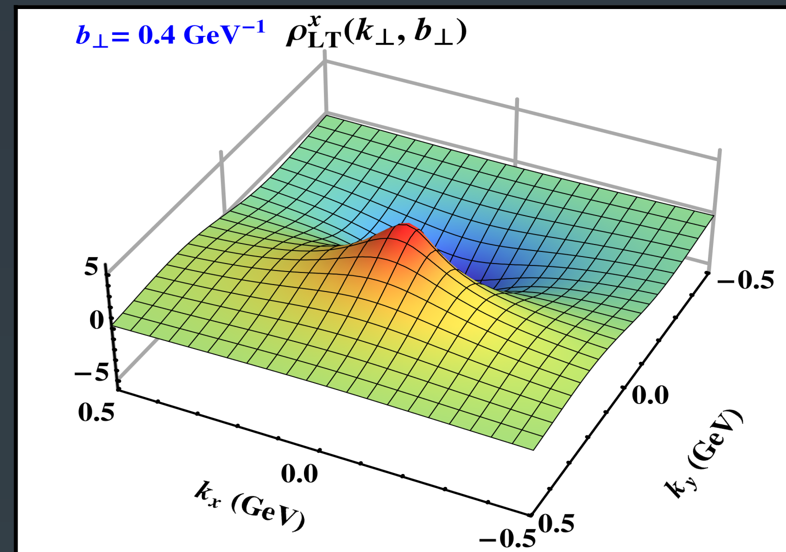
Transversely polarized quark in unpolarized target; quark polarization in x direction

Dipole behaviour in b space similar to spectator model, quadrupole behaviour in k space

Vanishes in TMD limit : as we have not considered the gauge link. Boer-Mulders function is zero in our model.

Numerical Result : ρ^{x}_{LT}



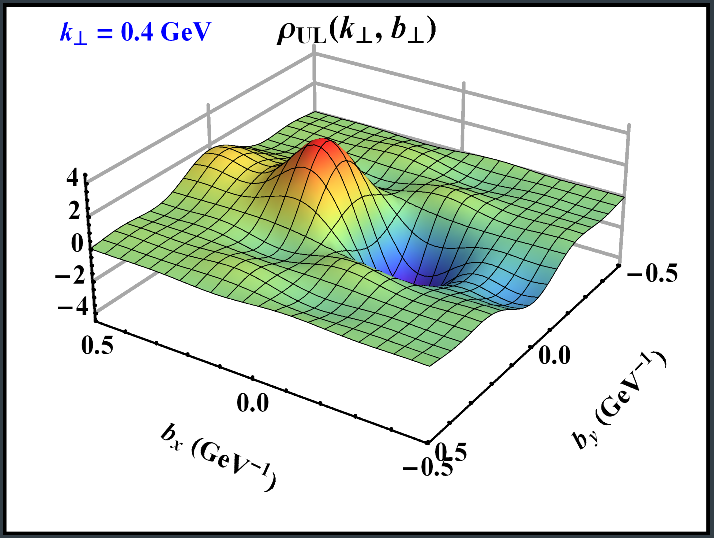


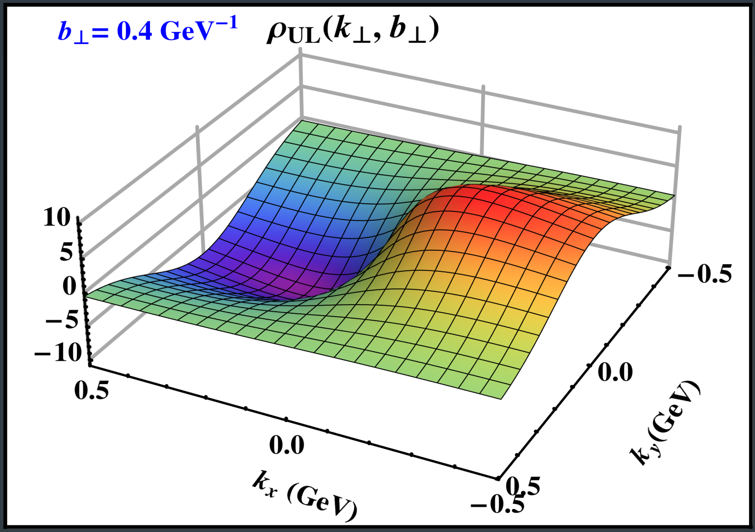
Transversely polarized quark in longitudinally polarized target, quark polarization in x direction : TMD limit related to worm gear function

Dipole behaviour in k space, similar to spectator model; different in b space

Similar to LFCQM (Lorce and Pasquini, PRD 93, 034040 (2016))

Numerical Result : ρ_{UL}





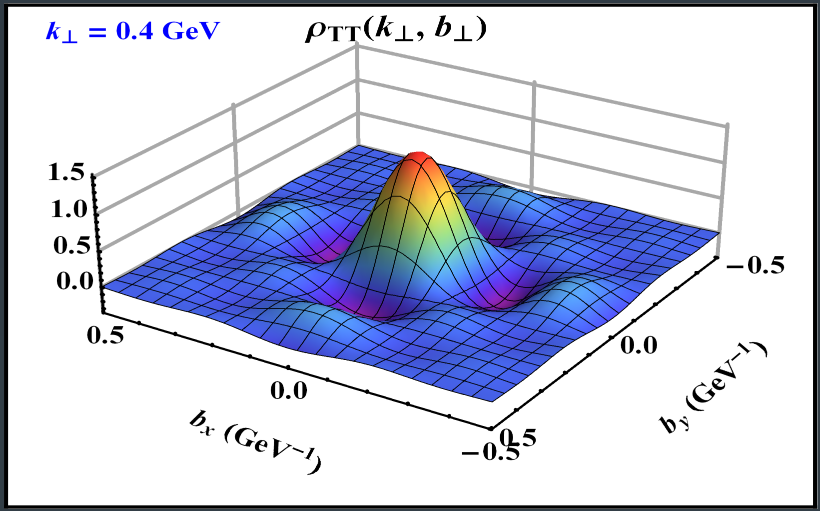
Dipole structure of ρ_{UL} as observed in other models

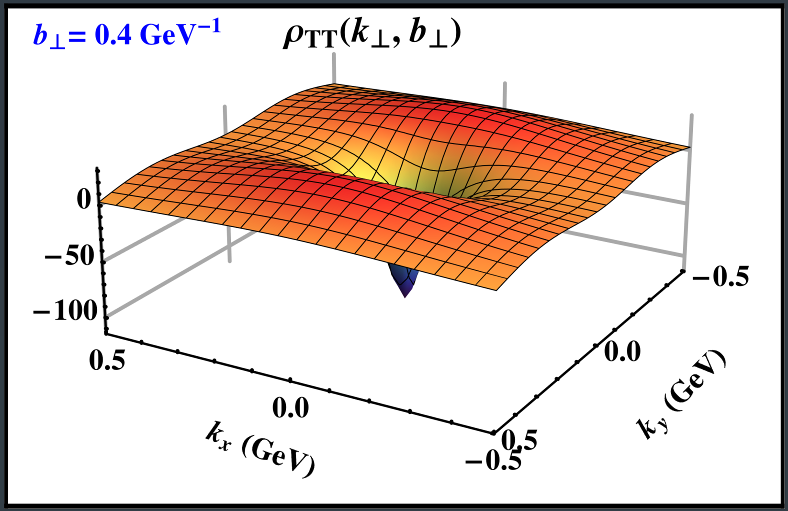
Lorce and Pasquini, PRD 84, 014015 (2011); Liu and Ma, PRD 91, 034019 (2015)

Represents spin-orbit correlation of the quark

Does not have TMD or GPD limit

Numerical Result : ρ_{TT}

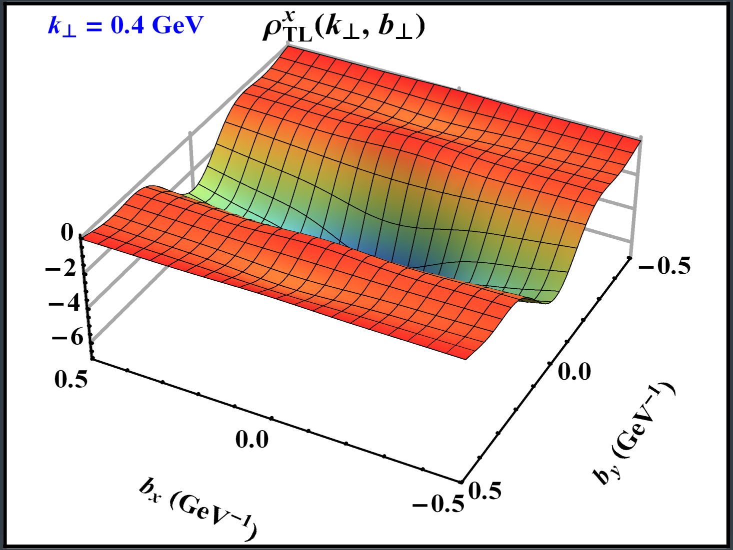


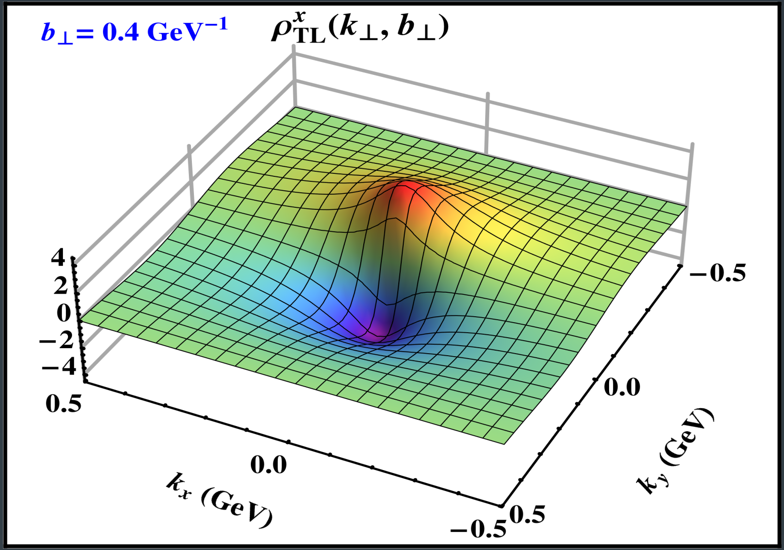


Behaviour of ρ_{TT} similar to ρ_{UU} and $\rho_{LL,}$ analytic result different

Similar behaviour as in light-front spectator model

Numerical Result : ρ^x_{TL}



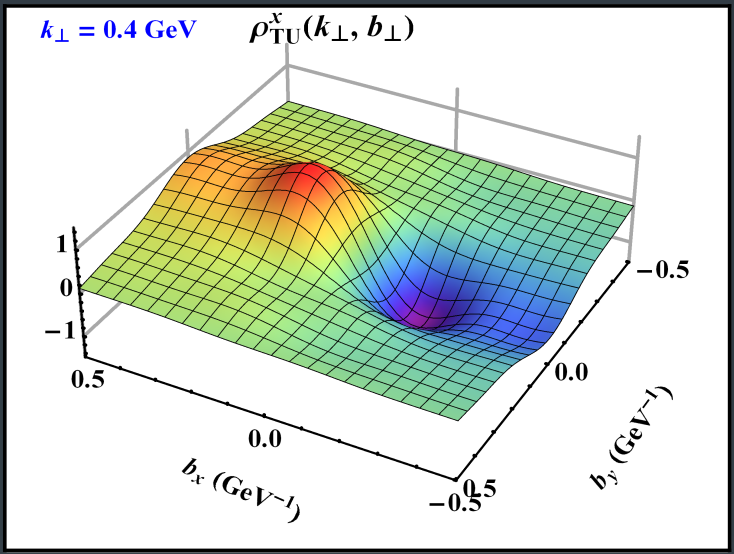


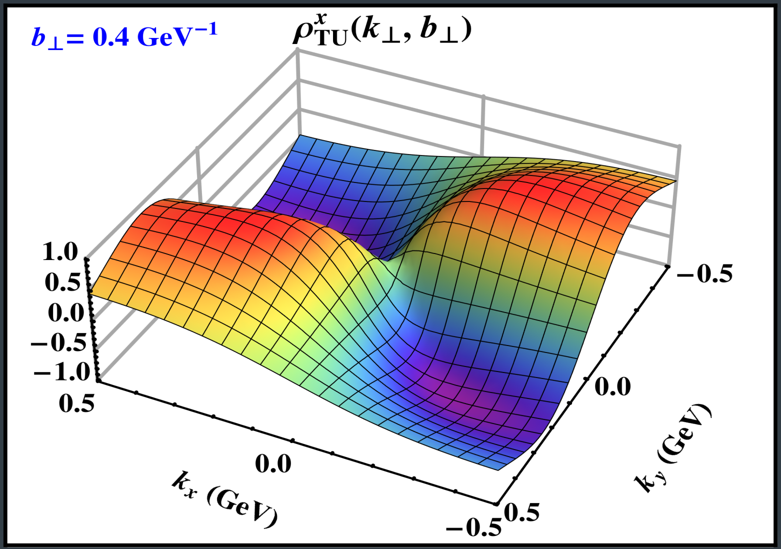
Longitudinally polarized quark in a transversely polarized target state

Behave similarly as $\rho_{LT,}$ with sign difference , analytic expression slightly different

TMD limit related to worm gear function g_{1T} (transverse helicity)

Numerical Result : ρ^x_{TU}





Unpolarized quark in transversely polarized target state

TMD limit related to Sivers function (T odd) , and GPD limit to H and E with other functions (T even). In our model Sivers function is zero

Dipole behaviour in b space similar to spectator model, behaves differently in k space

Summary and Conclusion

We calculated the Wigner distribution of quarks taking the state to be a quark dressed at one loop with a gluon, using overlaps of light-front wave functions

This is a simple composite spin ½ system, having a gluonic degree of freedom

We calculated the Wigner distributions for different polarization of the target and the quark : compared with other model calculations

In general behaviour in b space similar to other models

Numerical integration with better convergence : removal of the cutoff dependence of an earlier calculation

Work in progress : Wigner distribution for gluons