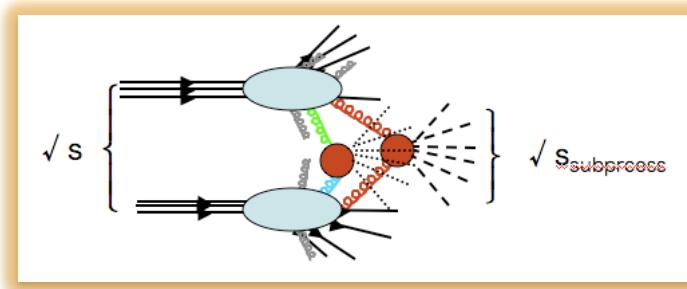
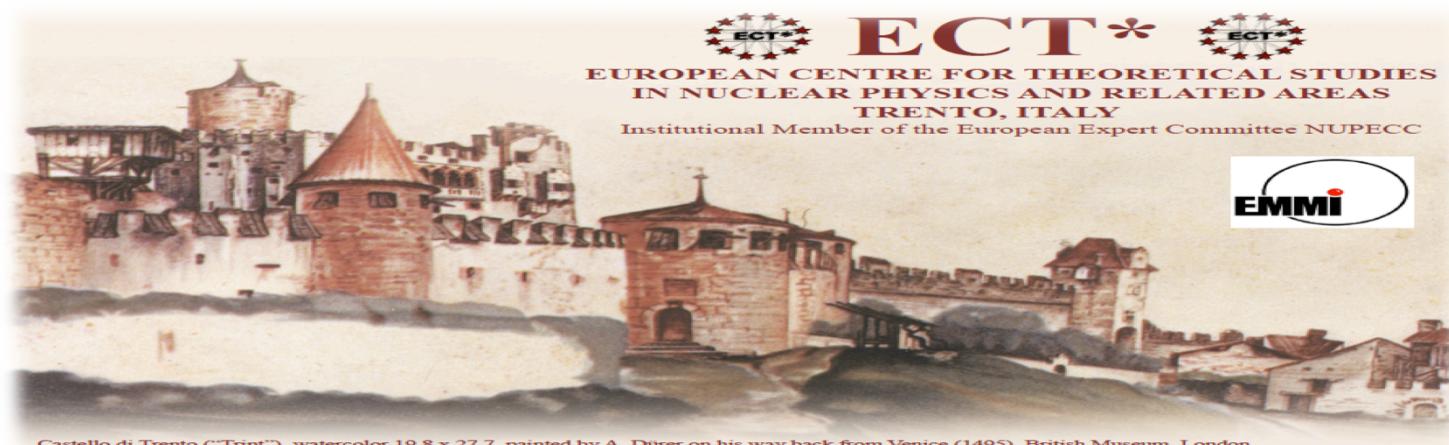


Mini-jets for total and inelastic pp cross-sections



Giulia Pancheri

With D. Fagundes, A. Grau, O.Shekhtsova, Y.N. Srivastava



Total, inelastic and elastic: can we understand them all with (perturbative) QCD?

- Not yet
- But lots of progress in σ_{total}

The total cross-section: then and now

1974

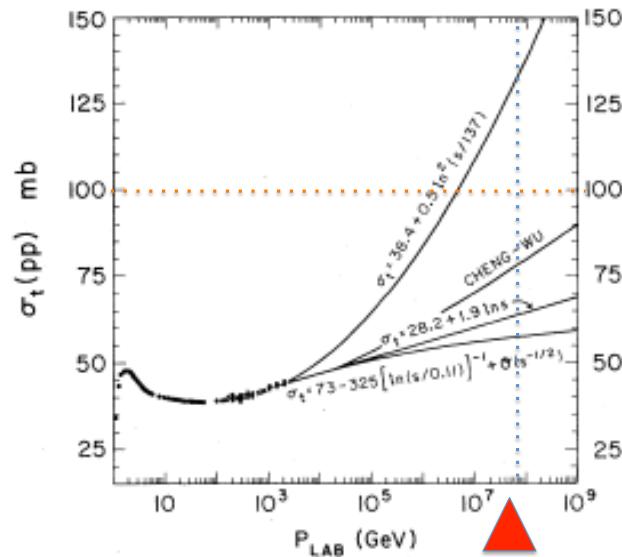
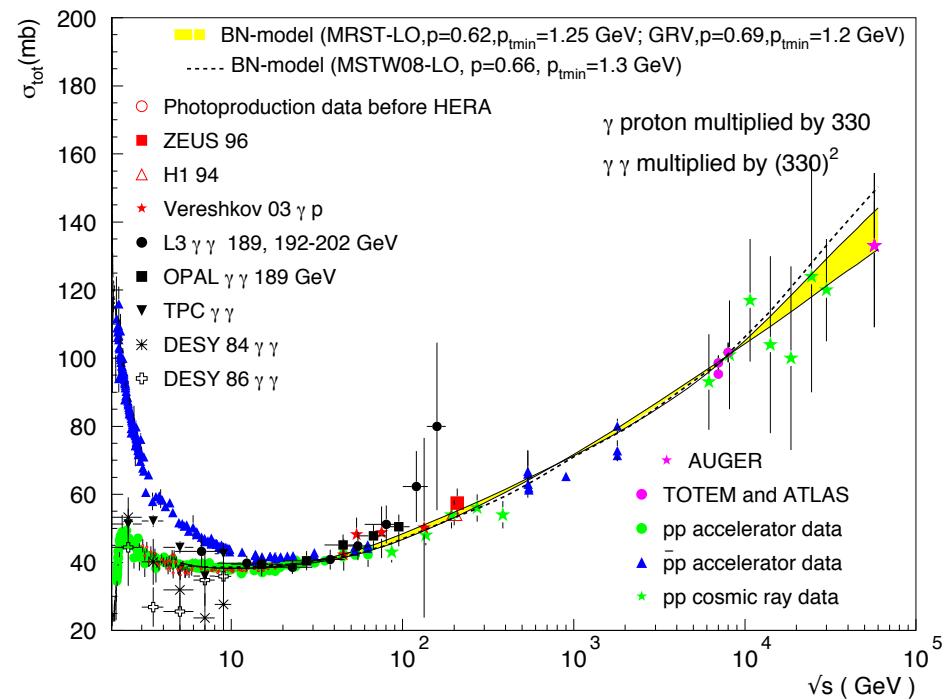


FIG. 1. Typical high-energy model extrapolations of the proton-proton total cross section to the energy range accessible to extensive-air-shower experiments.

Barger, Phillips,
Gaisser,
Noble, Yodh

9/26/16

2015
 $\gamma\gamma, \gamma p, \bar{p}p, pp, p - air$



F. Cornet, C. Garcia, A. Grau, GP, S. Sciuto
[10.1103/PhysRevD.92.114011](https://arxiv.org/abs/1609.05301)

Our strategy to understand inelastic and elastic

- presently we are adopting a double edge strategy
 - A model with pQCD with resummation in the infrared for
 - **total** cross-section
Phys.Lett. B382 (1996) 282-288 and Phys.Rev. D60 (1999) 114020
 - the **bulk** of the inelastic
Phys.Rev. D84 (2011) 094009 and Phys.Rev. D91 (2015) 114011
 - For **elastic** and full inelastic (including diffraction) :
 - an empirical parametrization
Phys.Rev. D88 (2013) 094019

Basic fact: All total cross-sections **rise**.

... but not too much ...

(**Froissart** dixit in 1961 + Martin 1962+Lukaszuk 1967)

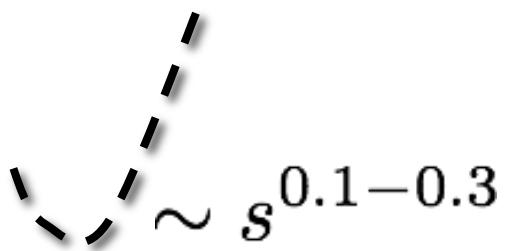
Asymptotically

$$\sigma_{tot} \lesssim \sum_{0,L} \simeq L_{max}^2 \longrightarrow \sigma_{total} \lesssim [\log s]^2$$

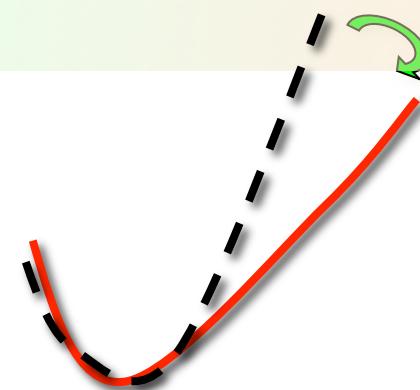
$$L_{max} = qb_{max} \sim \log s$$

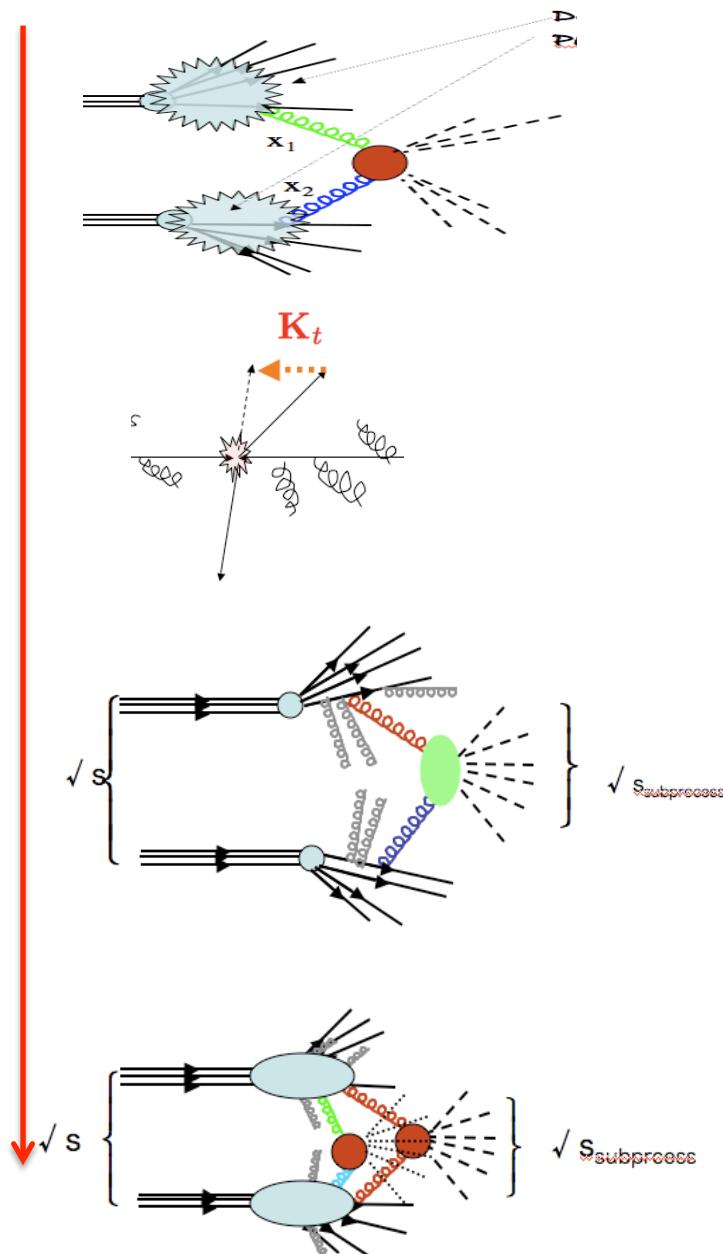
What generates
the rise?

What tames the rise into
a Froissart-like behavior?

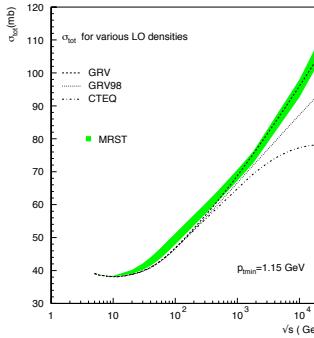
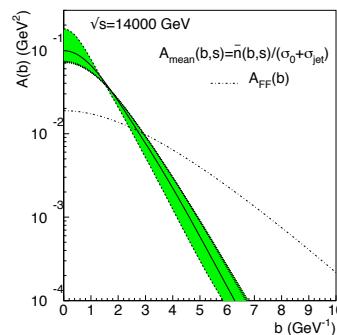
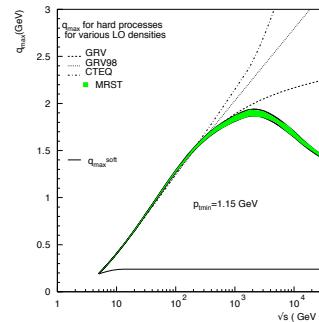
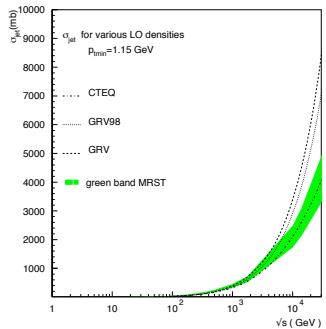

$$\sim s^{0.1-0.3}$$

How to go from a
power-law to $\log s$?





9/26/16



1. Calculate mini-jet cross-section
Choosing densities and p_{tmin}

$$\sigma_{\text{mini-jet}} \simeq s^\epsilon$$

$$\epsilon \simeq 0.3 - 0.4$$

2. Calculate q_{\max} : single soft gluon upper scale, for given PDF, p_{tmin}

$$q_{\max} \simeq p_{tmin}$$

$$\lesssim 2 - 3 \text{ GeV}$$

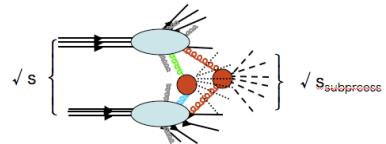
3. Calculate impact parameter distribution for given q_{\max} and given infrared parameter p

$$\chi(b, s) = \chi_{\text{low energy}} +$$

$$+ A(b, q_{\max}) \sigma_{\text{jet}}$$

4. Eikonalize

$$\sigma_{\text{total}} = 2 \int d^2 \mathbf{b} [1 - e^{-\chi(b, s)}]$$



Our QCD model for the total cross-section

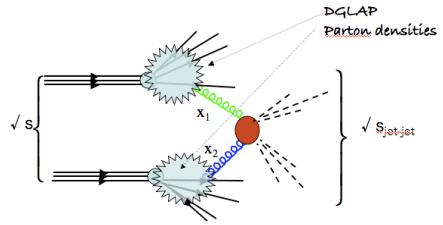
R. Godbole, A. Grau, GP, YN Srivastava

$$\sigma_{total} \simeq 2 \int d^2 \vec{b} [1 - e^{-\chi_I(b,s)}]$$

$$2\chi_I(b,s) = A_{FormFactor}(b,s)\sigma_{soft}(p_t < p_{tmin}) + \\ + A_{Resum}(b,s)\sigma_{mini-jet}(p_t > p_{tmin})$$

- **Minijets to drive the rise**
- **Soft kt-resummation** to tame the rise and introduce the cut-off in b-space needed to satisfy the Froissart bound
- Phenomenological **singular but integrable** soft gluon coupling to relate confinement (cut-off in b-space) with the rise
- Interpolation between soft and asymptotic freedom region

PLB1996-PRD2005



Minijets and the rise for $\sqrt{s} \approx 20$ GeV

- asymptotic freedom regime

$$\alpha_s(p_t) \rightarrow \alpha_{AF} = \frac{b_0}{\ln[p_t^2/\Lambda_{QCD}^2]} \quad \text{valid for parton-parton scattering with final parton } p_t \geq p_{tmin}$$

$$p_t \gg \Lambda_{QCD} \quad p_t \simeq 1 \text{ GeV}$$

$$\sigma_{\text{jet}}^{AB} = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^1 dx_1 \int_{4p_t^2/(x_1 s)}^1 dx_2 \\ \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t} \quad p_{tmin} \sim 1$$

for each initial parton

$$f(x) \sim 1/x \quad x \geq 2p_{tmin}/\sqrt{s}$$

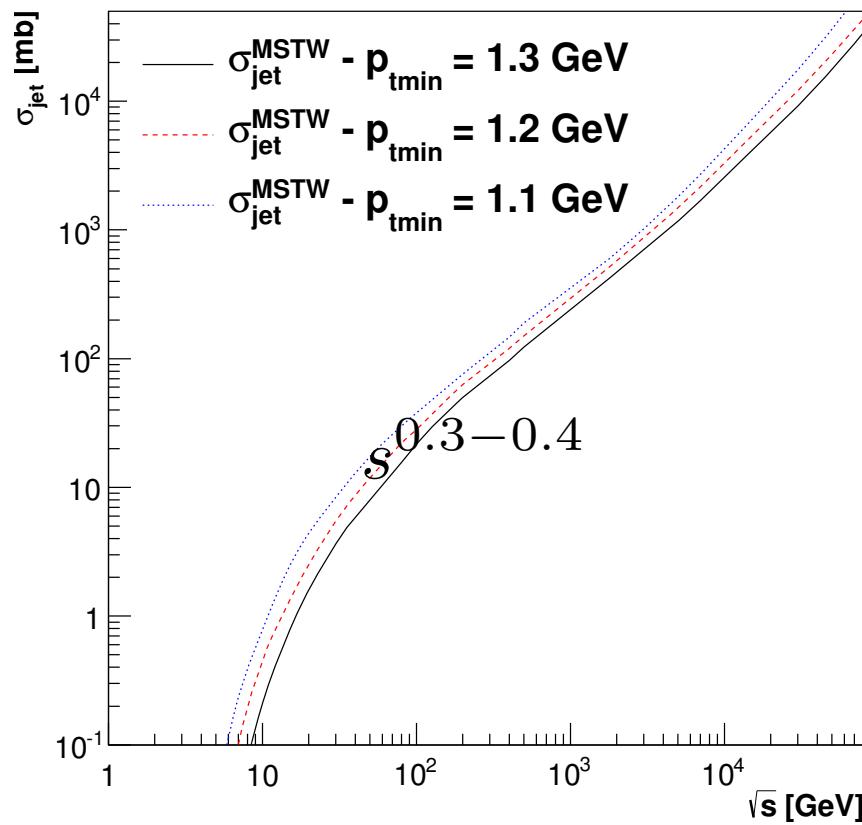
$x \downarrow \text{ as } \sqrt{s} \uparrow \text{ and } \sigma_{\text{mini-jet}} \uparrow$

$x \leq 0.1 - 0.2 \text{ and } \downarrow \sigma_{\text{mini-jet}} \uparrow$

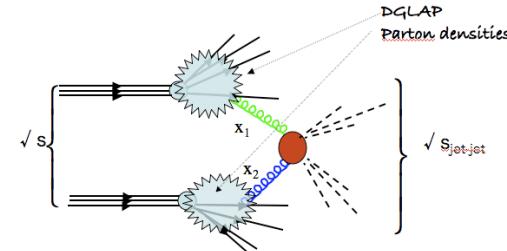
$$\sqrt{s} \gtrsim 10 - 20 \text{ GeV}$$

The QCD LO minijets

- It rises
too much
too fast

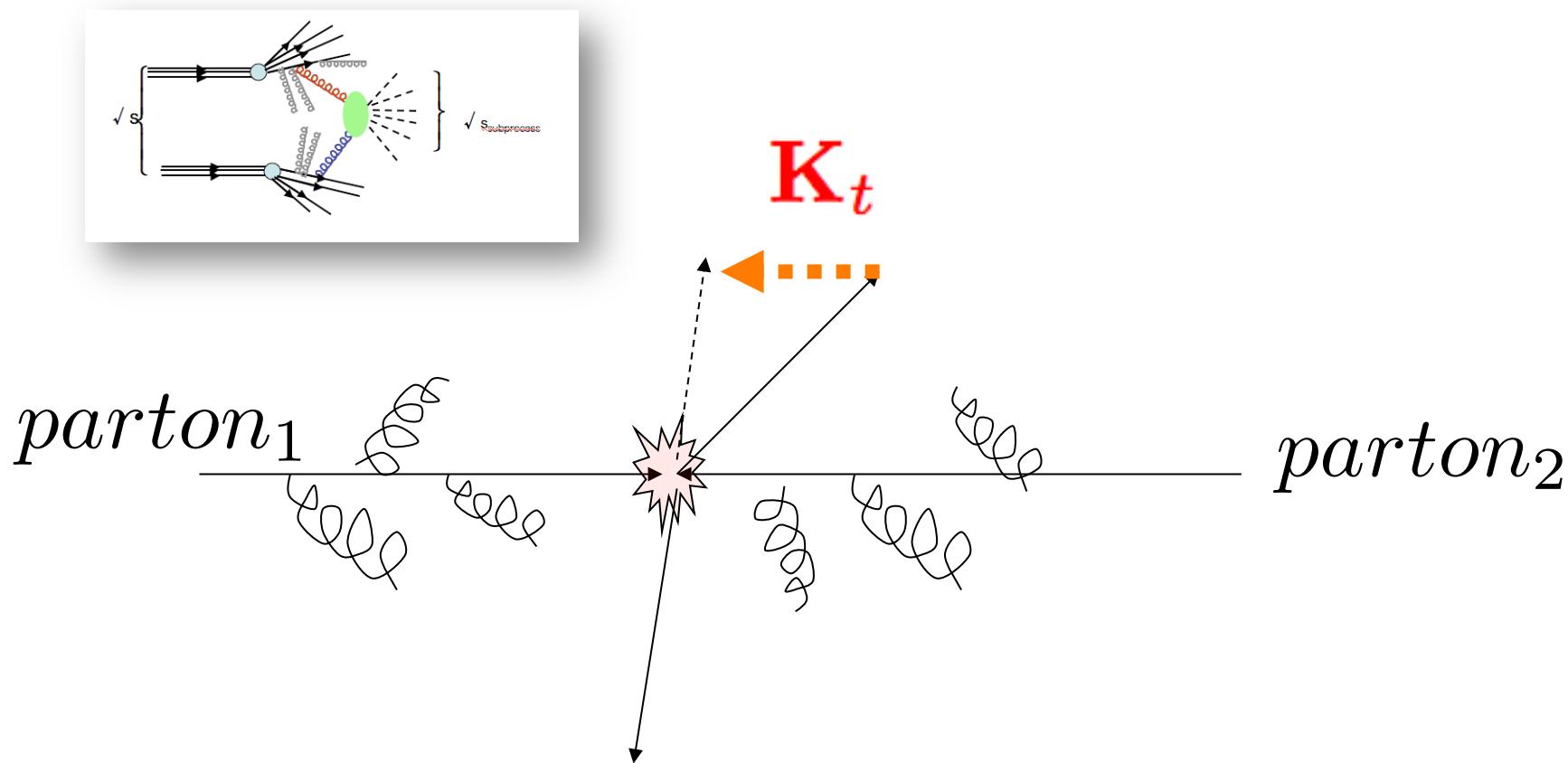


The mini-jet description is incomplete

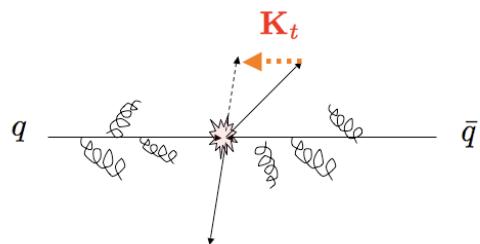


- Hard scattering of color objects requires soft gluon emission (+ other finite corrections)
 - Soft \Leftrightarrow many undistinguishable soft gluons
 - \Leftrightarrow resummation
 - exponentiation of regularized single spectrum obtained from integration in gluon momentum
- ⇒ Initial state gluon emission (and thus resummation) introduces an acollinearity \Rightarrow reduction of the x-section
- To comply with unitarity, we embed into an eikonal formalism in b-space (caveat later)

Soft gluon emission introduces acollinearity



Accidentality reduces the collision cross-section as
partons do not scatter head-on any more, also explained as the gluon cloud
Becoming too thick for partons to see each other : **gluon saturation**



How to resumQCD into the infrared?

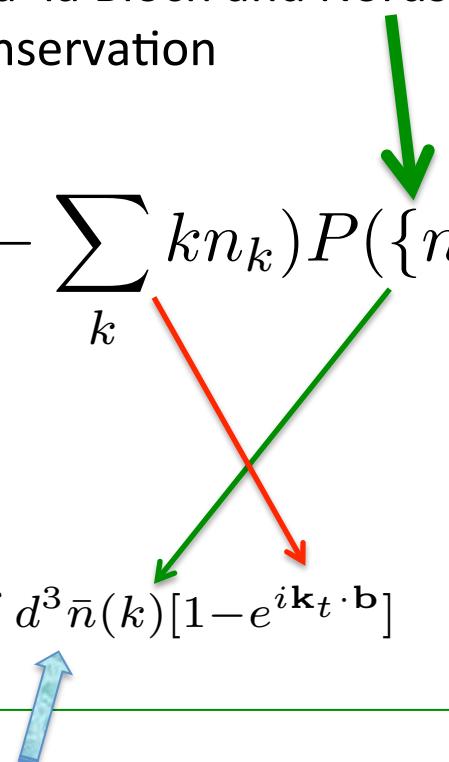
- Because of the unknown coupling as $k \rightarrow 0$, we **resum** the single gluon spectrum through a semi-classical formalism a' la Bloch and Nordsieck (Poisson distribution)+ energy-momentum conservation

→

$$d^4 P_{sum}(K) = d^4 K \sum_{n_k} \delta^4(K - \sum_k k n_k) P(\{n_k\})$$

In the continuum in \mathbf{K}_t

$$d^2 P_{resum}(\mathbf{K}_t) = d^2 \mathbf{K}_t \int d^2 \mathbf{b} e^{-i \mathbf{K}_t \cdot \mathbf{b} - \int d^3 \bar{n}(k) [1 - e^{i \mathbf{k}_t \cdot \mathbf{b}}]}$$



pQCD (mini-jets) + ResumQCD

- Minijets: for partons with $\alpha_s(p_t) < 1$
- \rightarrow

$$\frac{p_t}{\Lambda} \gg 1$$

Perturbative calculation, presently we work at LO

- ResumQCD : $\rightarrow k_t < \Lambda$
Soft gluons: indistinguishable, momentum distribution of soft gluons is continuous, must be resummed ~ Sudakov \rightarrow Bloch Nordsieck

Where the partons are as they collide :

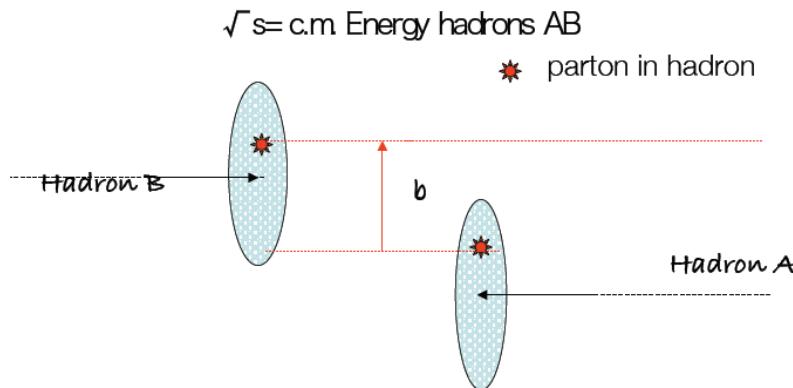
THE IMPACT PARAMETER DISTRIBUTION

We model the impact parameter distribution as the Fourier-transform of ISR soft k_t distribution and thus obtain a cut-off at large distances : Froissart bound?

$$A_{BN}(b, s) = N \int d^2\mathbf{K}_\perp e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \frac{d^2 P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b, q_{max})}}{\int d^2\mathbf{b} e^{-h(b, q_{max})}}$$

$$h(b, E) = \frac{16}{3\pi} \int_0^{q_{max}} \frac{dk_t}{k_t} \alpha_{eff}(k_t) \ln\left(\frac{2q_{max}}{k_t}\right) [1 - J_0(bk_t)]$$

$$\alpha_{eff}(k_t \rightarrow 0) \sim k_t^{-2p}$$



$$A_{BN}(b, s) \sim e^{-(b\bar{\Lambda})^{2p}}$$

q_{tmax}

?

Fuzzy factorization (as in QED)
Fixed by single
gluon emission kinematics

Our proposal for running $\alpha_s(k_t)$ in the infrared region



Vone gluon exchange $\sim r^{2p-1}$

$$\propto k_t^{-2p} \quad k_t \ll \Lambda$$

To reconcile with asymptotic Freedom

$$\propto \frac{1}{\log k_t^2/\Lambda^2} \quad k_t \gg \Lambda$$

A phenomenological interpolation



$$\alpha_{eff}(k_t) = \frac{12\pi}{11N_c - 2N_f} \frac{p}{\log[1 + p(k_t/\Lambda_{QCD})^{2p}]}$$

The infrared parameter p regulates
the large b -behaviour

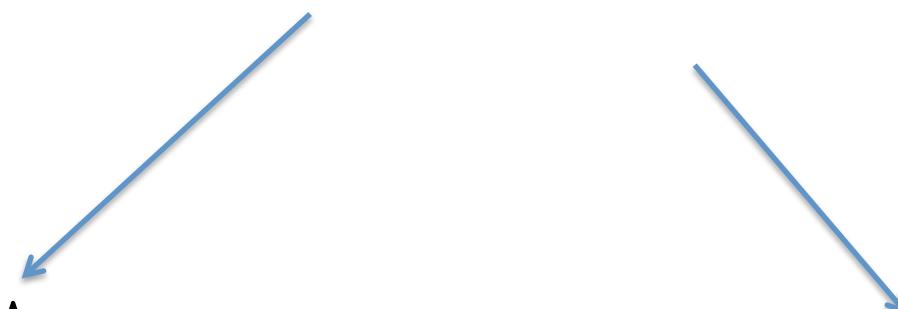
$$A(b)_{BN} = \mathcal{N} e^{- \int d^3 \bar{n}_g(k) [1 - e^{ik_t \cdot b}]}$$

$$d^3 \bar{n}_g(k) \propto \alpha_s(k_t)$$

$$k_t \gg \Lambda$$

$$\frac{11\pi}{11N_c - 2N_f} \log\left[\frac{k_t^2}{\Lambda^2}\right]$$

Asymptotic freedom expression for pQCD



Our ansatz for

$$k_t \rightarrow 0$$

$$\left(\frac{\Lambda^2}{k_t^2}\right)^p$$

In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering (mini-jets) and restores the Froissart bound

$$\sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s)} e^{-(b\bar{\Lambda})^{2p}}]$$

$$\sigma_{tot}(s) \rightarrow [\varepsilon \ln(s)]^{(1/p)}$$

$$\frac{1}{2} < p < 1$$

The infrared ansatz introduces a cut off in b-space

$$A(b) \sim e^{-(b^2 \bar{\Lambda}^2)^p} \quad \text{Large } b$$

$$\sigma_{tot} \simeq [\log s]^{1/p} \quad \sqrt{s} \rightarrow \infty$$

$$\frac{1}{2} \leq p < 1 \quad \text{For the integrated single soft gluon spectrum to be finite}$$

$$\frac{1}{2} \sim p \quad \rightarrow \text{exponential cut-off} \quad \rightarrow \quad \sigma_{tot} \simeq [\log s]^2 \quad \sqrt{s} \rightarrow \infty$$

$$p \simeq 1 \quad \rightarrow \quad \text{Gaussian cutoff} \quad \rightarrow \quad \sigma_{tot} \simeq [\log s] \quad \sqrt{s} \rightarrow \infty$$

An aside about the Bloch-Nordsieck
(BN) inspired model , its parameters
and present limitations

BN-model parameters

- The choice of PDF
- pQDC cutoff p_{tmin}
- The Infrared parameter p



pQCD parameters

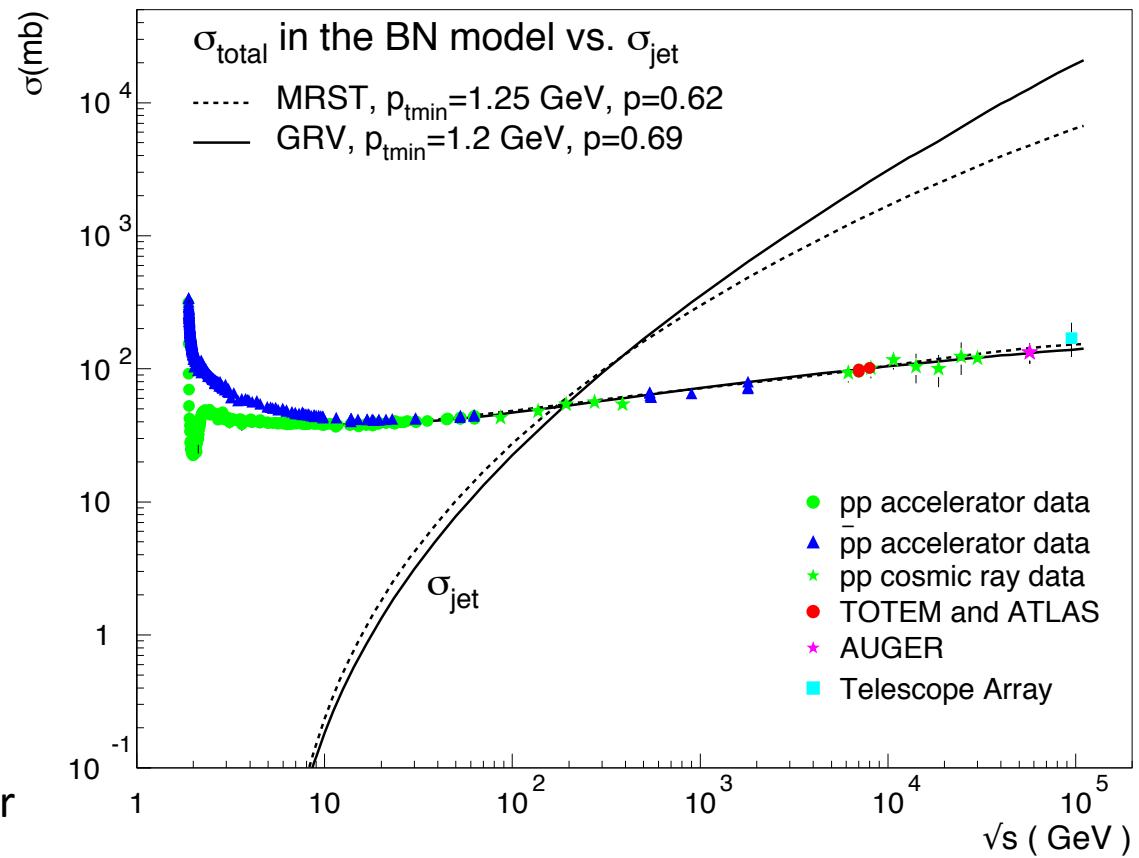
$\sigma_{mini-jet}(s, p_{tmin})$ vs σ_{total}

Conventional PDFs with LO parton parton cross-section

$\sigma_{mini-jet}(s, p_{tmin})$

Is sizeable around 5-10 GEV

Low-x behaviour different for different PDF set



Asymptotically

$$\sigma_{tot} \simeq [\log s]^{1/p} \quad \sqrt{s} \rightarrow \infty$$

Saturation of the Froissart bound

Logarithmic soft rise

$$p = \frac{1}{2}$$

$$p \lesssim$$

The total cross-section in **1974**

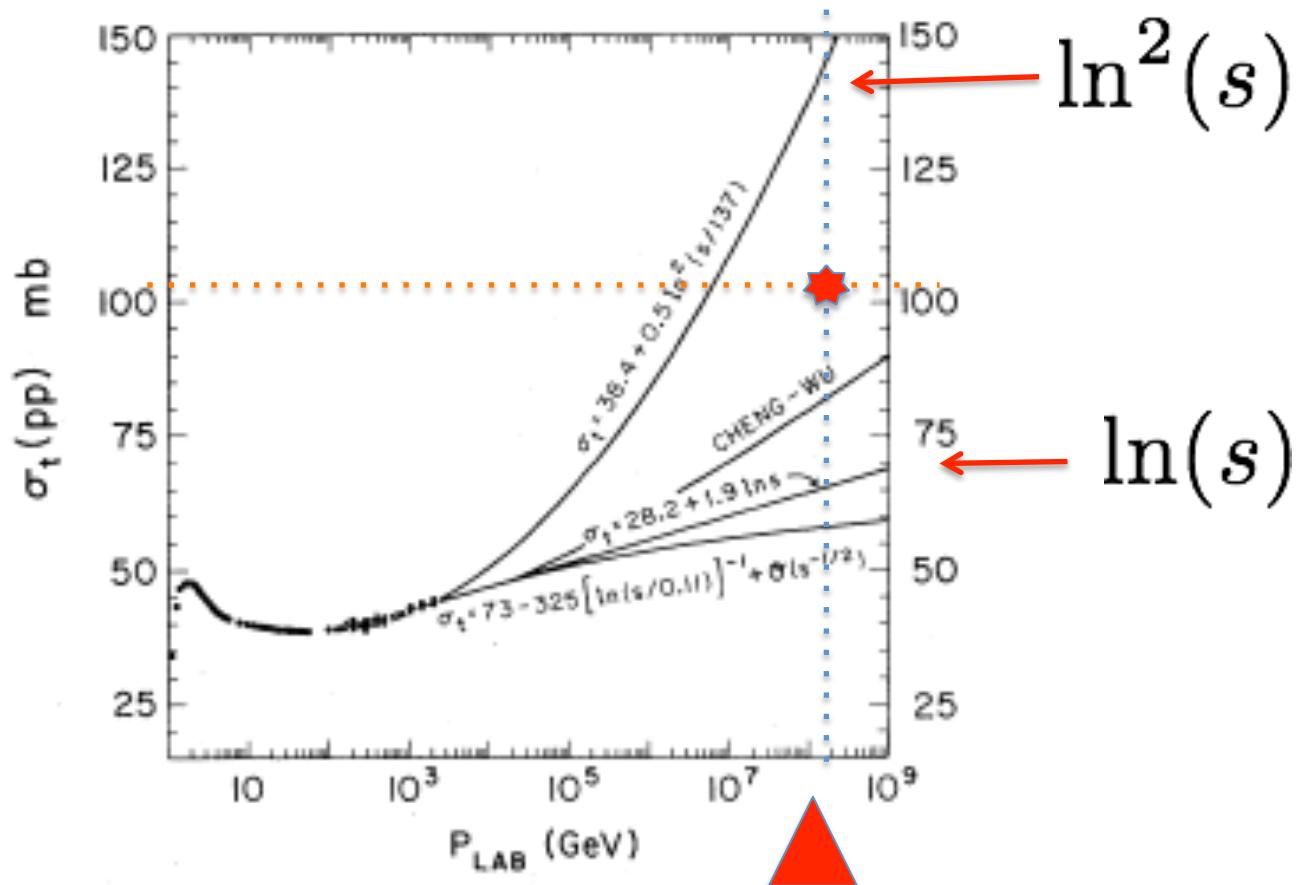


FIG. 1. Typical high-energy model extrapolations of the proton-proton total cross section to the energy range accessible to extensive-air-shower experiments.

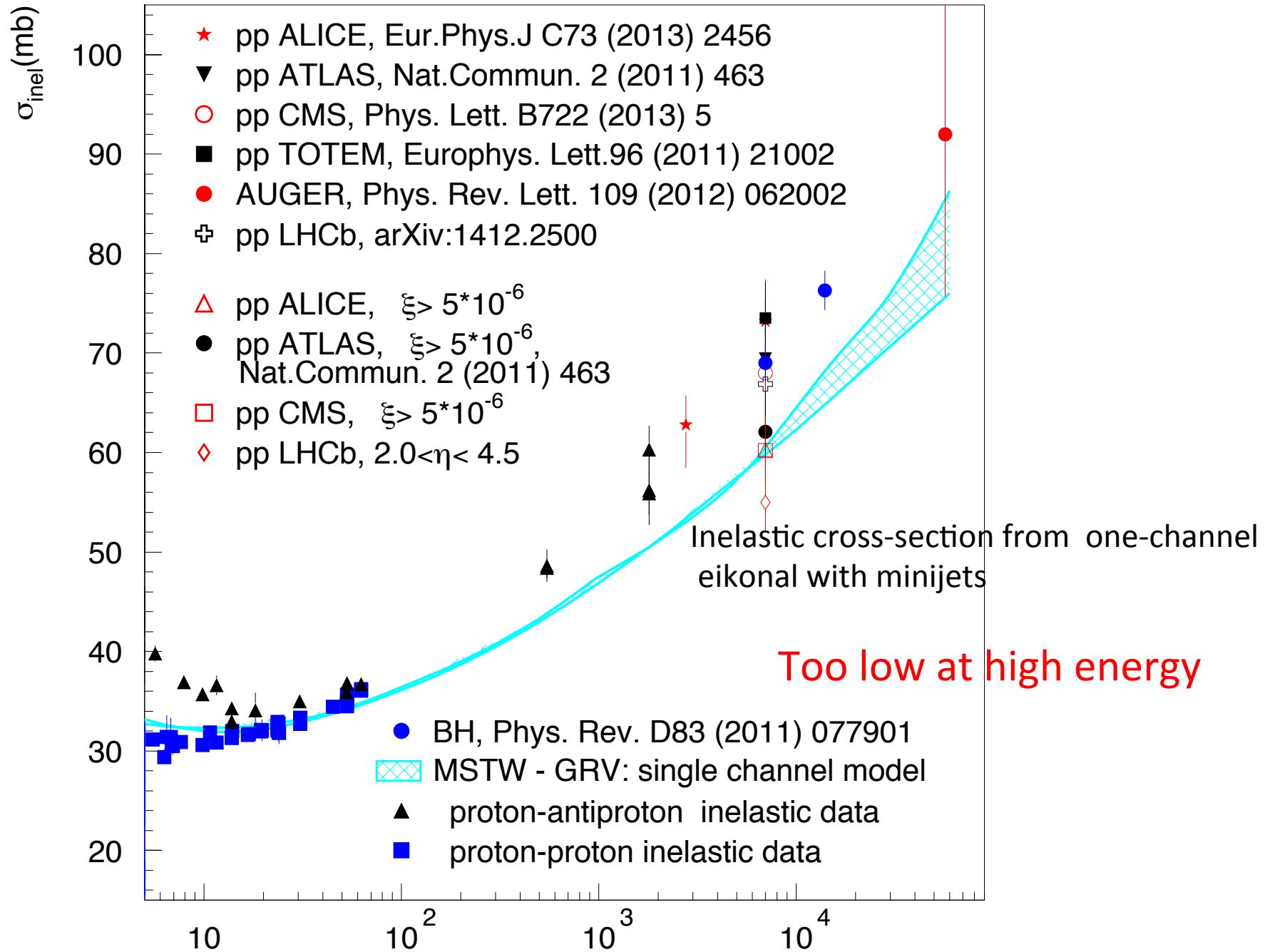
Barger Halzen, PRL1974
+Gaisser+Noble+Yodh ₂₄

Present limitation : amplitudes or cross-sections?

- Our formalism for **soft gluon resummation** in the infrared and **minijets** describes pp scattering through **cross-sections**, since the infrared cancellation requires a *probabilistic approach* (the semiclassical approach implies cancellation is order by order in the cross-section – as in QED)
- This is why so far the model has problems dealing with amplitudes, in particular the elastic amplitude.
- This is clearly seen with the inelastic cross-section.

The inelastic cross-section

- Basic definition $\sigma_{total} = \sigma_{elastic} + \text{everything else}$
 $\sigma_{inelastic}$
- In a one-channel eikonal model
$$A(s, -q^2) = \int d^2 b J_0(qb)[1 - e^{i\chi(s,b)}]$$
$$\sigma_{inel}^{one-ch}(s) = \int d^2 \mathbf{b}[1 - e^{-2\chi_I(b,s)}]$$
- $\sigma_{inelastic}$: to understand **extrapolations** and role of different regimes in central region and “diffraction “region, but **separation is ill defined**



The one-channel eikonal inelastic description can identify the mini-jet contribution

Uncorrelated independent events
Poisson distributed in b-space

$$\Pi_n(\bar{n}) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$$

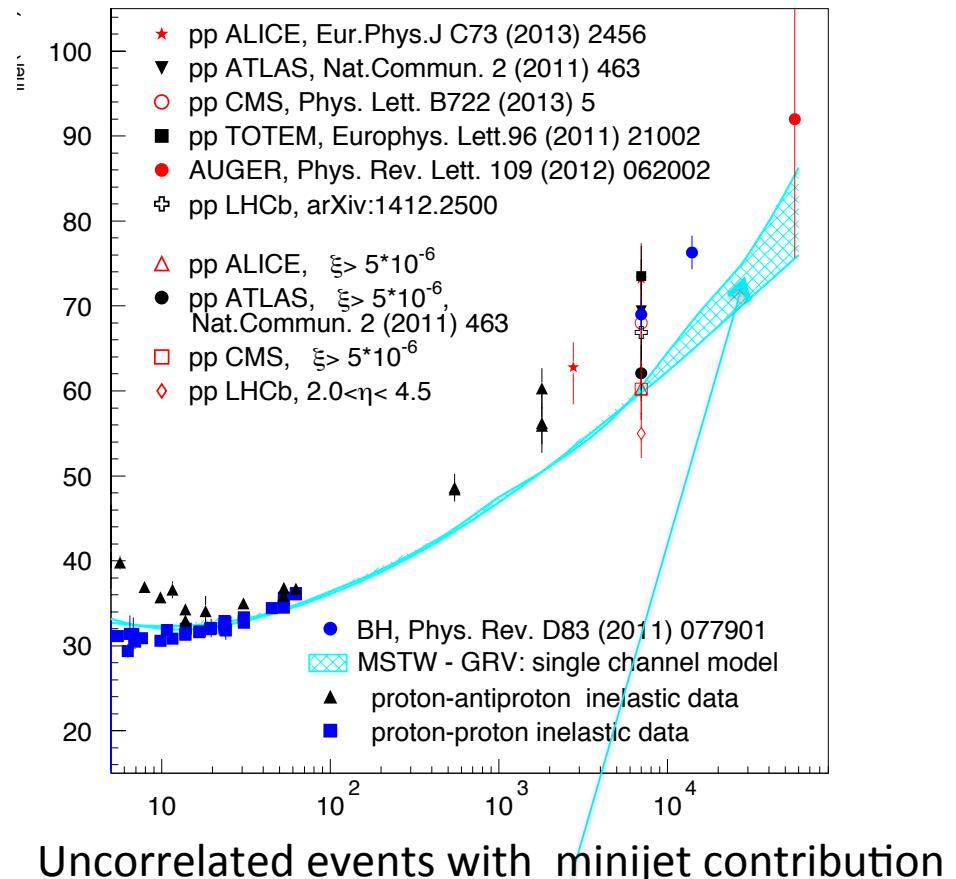
$$\sum_{1,\infty} \Pi_n = 1 - e^{-\bar{n}(b,s)}$$

$$\sigma_{inel}^{uncorrelated}(s) =$$

$$\int d^2b \Pi_n(\bar{n}) =$$

$$\int d^2b [1 - e^{-\bar{n}(b,s)}]$$

$$\bar{n}(b, s) \rightarrow \chi_I(b, s)/2$$



A strategy for understanding the total inelastic cross-section

1. for the minijet driven uncorrelated part:

- Obtain the $\chi_I(b, s)$ from model for total cross-section
- Input $\chi_I(b, s)$ to obtain

$$\sigma_{inelastic} = \sigma_{inel}^{correlated} + \sigma_{inel}^{uncorrelated} \simeq Poisson$$

2. for full inelastic:

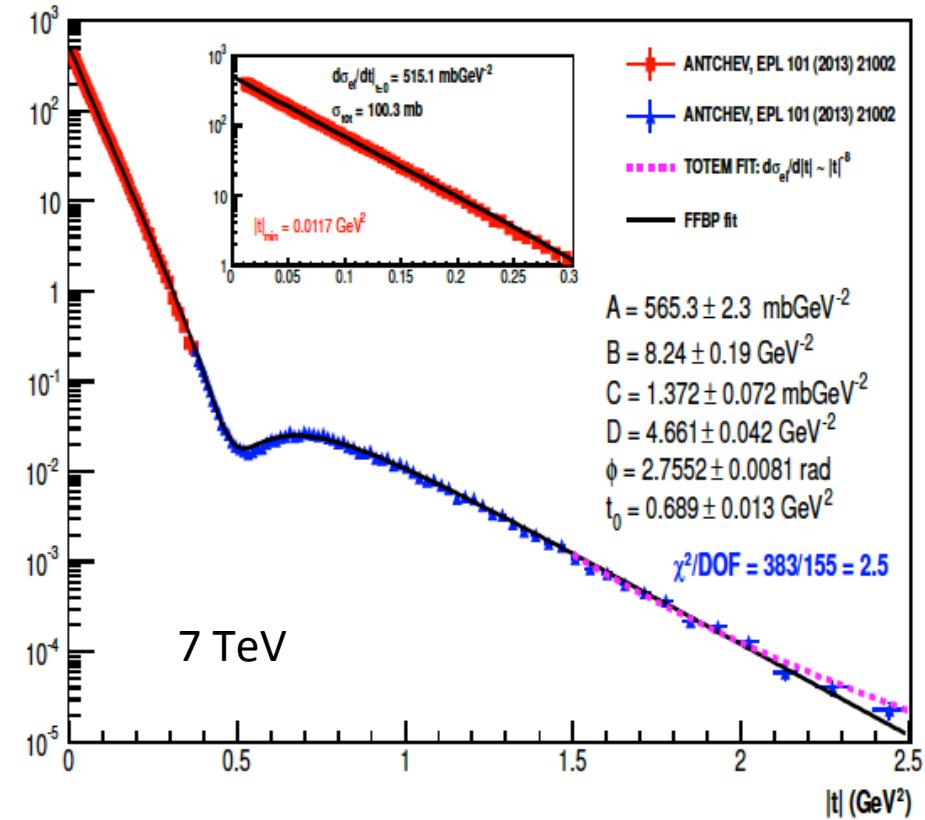
- go to multichannel amplitudes → more parameters (Gotsman et al, Khoze et al, Ostapchenko, etc.) ✗
- ✓ Use a convenient parametrization of the amplitude $d\sigma/dt$
→ $\sigma_{elastic}$ σ_{total}

The empirical model strategy

$$\mathcal{A}(s, t) = i[F_P^2(t)\sqrt{A(s)}e^{B(s)t/2} + e^{i\phi(s)}\sqrt{C(s)}e^{D(s)t/2}]$$

The amplitude is fitted
with 6 parameters

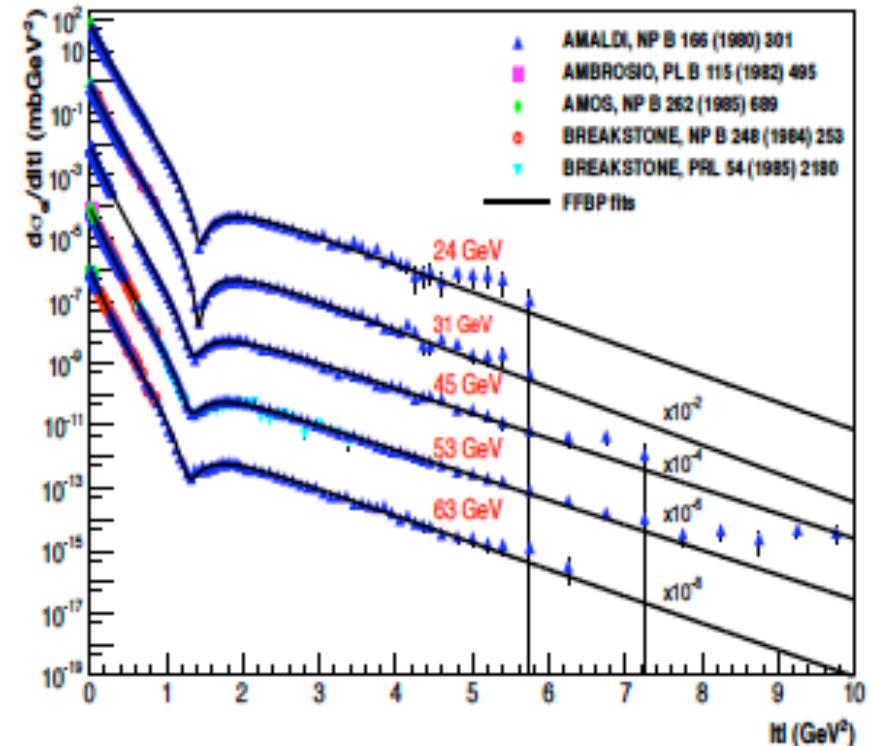
so as to reproduce $\frac{d\sigma}{dt}$



The empirical model : energy predictions for the parameters

- Fitting data from
ISR to LHC7
+ asymptotic theorems

- We can make an ansatz for the energy behaviour of the parameters



Asymptotics of the empirical parametrization

Phys. Rev. D88 (2013) 094019

$$4\sqrt{\pi A(s)}(mb) = 47.8 - 3.8 \ln s + 0.398(\ln s)^2$$
$$B(s)(GeV^{-2}) = 11.04 + 0.028(\ln s)^2 - \frac{8}{0.71} = -0.23 + 0.028(\ln s)^2$$
$$4\sqrt{\pi C(s)}(mb) = \frac{9.6 - 1.8 \ln s + 0.01(\ln s)^3}{1.2 + 0.001(\ln s)^3}$$
$$D(s)(GeV^{-2}) = -0.41 + 0.29 \ln s$$

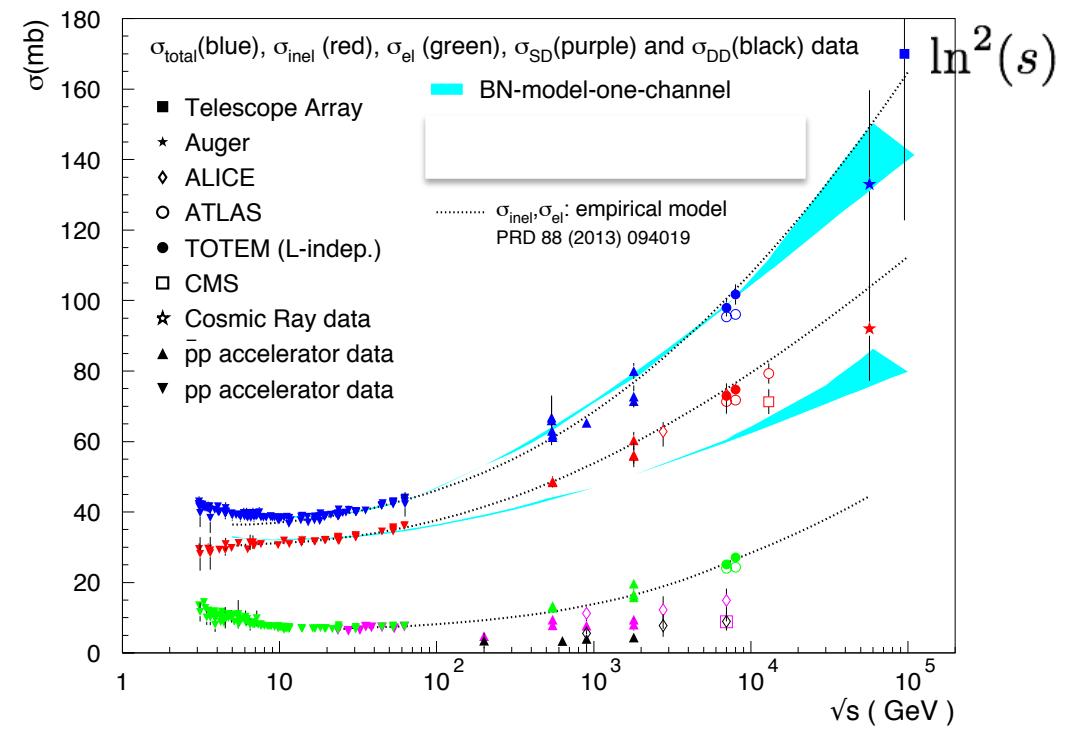
The empirical model strategy

$$\sigma_{inel} = \sigma_{total} - \sigma_{elastic}$$

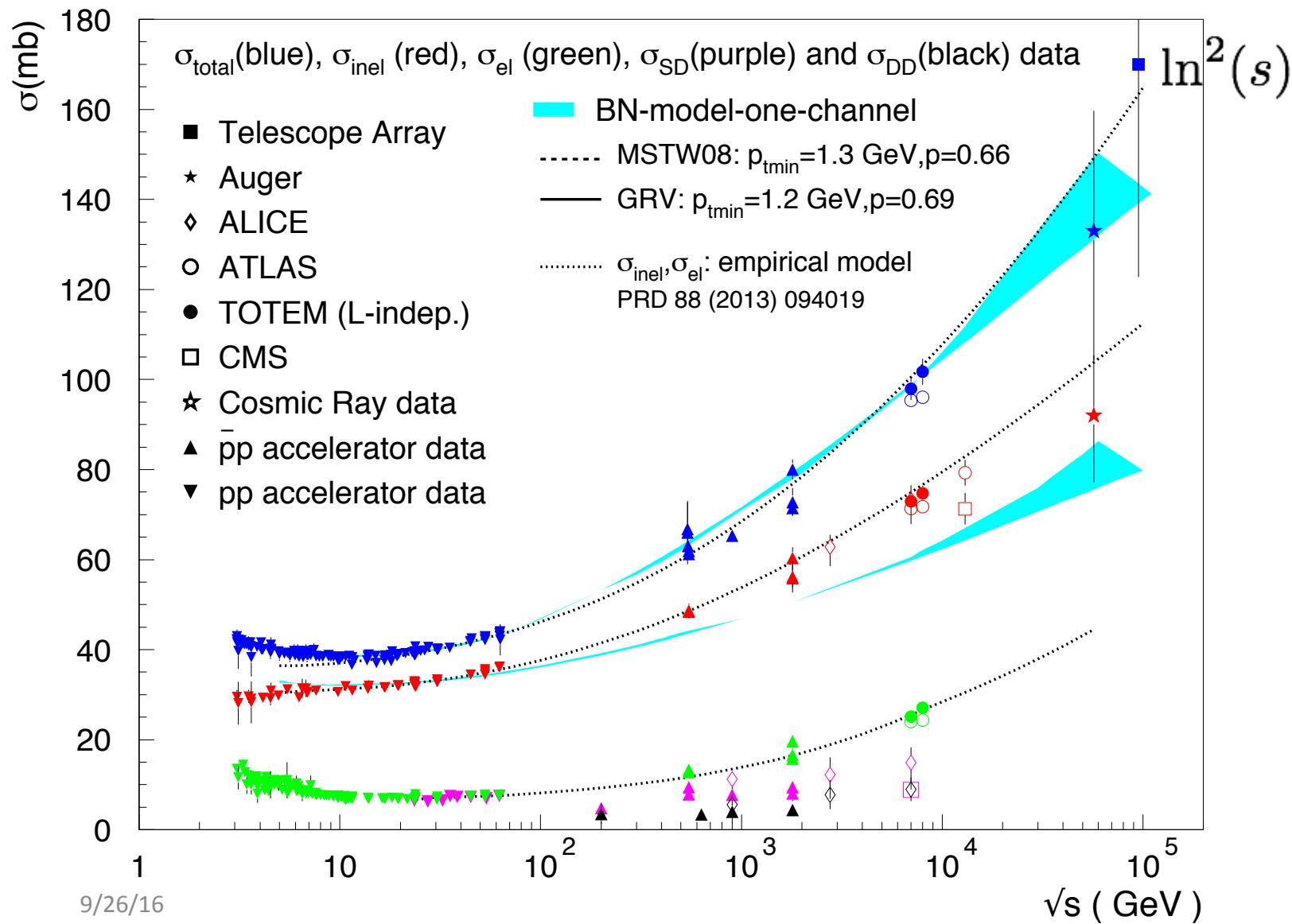
- From fit to $\frac{d\sigma}{dt}$

σ_{total} and $\sigma_{elastic}$

- Beyond 7 TeV, the energy dependence of the parameters is obtained from asymptotic theorems
 - $A(s)$ from Froissart bound
 - $B(s)$ from $\sigma_{el} \sim \frac{\sigma_{tot}^2}{B(s)}$
 - $C(s)$ and $D(s)$ from two asymptotic sum rules and Khuri – Kinoshita theorem
- And then obtain the full inelastic by subtraction



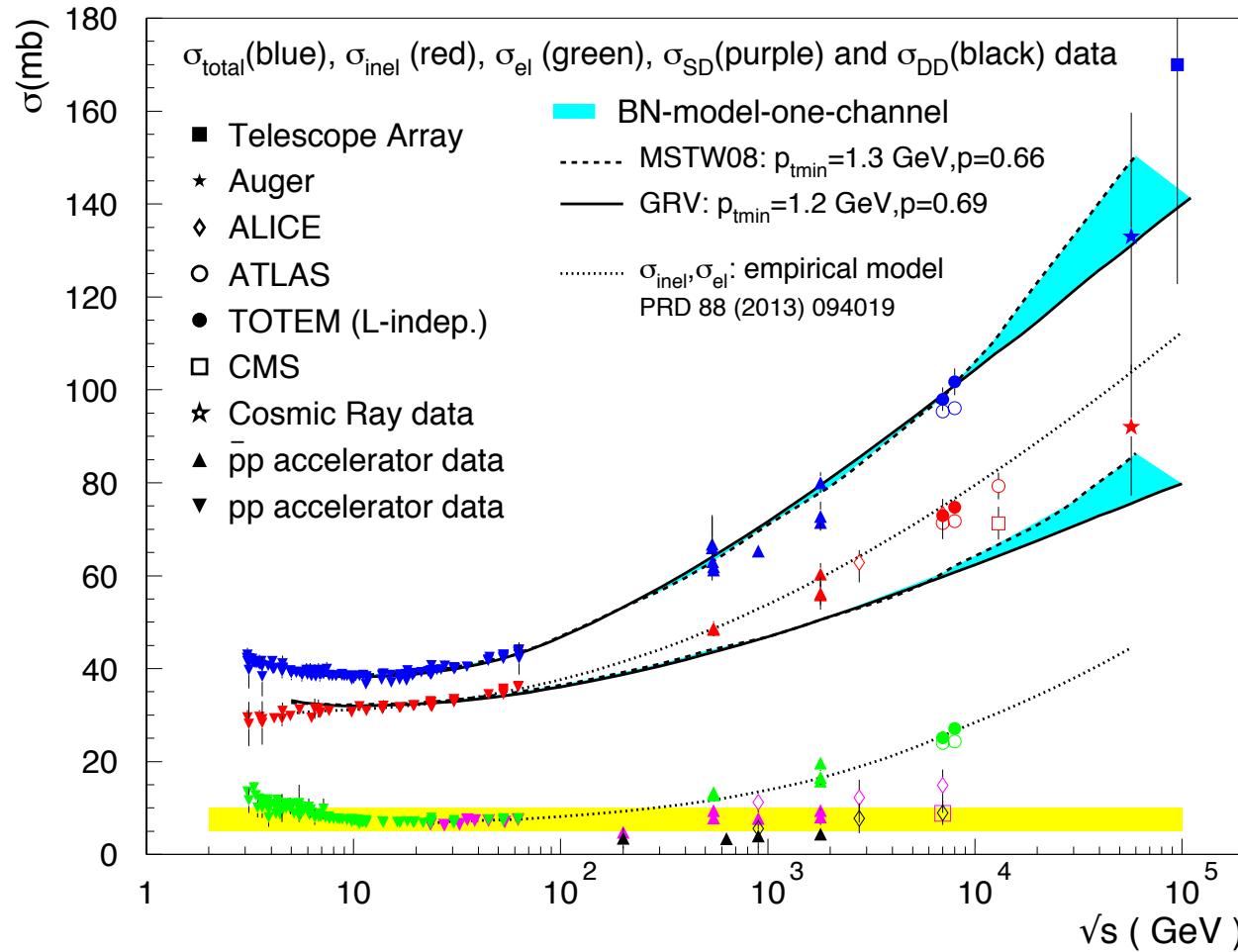
All together : pQCD with resum QCD + Empirical



A useless exercise?

- Our parametrization highlights
 - ATLAS elastic appears low, whereas TOTEM is within our “empirical” expectations
 - The total inelastic is somewhat higher than the data, most of which are obtained by extrapolation (some of them by subtraction)
 - The total would agree with our model with MSTW PDF, near saturation of the Foissart bound)

Conclusions: work in progress to include correlated events, as a diffraction



The end (for now...)

The regularized, integrated soft gluon spectrum

$$h(b, s) = \int_0^{q_{tmax}} d^3 \bar{n}_g(k) [1 - e^{i \vec{k}_t \cdot \vec{b}}]$$

depends on the maximum transverse momentum allowed to a single gluon at any given energy

And thus from PDFs through

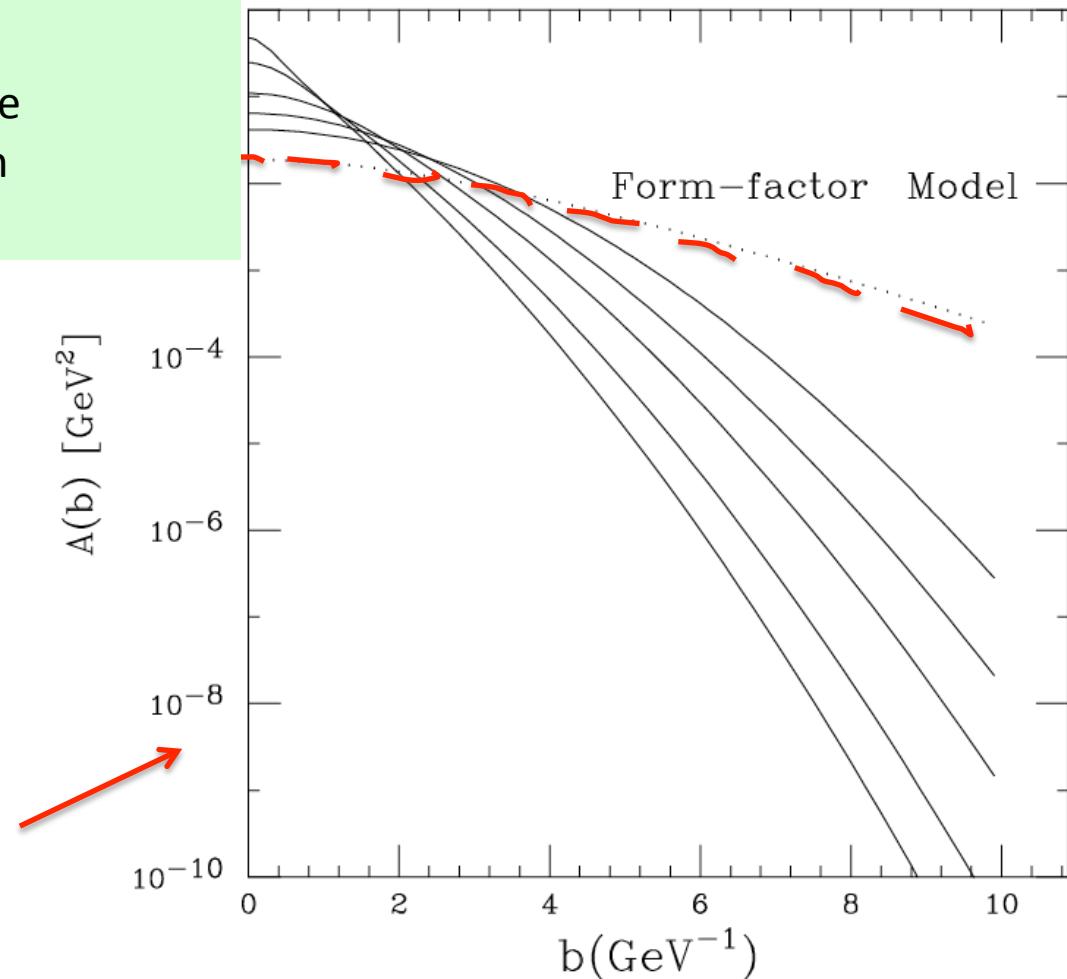
$$q_{tmax}(x_1, x_2; PDFs)$$

Parton b-distribution from form factor models vs resummation models

Corsetti, Grau, Pancheri, Srivastava, PLB 1996

$A(b, s)$ for mini-jet contribution

$q_{\text{max}} = 0.5, 1.0, 2.0, 5.0, 10.0 \text{ GeV}$



ResumQCD → impact parameter distribution

- b-distribution $\leftarrow \rightarrow$ How partons are distributed in b-space as a result of proton-proton collisions sheds light on the spatial distribution of hadronic matter, i.e. on the “hadronic cloud”, the gluon cloud, the region where gluons and quarks are confined.
- Our strategy: to resum in the infrared region and through a modelling of the coupling of gluon-to quark
- The infrared region is studied through the Fourier transform of soft gluon momentum distribution down to $k_t \rightarrow 0$

Basic fact: All total cross-sections **rise**... but not too much (**Froissart** dixit in 1961 + Martin 1962+Lukaszuk 1967)

$$\sigma_{total} \lesssim [\log s]^2 \quad \text{Asymptotically}$$

Where from?

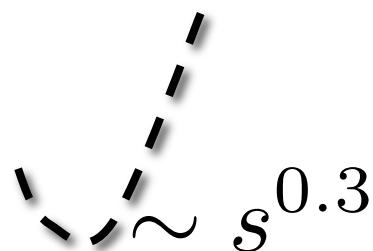
A cut off in maximum angular momentum in partial wave expansion and assumptions about large s behaviour of the pw amplitudes

$$\Rightarrow b < b_{max}$$

$$\sigma_{tot} \lesssim \sum_{0,L} \simeq L_{max}^2$$

$$L_{max} = qb_{max} \sim \log s$$

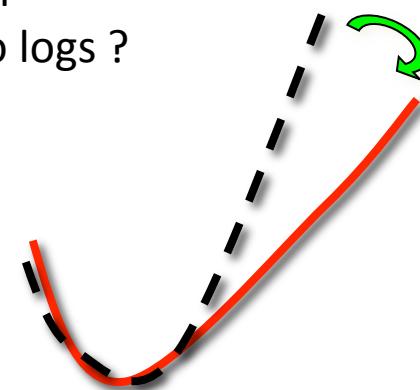
What generates the rise, which is very fast at the start (ISR)?

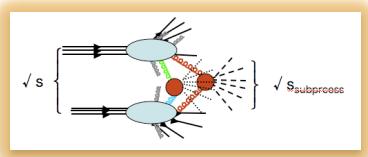


$$\sim s^{0.3}$$

What tames the rise into to a Froissart-like behavior?

How to go from power-law to logs ?





Eikonal model

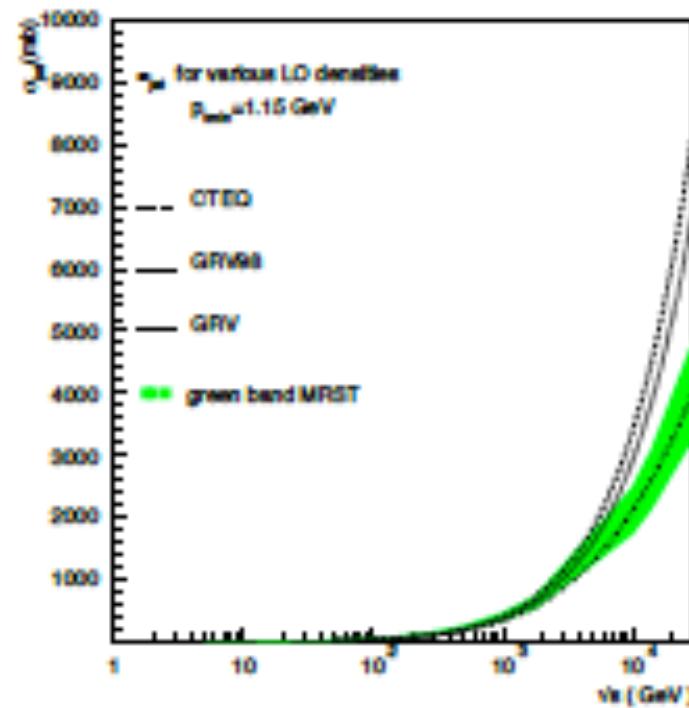
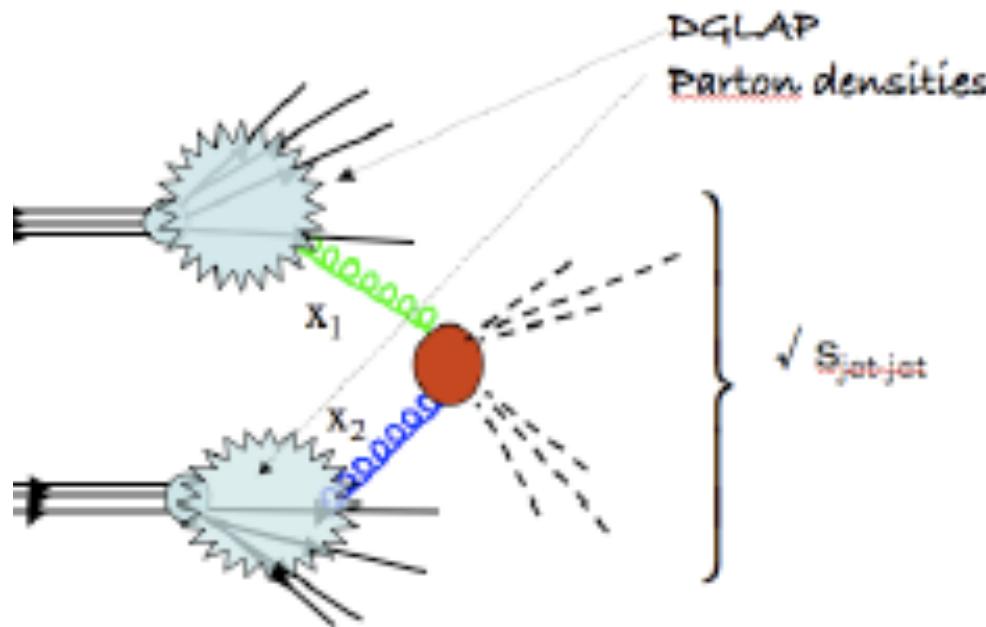
- Multiple collisions: eikonal form for the amplitude

$$\sigma_{total} = 2 \int d^2 b [1 - e^{-\chi_I(s, b)}]$$

- Begin with one-channel to minimize parameters and see if the model works

$$A(s, -q^2) = \int d^2 b J_0(qb) [1 - e^{i\chi(s, b)}]$$

Single Mini-jet scattering x-section rises very fast with energy



onic picture of mini-jet role in hadron-hadron scattering and representative mini-jet calcu

Soft gluon resummation is added

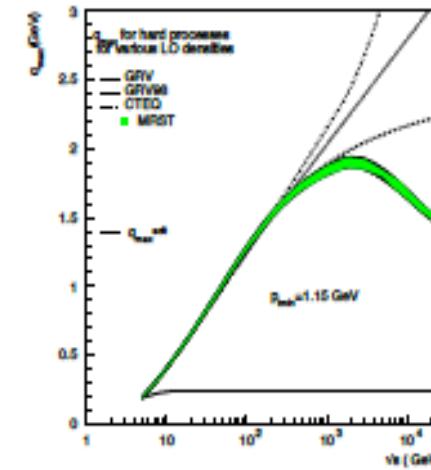
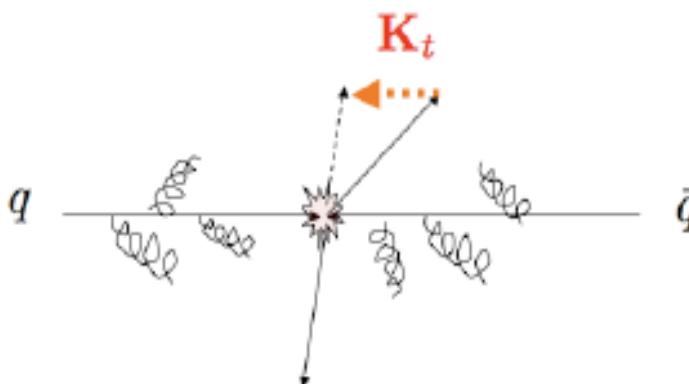


Fig. 48. Global soft K_t emission breaks the collinearity of partons. At right the maximum momentum, q_{\max} , allowed to a single gluon, averaged over different PDFs for valence quarks, [243]. Lower curve parametrizes effects at $\sqrt{s} < 10$ GeV.

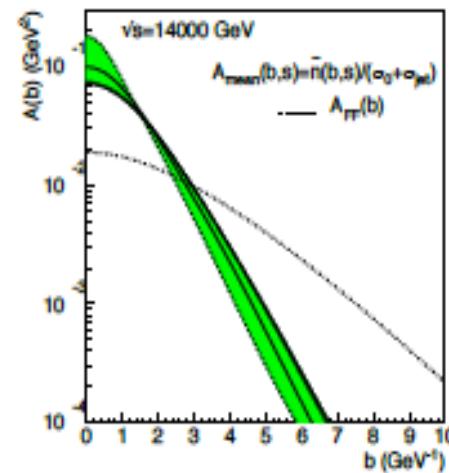
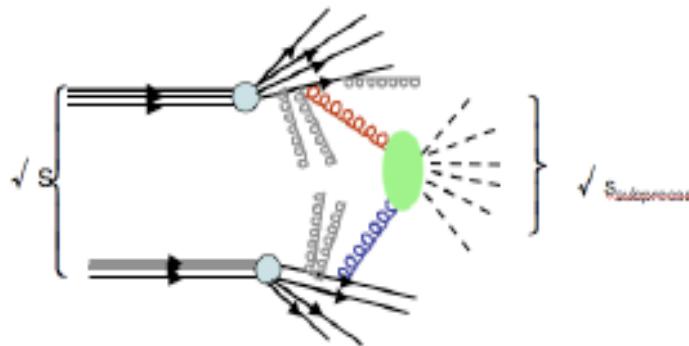
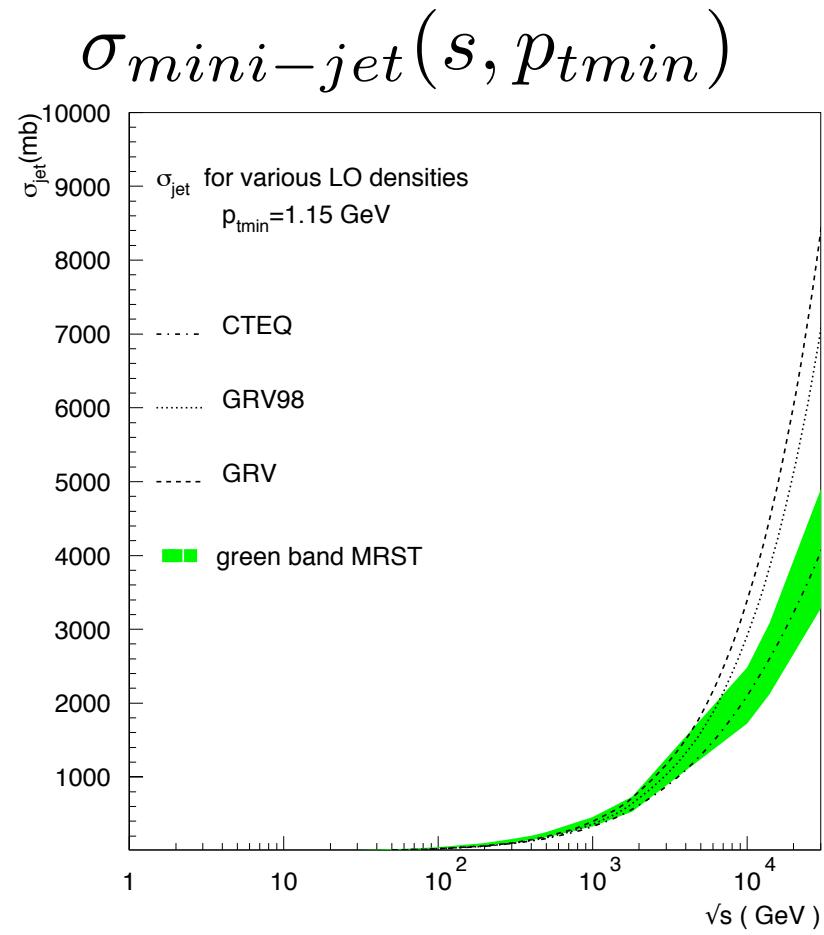
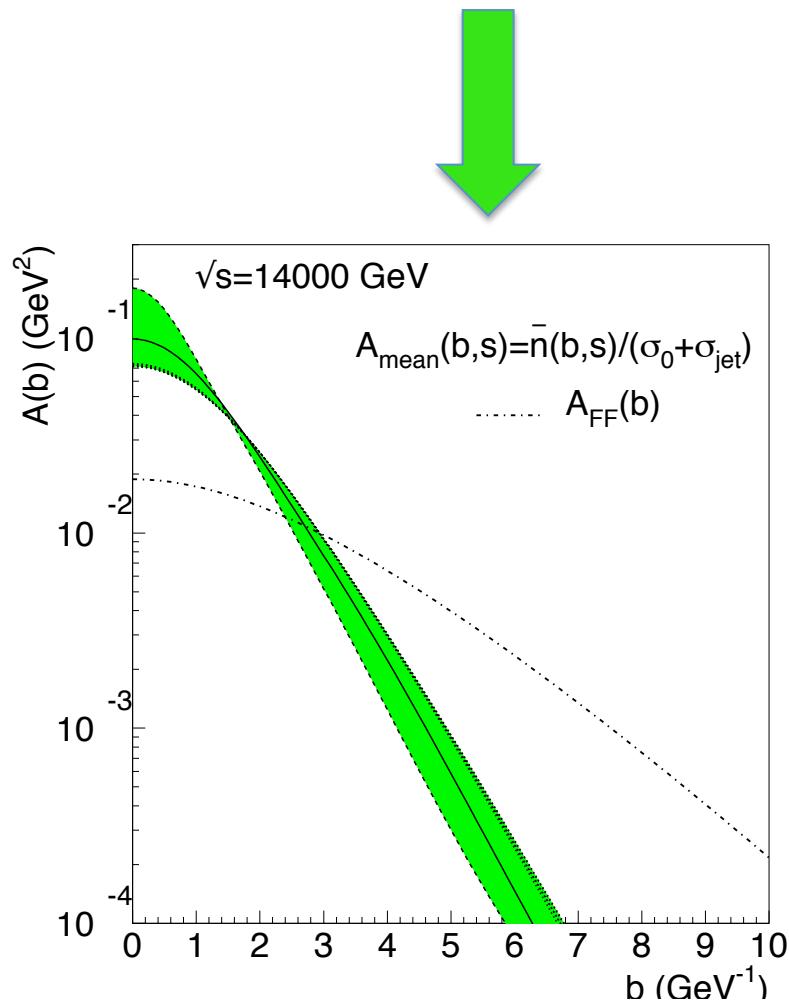


Fig. 49. Representative soft gluon radiation from initial quarks in LO picture of hadron-hadron scattering. At right, the impact

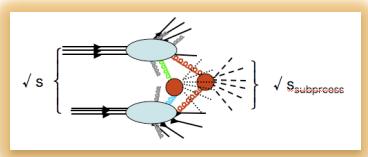
Low-x behaviour of PDF influences both
 $A(b)$ (through q_{\max}) and



Our model ansatz for $\alpha_{IR}(k_t^2 \rightarrow 0)$

$$\alpha_{IR}(k_t^2 \rightarrow 0) = \left(\frac{\Lambda}{k_t}\right)^{2b_0}$$

$$b_0 = \frac{11N_c - 2N_f}{12\pi}$$



Eikonal model

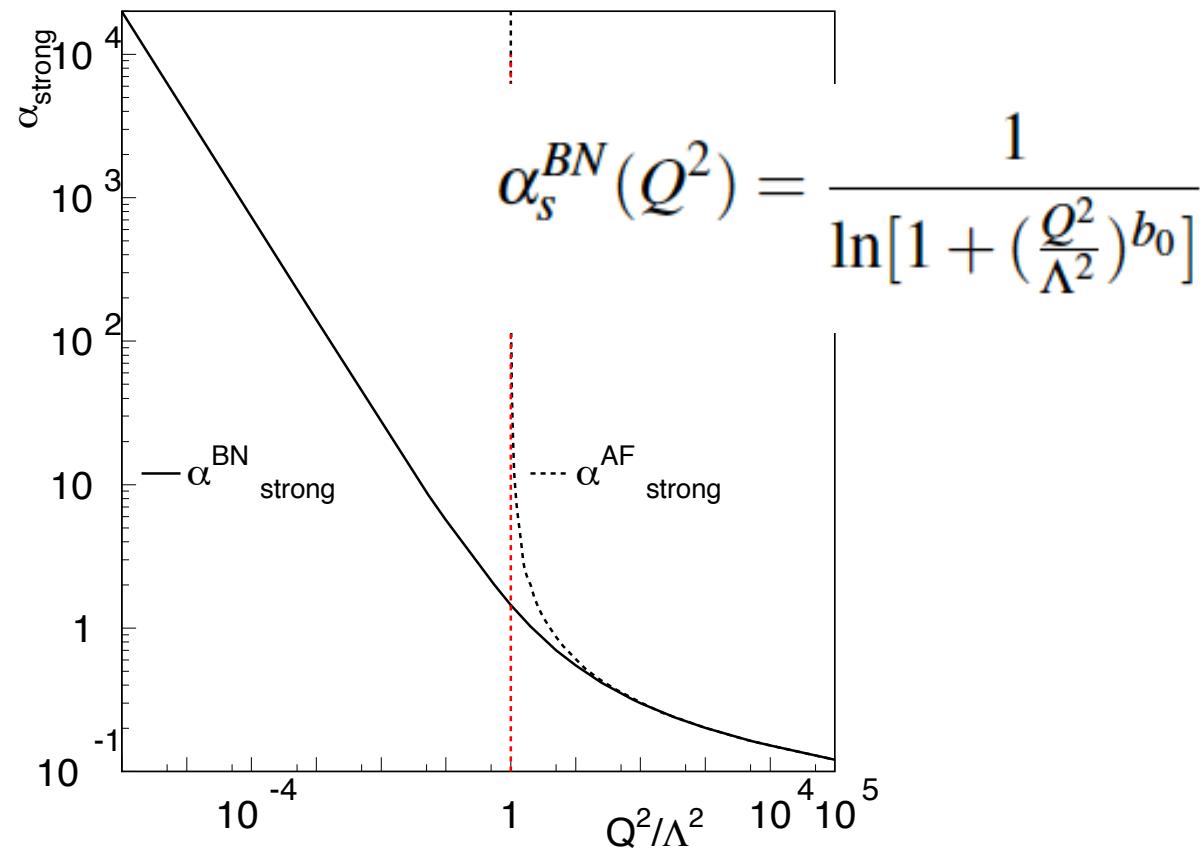
- Multiple collisions: eikonal form for the amplitude

$$\sigma_{total} = 2 \int d^2 b [1 - e^{-\chi_I(s, b)}]$$

- Begin with one-channel to minimize parameters and see if the model works

$$A(s, -q^2) = \int d^2 b J_0(qb) [1 - e^{i\chi(s, b)}]$$

Phenomenological behaviour used in our Bloch-Nordsieck model for total cross-section



What really is p

- Our ansatz for interpolation between AF and infrared

$$p \leftrightarrow b_0$$

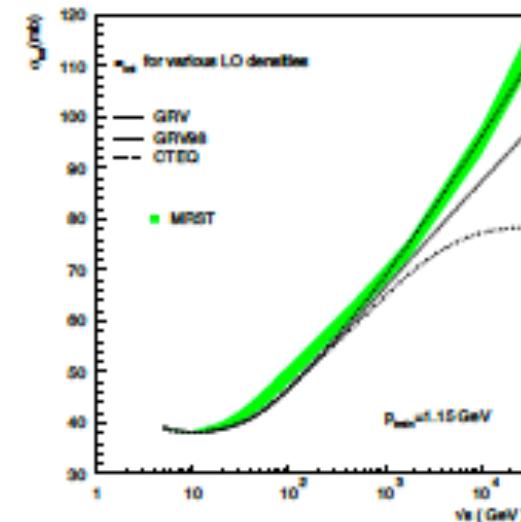
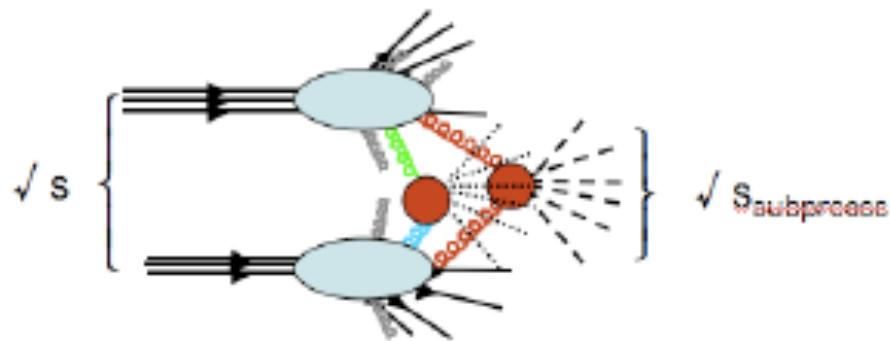
$$\alpha_s^{interpolation} = \frac{1}{\log[1 + (\frac{k^2}{\Lambda})^{b_0}]}$$

$$\frac{11\pi}{11N_c - 2N_f} \log\left[\frac{k_t^2}{\Lambda^2}\right] \quad \left(\frac{\Lambda^2}{k_t^2}\right)^p$$

Cartoon representation of actual calculation

Eikonalization of soft gluon corrected mini-jet scattering

The rise is tamed



The inelastic

Something is obviously missing:
correlated exchanges, the diffractive
part

All together now

