

$\mathcal{H}\mathcal{H} \chi s$ UPDATE

Michael Spira (PSI)

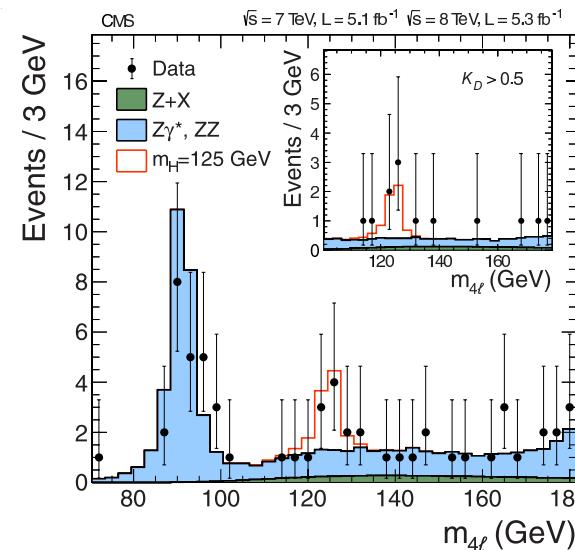
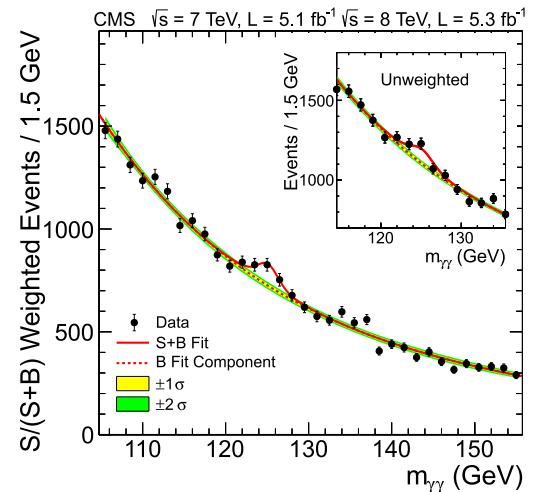
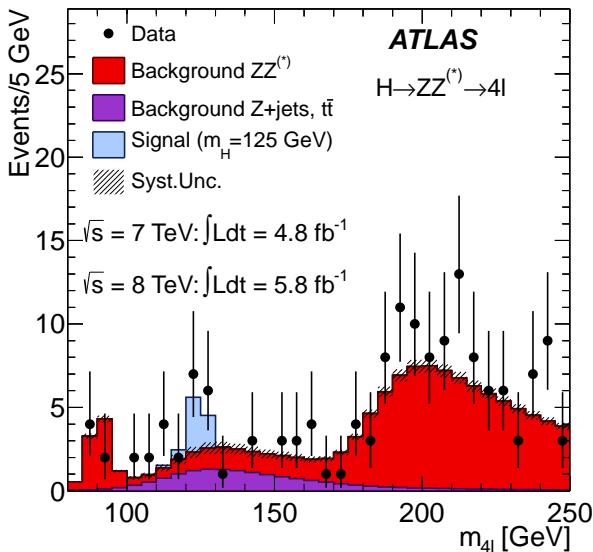
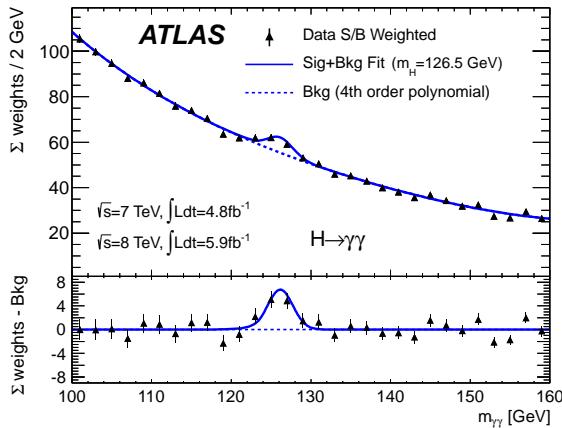
I Introduction

II Higgs Boson Pair Production

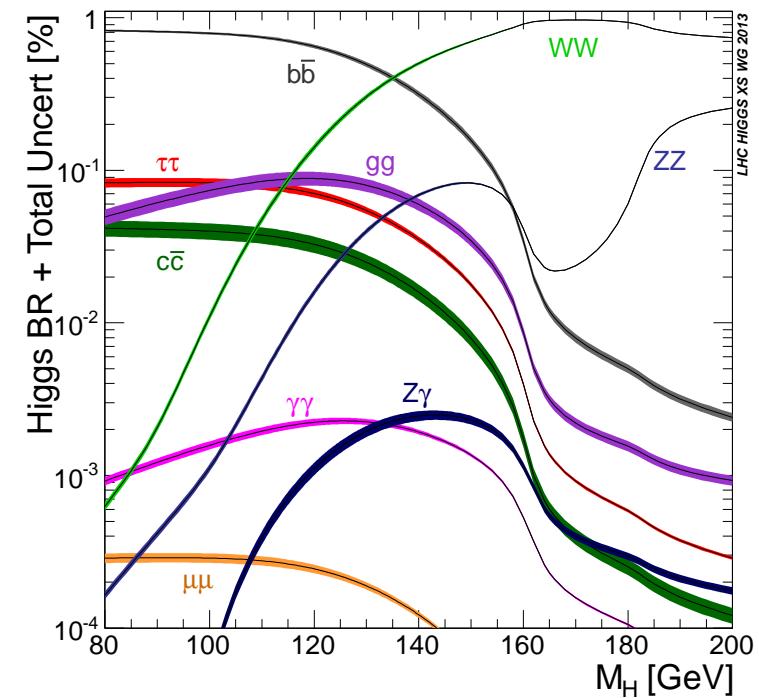
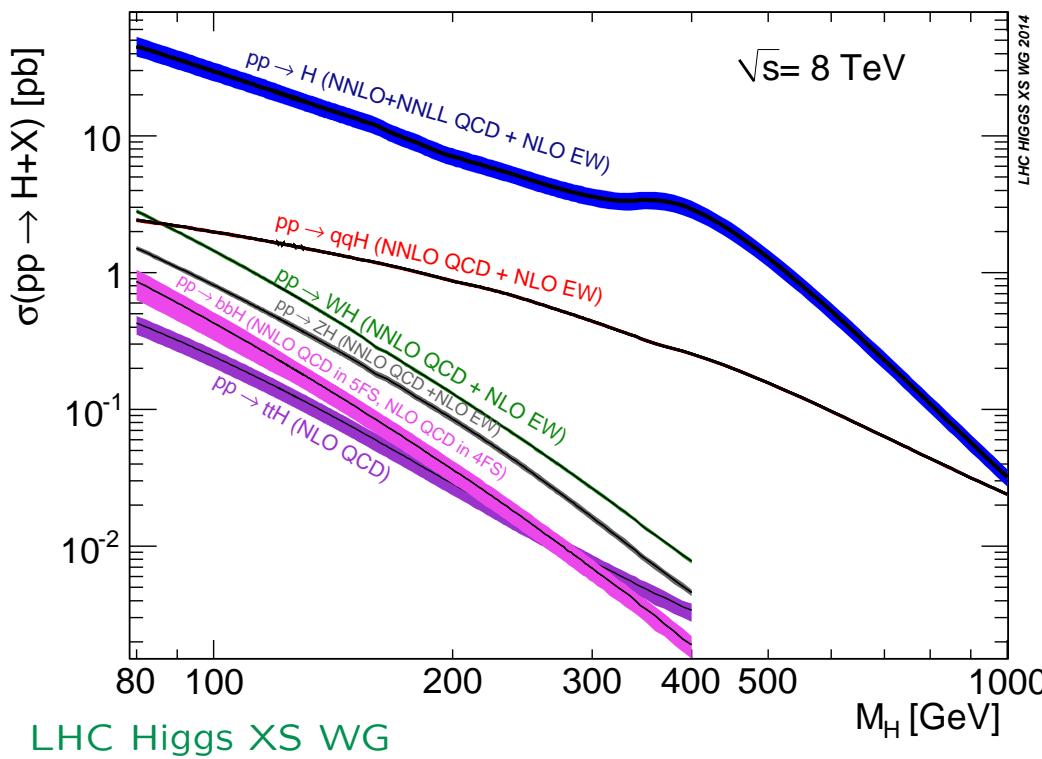
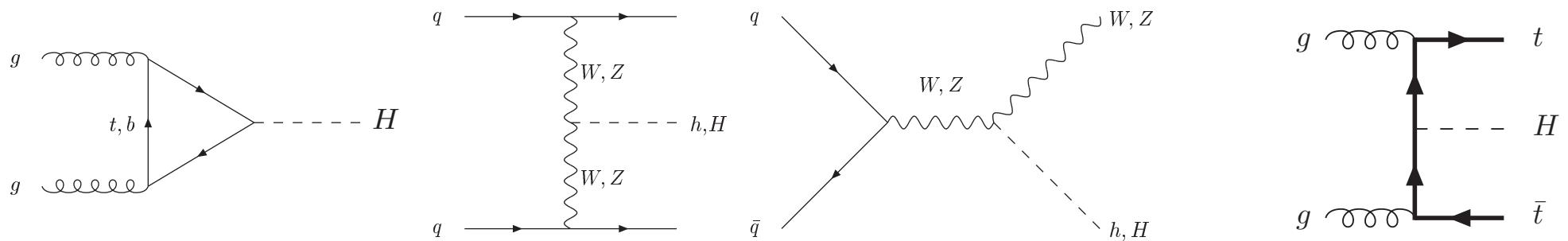
I INTRODUCTION

(i) Standard Model

- we have found the Higgs: $M_H \sim 125$ GeV
- $gg \rightarrow H$ dominant



● Higgs Boson Production



- Discovery: LHC [Tevatron]

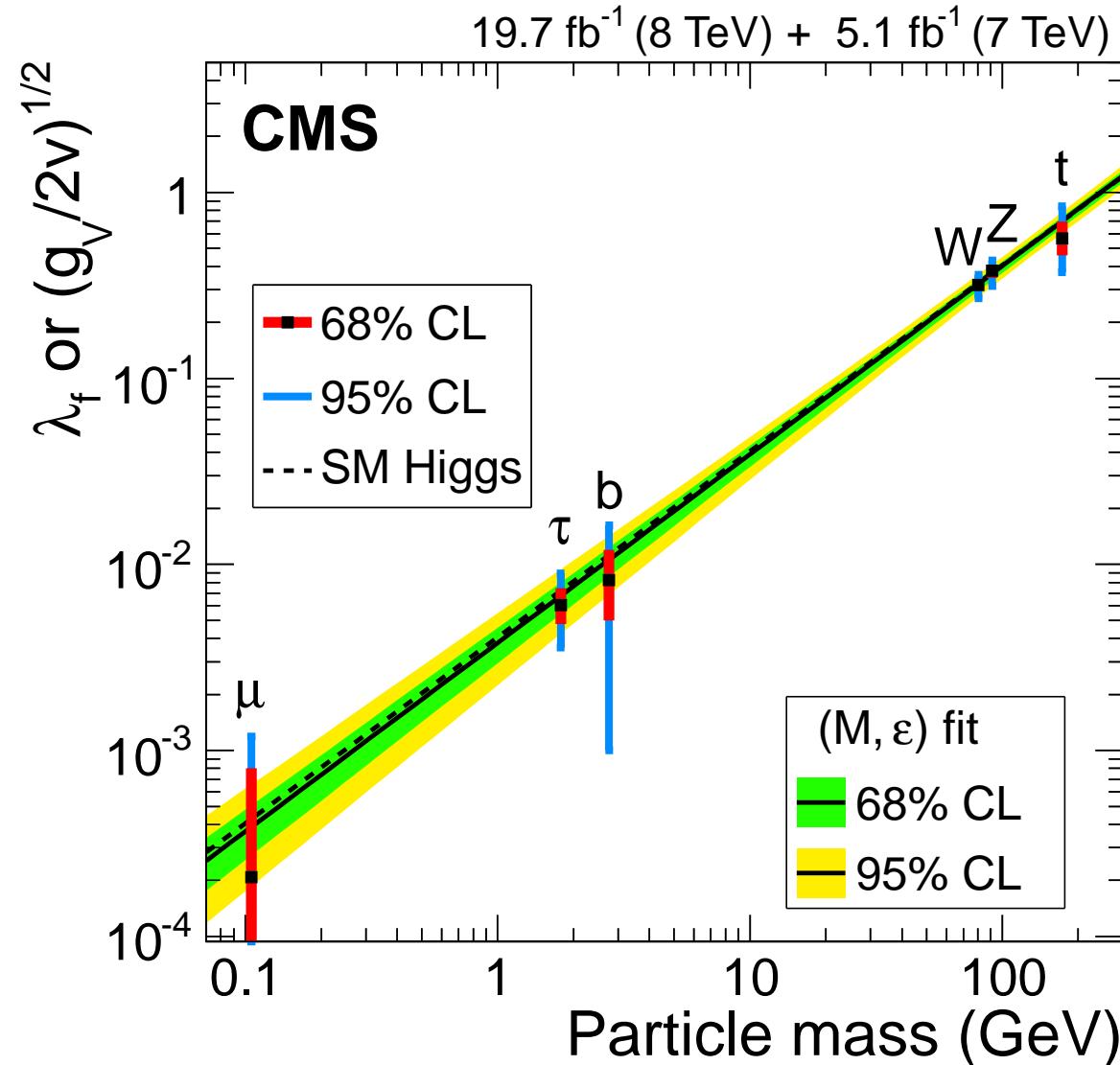
→ Higgs mass

couplings

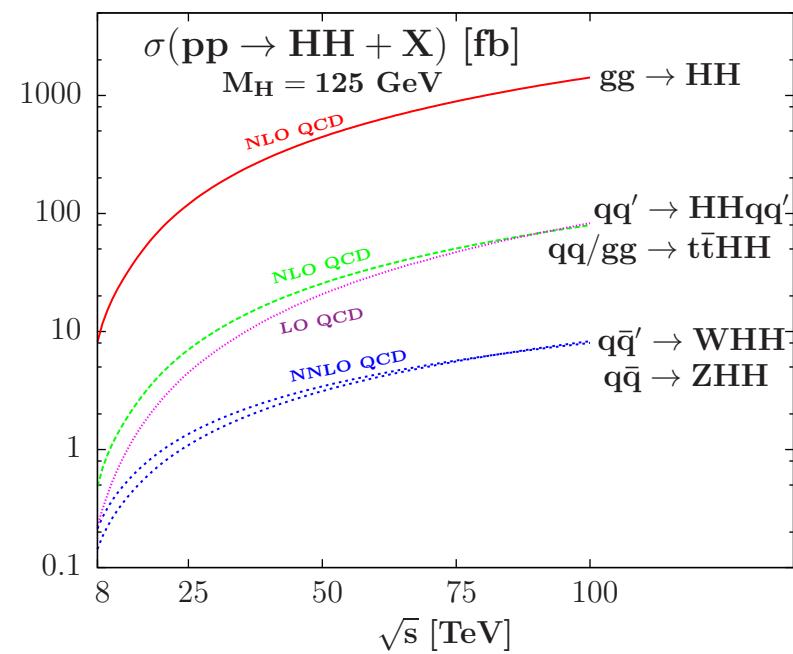
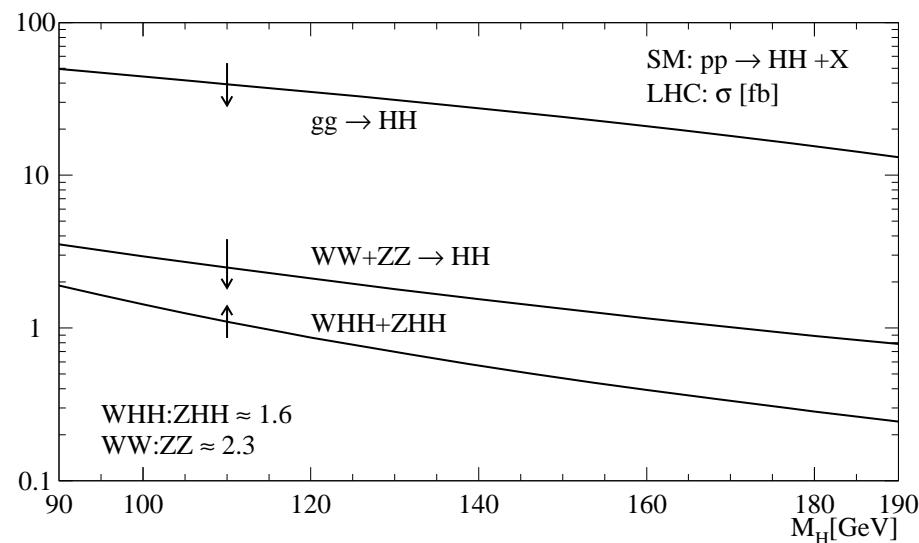
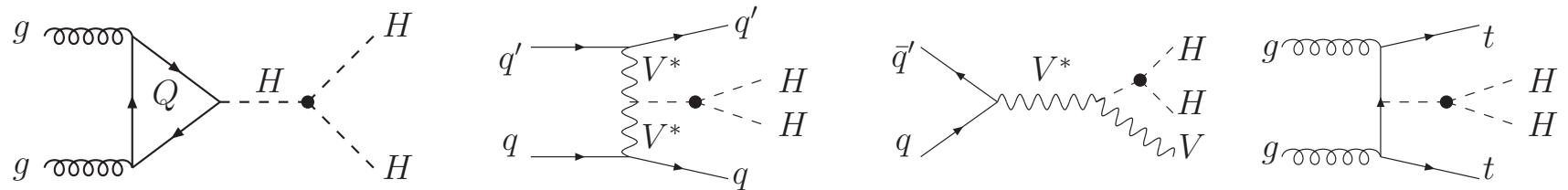
spin

\mathcal{CP}

λ ?



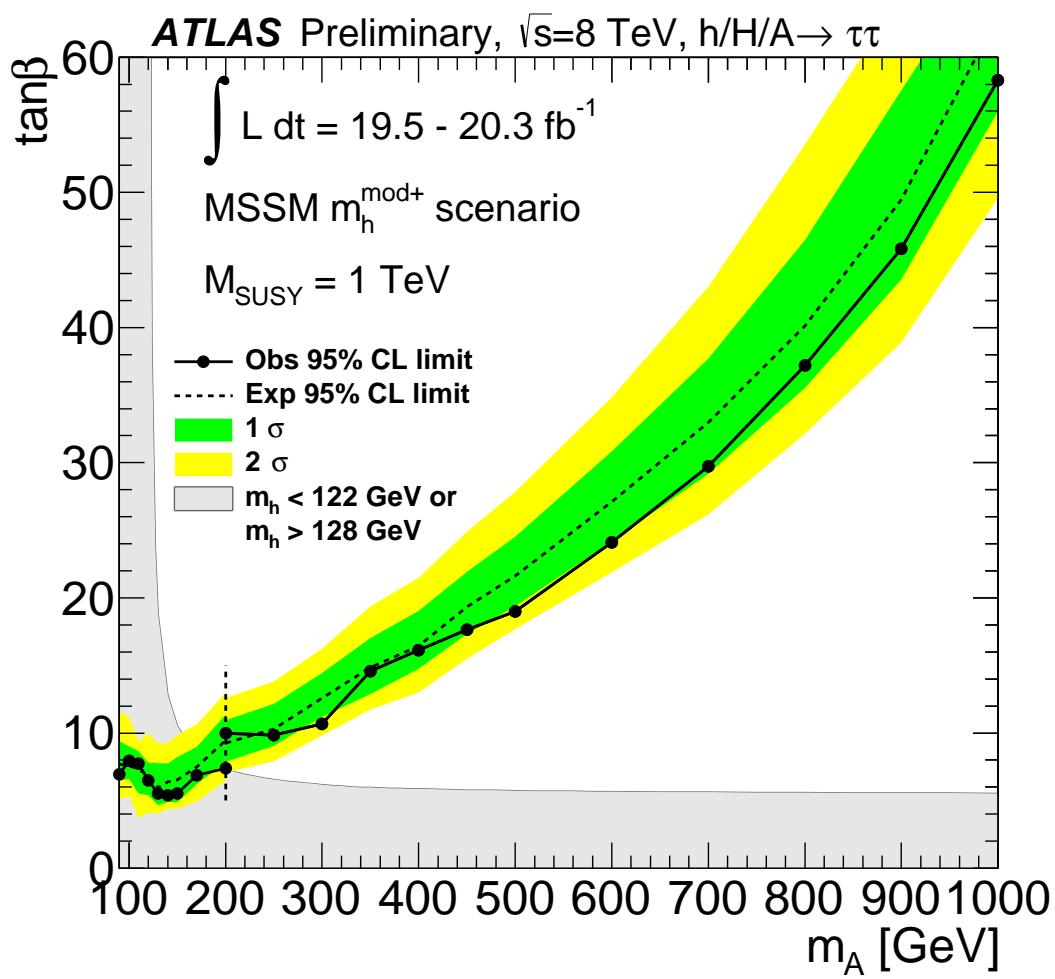
- Higgs boson pair production cross sections:



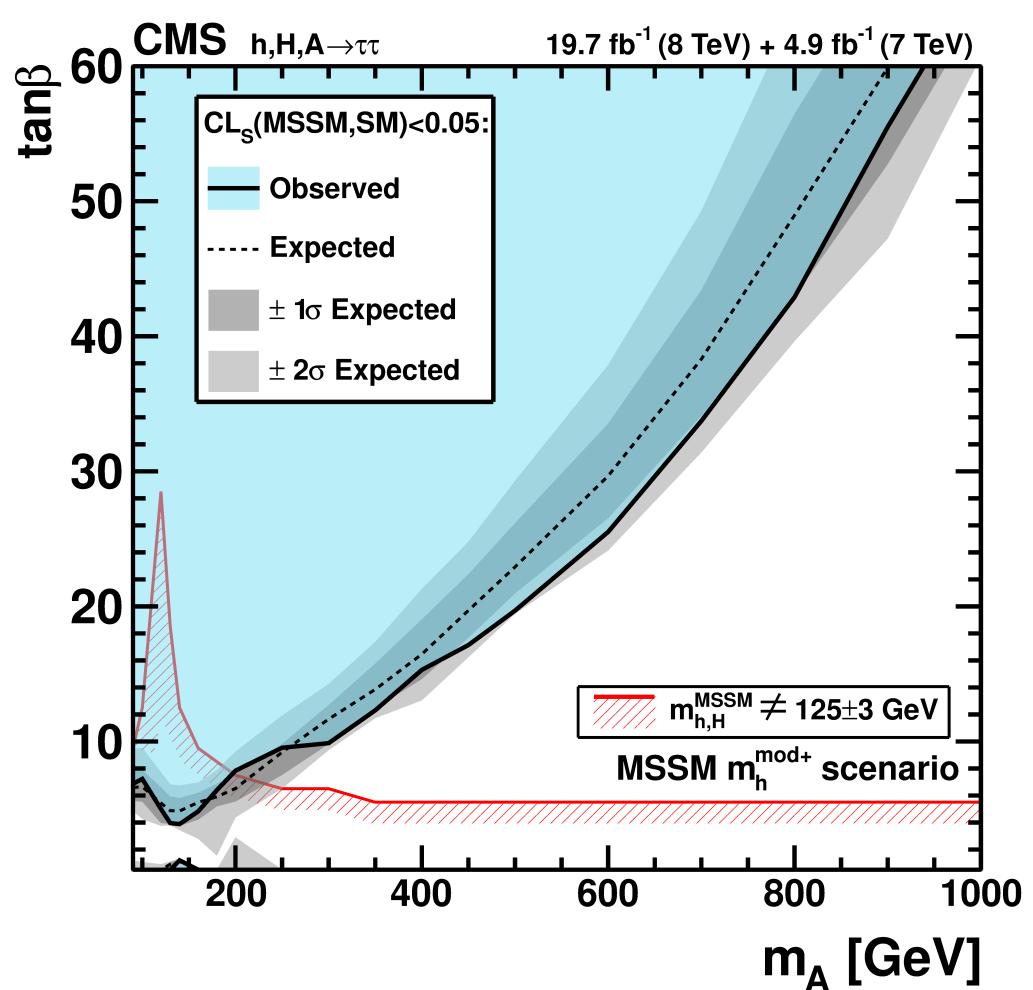
(ii) MSSM

- 2 Higgs doublets $\xrightarrow{\text{ESB}}$ 5 Higgs bosons: h, H, A, H^\pm
- LO: 2 input parameters: $M_A, \tan\beta = \frac{v_2}{v_1}$
- radiative corrections $\propto m_t^4 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$ $\rightarrow M_h \lesssim 135 \text{ GeV}$
 - Haber
 - Carena,...
 - Heinemeyer,...
 - Zhang
 - Slavich,...
 - ...
- Yukawa couplings: $\tan\beta \uparrow \Rightarrow g_u^\phi \downarrow \quad g_d^\phi \uparrow \quad g_V^\phi \downarrow$
- LHC: $gg \rightarrow \phi$ dominant for $\tan\beta \lesssim 10$
 $gg \rightarrow \phi b\bar{b}$ dominant for $\tan\beta \gtrsim 10$

$gg \rightarrow b\bar{b}\phi^0, \quad gg \rightarrow \phi^0$



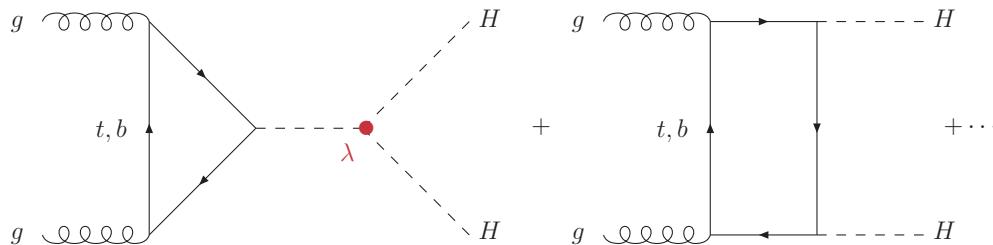
$\phi^0 \rightarrow \tau^+\tau^-$



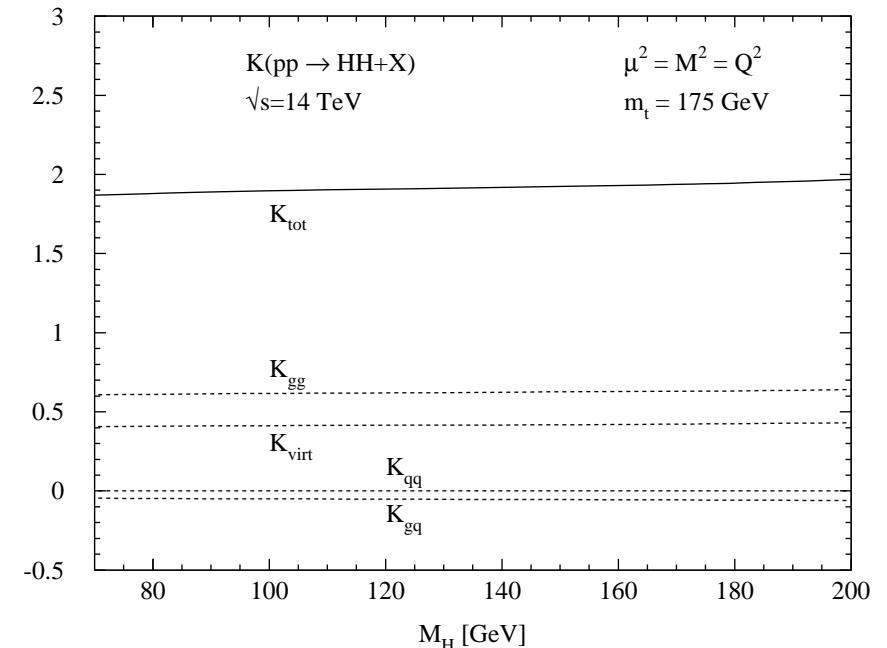
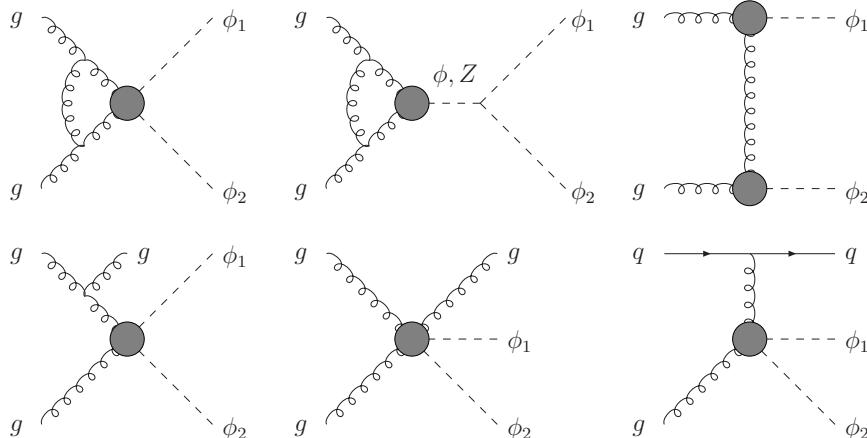
II *HIGGS BOSON PAIR PRODUCTION*

$gg \rightarrow HH$

SM



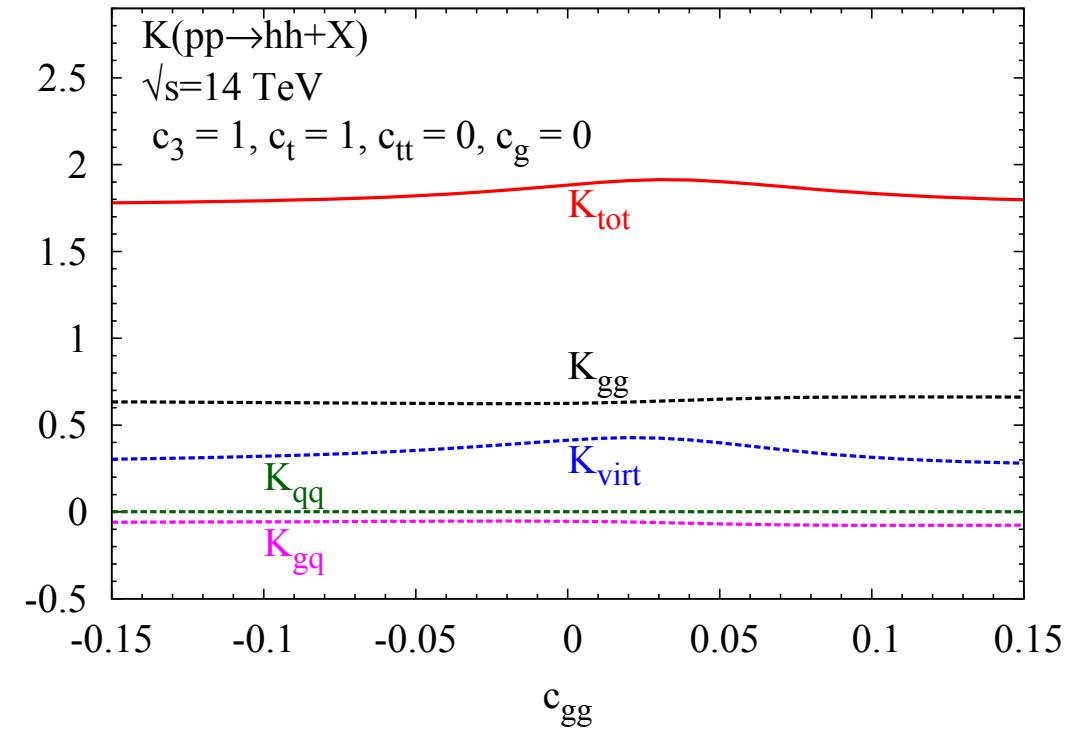
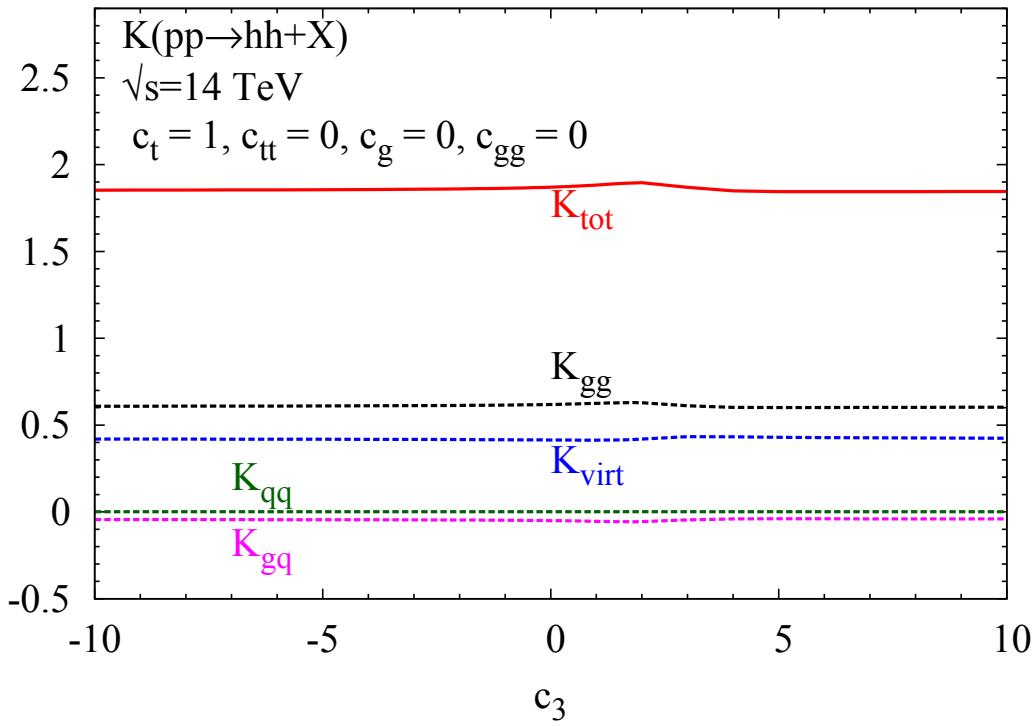
- third generation dominant $\rightarrow t, b$
- 2-loop QCD corrections: $\sim 90 - 100\%$
 $[M_H^2 \ll 4m_t^2, \quad \mu = M_{HH}]$



Dawson, Dittmaier, S.

- extended to dim6 → large impact on cxn [factor $\lesssim 10$]
small impact on K-factor

$$\mathcal{L}_{eff} = -m_t \bar{t}t \left(\textcolor{red}{c_t} \frac{h}{v} + \textcolor{red}{c_{tt}} \frac{h^2}{2v^2} \right) - \textcolor{red}{c_3} \frac{1}{6} \left(\frac{3M_h^2}{v} \right) h^3 + \frac{\alpha_s}{\pi} G^{a\mu\nu} G_{\mu\nu}^a \left(\textcolor{red}{c_g} \frac{h}{v} + \textcolor{red}{c_{gg}} \frac{h^2}{2v^2} \right)$$



Gröber, Mühlleitner, S., Streicher

$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

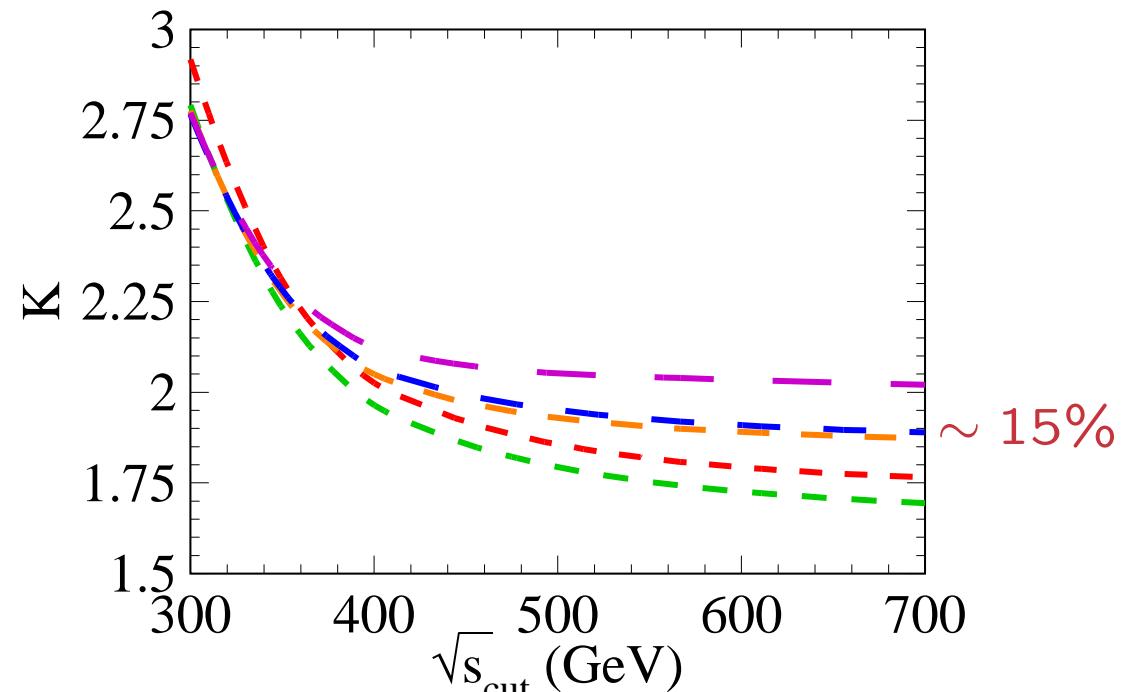
$$\begin{aligned}\sigma_{\text{LO}} &= \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \\ \Delta\sigma_{\text{virt}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \textcolor{red}{C} \\ \Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\ &\quad \left. + \textcolor{red}{d}_{gg}(z) + 6[1 + z^4 + (1 - z)^4] \left(\frac{\log(1 - z)}{1 - z} \right)_+ \right\} \\ \Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s(1 - z)^2} \right. \\ &\quad \left. + \textcolor{red}{d}_{gq}(z) \right\} \\ \Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \textcolor{red}{d}_{q\bar{q}}(z)\end{aligned}$$

$$C \rightarrow \pi^2 + \frac{11}{2} + C_{\Delta\Delta}, \quad d_{gg} \rightarrow -\frac{11}{2}(1-z)^3, \quad d_{gq} \rightarrow \frac{2}{3}z^2 - (1-z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27}(1-z)^3$$

- 2-loop QCD corrections:

$$\sigma = \sigma_0 + \frac{\sigma_1}{m_t^2} + \cdots + \frac{\sigma_4}{m_t^8}$$

Grigo, Hoff, Melnikov, Steinhauser



$$C = C^{(0)} + \frac{C^{(1)}}{m_t^2} + \cdots + \frac{C^{(4)}}{m_t^8}$$

$$d_{ij} = d_{ij}^{(0)} + \frac{d_{ij}^{(1)}}{m_t^2} + \cdots + \frac{d_{ij}^{(4)}}{m_t^8}$$

keep LO mass dependence **locally** $\Rightarrow \pm 10\%$

Grigo, Hoff, Steinhauser

$$C = C^{(0)}, \quad d_{ij} \quad \text{exact}$$

keep LO mass dependence **locally** $\Rightarrow -10\%$

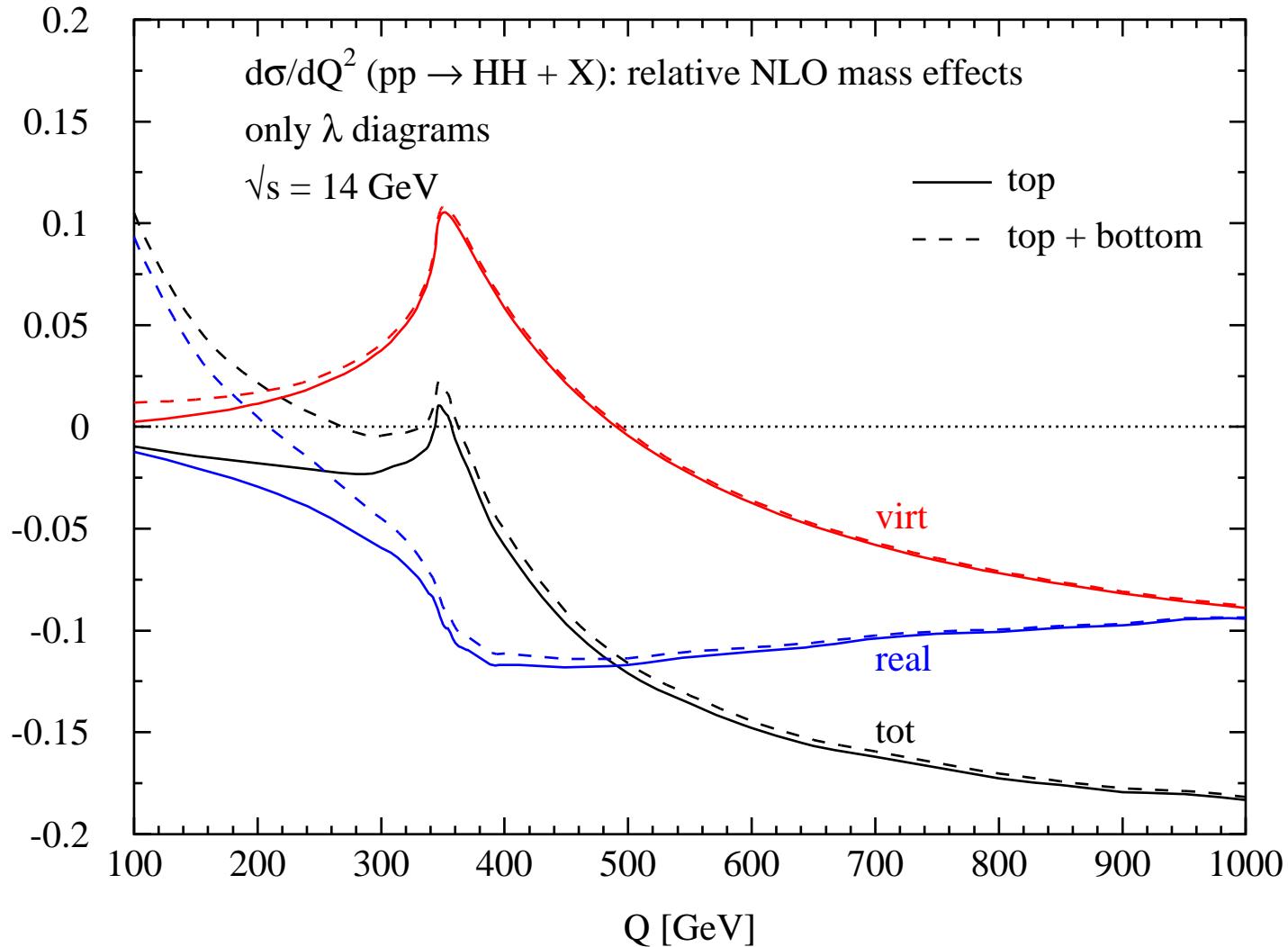
Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

exact result for double-blob diagrams [%-effect]

Degassi, Giardino, Gröber

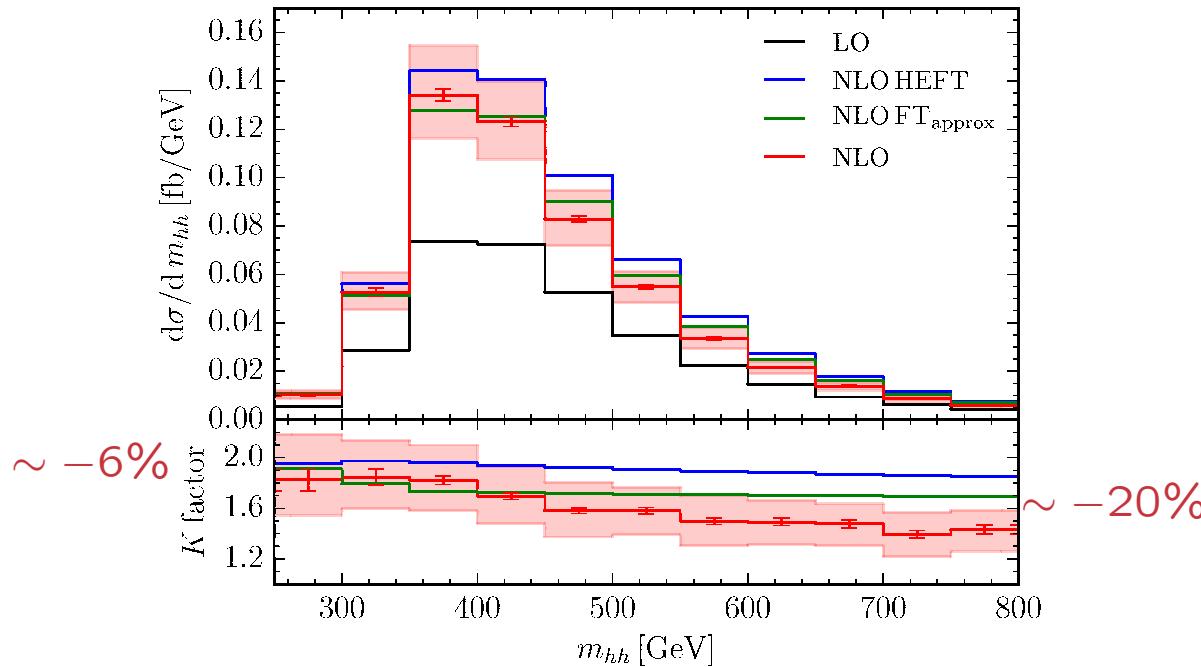
Diagrams with λ only:

$$\frac{d\sigma_m^{NLO} - d\sigma_\infty^{NLO}}{d\sigma_\infty^{NLO}}$$



Full NLO calculation: top only

Numerical integration, sector decomposition, contour deformation



Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke
Baglio, Campanario, Glaus, Mühlleitner, S., Streicher (in preparation)

- 13 TeV:

$$\begin{aligned}\sigma_{NLO} &= 27.80(8)^{+13.8\%}_{-12.8\%} \text{ fb} \\ \sigma_{NLO}^{HEFT} &= 32.22^{+18\%}_{-15\%} \text{ fb}\end{aligned}$$

⇒ -13.7% mass effects

- NNLO QCD corrections:

mainly obtained from NNLO single Higgs $\rightarrow \frac{d\sigma}{dQ^2}$

NLO corrections to double blob diagrams

de Florian, Mazzitelli

NNLO corrections to effective $HHgg$ coupling:

$$C_{HH}^{NNLO} = C_H^{NNLO} + \frac{35 + 16N_F}{12} \left(\frac{\alpha_s}{\pi} \right)^2$$

obtained from explicit diagrammatic calculation

Grigo, Hoff, Melnikov, Steinhauser

- NNLO corrections: $\sim 20\%$

$$[M_H^2 \ll 4m_t^2]$$

de Florian, Mazzitelli

Grigo, Melnikov, Steinhauser

- mass corrections: $\sim 5\%$

Grigo, Hoff, Steinhauser

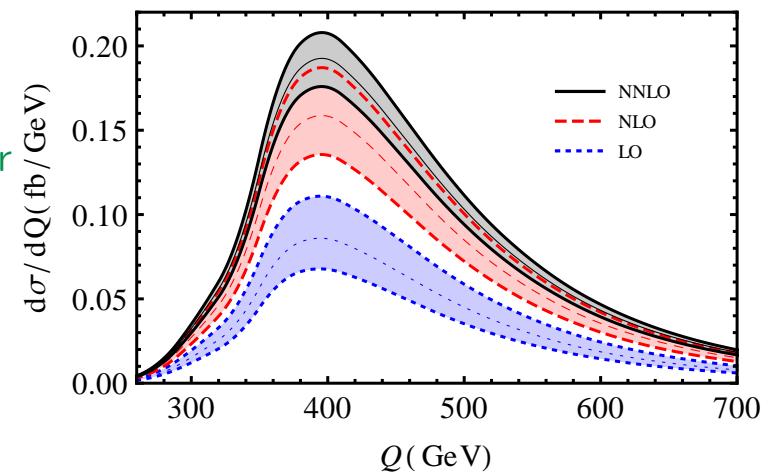
- soft gluon resummation: $\sim 10\%$

$$[M_H^2 \ll 4m_t^2]$$

de Florian, Mazzitelli

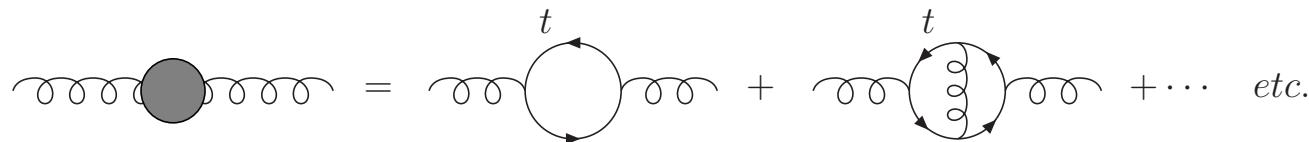
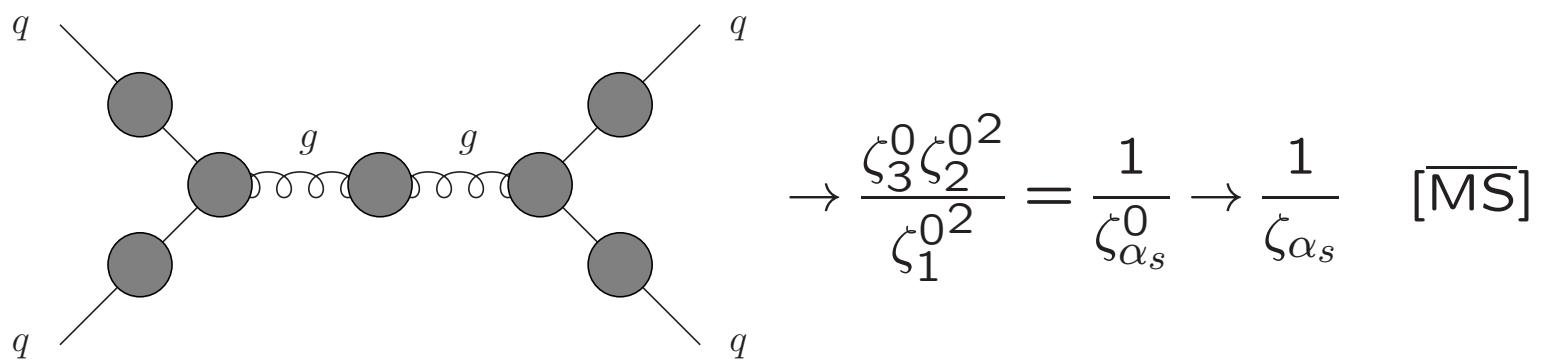
Shao, Li, Li, Wang

- uncertainties: $\sim 15 - 20\%$



Is there an easier way to obtain C_{HH}^{NNLO} ?

Yes...



$$\mathcal{L}_g = -\frac{1}{4\zeta_{\alpha_s}} \hat{G}^{a\mu\nu} \hat{G}_{\mu\nu}^a = -\frac{1-\Pi_t}{4} \hat{G}^{a\mu\nu} \hat{G}_{\mu\nu}^a \quad [\leftarrow (N_F + 1)]$$

$$\alpha_s^{(N_F)}(\mu_R^2) = \zeta_{\alpha_s} \alpha_s^{(N_F+1)}(\mu_R^2)$$

$$\Pi_t = 1 - \frac{1}{\zeta_{\alpha_s}} = \sum_n C_n \left(\frac{\alpha_s^{(N_F+1)}}{\pi} \right)^n$$

- $L_t = \log(\mu_R^2/\overline{m}_t^2(\mu_R^2))$

$$C_1 = -\frac{1}{6} L_t$$

$$C_2 = \frac{11}{72} - \frac{11}{24} L_t$$

$$C_3 = \frac{564731}{124416} - \frac{82043}{27648} \zeta_3 - \frac{2633}{31104} N_F - \frac{2777 - 201 N_F}{1728} L_t - \frac{35 + 16 N_F}{576} L_t^2$$

- shift the $\overline{\text{MS}}$ top mass

$$\begin{aligned}\overline{m}_t(\mu_R^2) &\rightarrow \overline{m}_t(\mu_R^2) \left(1 + \frac{H}{v}\right) \\ L_t &\rightarrow \bar{L}_t = L_t - 2 \log \left(1 + \frac{H}{v}\right)\end{aligned}$$

- keep only H -dependent terms
- reexpress $(N_F + 1)-$ by N_F -flavour quantities

$$\begin{aligned}\hat{G}^{a\mu\nu} \hat{G}_{\mu\nu}^a &= \zeta_{\alpha_s} G^{a\mu\nu} G_{\mu\nu}^a \\ \alpha_s^{(N_F+1)}(\mu_R^2) &= \alpha_s^{(N_F)}(\mu_R^2) \left\{ 1 + \frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi} \frac{L_t}{6} + \left(\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi} \right)^2 \left[-\frac{11}{72} + \frac{11}{24} L_t + \frac{L_t^2}{36} \right] \right\}\end{aligned}$$

Bernreuther, Wetzel
Chetyrkin, Kniehl, Steinhauser

$$\begin{aligned}
\mathcal{L}_{eff} &= \frac{\alpha_s}{12\pi} \left\{ (1+\delta) \log \left(1 + \frac{H}{v} \right) - \frac{\eta}{2} \log^2 \left(1 + \frac{H}{v} \right) \right. \\
&\quad \left. + \frac{\rho}{3} \log^3 \left(1 + \frac{H}{v} \right) - \frac{\sigma}{4} \log^4 \left(1 + \frac{H}{v} \right) \right\} G^{a\mu\nu} G_{\mu\nu}^a \\
\delta &= \delta_1 \frac{\alpha_s}{\pi} + \delta_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \delta_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \delta_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5) \\
\eta &= \eta_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \eta_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \eta_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5) \\
\rho &= \rho_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \rho_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5) \\
\sigma &= \sigma_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5)
\end{aligned}$$

$$\begin{aligned}
\delta_1 &= \frac{11}{4} \\
\delta_2 &= \frac{2777}{288} + \frac{19}{16} L_t + N_F \left(\frac{L_t}{3} - \frac{67}{96} \right) \\
\eta_2 &= \frac{35 + 16N_F}{24}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{eff} &= \frac{\alpha_s}{12\pi} \left\{ (1+\delta) \log \left(1 + \frac{H}{v} \right) - \frac{\eta}{2} \log^2 \left(1 + \frac{H}{v} \right) \right. \\
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\delta &= \delta_1 \frac{\alpha_s}{\pi} + \delta_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \delta_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \delta_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5) \\
\eta &= \eta_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \eta_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \eta_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5) \\
\rho &= \rho_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \rho_4 \left(\frac{\alpha_s}{\pi} \right)^4 + \mathcal{O}(\alpha_s^5) \\
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\end{aligned}$$

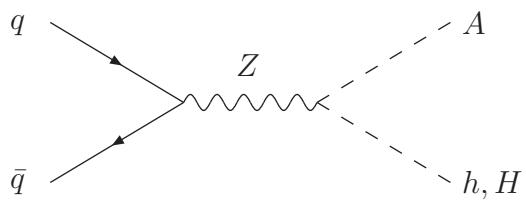
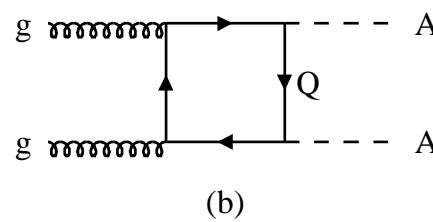
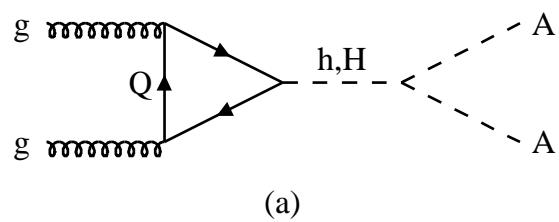
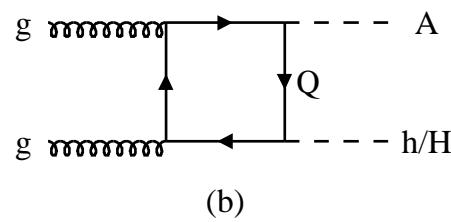
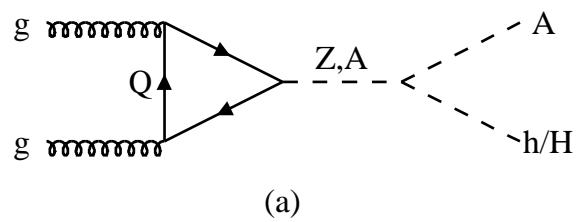
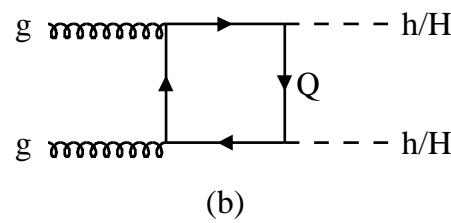
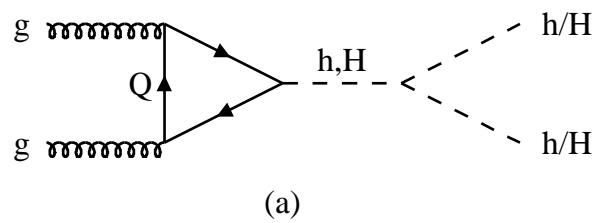
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\eta_2 &= \frac{35 + 16N_F}{24}
\end{aligned}$$

- known now up to $N^4\text{LO}$:

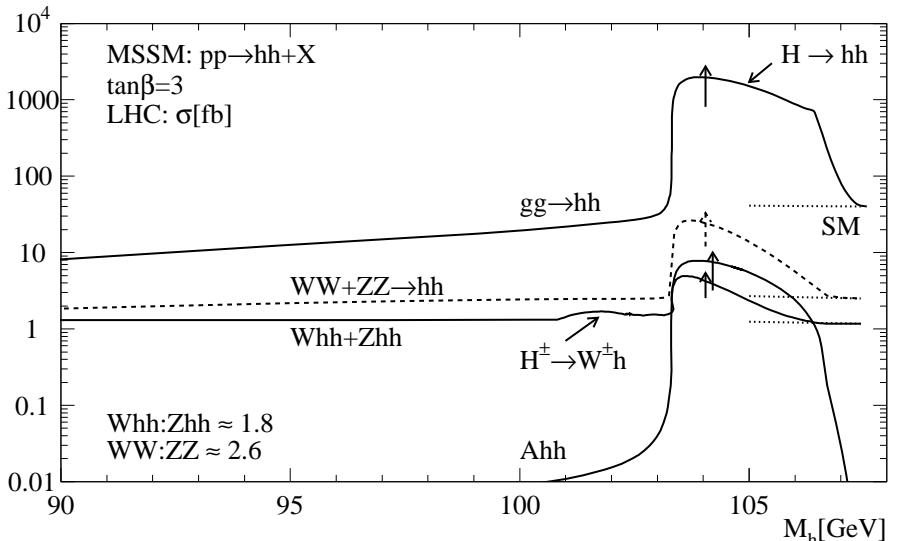
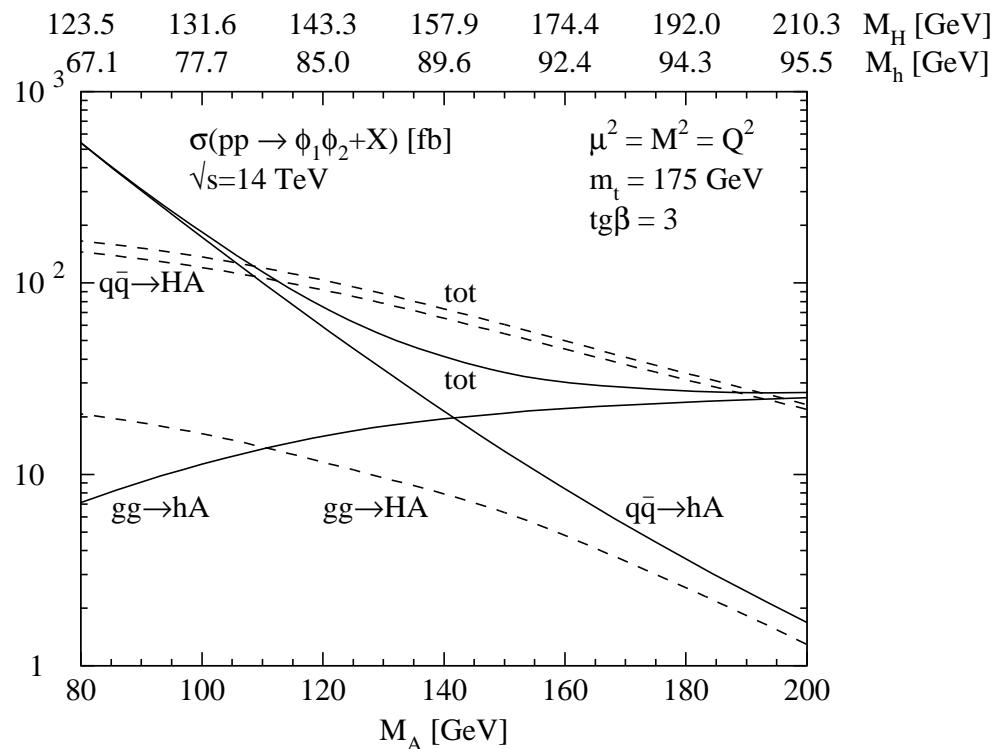
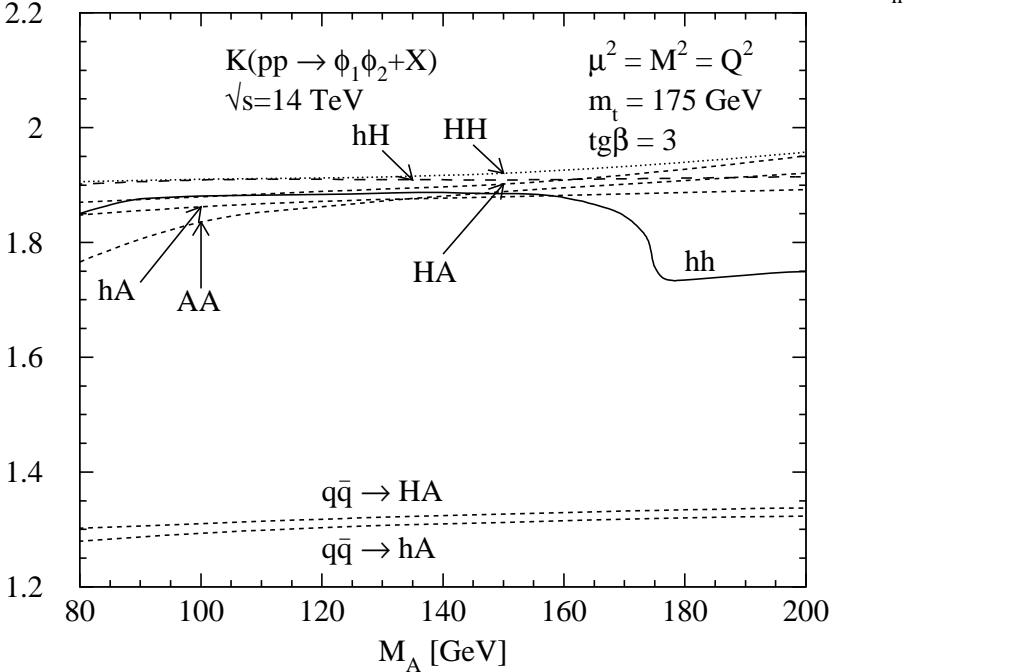
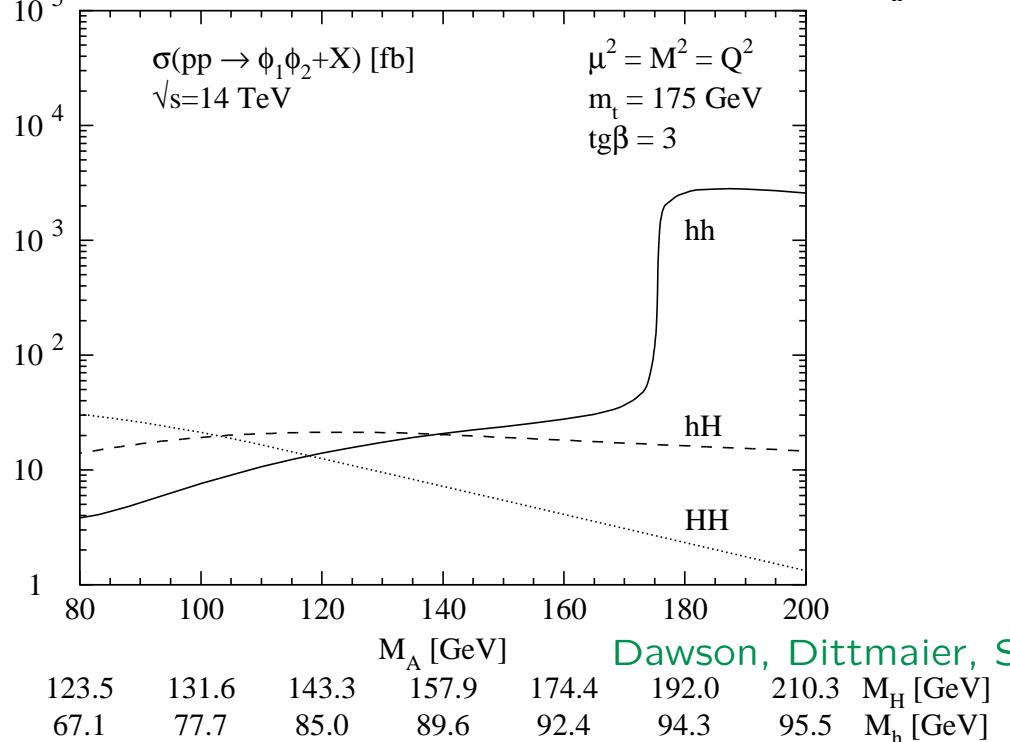
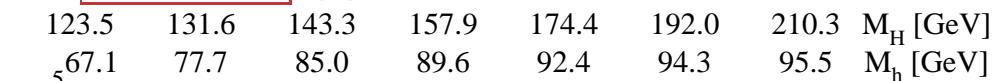
$$\begin{aligned}
\delta_3 &= \frac{897943}{9216} \zeta_3 - \frac{2892659}{41472} + \frac{209}{64} L_t^2 + \frac{1733}{288} L_t \\
&+ N_F \left(\frac{40291}{20736} - \frac{110779}{13824} \zeta_3 + \frac{23}{32} L_t^2 + \frac{55}{54} L_t \right) + N_F^2 \left(-\frac{L_t^2}{18} + \frac{77}{1728} L_t - \frac{6865}{31104} \right) \\
\delta_4 &= -\frac{121}{1440} N_F \log^5 2 + \frac{3751}{2880} \log^5 2 + \frac{685}{41472} N_F^2 \log^4 2 + \frac{11679301}{1741824} N_F \log^4 2 \\
&- \frac{93970579}{870912} \log^4 2 + \frac{121}{144} N_F \zeta_2 \log^3 2 - \frac{3751}{288} \zeta_2 \log^3 2 - \frac{685}{6912} N_F^2 \zeta_2 \log^2 2 \\
&- \frac{11679301}{290304} N_F \zeta_2 \log^2 2 + \frac{93970579}{145152} \zeta_2 \log^2 2 + \frac{2057}{192} N_F \zeta_4 \log 2 - \frac{63767}{384} \zeta_4 \log 2 \\
&+ \frac{685}{1728} N_F^2 a_4 + \frac{11679301}{72576} N_F a_4 - \frac{93970579}{36288} a_4 + \frac{121}{12} N_F a_5 - \frac{3751}{24} a_5 + \frac{211}{1728} N_F^3 \zeta_3 \\
&- \frac{270407}{1492992} N_F^3 + \frac{4091305}{331776} N_F^2 \zeta_3 - \frac{576757}{55296} N_F^2 \zeta_4 - \frac{115}{384} N_F^2 \zeta_5 - \frac{48073}{27648} N_F^2 \\
&- \frac{151369}{725760} N_F X_0 - \frac{12171659669}{38707200} N_F \zeta_3 + \frac{608462731}{11612160} N_F \zeta_4 + \frac{313489}{6912} N_F \zeta_5 \\
&+ \frac{76094378783}{522547200} N_F + \frac{4692439}{1451520} X_0 + \frac{28121193841}{19353600} \zeta_3 + \frac{4674213853}{2903040} \zeta_4 - \frac{807193}{1728} \zeta_5 \\
&- \frac{854201072999}{522547200} + \left(\frac{481}{5184} N_F^3 + \frac{28297}{9216} N_F^2 \zeta_3 - \frac{21139}{3456} N_F^2 - \frac{32257}{288} N_F \zeta_3 \right. \\
&\quad \left. + \frac{5160073}{41472} N_F + \frac{9364157}{12288} \zeta_3 - \frac{49187545}{55296} \right) L_t + \left(-\frac{77}{6912} N_F^3 - \frac{1267}{13824} N_F^2 + \frac{4139}{2304} N_F \right. \\
&\quad \left. + \frac{8401}{384} \right) L_t^2 + \left(\frac{1}{108} N_F^3 - \frac{157}{576} N_F^2 + \frac{275}{192} N_F + \frac{2299}{256} \right) L_t^3
\end{aligned}$$

$$\begin{aligned}
\eta_3 &= \frac{1333}{432} + \frac{589}{48}L_t + N_F \left(\frac{1081}{432} + \frac{191}{72}L_t \right) + N_F^2 \left(\frac{77}{864} - \frac{2}{9}L_t \right) \\
\eta_4 &= \frac{481}{2592}N_F^3 + N_F^2 \left(\frac{28297}{4608}\zeta_3 - \frac{373637}{31104} \right) + N_F \left(\frac{429965}{1728} - \frac{2985893}{13824}\zeta_3 \right) \\
&+ \frac{26296585}{18432}\zeta_3 - \frac{143976701}{82944} + \left(-\frac{77}{1728}N_F^3 - \frac{1421}{3456}N_F^2 + \frac{9073}{1728}N_F + \frac{45059}{576} \right) L_t \\
&+ \left(\frac{N_F^3}{18} - \frac{455}{288}N_F^2 + \frac{63}{8}N_F + \frac{6479}{128} \right) L_t^2 \\
\rho_3 &= \frac{1697}{144} + \frac{175}{72}N_F - \frac{2}{9}N_F^2 \\
\rho_4 &= \frac{130201}{1728} + \frac{18259}{192}L_t + N_F \left(\frac{5855}{1728} + \frac{2077}{144}L_t \right) - N_F^2 \left(\frac{175}{384} + \frac{439}{144}L_t \right) \\
&+ N_F^3 \left(\frac{L_t}{9} - \frac{77}{1728} \right) \\
\sigma_4 &= \frac{51383}{864} + \frac{317}{36}N_F - \frac{47}{24}N_F^2 + \frac{2}{27}N_F^3
\end{aligned}$$

MSSM $gg \rightarrow hh, hH, HH, hA, HA, AA$ and $q\bar{q} \rightarrow hA, HA$

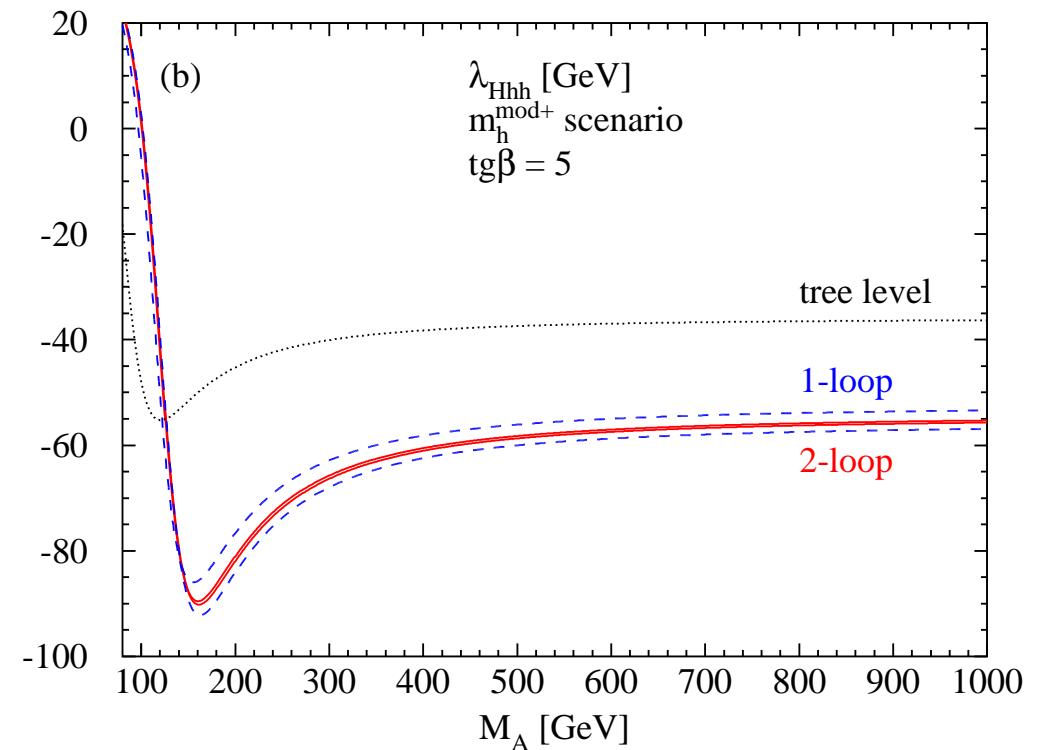
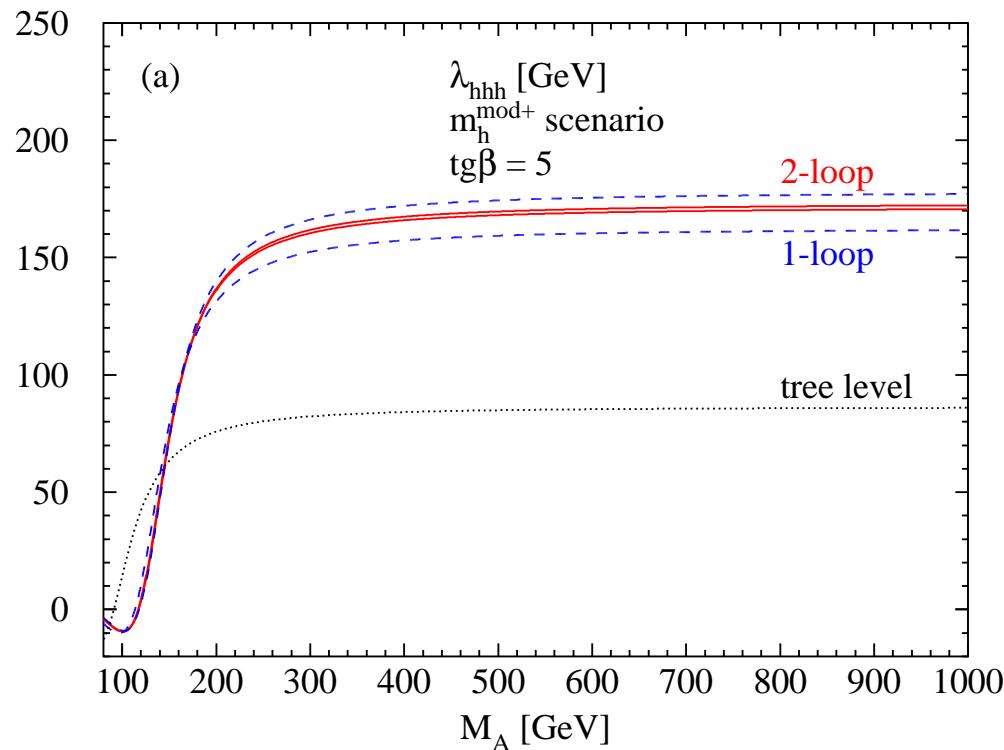


- resonant contributions: $H \rightarrow hh, AA$
- QCD corrections: $\lesssim 100\% [m_t^2 \gg Q^2 \Rightarrow \text{small } \tan\beta]$
- partial SUSY–QCD corrections: $\lesssim 100\% [\text{heavy SUSY particles}]$

MSSM $gg \rightarrow hh, hH, HH, hA, HA, AA$ and $q\bar{q} \rightarrow hA, HA$ 

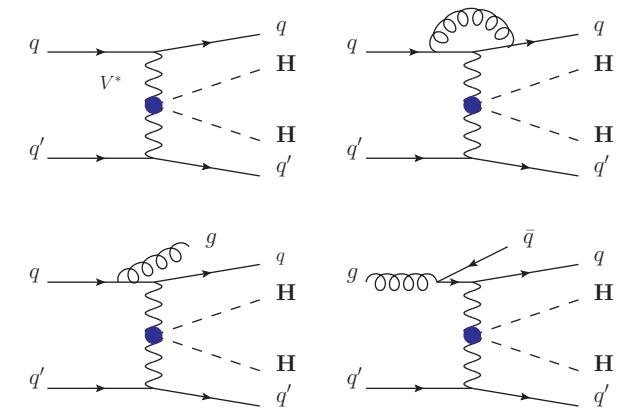
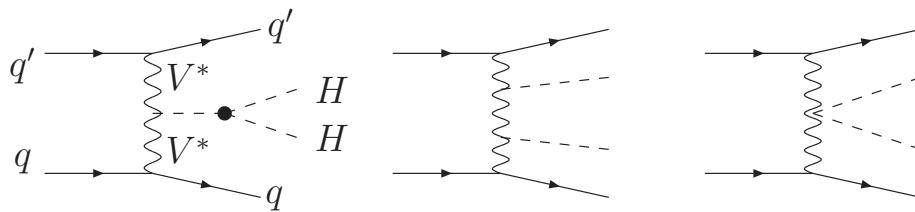
Djouadi, Kilian, Mühlleitner, Zerwas

MSSM trilinear couplings up to $\mathcal{O}(\alpha_t \alpha_s)$



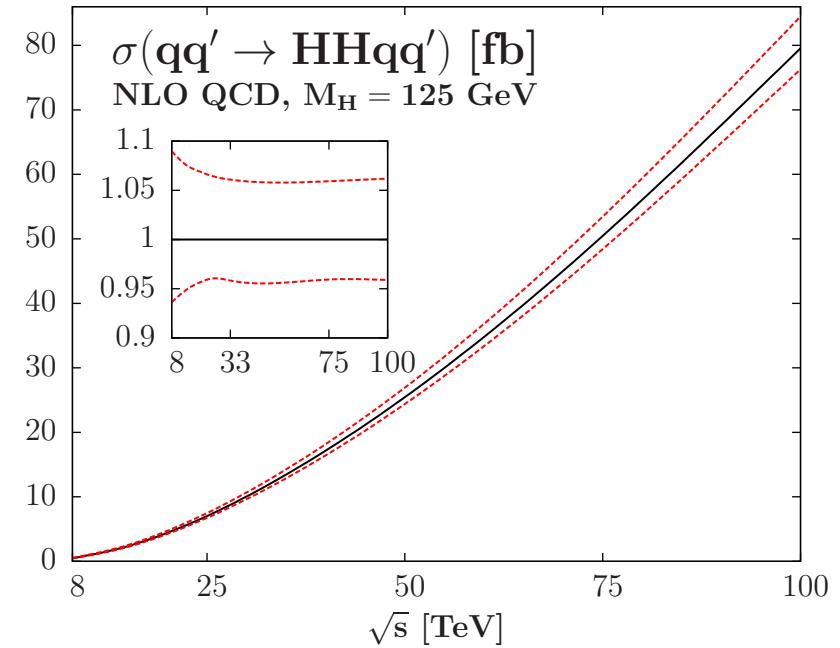
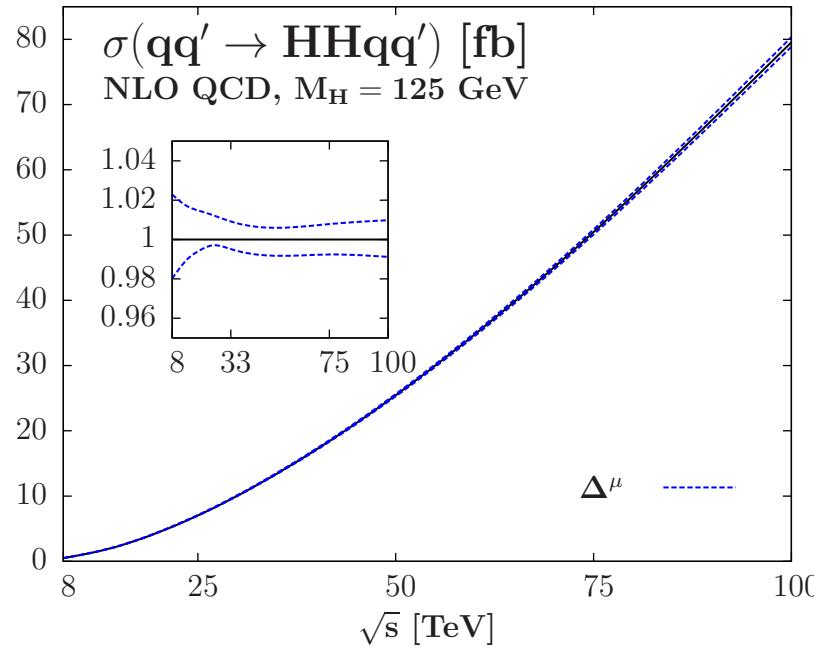
Brucherseifer, Gavin, S.

(ii) $VV \rightarrow HH$



- NLO QCD corrections [\leftarrow DIS]: $\sim 10\%$
[$\mu = Q_{V^*}$]
- NNLO QCD corrections [\leftarrow DIS]: $\lesssim 1\%$

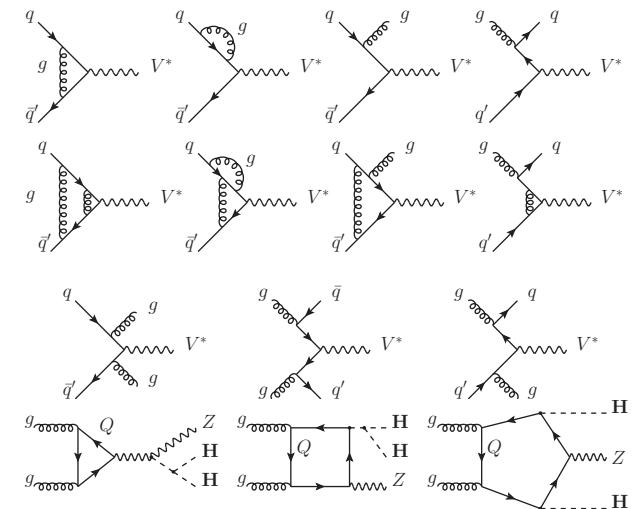
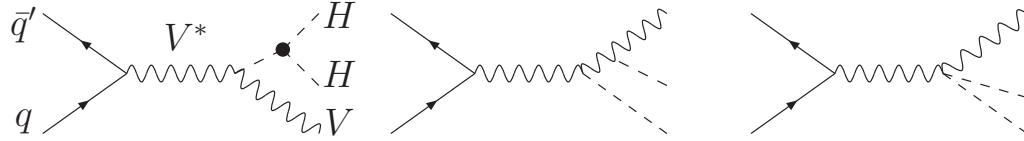
Sheng, Ren-You, Wen-Gan, Lei, Wei-Hua, Xiao-Zhou



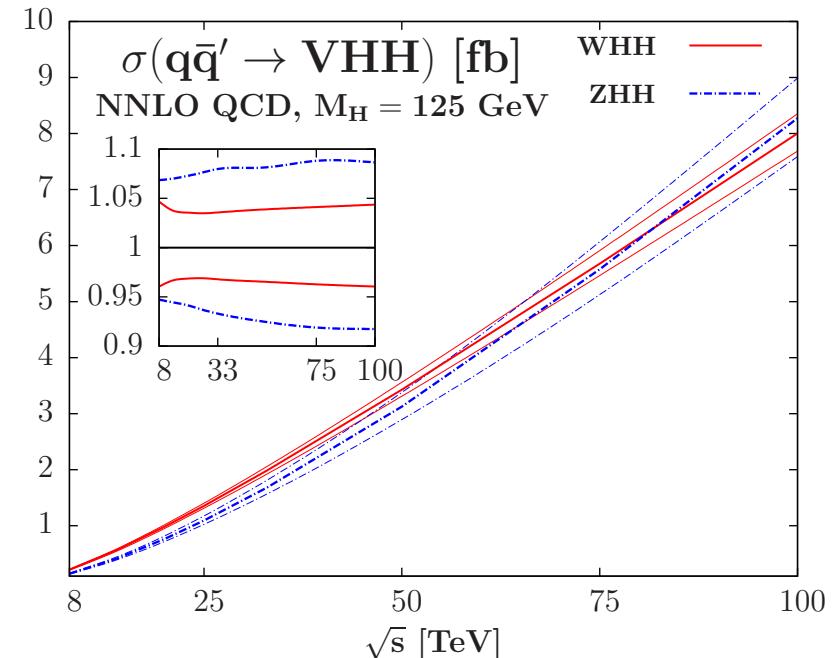
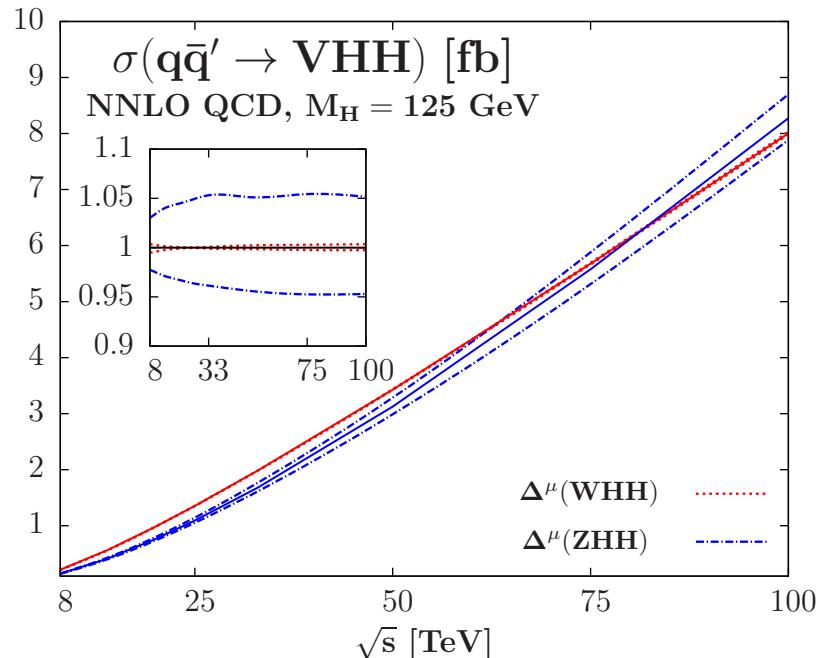
- uncertainties: $\sim 5 - 7\%$

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.

(iii) $q\bar{q} \rightarrow VHH$



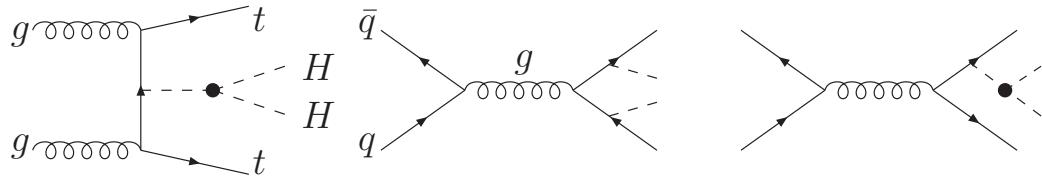
- NNLO QCD corrections [\leftarrow DIS]: $\sim 30\%$
 $[\mu = M_{VHH}]$ $[gg \rightarrow ZHH : 20 - 30\%]$



- uncertainties: $\sim 2 - 6\%$

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.

(iv) $q\bar{q} \rightarrow t\bar{t}HH$

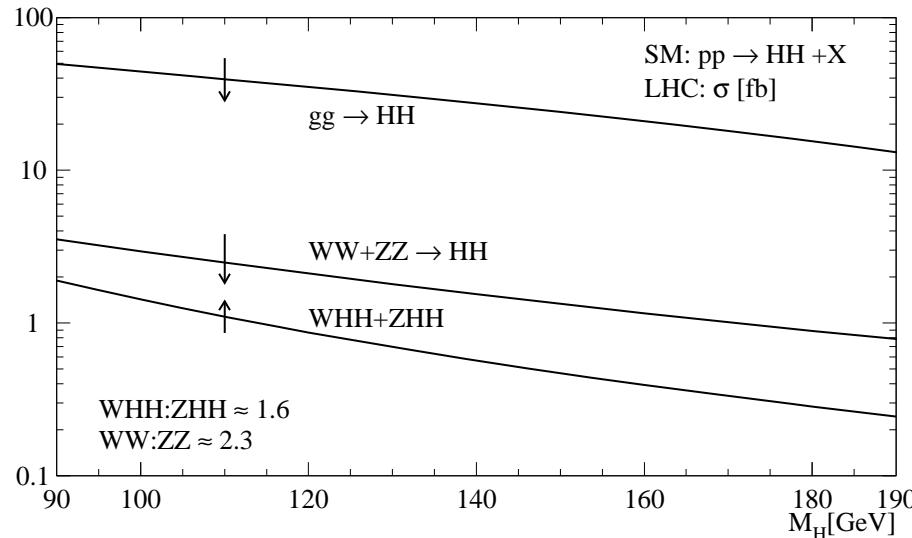


- NLO QCD corrections moderate $\sim 20\%$
Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro
- uncertainties: $\sim 10 - 15\%$

← very difficult channel

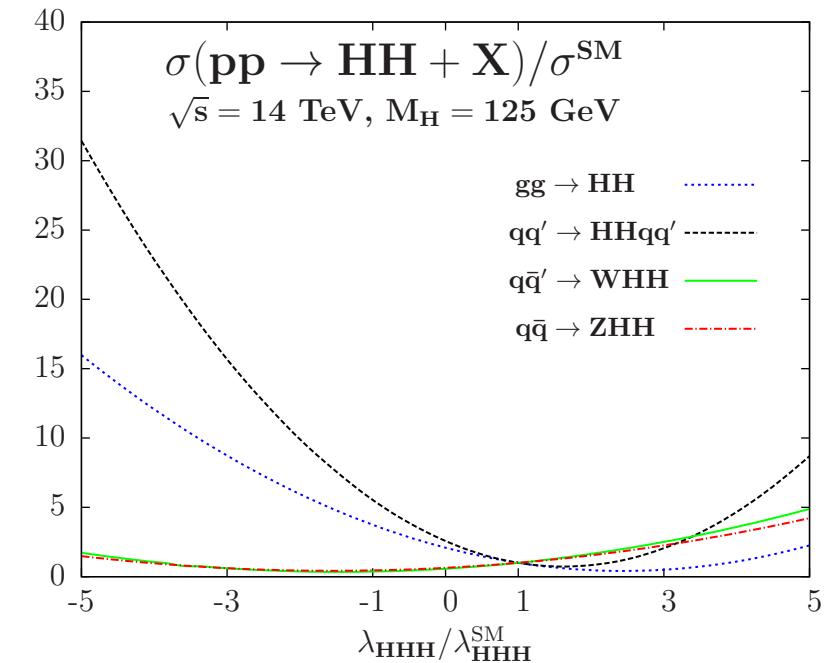
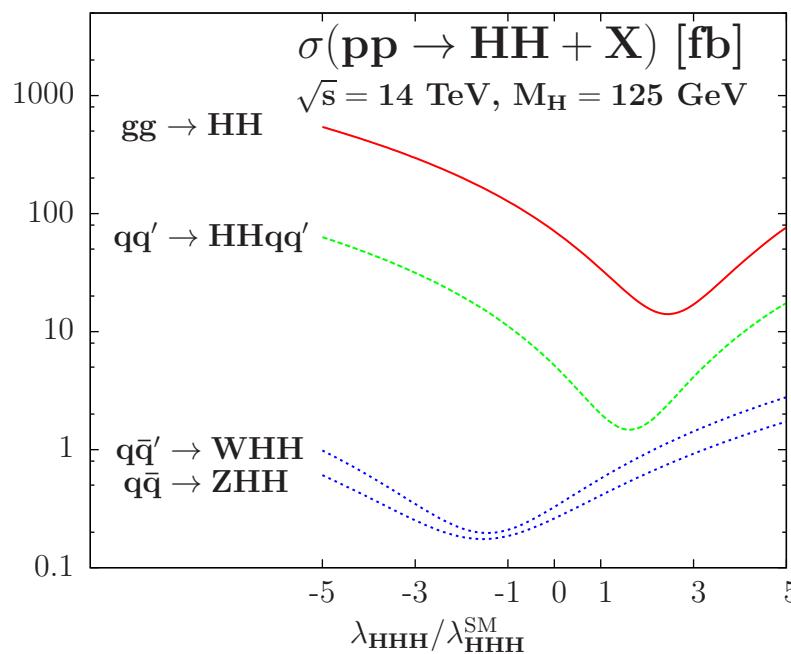
SENSITIVITY TO TRILINEAR COUPLING

SM



Djouadi, Kilian, Mühlleitner, Zerwas

$$gg \rightarrow HH : \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$



Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, S.

Channels: need 3 ab^{-1}

$HH \rightarrow b\bar{b}\gamma\gamma$: low signal rate, most promising?

$HH \rightarrow b\bar{b}\tau^+\tau^-$: mass reconstruction difficult

$HH \rightarrow b\bar{b}W^*W^{(*)}$: hopeless?

- $b\bar{b}\gamma\gamma$: $\begin{array}{c} +160\% \\ -190\% \end{array} @ 600 fb^{-1}$

$\begin{array}{c} +74\% \\ -62\% \end{array} @ 6 ab^{-1}$

Baur, Plehn, Rainwater

- boosted kinematics: less sensitive to λ

Dolan, Englert, Spannowsky

- $HH + jet \rightarrow b\bar{b}b\bar{b}j, b\bar{b}\tau^+\tau^-j$ promising?

Dolan, Englert, Spannowsky

- reduction of THUs by ratio $\frac{\sigma(gg \rightarrow HH)}{\sigma(gg \rightarrow H)} \rightarrow \frac{\delta\lambda}{\lambda} \sim 30\% ?$
[different $Q^2!$]

Goertz, Papaefstathiou, Yang, Zurita