Towards a theory for the nonperturbative Pomeron

LHC Forward - Physics - 2016

Collaboration with C.Contreras and G.P.Vacca

JB, Contreras, Vacca,
JHEP 1603 (2016) 201 and
hep-th/1608.08836

- Introduction
- Search for fixed points: first results
- Short glimpse at phenomenology
- Conclusions

Introduction

Goal:

try to connect the Regge limit of pQCD with nonperturbative strong interaction

pQCD: short transverse distances, BFKL

$$\alpha(0) = 1 + \omega_{BFKL} > 0$$

α' very small

power-like large-b behavior

Odderon with $\alpha_O(0) = 1, \ \alpha'_O$ small

ultraviolet

nonperturbative: pp scattering at LHC

$$\alpha(0) \approx 1.1$$

$$\alpha' \approx 0.25 \text{ GeV}^{-2}$$

exponential large-behavior

Some evidence for Odderon

infrared

Framework: Reggeon field theory

This talk: first step, only infrared limit.

Method:

renormalization group, flow equations:

integrate over large momentum modes, investigate the infrared limit

What do we need:

- Nonperturbative Pomeron at LHC:
  intercept slightly above one $\alpha_P(0) \approx 1.1$
  at very high energies intercept must go to zero (unitarity)

- There exists UV region (IR physics cutoff):
  total cross sections at LEP, DIS at HERA, BFKL tests at HERA, LHC
  'hard' Pomeron with intercept clearly above one $\alpha_{BFKL} \approx 1.25$

Attractive idea:

use reggeon field theory (2+1-dim field theory) and renormalization group, construct a flow from UV scale to IR scale

$$S = \int dy d^2x \mathcal{L}(\psi, \psi^\dagger)$$

e.g. local approximation: $\mathcal{L} = (\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_y} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$

$$V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi)\psi$$
$$+ g(\psi^\dagger \psi)^2 + g' \psi^\dagger ({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

Study the flow as function of IR cutoff k in transverse momentum,
all fields and parameters become k-dependent,
IR limit: infinite transverse momenta, infinite energies

The formalism: functional renormalization, flow equations

Reminder: Wilson approach

The standard Wilsonian action is defined by an iterative change in the UV-cutoff induced by a partial integration of quantum fluctuations:

$$\Lambda \to \Lambda' < \Lambda$$

$$\int [\mathrm{d}\varphi]^{\Lambda} e^{-S^{\Lambda}[\varphi]} = \int [\mathrm{d}\varphi]^{\Lambda'} e^{-S^{\Lambda'}[\varphi]} \qquad k < \Lambda$$

Alternatively: ERG-approach (Wetterich), sequence of theories, IR cutoff

(successful use in statistical mechanics and in gravity)

define a bare theory at scale Λ.

The integration of the modes in the interval $[k, \Lambda]$ defines a k-dependent average functional.

Letting k flowing down to 0 defines a flow for the functional which leads to full theory. k-dependent effective action:

regulator

$$e^{-\Gamma_k[\phi]} = \int [d\varphi]\mu_k e^{-S[\varphi]+\int_x (\varphi-\phi)_x \frac{\delta \Gamma_k[\phi]}{\delta \phi_x} - \Delta S_k[\varphi-\phi]}$$

Taking a derivative with respect the RG time t=log (k/k_0) one obtains flow equation:

$$\partial_t \Gamma_k = \frac{1}{2} Tr\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k\right] - \frac{\dot{\mu}_k}{\mu_k}$$

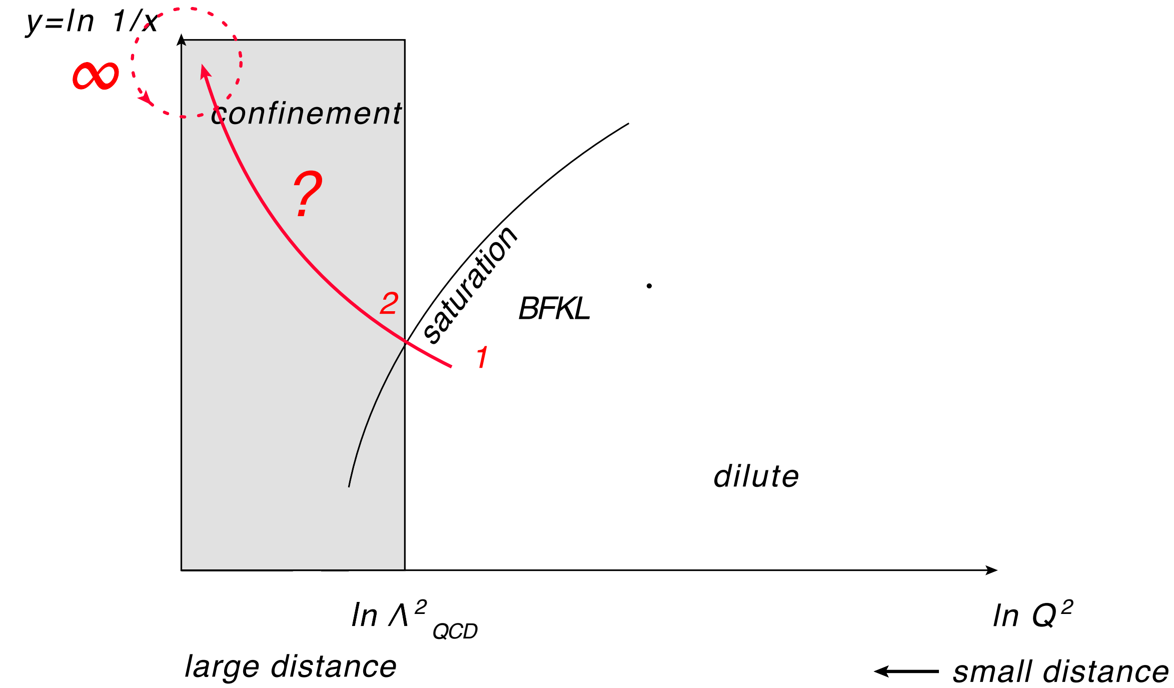
$$\mathcal{R} = \text{regulator operator}$$

which is UV and IR finite
From this derive coupled differential equations for Green's and vertex functions (see below)

A comment on the role of transverse distances and cutoff in transverse momentum:

1) pp scattering at present energies: transverse extension grows with s
2) growth of total cross section varies with transverse size of projectiles
BFKL in $\gamma^*\gamma^*$, $\gamma^* p$ in DIS, pp
Trend: transverse size grows with energy,
intercept decreases with size
$\rightarrow$ IR cutoff in transverse momentum is physical

This talk:
only the first steps



1) Existence of a theory in the IR limit:
fixed point in the space of reggeon field theories: existence of theory
Properties of the fixed point theory

2) How to approach the fixed point

Serious approximation: local approximation

in the UV, BFKL -Pomeron is composite field, nonlocal kernels

Solve flow equations, search for fixed points

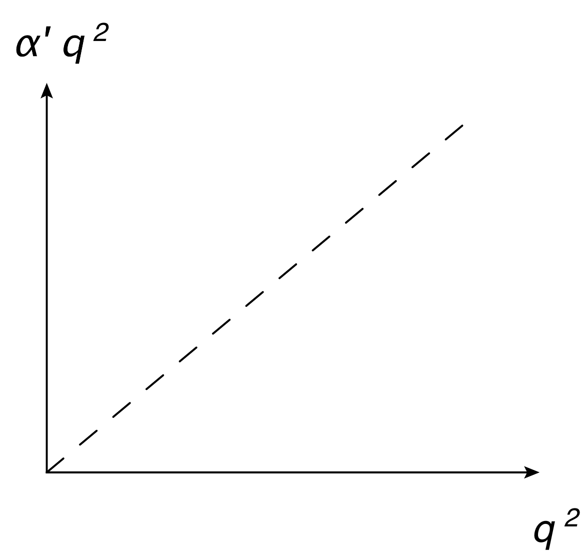
$$\Gamma[\psi^\dagger,\psi] = \int \mathrm{d}^2x\,\mathrm{d}\tau \left(Z(\frac{1}{2}\psi^\dagger \overset{\leftrightarrow}{\partial_\tau}\psi - \alpha'\psi^\dagger\nabla^2\psi) + V[\psi^\dagger,\psi] \right), \qquad \alpha(0) - 1 = \mu/Z$$

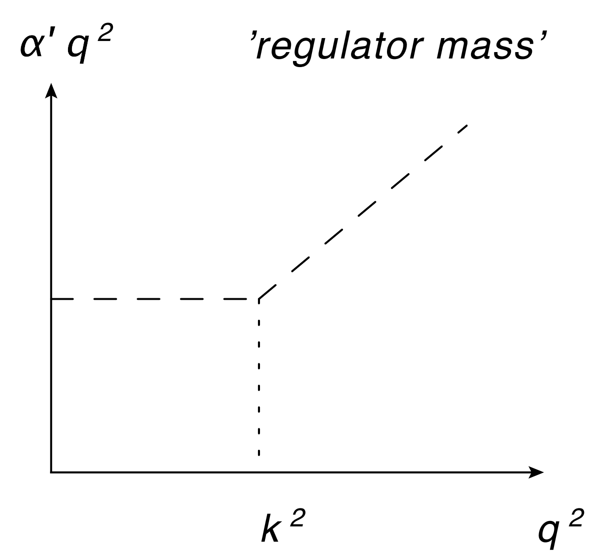
$$V[\psi^\dagger,\psi] = -\mu\psi^\dagger\psi + i\lambda\psi^\dagger(\psi^\dagger+\psi)\psi + g(\psi^\dagger\psi)^2 + g'\psi^\dagger({\psi^\dagger}^2+\psi^2)\psi$$
$$+ i\lambda_5{\psi^\dagger}^2\left(\psi^\dagger+\psi\right)\psi^2 + i\lambda_5'\psi^\dagger\left({\psi^\dagger}^3+\psi^3\right)\psi + \ldots$$

After introducing a regulator: all parameters become k-dependent

$$\Gamma_k[\psi^\dagger,\psi] = \int d^2x\,d\tau \left(Z_k(\frac{1}{2}\psi^\dagger \overset{\leftrightarrow}{\partial_\tau}\psi - \alpha_k'\psi^\dagger\nabla^2\psi) + \psi^\dagger R_k\psi + V_k[\psi,\psi^\dagger] \right)$$

There is freedom in choosing a regulator, for example:





Concretely: partial differential equation for potential $V(\psi, \psi^\dagger)$:

$$\dot{\tilde{V}}_k[\tilde{\psi}^\dagger, \tilde{\psi}] = (-(D+2) + \zeta_k)\tilde{V}_k[\tilde{\psi}^\dagger, \tilde{\psi}] + (D/2 + \eta_k/2)(\tilde{\psi}\frac{\partial \tilde{V}_k}{\partial \tilde{\psi}}|_t + \tilde{\psi}^\dagger \frac{\partial \tilde{V}_k}{\partial \tilde{\psi}^\dagger}|_t) + \frac{\dot{V}_k}{\alpha' k^{D+2}}.$$

$$\dot{V}_k = N_D A_D(\eta_k, \zeta_k)\alpha'_k k^{2+D} \frac{1 + \tilde{V}_{k\tilde{\psi}\tilde{\psi}^\dagger}}{\sqrt{1 + 2\tilde{V}_{k\tilde{\psi}\tilde{\psi}^\dagger} + \tilde{V}^2_{k\tilde{\psi}\tilde{\psi}^\dagger} - \tilde{V}_{k\tilde{\psi}\tilde{\psi}}\tilde{V}_{k\tilde{\psi}^\dagger\tilde{\psi}^\dagger}}}.$$

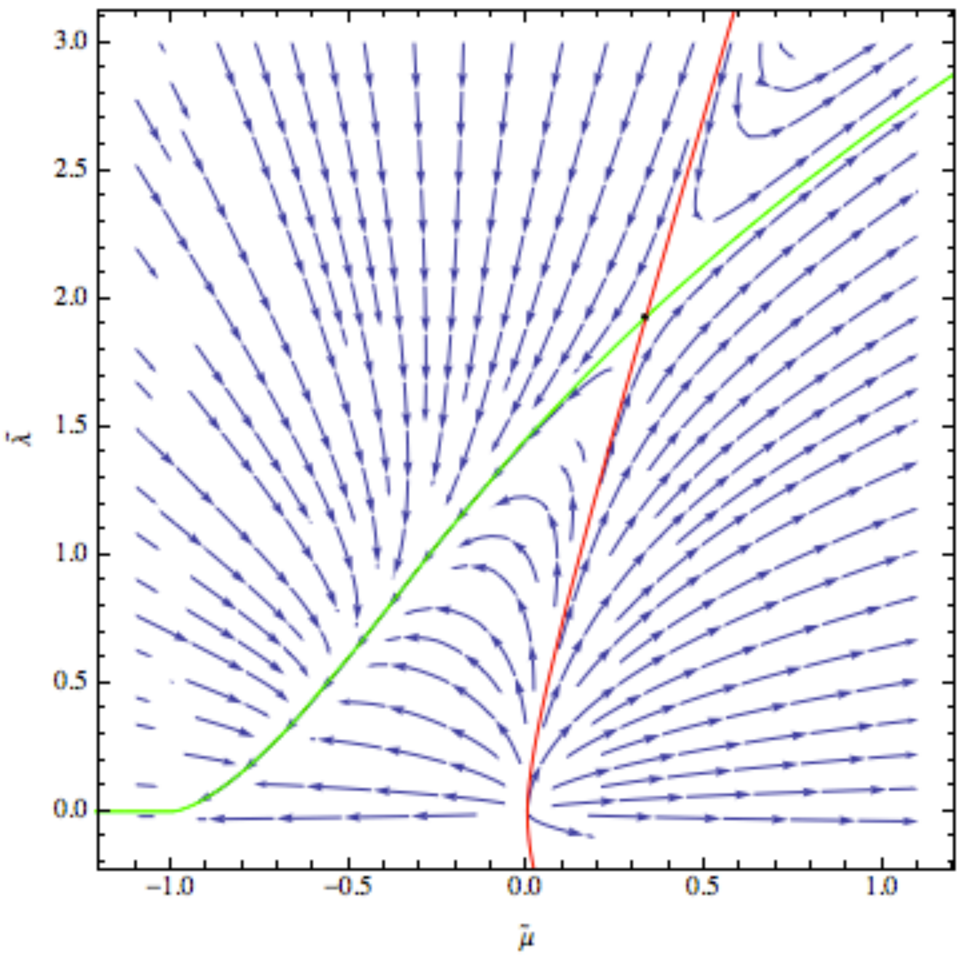
Fixed point: put rhs =0

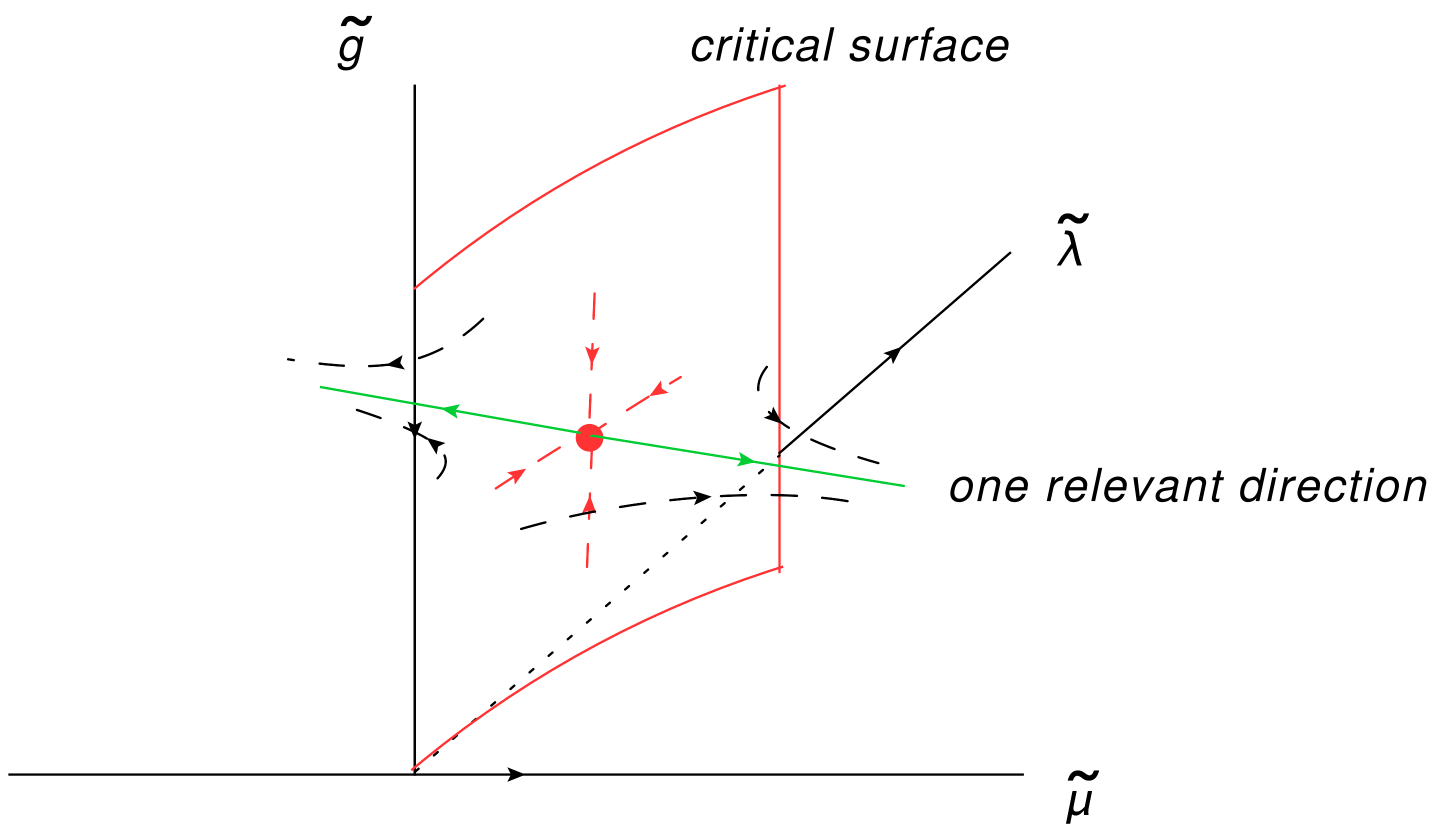
Possible ways to solve (for constant fields, approximately):

- polynomial expansion in fields around zero (beta-functions)
- polynomial expansion in fields around stationary point
- solve differential equations in the region of large fields

Results of fixed point analysis

1) Existence of a fixed point with one relevant direction





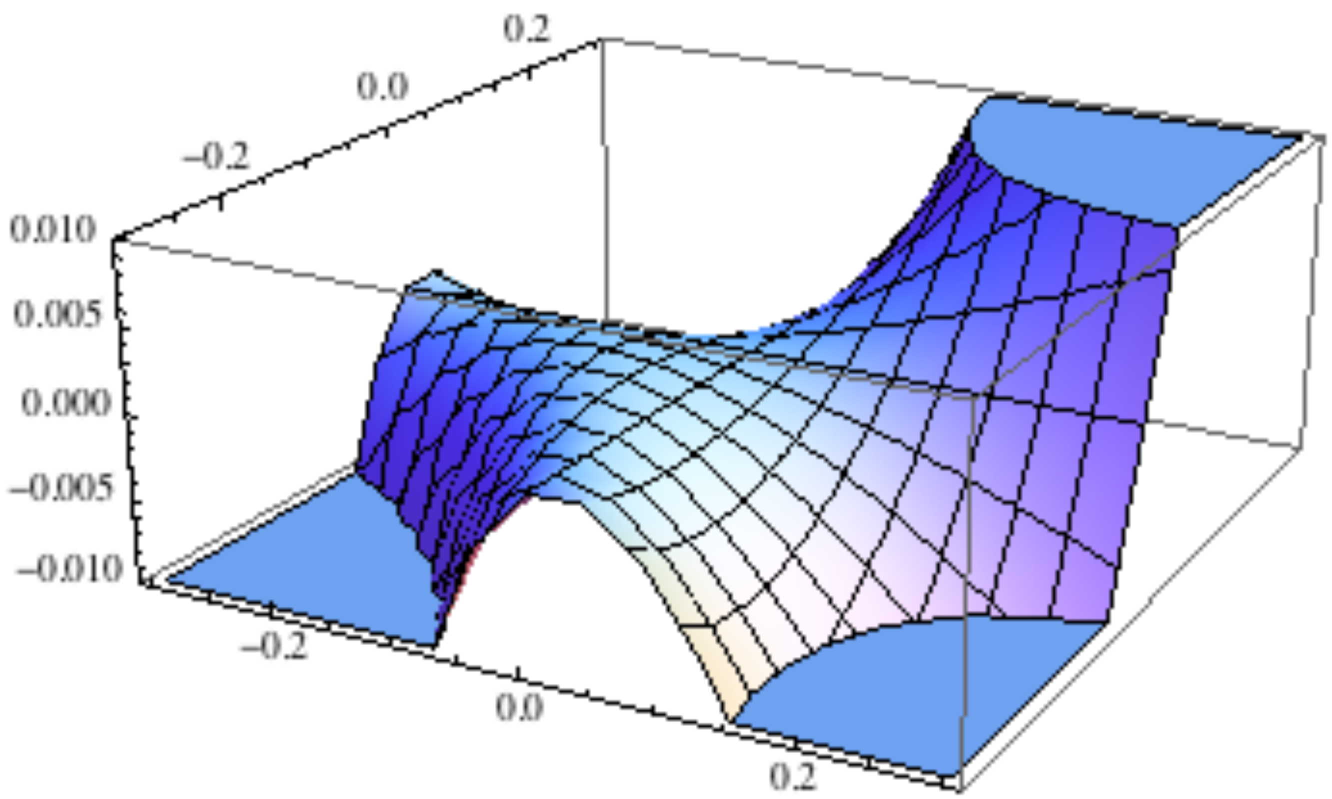
Flow in the space of parameters of the potential (couplings) :
reggeon mass (intercept) $\alpha(0) - 1 = \tilde{\mu}/Z$, triple coupling $\tilde{\lambda}$
fixed point IR attractive inside critical surface (red),
repulsive along one-dimensional relevant direction (green)

Convergence for higher truncations (expansion around nonzero stationary point) :

truncation	3	4	5	6	7	8
exponent ν	0.74	0.75	0.73	0.73	0.73	0.73
mass $\tilde{\mu}_{eff}$	0.33	0.362	0.384	0.383	0.397	0.397
$i\psi_{0,diag}$	0.058	0.072	0.074	0.074	0.0.074	0.074
iu_0	0.173	0.213	0.218	0.218	0.218	0.218

Compare with Monte Carlo result for Directed Percolation
(same universality class): $\nu = 0.73$

Shape of the effective potential (in the subspace of imaginary fields):



Extrema, location at lowest truncation:

$$(\tilde{\psi}_0, \tilde{\psi}_0^\dagger) = (0,0), \qquad (\frac{\tilde{\mu}}{i\tilde{\lambda}}, 0), \qquad (0, \frac{\tilde{\mu}}{i\tilde{\lambda}}), \qquad (\frac{\tilde{\mu}}{3i\tilde{\lambda}}, \frac{\tilde{\mu}}{3i\tilde{\lambda}}).$$

No further structure for larger fields

Main result of this part:

- found a candidate for fixed point (IR stable except for one relevant direction
- robust when changing truncations
- know the effective potential
- Include Odderon: IR stable with two (three) relevant directions
  at the fixed point:
  Pomeron does not feel the Odderon, whereas Odderon has strong absorption.

First glimpse at physics

Need to find out: on which trajectory is real physics?

Look at flow of physical physical observable: Pomeron intercept $\alpha(0) - 1 = \mu/Z$:

So far: fixed point analysis was done in terms of dimensionless variables: reggeon energy and momentum have different dimensions

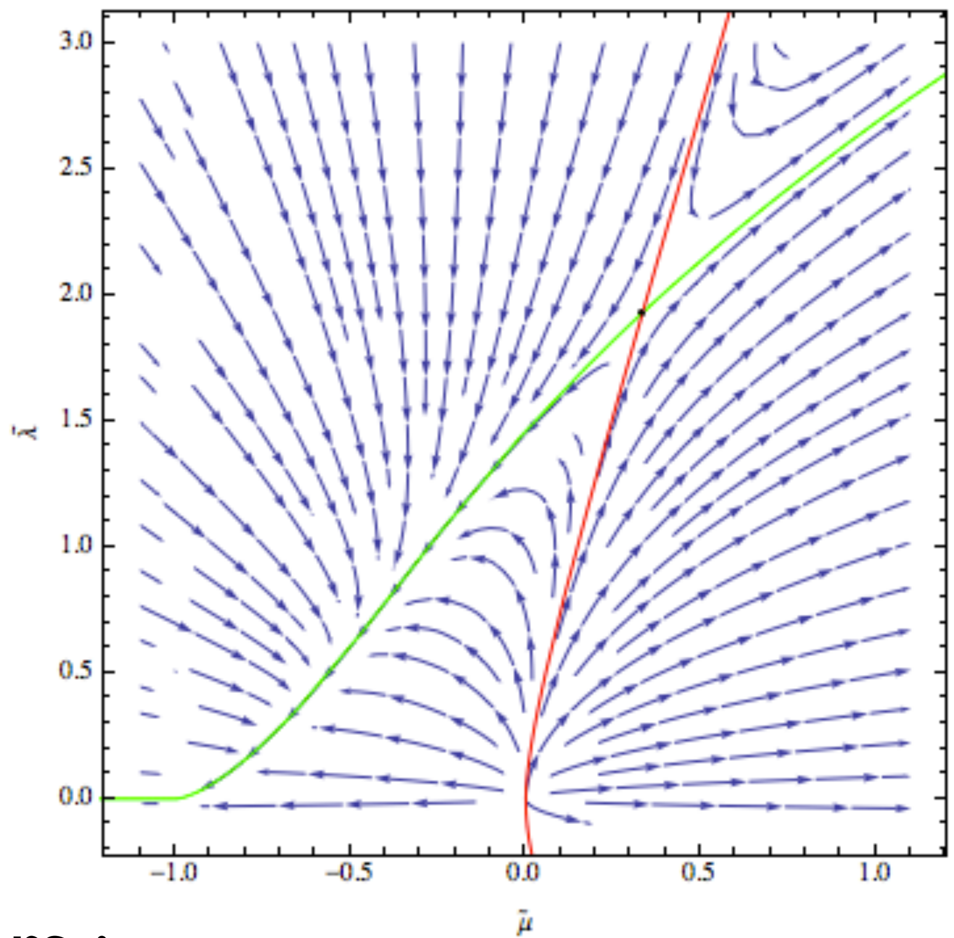
$$S = \int d^2x\, d\tau \left(Z(\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_\tau} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V[\psi^\dagger, \psi] \right), \qquad [\psi] = [\psi^\dagger] = k^{D/2}, \qquad [\alpha'] = E k^{-2}.$$

$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$

$$\tilde{\lambda}_k = \frac{\lambda_k}{Z_k^{\frac{3}{2}} \alpha'_k k^2} k^{D/2}$$

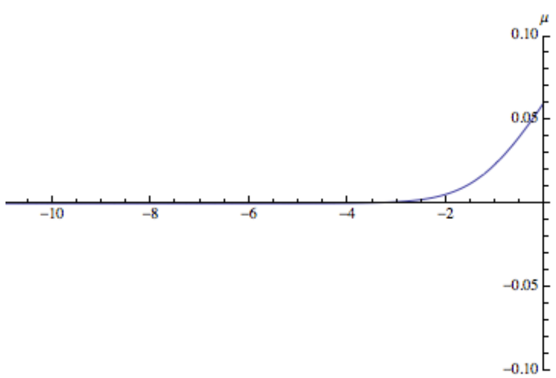
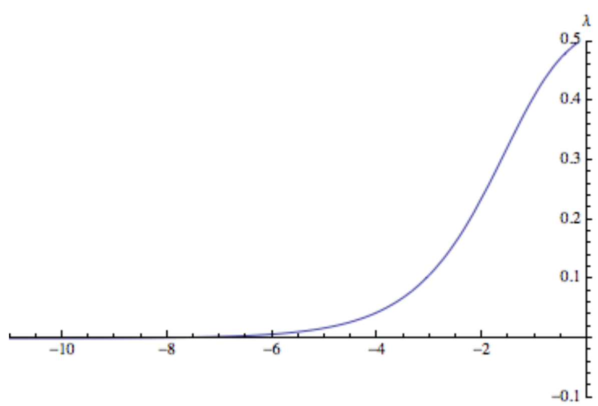
Evolution of physical (=dimensionful) parameters $\mu_k, \lambda_k, ...$ looks quite different from dimensionless ones $\tilde{\mu}_k, \tilde{\lambda}_k, ...$

dimensionless parameters



physical parameters :

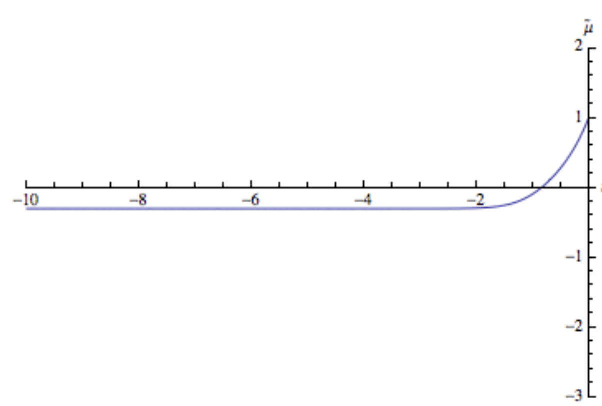
Critical subspace (red):

$\alpha(0) \to 1$ $\qquad$ $\lambda_{triple} \to 0$

But: theory not free!

Near critical subspace (blue): several possibilities, e.g.



$\alpha_k(0) \to \alpha_{k=0} < 1$

Main result: theory allows for different possibilities:

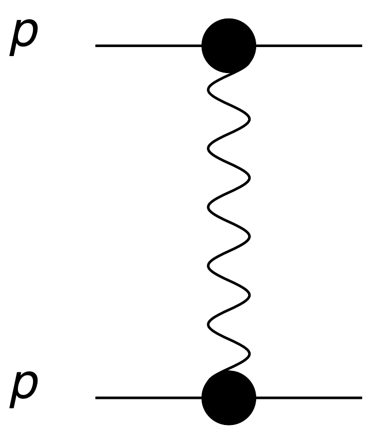
1) inside critical subspace: infrared stable fixed point with intercept one. But: need constraint at starting point in UV region
2) near critical surface: falling or rising total cross section. Need further study

In the following: consider a scenario inside the critical subspace

A simple model:
single Pomeron exchange - a scaling law

$$T_{el}(s,t) = is \int \frac{d\omega}{2\pi} s^{i\omega} \beta_p(t) \frac{1}{Z_k(i\omega + \alpha_k' q^2) - \mu_k} \beta_p(t)$$

$$= is\beta_p(t) Z_k^{-1} s^{\mu_k/Z - \alpha_k' q^2} \beta_p(t).$$



For small k:

$$T_{el}(s,t) \sim isk^{\eta} s^{k^{(2-\zeta)}\tilde{\mu_k}} f(\ln s\, q^2 k^{-\zeta})$$

$$\eta \approx -0.331\ (-0.6),\ \ \zeta \approx 0.172\ (0.28).$$

anomalous dimensions : directed percolation

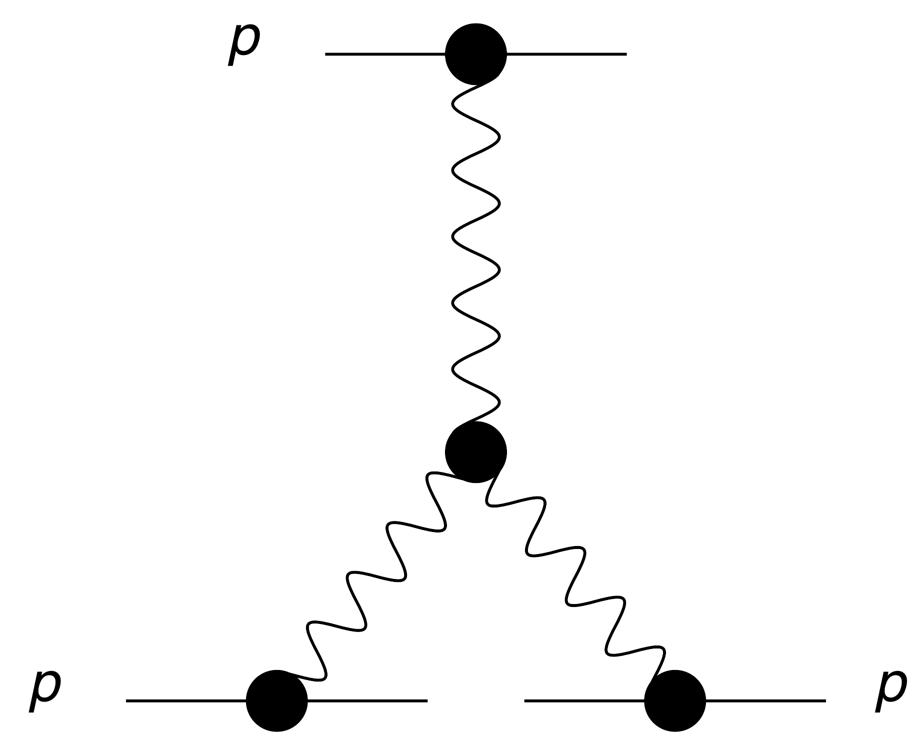
Assume: for very large energies $\alpha_k' k^2 \sim \dfrac{1}{\ln s}$ $\quad \left(R^2 \sim \dfrac{1}{k^2} \sim R_0^2 + \alpha_k' \ln s\right)$

$$T_{el}(s,t) \sim is(\ln s)^{-\eta/(2-\zeta)} s^{(\ln s)^{-1}\tilde{\mu}_{fp}} f(t(\ln s)^{2/(2-\zeta)})$$

Triple Pomeron cross section:

$$\frac{d\sigma}{dtdM^2} = \frac{1}{16\pi M^2} \int \frac{d\omega}{2\pi i} \int \frac{d\omega_1}{2\pi i} \int \frac{d\omega_2}{2\pi i} \left(\frac{s}{M^2}\right)^{\omega_1+\omega_2} \left(\frac{M^2}{M_0^2}\right)^{\omega}$$

$$\beta(0) \frac{1}{Z_k i\omega - \mu_k} \lambda_k \frac{1}{Z_k(i\omega_1 + \alpha'_k q^2) - \mu_k} \frac{1}{Z_k(i\omega_2 + \alpha'_k q^2) - \mu_k} \beta(t)^2.$$



Additional energy dependence:

$$\lambda_k / Z_k^3 \sim (\ln s)^{-1+\frac{1-3/2\eta}{2-\zeta}}$$

Comparison with previous work:

$\rightarrow$ 2 x Gribov, Migdal
Abarbanel, Bronzan
Migdal, Polyakov, Ter-Martirosyan

Question: how could a truly asymptotic theory of Pomerons look like?
Impose obvious condition: (renormalized) intercept must be at one

RG analysis of RFT with triple coupling near D=4:

$$T_{el}(s,t) \sim is(\ln s)^{\eta_O} F(t(\ln s)^{z_O}) \qquad \eta_O = -\frac{\eta}{z},\ z = 2-\zeta,\ z_O = \frac{2}{z}$$

$$= is(\ln s)^{-\eta/(2-\zeta)} F(t(\ln s)^{2/(2-\zeta)})$$

For comparison: we did not impose condition on intercept

$$T_{el}(s,t) \sim is(\ln s)^{-\eta/(2-\zeta)} s^{(\ln s)^{-1}\tilde{\mu}_{fp}} f(t(\ln s)^{2/(2-\zeta)})$$

$\uparrow$

Difference in intercept

Closer to real physics!

Conclusions

1) Defined the framework (ERG) for reggeon field theory

2) Have studied the IR (long distance) limit of a general class of Reggeon Field Theories:
there exists a fixed point which describes an acceptable effective theory
Desirable improvements: get away from the local approximation

3) First attempt to connect with reality:

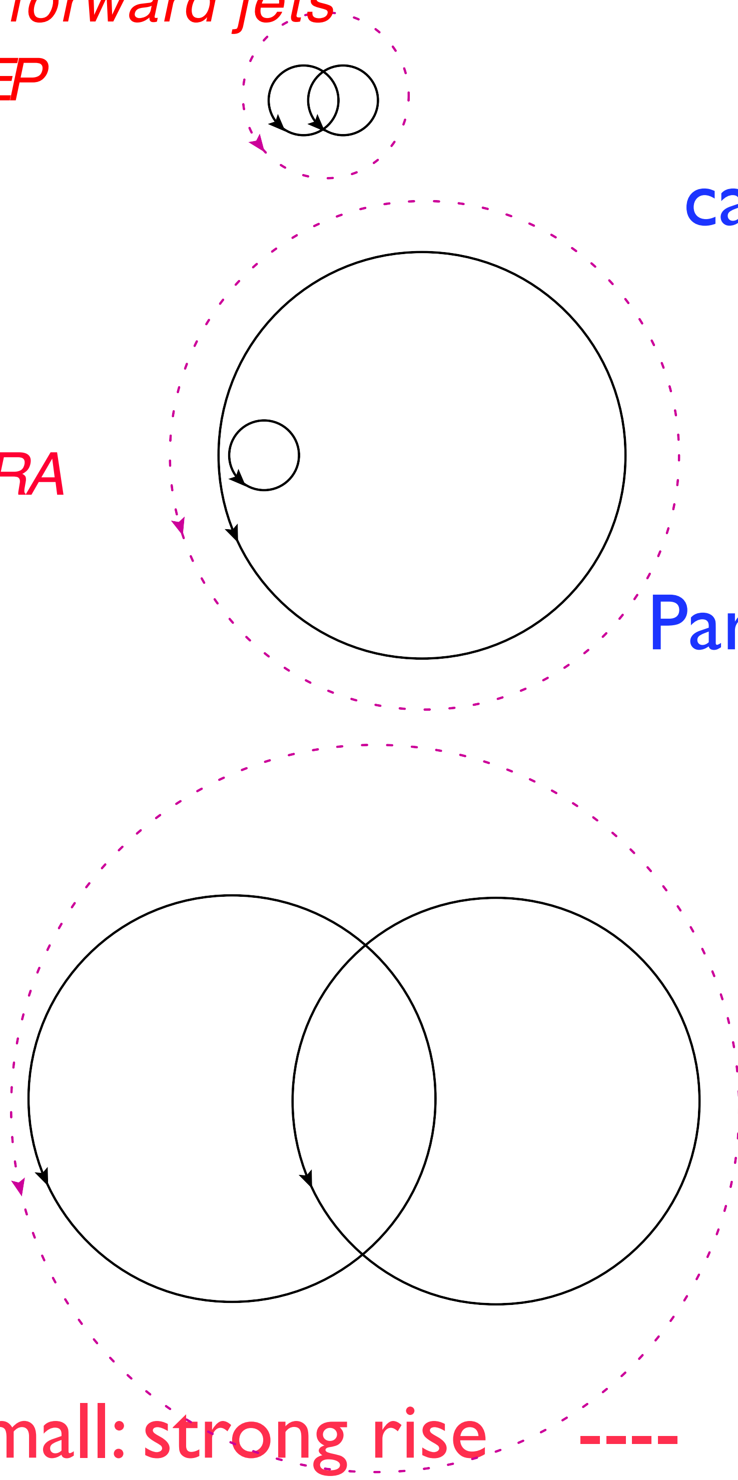
Several possibilities:
e.g. intercept at finite energies above one, approaches zero at infinite energies,
qualitative agreement with real physics. Prediction for Odderon slope
Takes care of finite transverse size (confinement)
Phenomenology needed.

4) Next step: which possibility is realized in QCD?

go to the UV region (BFKL, perturbative QCD reggeon field theory),
connect with IR region.

Backup slides

Energy dependence of total cross sections varies with transverse size:



HERA forward jets

LEP

$\gamma^*\gamma^* \qquad \sigma_{tot} \approx \ s^{\,\omega_{BFKL}}$

calculable in pQCD

HERA

$\gamma^* p \qquad \sigma_{tot} \approx (W^2)^{\lambda}$

Partly calculable in pQD

LHC

$p\,p \qquad \sigma_{tot} \approx s^{\,0.08}$

nonperturbative

Small: strong rise ---- large: slow rise

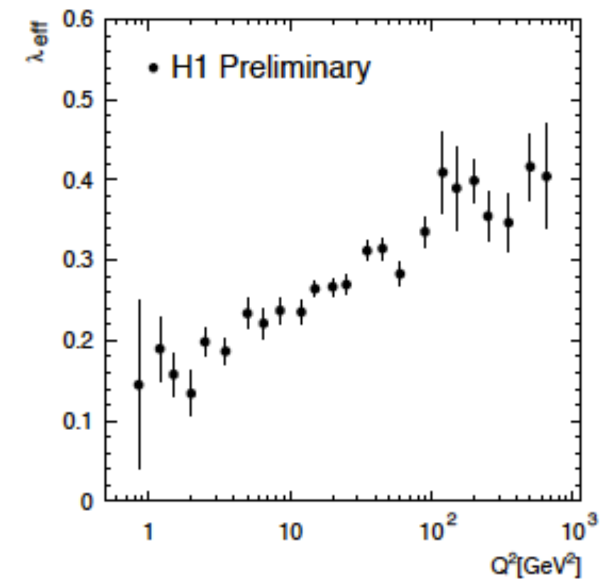
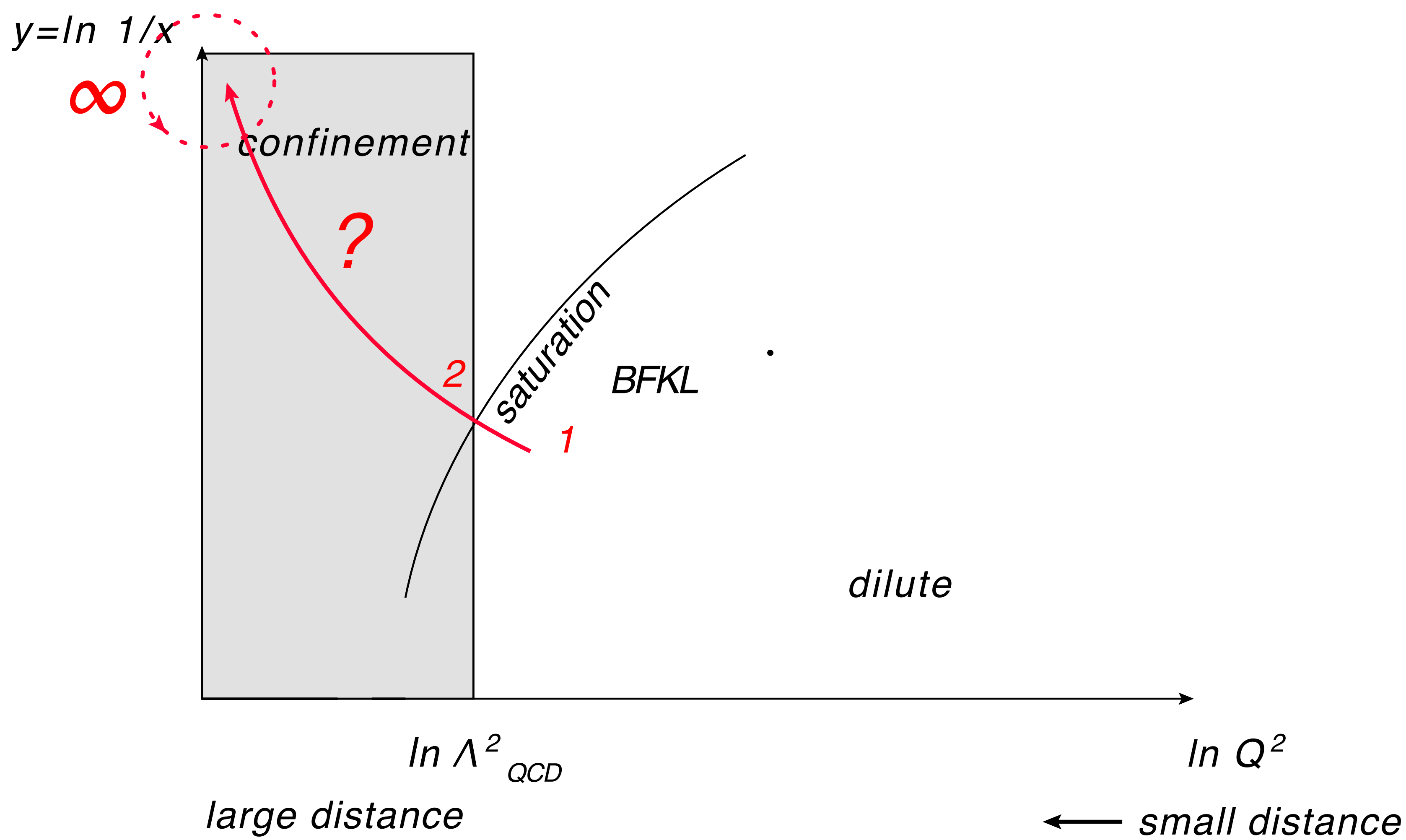


Figure 6: The slope $\lambda_{\rm eff}$ of F_2 as a function of Q^2 .

Introduction

Question: how to continue small-x physics from pQCD to the nonperturbative region?

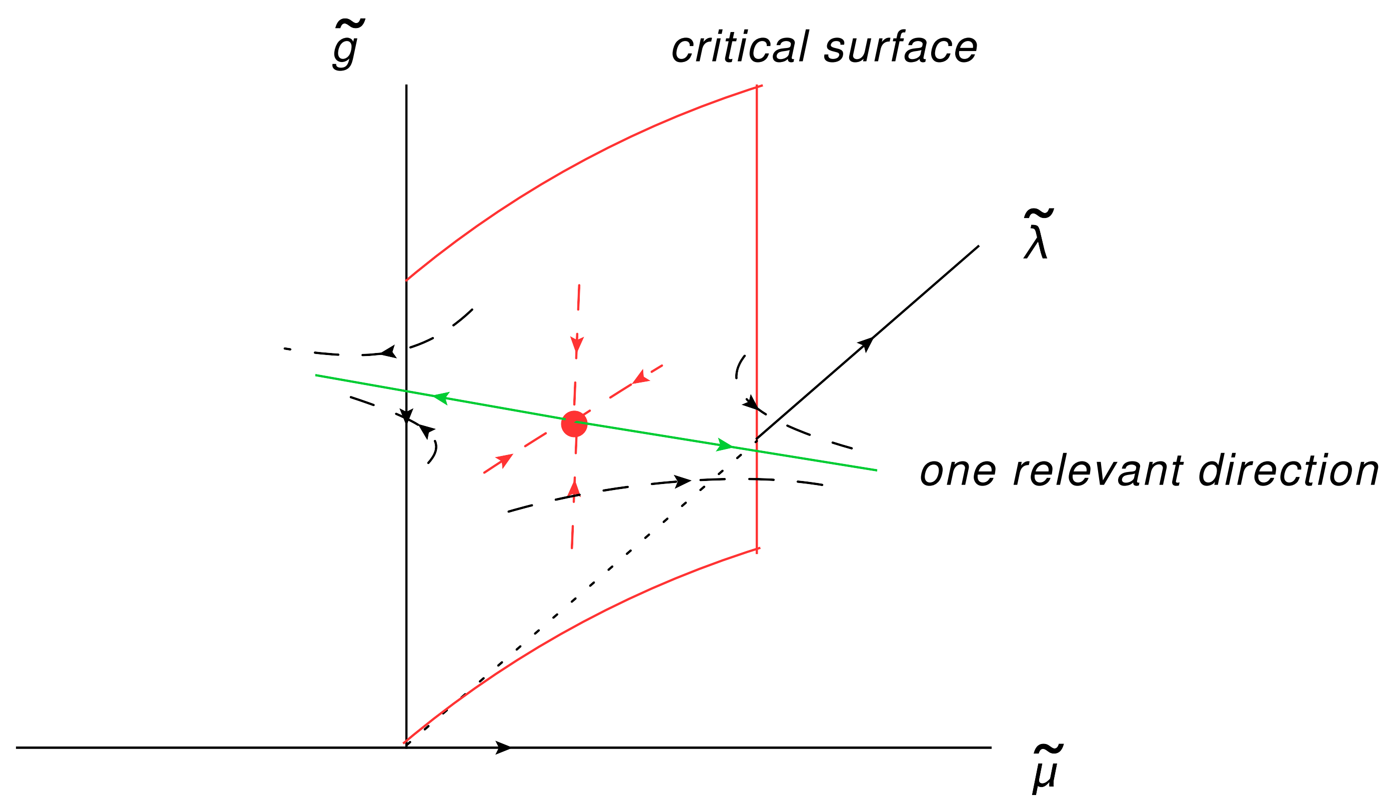


Regge description: 2+1-dimensional field theory

- (1) short distance region: BFKL, reggeon field theory of reggeized gluons
- (2) saturation: nonlinear BK-equation (fan diagrams)
- (∞) soft region (large distances): Regge poles (DL, Kaidalov, Tel Aviv, Durham)

Tentative interpretation: different phases:

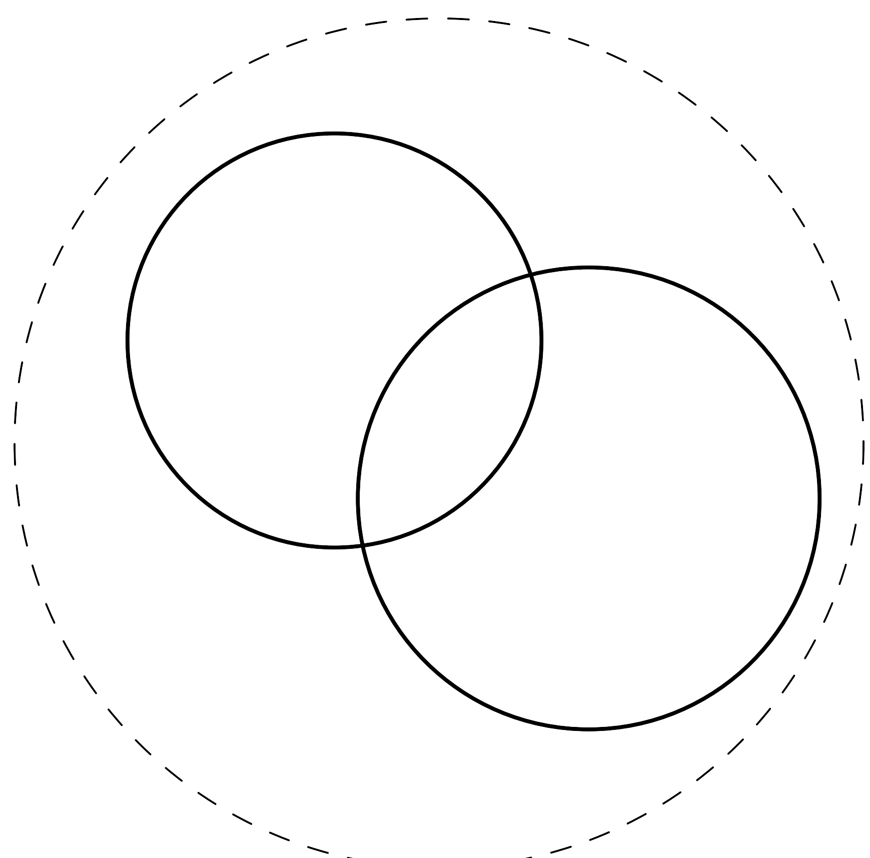
n-1 dim. critical subspace: massless
divides the n-dimensional space into
two (subcritical, supercritical) half spaces



Which phase: depends upon starting point at k=0 (UV)

Possible interpretation of IR cutoff, evolution time $\tau = \ln k/k_0$:

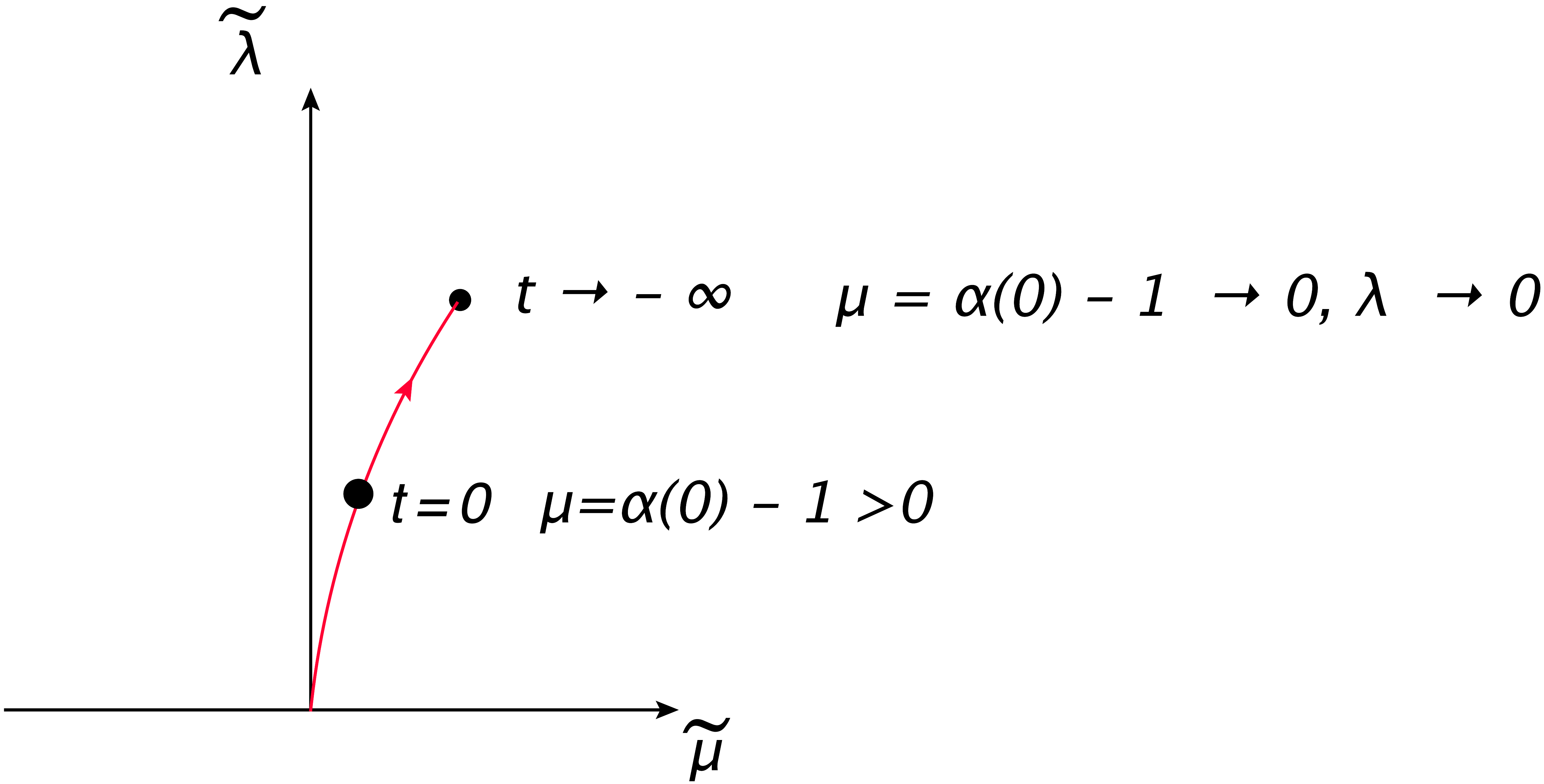
IR-cutoff: $k^2 \sim 1/\text{transverse distance}^2 \sim 1/\ln s$



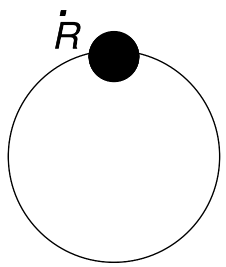
transverse plane

$\longleftarrow \quad R^2 = R_a^{\,2} + R_b^{\,2} + \alpha' \ln s \quad \longrightarrow$

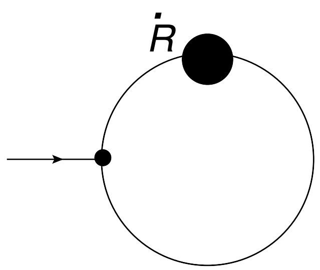
Possible physical scenario:



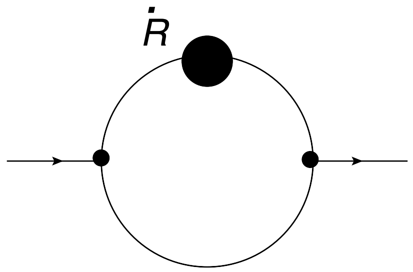
Vertex functions, Green's functions, physical observables:
take functional derivatives w.r.t. the fields:



$$\partial_t \Gamma_k = \frac{1}{2} G_{k;AB}\, \partial_t \mathcal{R}_{k;BA}$$



$$\partial_t \Gamma^{(1)}_{k;A_1} = -\frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}$$



$$\begin{aligned}\partial_t \Gamma^{(2)}_{k;A_1A_2} = & \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_1BC} G_{k;CD} \Gamma^{(3)}_{k;A_2DE} G_{k;EF}\, \partial_t \mathcal{R}_{k;FA} \\ & + \frac{1}{2} G_{k;AB} \Gamma^{(3)}_{k;A_2BC} G_{k;AB} \Gamma^{(3)}_{k;A_1BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA} \\ & - \frac{1}{2} G_{k;AB} \Gamma^{(4)}_{k;A_1A_2BC} G_{k;CD}\, \partial_t \mathcal{R}_{k;DA}\end{aligned}$$

coupled partial differential equations

First step:

Expand the potential in powers of fields, derive beta-functions for parameters of the potential (coupling constants):

$$\dot{\tilde{\mu}} = \tilde{\mu}(-2+\zeta+\eta) + 2N_D A_D(\eta_k,\zeta_k)\frac{\tilde{\lambda}^2}{(1-\tilde{\mu})^2},$$

$$\dot{\tilde{\lambda}} = \tilde{\lambda}\left((-2+\zeta+\frac{D}{2}+\frac{3\eta}{2}) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{4\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{(\tilde{g}+3\tilde{g}')}{(1-\tilde{\mu})^2}\right)\right),$$

$$\dot{\tilde{g}} = \tilde{g}(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{27\tilde{\lambda}^4}{(1-\tilde{\mu})^4} + \frac{(16\tilde{g}+24\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{\left(\tilde{g}^2+9\tilde{g}'^2\right)}{(1-\tilde{\mu})^2}\right)$$

$$\dot{\tilde{g}}' = \tilde{g}'(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{12\tilde{\lambda}^4}{(1-\tilde{\mu})^4} + \frac{(4\tilde{g}+18\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{3\tilde{g}\tilde{g}'}{(1-\tilde{\mu})^2}\right)$$

Fixed points: zeroes of the beta-functions

First results: fixed points

Local reggeon field theory:

$\mu = \alpha(0) - 1$

$$\mathcal{L} = (\tfrac{1}{2}\psi^\dagger \overleftrightarrow{\partial_y} \psi - \alpha' \psi^\dagger \nabla^2 \psi) + V(\psi, \psi^\dagger)$$

$$V(\psi, \psi^\dagger) = -\mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi)\psi + g(\psi^\dagger \psi)^2 + g' \psi^\dagger ({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

some universal symmetry properties

Some history:

Gribov, Migdal; Abarbanel, Bronzan; Migdal, Polyakov, Ter-Martirosyan

In early seventies : first studies of RFT with triple couplings, expansion near D=4 (ϵ- expansion). IR-fixed point.

In 1980: J. Cardy and R. Sugar noticed that the RFT is in the same universality class of a Markov process known as Directed Percolation (DP). Critical exponents can then be accessed also with numerical montecarlo computations.

This attempt:
search in the full space of theories, no restriction to D=4

Effective action with local potential:

$$\Gamma_k = \int \mathrm{d}y\, \mathrm{d}^D x \left[Z_k (\frac{1}{2} \psi^{dagger} \overleftrightarrow{\partial_y} \psi - \alpha'_k \psi^{dagger} \nabla^2 \psi) + V_k(\psi, \psi^\dagger) \right]$$

Propagator of flow equations:

$$\Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi\psi^{dagger}} \\ iZ_k\omega + Z_k\alpha'_k q^2 + R_k + V_{k\psi^{dagger}\psi} & V_{k\psi^{dagger}\psi^{dagger}} \end{pmatrix}$$

Flow equation for potential:

$$\dot{V}_k(\psi, \psi^\dagger) = \frac{1}{2} \mathrm{tr} \left\{ \int \frac{\mathrm{d}\omega\, \mathrm{d}^D q}{(2\pi)^{D+1}} \left[\left(\Gamma_k^{(2)} + \mathbb{R} \right)^{-1} \dot{\mathcal{R}}_k \right] \right\}$$

Coarse graining in momentum space: $R_k(q) = Z_k \alpha'_k (k^2 - q^2) \Theta(k^2 - q^2)$