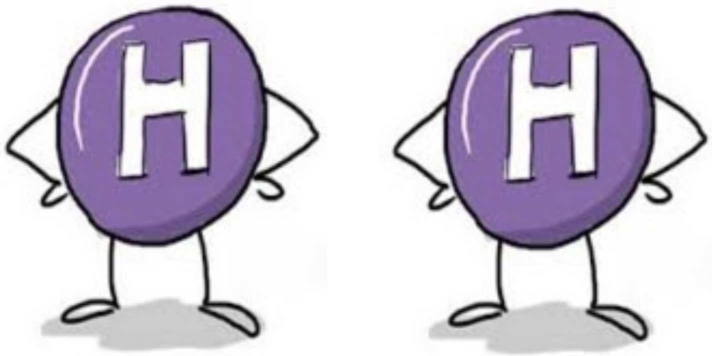
Alignment without decoupling in the MSSM



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Higgs Days at Santander

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SANTA CRUZ

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- Regions of approximate alignment without decoupling in a pMSSM scan

- Conclusions

References

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H.E. Haber and D. O'Neil, Phys. Rev. D **74**, 015018 (2006) [Erratum: ibid., **74**, 059905 (2006)] [hep-ph/0602242].

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P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638 [hep-ph].

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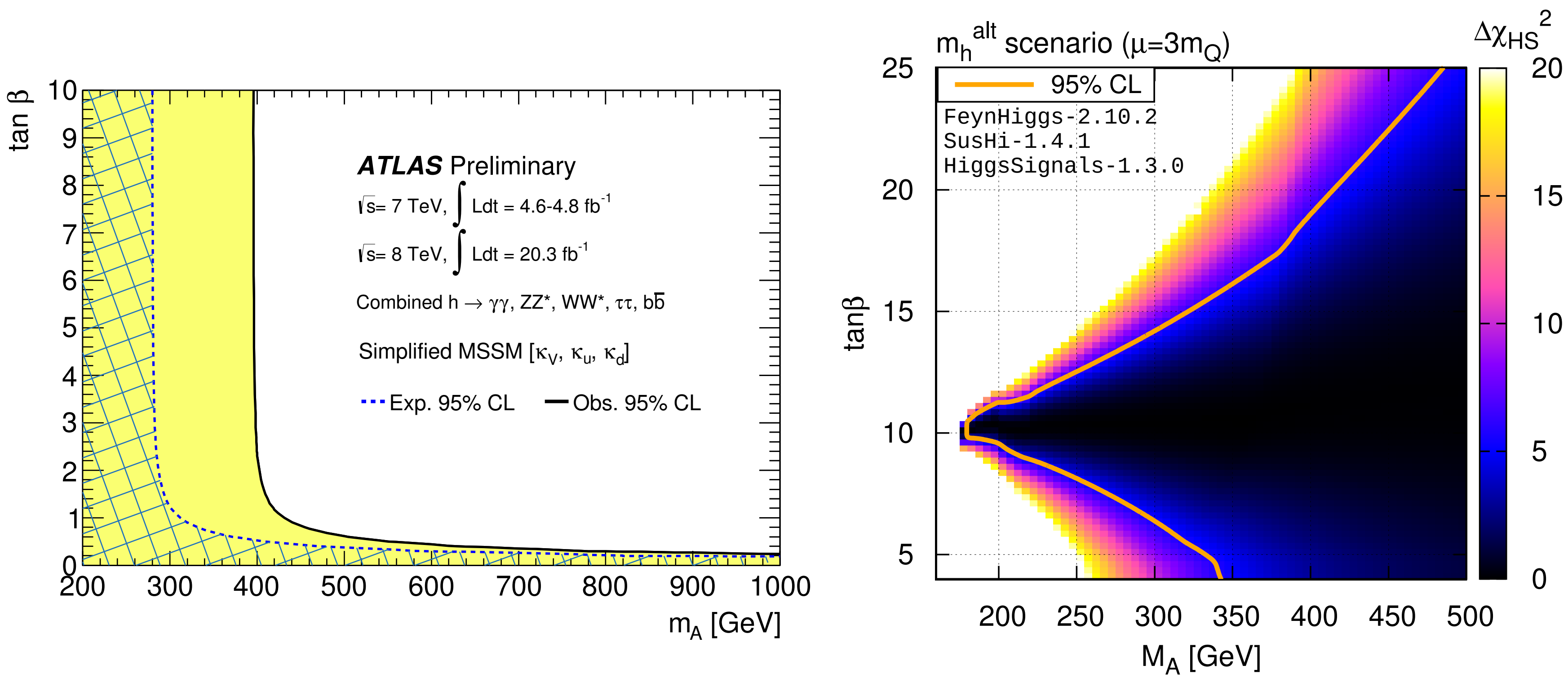
The MSSM Higgs sector in light of precision Higgs data

The observed Higgs boson at 125 GeV is SM-like (to within roughly an accuracy of 20%). The common wisdom is that this observation implies that additional Higgs states of the MSSM Higgs sector must be rather heavy (corresponding to the decoupling limit).

Indeed, ATLAS has claimed to rule out $m_A \lesssim 400$ GeV based on Run 1 precision Higgs data. But, one needs to be careful about the underlying assumptions...

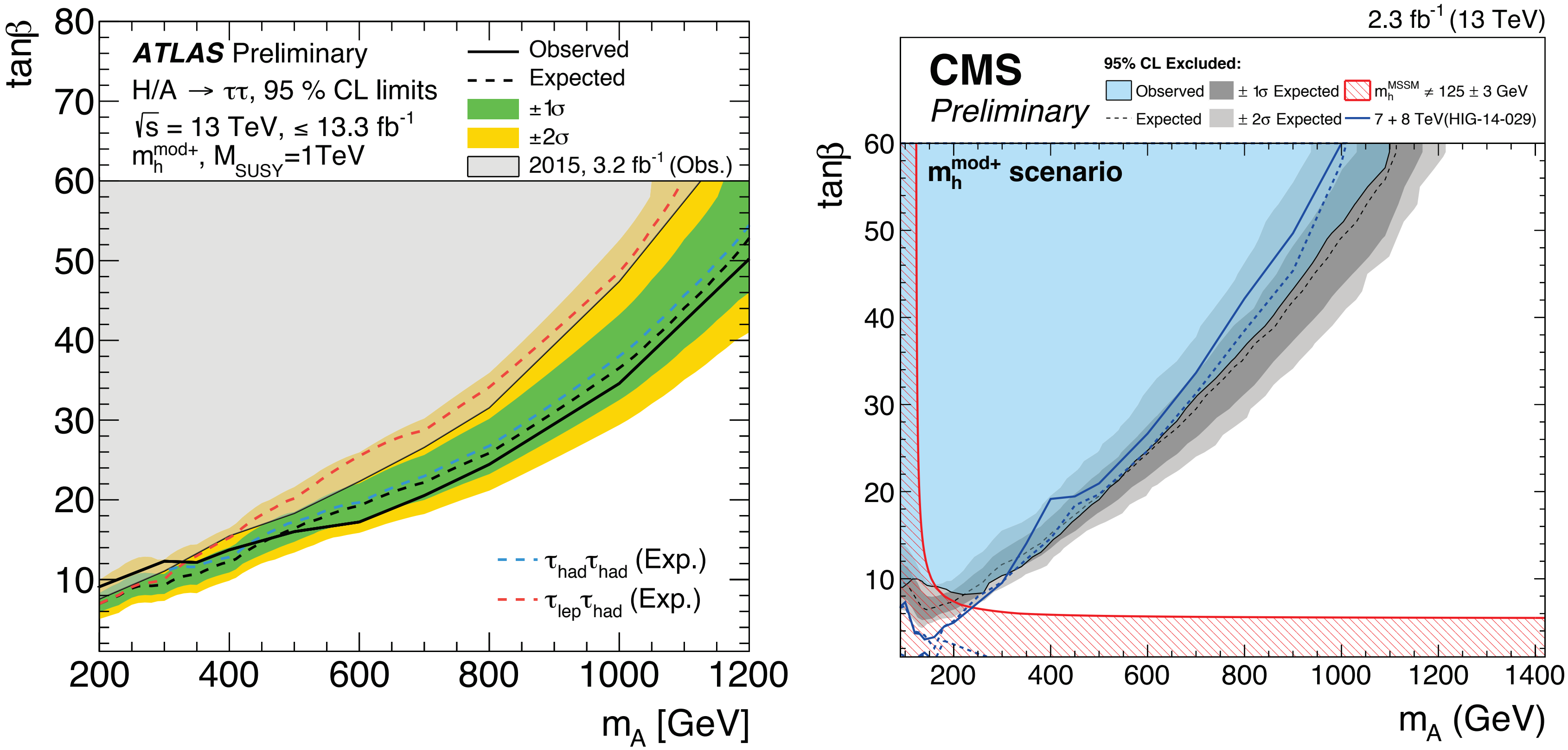
For example, in the so called MSSM $m_h^{\rm alt}$ benchmark scenario introduced in M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D **91**, 035003 (2015), the Run 1 precision Higgs data places virtually no bound on m_A if $\tan\beta \sim 10$.



<u>Left panel</u>: Regions of the $(m_A, \tan\beta)$ plane excluded in a simplified MSSM model via fits to the measured rates of the production and decays of the SM-like Higgs boson h. Taken from ATLAS-CONF-2014-010.

<u>Right panel</u>: Likelihood distribution, $\Delta\chi^2_{\mathrm{HS}}$ obtained from testing the signal rates of h against a combination of Higgs rate measurements from the Tevatron and LHC experiments, obtained with `HiggsSignals`, in the alignment benchmark scenario of Carena et al. (op. cit.). From P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, EPJC **75**, 421 (2015).

Direct searches for the additional Higgs states also suggest that these states must be heavy, although the sensitivity of these searches are limited if $\tan\beta \lesssim 10$.



The observed and expected 95% CL limits on $\tan\beta$ as a function of m_A in the MSSM $m_h^{\text{mod}+}$ benchmark scenario. Left panel: ATLAS results taken from ATLAS-CONF-2016-085. Right panel: CMS results taken from CMS-PAS-HIG-16-006.

Theoretical structure of the 2HDM

Consider the most general renormalizable 2HDM potential,

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ & + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right\}.\end{aligned}$$

After minimizing the scalar potential, $\langle\Phi_i^0\rangle = v_i/\sqrt{2}$ (for $i = 1,2$) with $v \equiv (|v_1|^2 + |v_2|^2)^{1/2} = 2m_W/g = 246$ GeV.

Define the scalar doublet fields of the Higgs basis,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^*\Phi_1 + v_2^*\Phi_2}{v}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v},$$

such that $\langle H_1^0\rangle = v/\sqrt{2}$ and $\langle H_2^0\rangle = 0$. The Higgs basis is uniquely defined up to an overall rephasing, $H_2 \to e^{i\chi}H_2$.

In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\ & + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \big[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \big] H_1^\dagger H_2 + \text{h.c.} \right\} ,\end{aligned}$$

where Y_1, Y_2 and $Z_1, \ldots, Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$[Y_3, Z_6, Z_7] \to e^{-i\chi} [Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi} Z_5 \, .$$

Physical observables must be independent of χ.

After minimizing the scalar potential, $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

Remark: Generically, the Z_i are $\mathcal{O}(1)$ parameters.

The alignment limit in the general 2HDM

The neutral Higgs mass-eigenstates, denoted by $\{h_1, h_2, h_3\}$, are linear combinations of $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\ \sqrt{2}\,\mathrm{Re}\,H_2^0\,,\ \sqrt{2}\,\mathrm{Im}\,H_2^0\}$, and are determined by diagonalizing the 3×3 real symmetric squared-mass matrix,

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$. The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}, such that θ_{12} and θ_{13} are invariant whereas $\theta_{23} \to \theta_{23} - \chi$ under the rephasing of H_2.*

The couplings of $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$ coincide with those of the SM Higgs boson. Thus, the alignment limit corresponds to two limiting cases:

1. $Y_2 \gg v^2$, corresponding to the decoupling limit.
2. $|Z_6| \ll 1$, corresponding to alignment with or without decoupling.

We identify the SM-like Higgs boson, $h_1 \simeq \sqrt{2}\,\mathrm{Re}\,H_1^0 - v$, with $m_h^2 \simeq Z_1 v^2$.

*See H.E. Haber and D. O'Neil, Phys. Rev. **D74**, 015018 (2006) [Erratum: ibid., **D74**, 059905 (2006)].

The alignment limit of the general 2HDM in equations

To obtain the conditions in which h_1 is the SM-like Higgs boson, noting that:

$$\frac{g_{h_1VV}}{g_{h_{\rm SM}VV}} = c_{12}c_{13}\,, \qquad \text{where } V = W \text{ or } Z\,,$$

where $h_{\rm SM}$ is the SM Higgs boson, we demand that

$$s_{12}\,,\, s_{13} \ll 1\,.$$

Here, $s_{12} \equiv \sin\theta_{12}$, $c_{12} \equiv \cos\theta_{12}$, etc. We denote the masses of the neutral Higgs mass eigenstates by m_1, m_2 and m_3. It follows that:

$$Z_1 v^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2\,,$$

$$\begin{aligned}
{\rm Re}(Z_6\, e^{-i\theta_{23}})\, v^2 &= c_{13}s_{12}c_{12}(m_2^2 - m_1^2)\,,\\
{\rm Im}(Z_6\, e^{-i\theta_{23}})\, v^2 &= s_{13}c_{13}(c_{12}^2 m_1^2 + s_{12}^2 m_2^2 - m_3^2)\,,\\
{\rm Re}(Z_5\, e^{-2i\theta_{23}})\, v^2 &= m_1^2(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2^2(c_{12}^2 - s_{12}^2 s_{13}^2) - m_3^2 c_{13}^2\,,\\
{\rm Im}(Z_5\, e^{-2i\theta_{23}})\, v^2 &= 2s_{12}c_{12}s_{13}(m_2^2 - m_1^2)\,.
\end{aligned}$$

Assuming no mass degeneracies in the neutral scalar sector, it then follows that in the alignment limit,

$$\boxed{\begin{aligned} s_{12} &\equiv \sin\theta_{12} \simeq \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{m_2^2 - m_1^2} \ll 1\,, \\ s_{13} &\equiv \sin\theta_{13} \simeq -\frac{\mathrm{Im}(Z_6 e^{-i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,, \end{aligned}}$$

One additional small quantity characterizes the alignment limit,

$$\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \simeq \frac{(m_2^2 - m_1^2)s_{12}s_{13}}{v^2} \simeq -\frac{2\,\mathrm{Im}(Z_6^2 e^{-2i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,.$$

Finally, the following mass relations in the alignment limit are noteworthy,

$$\begin{aligned} m_1^2 &\simeq Z_1 v^2\,, \\ m_2^2 - m_3^2 &\simeq \mathrm{Re}(Z_5 e^{-2i\theta_{23}})v^2\,. \end{aligned}$$

The alignment limit in the CP-conserving 2HDM

In the case of a CP-conserving scalar potential, one can choose χ such that $\mathrm{Im}Z_5 = \mathrm{Im}Z_6 = \mathrm{Im}Z_7 = 0$, corresponding to a *real Higgs basis*. We identify the CP-odd Higgs boson as $A = \sqrt{2}\,\mathrm{Im}\,H_2^0$, with $m_A^2 = Y_2 + \frac{1}{2}(Z_3 + Z_4 - Z_5)v^2$. After eliminating Y_2 in favor of m_A^2, the CP-even Higgs squared-mass matrix with respect to the Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\, \sqrt{2}\,\mathrm{Re}\,H_2^0\}$ is given by,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} .$$

The CP-even Higgs bosons are h and H with $m_h \leq m_H$. The couplings of $\sqrt{2}\,\mathrm{Re}\;H_1^0 - v$ coincide with those of the SM Higgs boson. Thus, the alignment limit corresponds to two limiting cases:

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where h is SM-like and $m_A \sim m_H \sim m_{H^\pm} \gg m_h$.
2. $|Z_6| \ll 1$. h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$. Otherwise, H is SM-like.

In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\, H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\, H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle α that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the original basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2\}$, and $\tan\beta \equiv v_2/v_1$.

Since the SM-like Higgs must be approximately $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$, it follows that

- h is SM-like if $|c_{\beta-\alpha}| \ll 1$,
- H is SM-like if $|s_{\beta-\alpha}| \ll 1$.

Alignment *without* decoupling is required to have a SM-like H.

Remark: Although the tree-level couplings of $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$ coincide with those of the SM Higgs boson, the one-loop couplings can differ due to the exchange of non-minimal Higgs states (if not too heavy). For example, the $H^\pm$ loop contributes to the decays of the SM-like Higgs boson to $\gamma\gamma$ and γZ.

The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

$$
\begin{aligned}
Z_1 v^2 &= m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2\,,\\
Z_6 v^2 &= (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha}\,,\\
Z_5 v^2 &= m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2\,.
\end{aligned}
$$

If h is SM-like, then $m_h^2 \simeq Z_1 v^2$ and

$$
|c_{\beta-\alpha}| = \frac{|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1\,,
$$

If H is SM-like, then $m_H^2 \simeq Z_1 v^2$ and

$$
|s_{\beta-\alpha}| = \frac{|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1\,.
$$

The MSSM Higgs Sector at tree-level

The MSSM Higgs sector is a CP-conserving 2HDM. The dimension-four terms of the scalar potential constrained by supersymmetry. At tree level,

$$\lambda_1 = \lambda_2 = -\lambda_3 - \lambda_4 - \lambda_5 = \tfrac{1}{4}(g^2 + g'^2)\,, \quad \lambda_4 = -\tfrac{1}{2}g^2\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0\,.$$

The corresponding real Higgs basis parameters of interest are:

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2\,, \qquad Z_5 v^2 = m_Z^2 s_{2\beta}^2\,, \qquad Z_6 v^2 = -m_Z^2 s_{2\beta} c_{2\beta}\,.$$

in a convention where $\tan\beta \geq 0$. It follows that,

$$\cos^2(\beta - \alpha) = \frac{m_Z^4\, s_{2\beta}^2 c_{2\beta}^2}{(m_H^2 - m_h^2)(m_H^2 - m_Z^2 c_{2\beta}^2)}\,.$$

The decoupling limit is achieved when $m_H \gg m_h$ as expected. Alignment without decoupling is (naively) possible at tree-level when $Z_6 = 0$, which yields $\sin 4\beta \simeq 0$. However, this limit is not phenomenologically viable. In any case, radiative corrections are required to obtain the observed Higgs mass.

Tree-level MSSM Higgs couplings to quarks and squarks

The MSSM employs the Type–II Higgs–fermion Yukawa couplings. Employing the more common MSSM notation, $H_D^i \equiv \epsilon_{ij}\Phi_1^{j\,*}$ and $H_U^i = \Phi_2^i$ (where $i,j=1,2$ are weak SU(2) indices), the tree-level Yukawa couplings are:

$$-\mathscr{L}_{\rm Yuk} = \epsilon_{ij}\big[h_b\bar{b}_R H_D^i Q_L^j + h_t\bar{t}_R Q_L^i H_U^j\big] + {\rm h.c.}\,,$$

which yields

$$m_b = h_b v c_\beta/\sqrt{2}\,, \qquad\qquad m_t = h_t v s_\beta/\sqrt{2}\,.$$

The leading terms in the coupling of the Higgs bosons to third generation squarks are proportional to the Higgs–top quark Yukawa coupling, h_t,

$$\mathscr{L}_{\rm int} \ni h_t\big[\mu^*(H_D^\dagger\widetilde{Q})\widetilde{U} + A_t\epsilon_{ij}H_U^i\widetilde{Q}^j\widetilde{U} + {\rm h.c.}\big] - h_t^2\big[H_U^\dagger H_U(\widetilde{Q}^\dagger\widetilde{Q} + \widetilde{U}^*\widetilde{U}) - |\widetilde{Q}^\dagger H_U|^2\big]\,,$$

where $\widetilde{Q} = \begin{pmatrix}\widetilde{t}_L \\ \widetilde{b}_L\end{pmatrix}$ and $\widetilde{U} \equiv \widetilde{t}_R^*$.

In terms of the Higgs basis fields H_1 and H_2,

$$
\begin{aligned}
\mathscr{L}_{\rm int} \ni h_t\epsilon_{ij}\big[(\sin\beta X_t H_1^i+\cos\beta Y_t H_2^i)\widetilde{Q}^j\widetilde{U}+{\rm h.c.}\big] \\
-h_t^2\bigg\{\bigg[s^2_\beta|H_1|^2+c^2_\beta|H_2|^2+\sin\beta\cos\beta(H_1^\dagger H_2+{\rm h.c.})\bigg](\widetilde{Q}^\dagger\widetilde{Q}+\widetilde{U}^*\widetilde{U}) \\
-s^2_\beta|\widetilde{Q}^\dagger H_1|^2-c^2_\beta|\widetilde{Q}^\dagger H_2|^2-\sin\beta\cos\beta\big[(\widetilde{Q}^\dagger H_1)(H_2^\dagger\widetilde{Q})+{\rm h.c.}\big]\bigg\}\,,
\end{aligned}
$$

where

$$
X_t\equiv A_t-\mu^*\cot\beta\,,\qquad\qquad Y_t\equiv A_t+\mu^*\tan\beta\,.
$$

Assuming CP-conservation for simplicity, we shall henceforth take μ, A_t real.

The radiatively corrected MSSM Higgs Sector

We are most interested in the limit where m_h, m_A, m_H, $m_{H^\pm} \ll M_S$, where M_S characterizes the scale of the squark masses. In this case, we can formally integrate out the squarks and generate a low-energy effective 2HDM Lagrangian. This Lagrangian will no longer be of the tree-level MSSM form but rather a completely general 2HDM Lagrangian. If we neglect CP-violating phases that could appear in the MSSM parameters such as μ and A_t, then the resulting 2HDM Lagrangian contains all possible CP-conserving terms of dimension-four or less.

At one-loop, leading log corrections are generated for $\lambda_1, \ldots \lambda_4$. In addition, threshold corrections proportional to A_t, A_b and μ can contribute significant corrections to all the scalar potential parameters $\lambda_1 \ldots, \lambda_7$.

Computational technique[†]

- Employ the renomalization group equations (RGEs) for λ_1, $\lambda_2, \ldots \lambda_7$ of the general 2HDM without supersymmetry for $m_Z \leq \mu \leq M_S$.

- Impose supersymmetric boundary conditions at the scale $\mu = M_S$ for the λ_i. Use the RGEs to obtain the λ_i at the electroweak scale (either m_t or m_Z). In this approximation, $\lambda_5 = \lambda_6 = \lambda_7 = 0$ at all scales.

- Include threshold effects proportional to A_t, A_b and μ directly into the boundary conditions for the λ_i at the scale $\mu = M_S$. Use the RGE to obtain the λ_i at the electroweak scale (either m_t or m_Z).[‡]

- Use the radiatively corrected λ_i to compute the corresponding Higgs basis parameters Z_i

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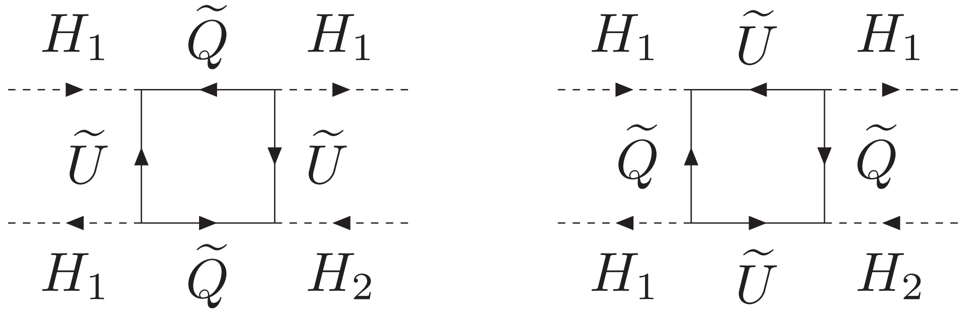
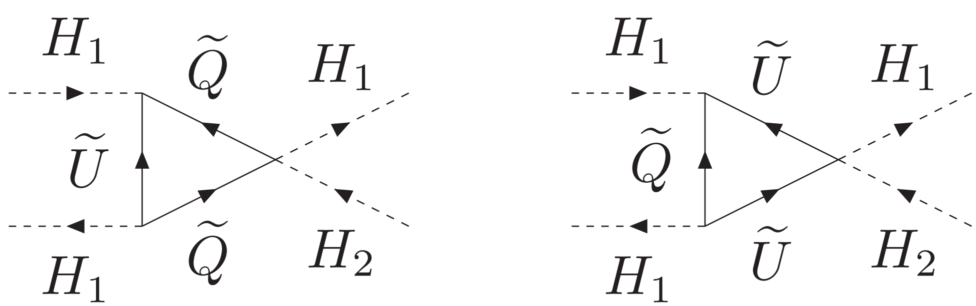
[†] Explicit one-loop formulae can be found in H.E. Haber and R. Hempfling, Phys. Rev. **D48**, 4280 (1993).

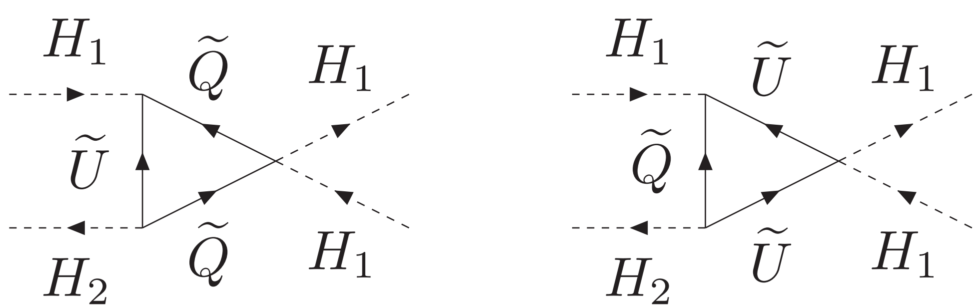
[‡] Multiple SUSY mass thresholds can be taken into account with suitable modifications of the RGEs.



$\propto s_\beta^3 c_\beta X_t^3 Y_t$



$\propto s_\beta^3 c_\beta X_t^2$



$\propto s_\beta^3 c_\beta X_t Y_t$

Example: One-loop threshold corrections to Z_6

The leading one-loop corrected expressions for Z_1, Z_5 and Z_6 are given by

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_5 v^2 = s_{2\beta}^2\left\{m_Z^2 + \frac{3v^2 h_t^4}{32\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t Y_t}{M_S^2}\left(1 - \frac{X_t Y_t}{12M_S^2}\right)\right]\right\},$$

$$Z_6 v^2 = -s_{2\beta}\left\{m_Z^2 c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t + Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\}.$$

where $M_S^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$, $X_t \equiv A_t - \mu\cot\beta$ and $Y_t = A_t + \mu\tan\beta$.

Note that $m_h^2 \simeq Z_1 v^2$ is consistent with $m_h \simeq 125$ GeV for suitable choices for M_S and X_t. Exact alignment (i.e., $Z_6 = 0$) can now be achieved due to an accidental cancellation between tree-level and loop contributions.

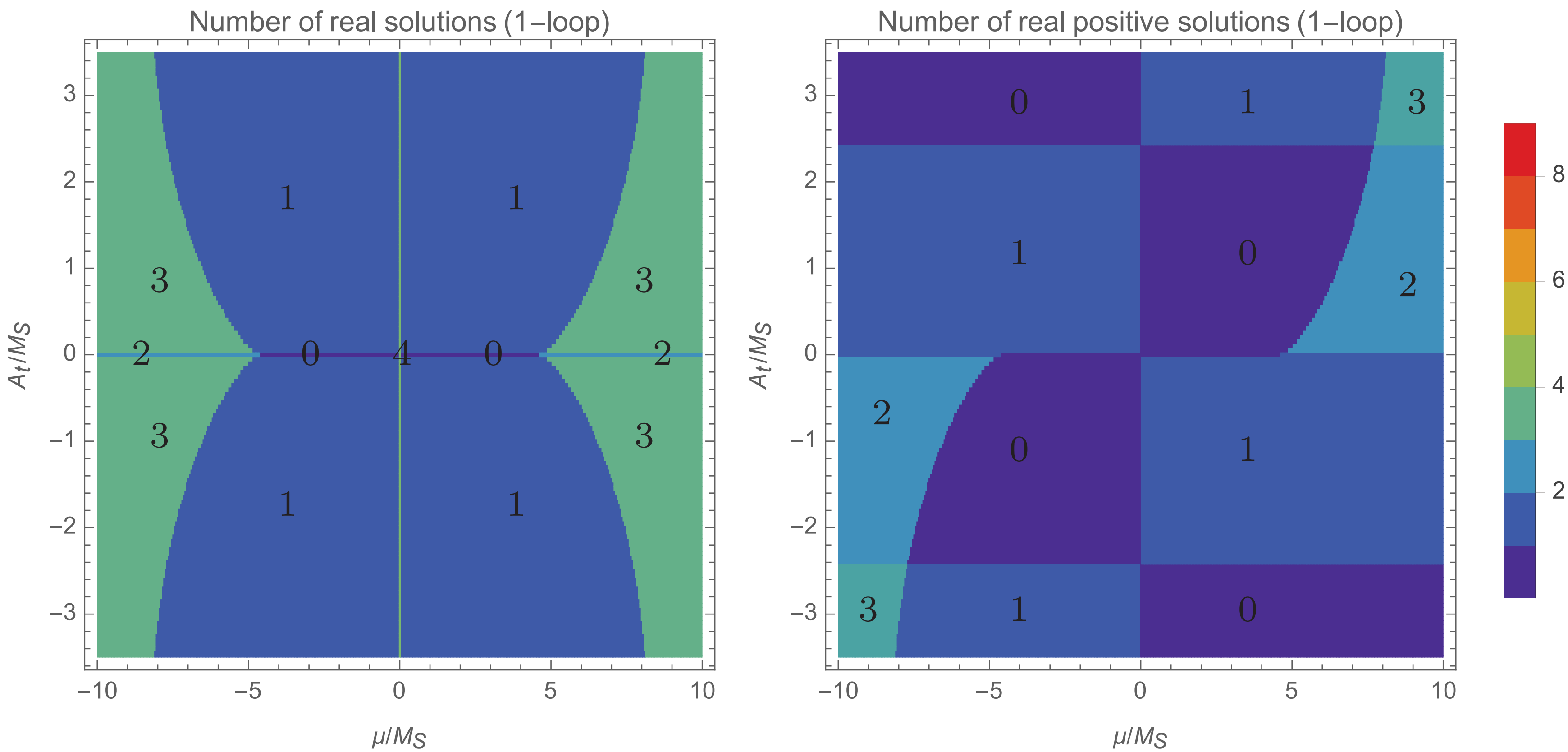
In equations,

$$m_Z^2 c_{2\beta} = \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right].$$

That is, $Z_6 \simeq 0$ for particular choices of $\tan\beta$. The alignment condition is then achieved by (numerically) solving a 7th order polynomial equation for positive real solutions of $t_\beta \equiv \tan\beta$ (where $\widehat{A}_t \equiv A_t/M_S$ and $\widehat{\mu} \equiv \mu/M_S$),

$$M_Z^2 t_\beta^4(1-t_\beta^2) - Z_1 v^2 t_\beta^4(1+t_\beta^2) + \frac{3m_t^4\widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu})(1+t_\beta^2)^2}{4\pi^2 v^2}\left[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\right] = 0\,.$$

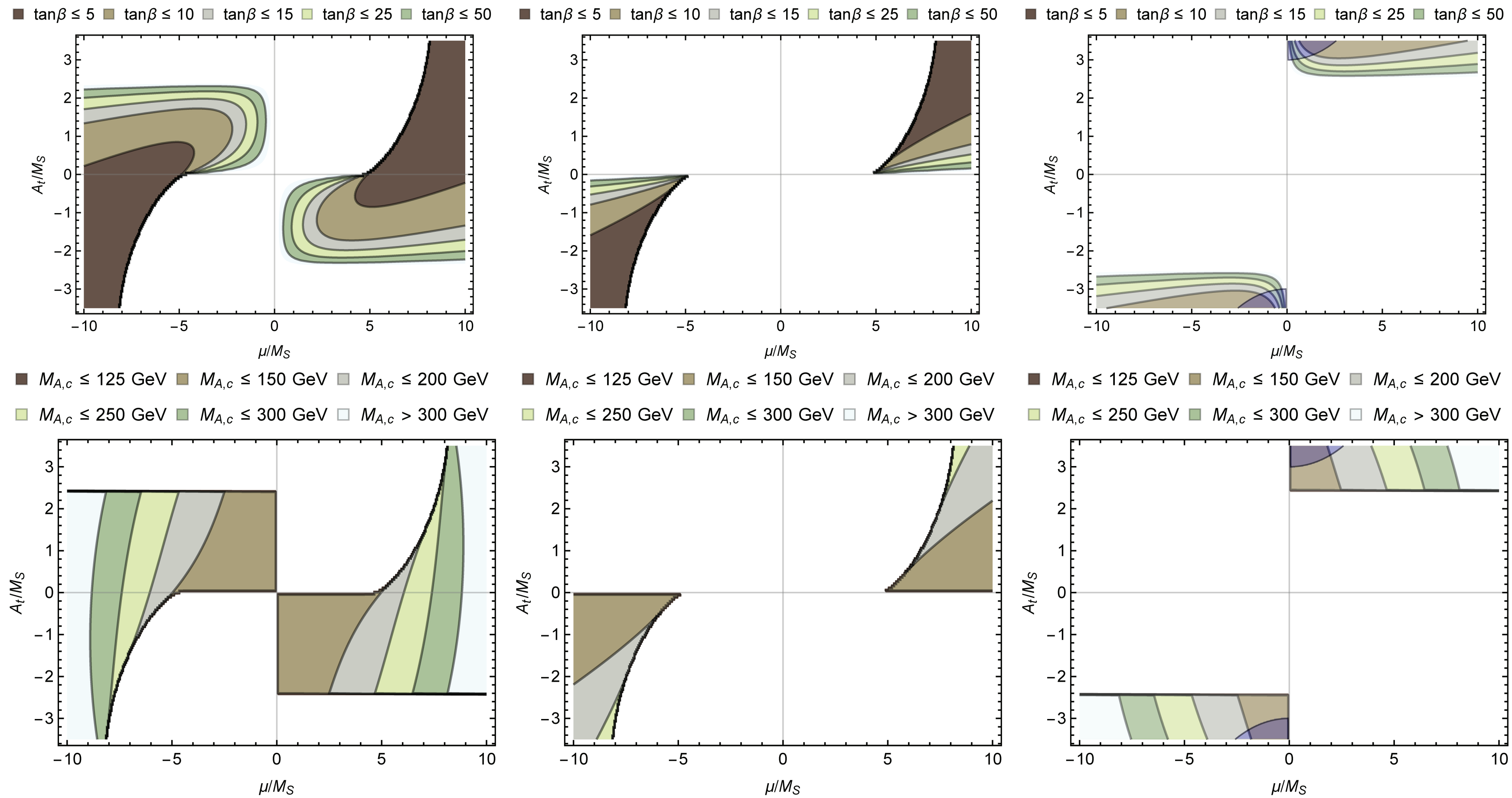
REMARK: Typically, we identify h as the SM-like Higgs boson. However, in the alignment limit there exist parameter regimes, corresponding to the case of $m_A^2 + (Z_5 - Z_1)v^2 < 0$ (where the radiatively corrected Z_1 and Z_5 are employed), in which H is the SM-like Higgs boson. In either case, $Z_1 v^2$ is the (approximate) squared mass of the SM-like Higgs boson.



Negative and positive solutions for $\tan\beta$ are interchanged under $\mu \to -\mu$.

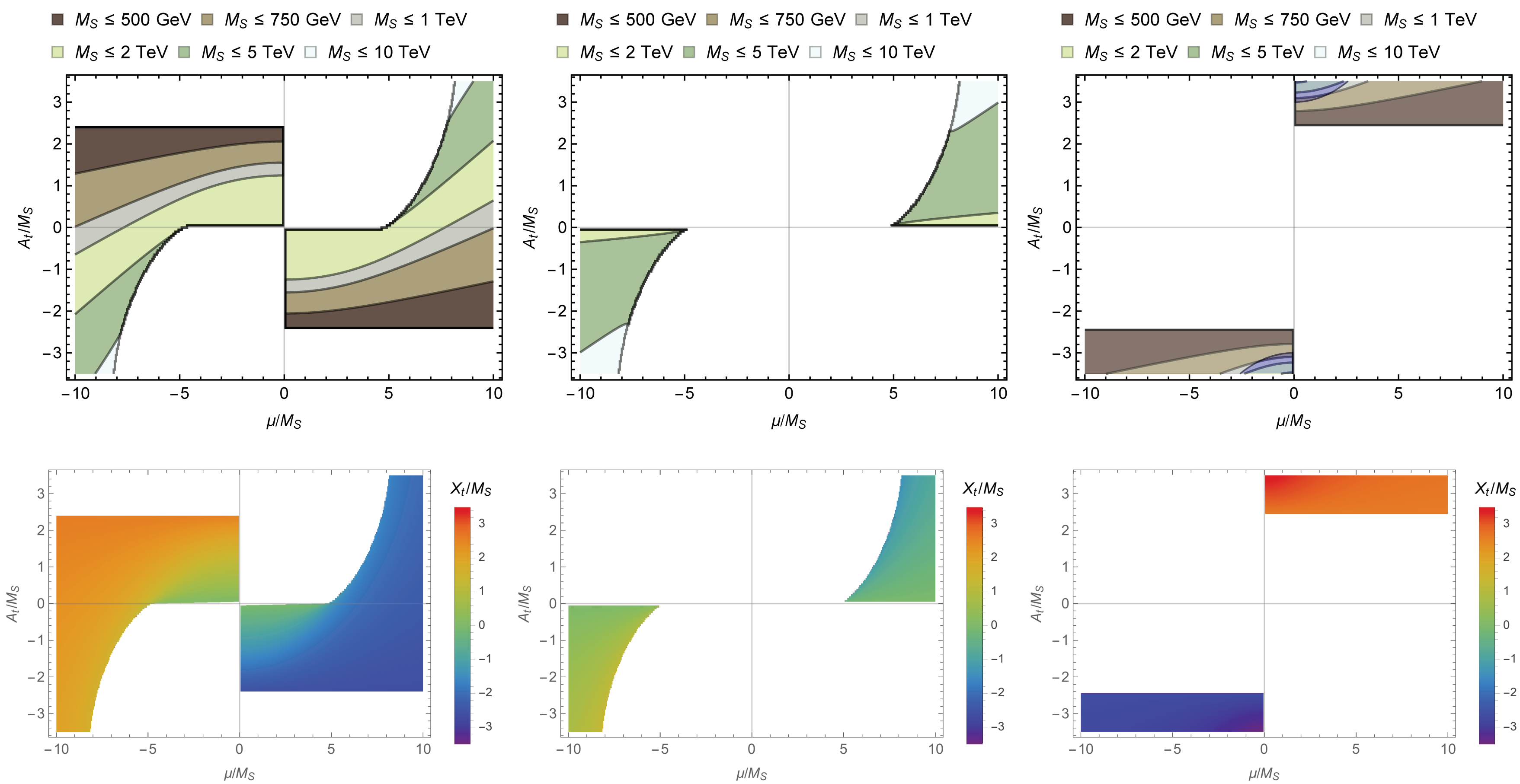
The number of real solutions decreases by two across a boundary when two solutions coalesce and then move off the real axis as a complex conjugate pair.

A positive solution changes to a negative solution across a boundary (right panel) by growing to $+\infty$ and then "jumping" to $-\infty$.



Top panels: Contours of positive values of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane, including the leading terms of the one-loop approximation. Z_1 is adjusted to give the correct Higgs mass. Taking the three top panels together, one can immediately discern the regions of zero, one, two and three values of $\tan\beta$ in which exact alignment is realized. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

Bottom panels: Contours of the critical value $M_{A,c}$, corresponding to the $\tan\beta$ solutions found in the top panels. If $m_A < M_{A,c}$, then the heavier of the CP-even Higgs bosons, H, is SM-like.



Top panels: Contours of the top squark mass parameter M_S, which depends on the values of μ/M_S and A_t/M_S, needed to obtain the correct Higgs squared-mass in the alignment limit, $Z_1 v^2 = 125$ GeV. The three figures correspond to the three $\tan\beta$ solutions of exact alignment previously exhibited.

Bottom panels: Value of $X_t/M_S \equiv \widehat{A}_t - \widehat{\mu}/\tan\beta$, as a function of $\widehat{\mu} \equiv \mu/M_S$ and $\widehat{A}_t \equiv A_t/M_S$ using the corresponding $\tan\beta$ solutions of exact alignment.

Leading two-loop corrections of $\mathcal{O}(\alpha_s h_t^2)$

The dominant part of the two-loop corrections to the CP-even Higgs squared-mass matrix can be obtained from the corresponding one-loop formulae with the following very simple two step prescription.[§] First, we replace

$$m_t^4 \ln\left(\frac{M_S^2}{m_t^2}\right) \longrightarrow m_t^4(\lambda)\ln\left(\frac{M_S^2}{m_t^2(\lambda)}\right)\,, \qquad \text{where } \lambda \equiv \left[m_t(m_t)M_S\right]^{1/2},$$

where $m_t(m_t) \simeq 165.6$ GeV is the $\overline{\rm MS}$ top quark mass, and the running top quark mass in the one-loop approximation is given by

$$m_t(\lambda) = m_t(m_t)\left[1 + \frac{\alpha_s}{\pi}\ln\left(\frac{m_t^2(m_t)}{\lambda^2}\right)\right]\,.$$

In our numerical analysis, we take $\alpha_s = \alpha_s(m_t(m_t)) \simeq 0.10826$.

[§] M. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C.E.M. Wagner and G. Weiglein, Nucl. Phys. B **580**, 29 (2000).

Second, when m_t^4 multiplies that threshold corrections due to stop mixing (i.e., the one-loop terms proportional to X_t and Y_t), then we make the replacement,

$$m_t^4 \longrightarrow m_t^4(M_S)\,,$$

where

$$m_t(M_S) = m_t(m_t)\left[1 + \frac{\alpha_s}{\pi}\ln\left(\frac{m_t^2(m_t)}{M_S^2}\right) + \frac{\alpha_s}{3\pi}\frac{X_t}{M_S}\right]\,.$$

Note that the running top-quark mass evaluated at M_S includes a threshold correction proportional to X_t that enters at the scale of supersymmetry breaking. Here, we only keep the leading contribution to the threshold correction under the assumption that $m_t \ll M_S$.

In applying the prescription outlined above, we formally work to $\mathcal{O}(\alpha_s)$ while dropping terms of $\mathcal{O}(\alpha_s^2)$ and higher. For example,

$$\ln\left(\frac{M_S^2}{m_t^2(\lambda)}\right) \simeq \left[1 + \frac{\alpha_s}{2\pi}\right]\ln\left(\frac{M_S^2}{m_t^2(m_t)}\right)\,.$$

We can now obtain more precise approximations for Z_1, Z_5 and Z_6.

$$Z_1 v^2 = M_Z^2 c_{2\beta}^2 + CL\big(1 - 2\overline{\alpha}_s L + \overline{\alpha}_s\big) + CX_1\big(1 - 4\overline{\alpha}_s L + \tfrac{4}{3}\overline{\alpha}_s x_t\big)\,,$$

$$Z_5 v^2 = s_{2\beta}^2\left[M_Z^2 + \frac{CL}{4s_\beta^4}\big(1 - 2\overline{\alpha}_s L + \overline{\alpha}_s\big) + \frac{C}{4s_\beta^4} X_5\big(1 - 4\overline{\alpha}_s L + \tfrac{4}{3}\overline{\alpha}_s x_t\big)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left[M_Z^2 c_{2\beta} - \frac{CL}{2s_\beta^2}\big(1 - 2\overline{\alpha}_s L + \overline{\alpha}_s\big) - \frac{C}{2s_\beta^2} X_6\big(1 - 4\overline{\alpha}_s L + \tfrac{4}{3}\overline{\alpha}_s x_t\big)\right],$$

where we have defined,

$$C \equiv \frac{3m_t^4}{2\pi^2 v^2}\,, \quad \overline{\alpha}_s \equiv \frac{\alpha_s}{\pi}\,, \quad x_t \equiv X_t/M_S\,, \quad y_t \equiv Y_t/M_S\,, \quad L \equiv \ln\left(\frac{M_S^2}{m_t^2}\right)\,,$$

and

$$X_1 \equiv x_t^2\big(1 - \tfrac{1}{12}x_t^2\big)\,, \qquad X_5 \equiv x_t y_t\big(1 - \tfrac{1}{12}x_t y_t\big)\,, \qquad X_6 \equiv \tfrac{1}{2}x_t(x_t + y_t) - \tfrac{1}{12}x_t^3 y_t\,.$$

In the above equations, $m_t \equiv m_t(m_t)$ is the $\overline{\rm MS}$ top quark mass.

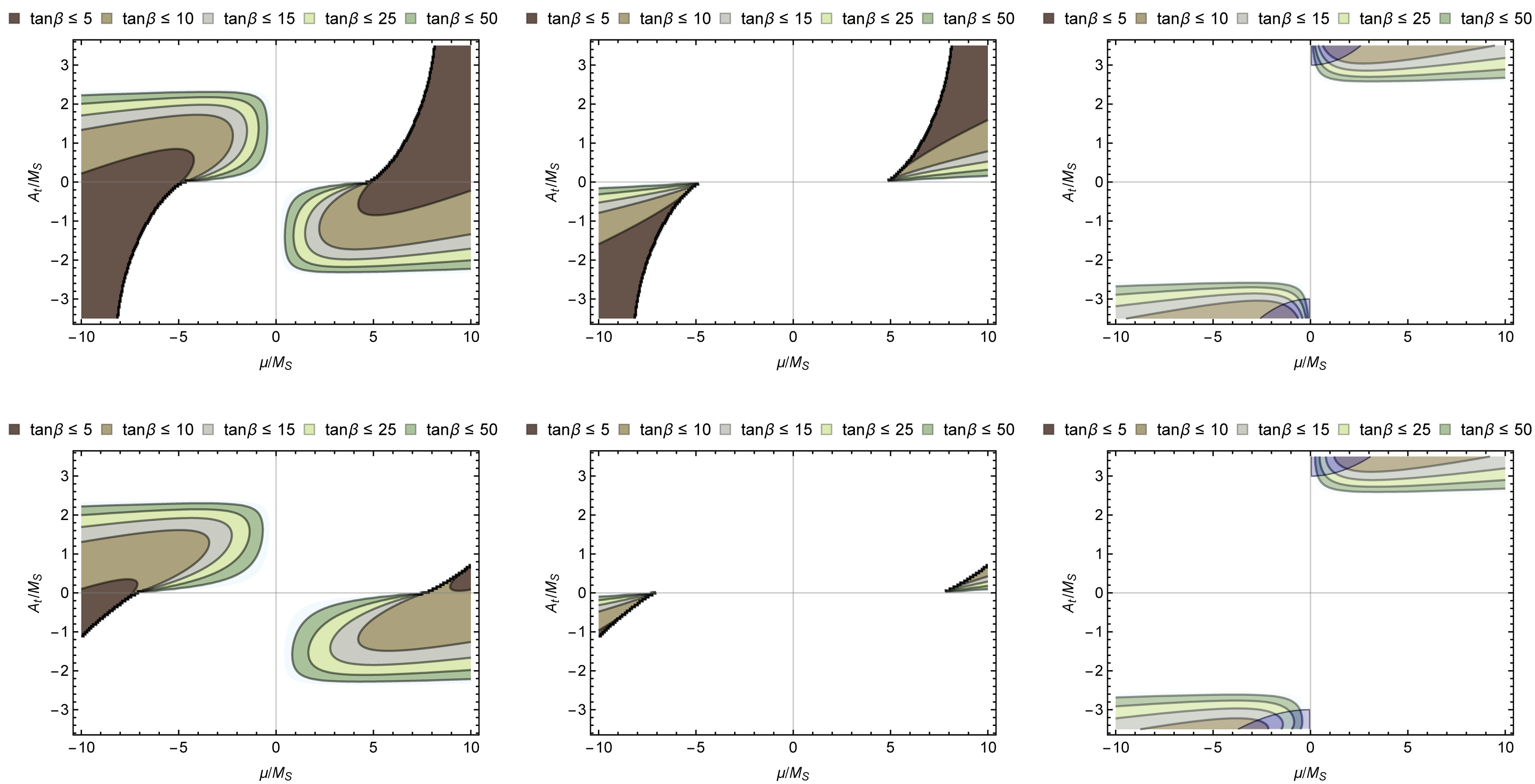
Exact alignment, $Z_6 = 0$, now yields an 11th order polynomial equation,

$$
\begin{aligned}
& M_Z^2 t_\beta^8 (1-t_\beta^2) - Z_1 v^2 t_\beta^8 (1+t_\beta^2) + \frac{3m_t^4 \widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu}) t_\beta^4 (1+t_\beta^2)^2}{4\pi^2 v^2} \bigl[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\bigr] \\
& \quad + 2\overline{\alpha}_s t_\beta^4 \bigl[M_Z^2 (1-t_\beta^2)^2 - Z_1 v^2 (1+t_\beta^2)^2\bigr] \widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu}) \bigl[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\bigr] \\
& \quad + \frac{\overline{\alpha}_s m_t^4 \widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu})^2 (1+t_\beta^2)^2}{\pi^2 v^2} \bigl[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\bigr] \\
& \qquad\qquad\qquad \times \bigl[t_\beta^3 + 3t_\beta^2 (\widehat{A}_t t_\beta - \widehat{\mu}) - \tfrac{1}{4}(\widehat{A}_t t_\beta - \widehat{\mu})^3\bigr] = 0 .
\end{aligned}
$$

As previously noted, solutions to this equation for negative $\tan\beta$ at a point in the $(\widehat{\mu}\,,\,\widehat{A}_t)$ plane can be reinterpreted as positive $\tan\beta$ solutions at the point $(-\widehat{\mu}\,,\,\widehat{A}_t)$.

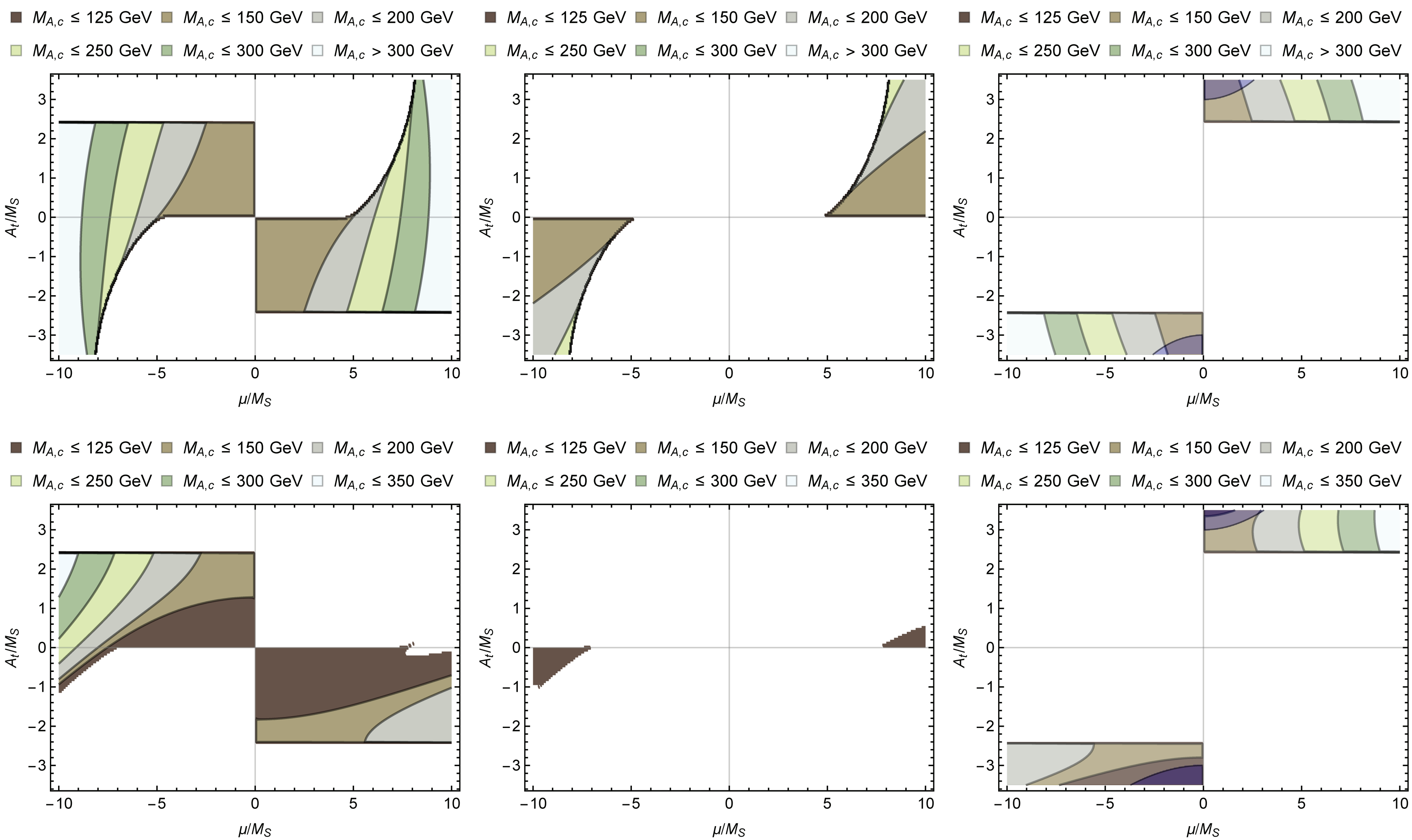
In the region of interest in the $(\mu/M_S, A_t/M_S)$ plane, we find that the previous one-loop real $\tan\beta$ solutions are still present (appropriately perturbed at the two-loop level). In addition, another real $\tan\beta$ solution emerges with $|X_t/M_S| \gtrsim 3$, and is therefore discarded.

Comparing the one-loop results for $\tan\beta$ solutions at exact alignment (top panels) to the corresponding two-loop improved results (bottom panels).



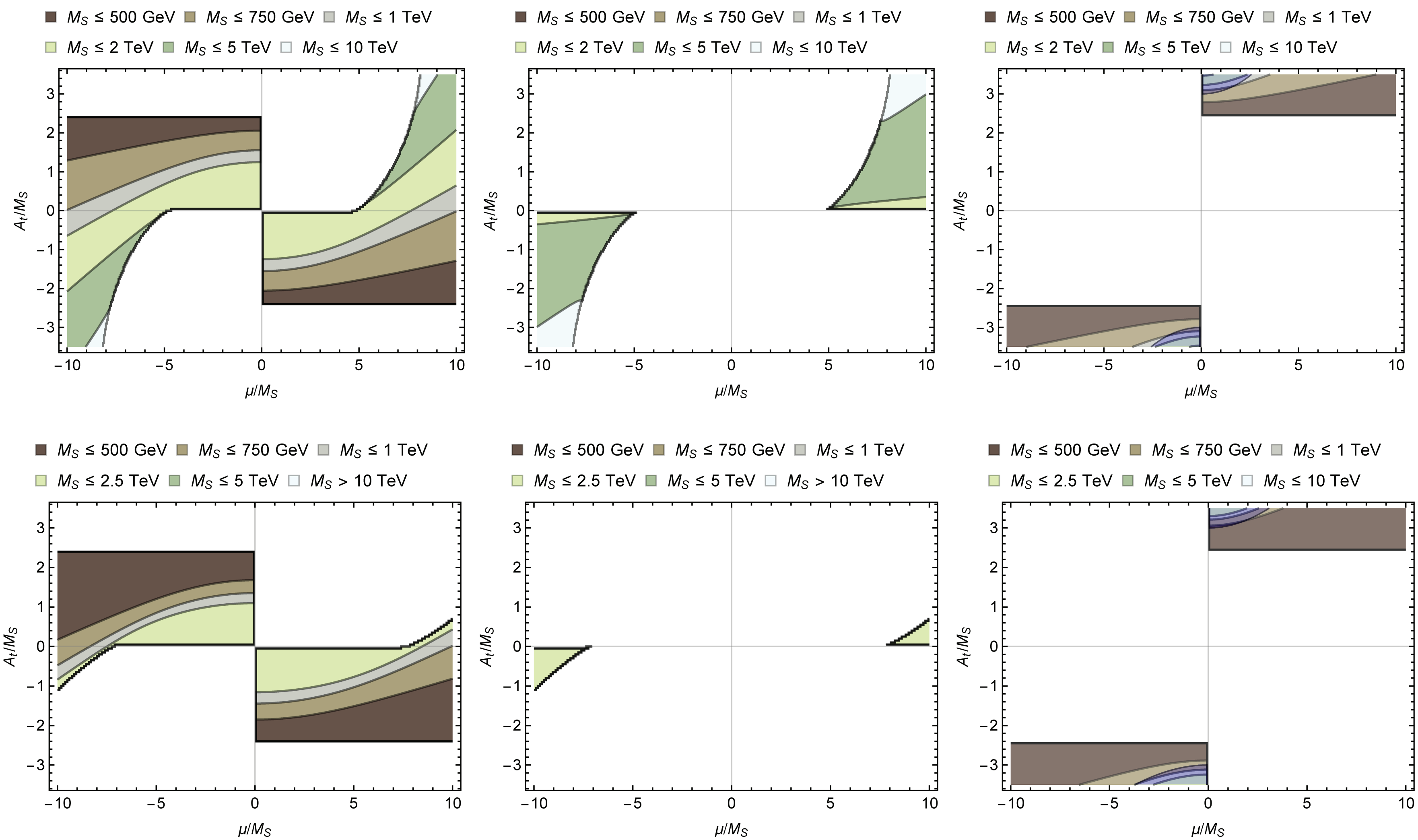
Contours of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane. Z_1 is adjusted to give the correct Higgs mass. Top panels: Approximate one-loop result. Bottom panels: Two-loop improved result. Taking the top (bottom) three panels together, one can immediately discern the regions of zero, one, two and three values of $\tan\beta$ in which exact alignment is realized. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

Comparing the one-loop results for $M_{A,c}$ solutions (top panels) to the corresponding two-loop improved results (bottom panels). If $m_A < M_{A,c}$, then the heavier of the CP-even Higgs bosons, H, is SM-like.



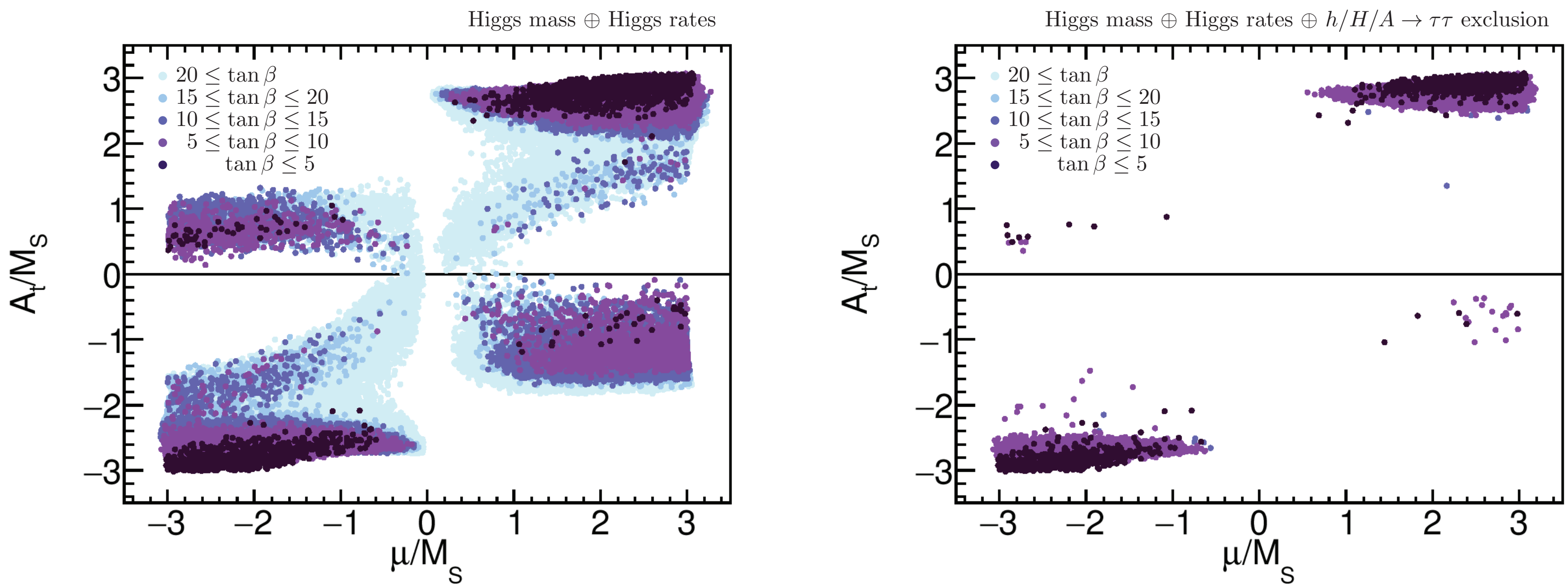
Contours of the critical value $M_{A,c}$ associated with the $\tan\beta$ solutions for exact alignment previously found. Top panels: Approximate one-loop result. Bottom panels: Two-loop improved result. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

Comparing the value of M_S needed to obtain the correct Higgs squared-mass in the alignment limit, $Z_1 v^2 = 125$ GeV, in the one loop approximation (top panels) and the corresponding two-loop improved results (bottom panels).



Contours of the top squark mass parameter M_S associated with the $\tan\beta$ solutions for exact alignment previously found. Top panels: Approximate one-loop result. Bottom panels: Two-loop improved result.

How well do the approximate two-loop results for the exact alignment limit¶ match a comprehensive scan over the MSSM parameter space? In a recent paper,‖ an 8-parameter pMSSM scan was performed to determine allowed parameter regimes which contain a light CP-odd Higgs boson A. Typically, h is SM-like, although one cannot yet rule out the possibility of a SM-like H.

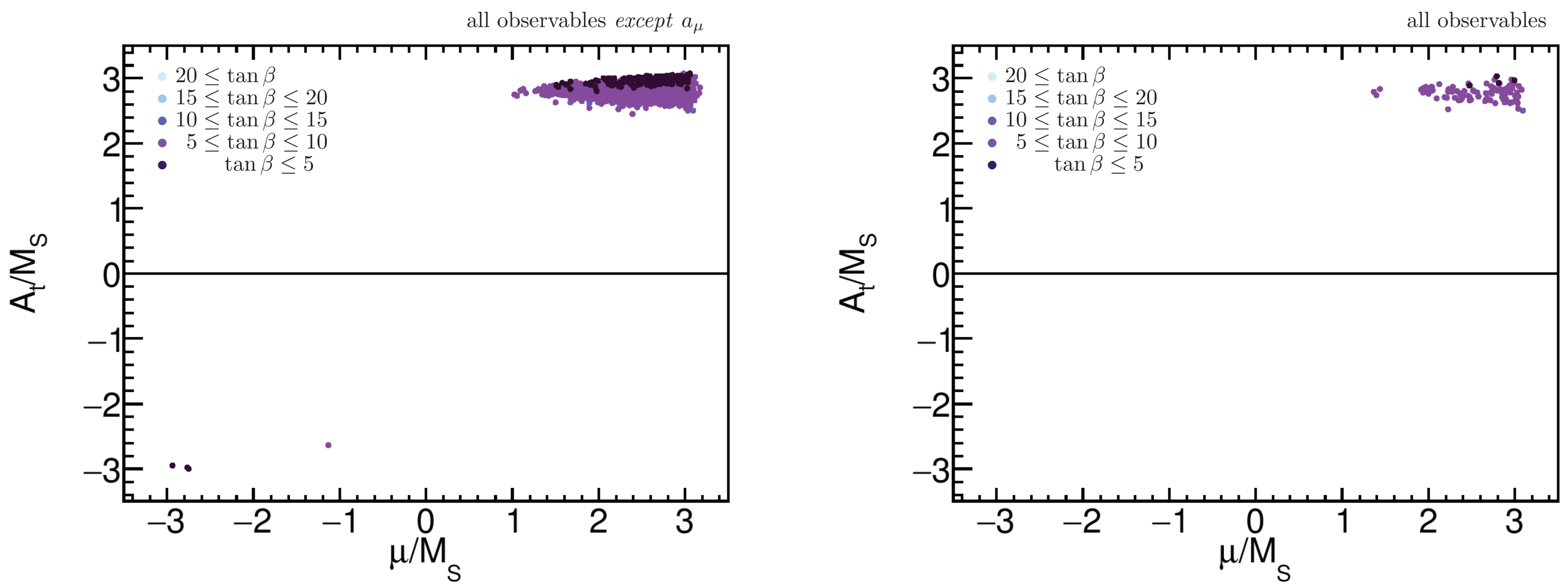


Preferred points of the pMSSM-8 scan with low $m_A \leq 350~\mathrm{GeV}$ for different selections of observables. The points are within the (approximate) 95% CL region, based on the following observables. Left panel: only Higgs mass and signal rates; Right panel: Higgs mass, signal rates and $h/H/A \to \tau^+\tau^-$ exclusion likelihood.

¶Of course, the precision Higgs data only requires that the condition of alignment is approximately satisfied.

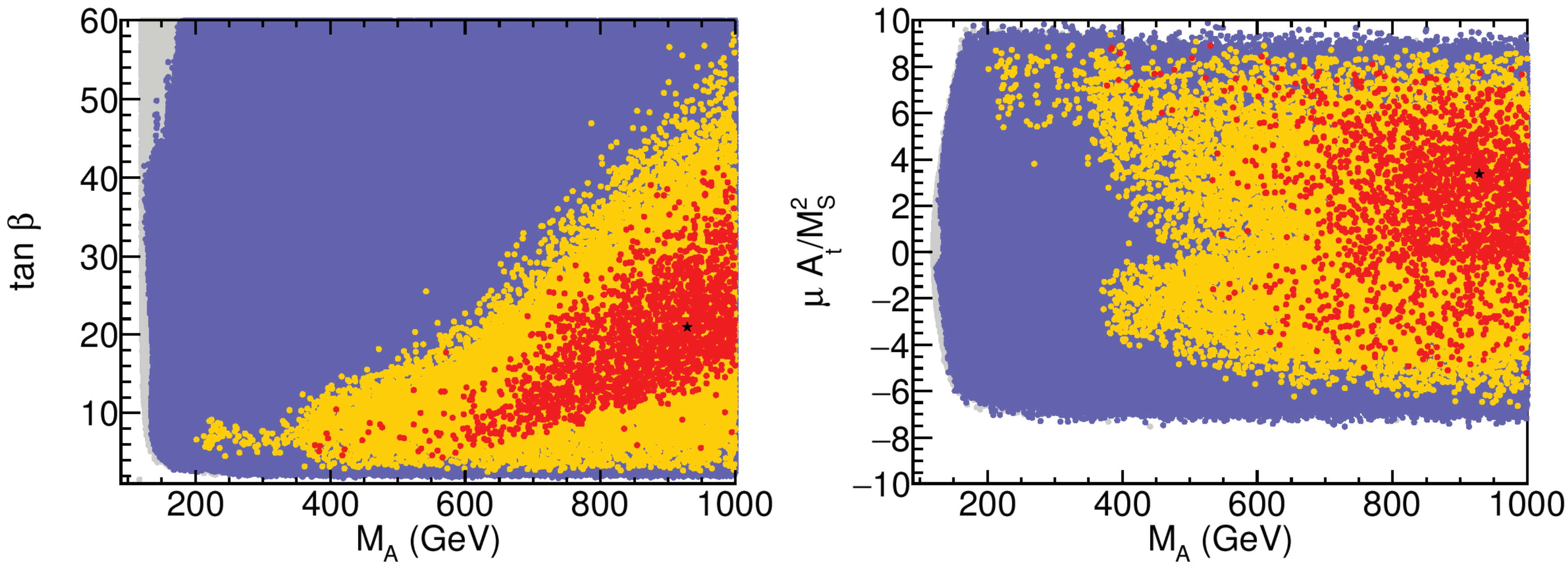
‖P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638.

Including additional constraints from SUSY particle searches and the impact of SUSY radiative corrections on SM observables, the allowed parameter regions of the pMSSM-8 scan shrinks further. For example, the negative μ region is mostly disfavored by BR($B \to X_s \gamma$), whereas the negative A_t region is disfavored by BR($B_s \to \mu^+\mu^-$).



Preferred points of the pMSSM-8 scan with low $m_A \le 350$ GeV for all observables except a_μ (left panel), and for all observables (right panel).

Bottom line: m_A values as low as 200 GeV are still allowed in the MSSM.



Preferred parameter regions in the $(M_A,\ \tan\beta)$ plane (left) and $(M_A,\ \mu A_t/M_S^2)$ plane (right), where $M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$ and h is the SM-like Higgs boson, in a pMSSM-8 scan. Points that do not pass the direct constraints from Higgs searches from `HiggsBounds` and from LHC SUSY particle searches from `CheckMATE` are shown in gray. Applying a global likelihood analysis to the points that pass the direct constraints, the color code employed is red for $\Delta\chi_h^2 < 2.3$, yellow for $\Delta\chi_h^2 < 5.99$ and blue otherwise. The best fit point is indicated by a black star.

Conclusions

- In light of the precision Higgs data, the condition of alignment (i.e., one of the Higgs mass eigenstates is aligned with the Higgs vacuum expectation value) is approximately satisfied.

- The alignment limit is approximately satisfied in the decoupling regime where $m_A \gg m_h$. But, approximate alignment can also be achieved without decoupling if the Higgs basis parameter $|Z_6| \ll 1$.

- Alignment without decoupling is possible in the MSSM, but it is achieved in a parameter regime in which there is an accidental approximate cancellation between tree-level and loop-level contributions to Z_6. (No symmetry exists that can enforce such a cancellation.

- Regions of approximate alignment without decoupling must necessarily appear in any comprehensive scan of the MSSM parameter space.

- Using all relevant data to constrain the MSSM Higgs sector, it is still possible that: (i) m_A is as low as 200 GeV, and (ii) H (rather than h) can be identified as the observed (SM-like) Higgs boson.