Single diffraction in the dipole picture: an outlook

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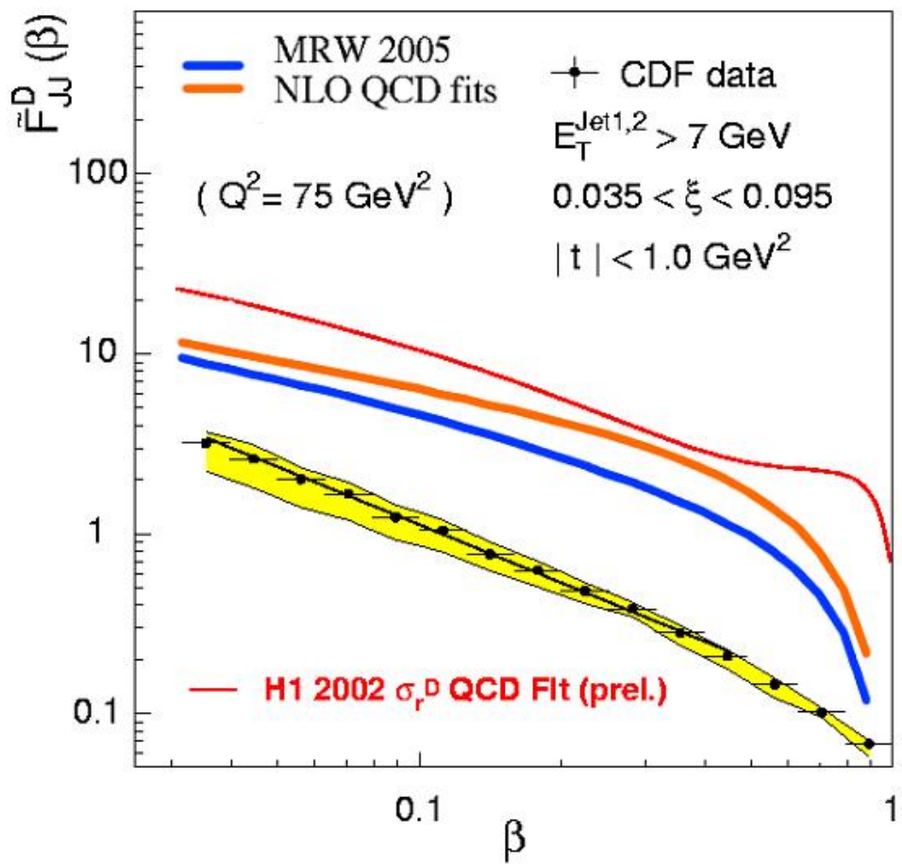
Diffractive factorisation breaking in pp collisions

Incoming hadrons are not elementary — experience soft interactions dissolving them leaving much fewer rapidity gap events than in ep scattering



Sources of diffractive factorisation breaking:

✓ soft survival (=absorptive) effects (Khoze-Martin-Ryskin and Gotsman-Levin-Maor) affecting e.g. the Pomeron flux (Goulianos)

✓ interplay of hard and soft fluctuations in incoming hadron wave function

✓ saturated shape of the universal dipole cross section for large dipole sizes

Two distinct approaches treating the above effects:

✓ **Regge-corrected (KMR) approach** — the first source of the factorisation breaking is accounted at the cross section level by "dressing" the factorisation formula by soft Pomeron exchanges

✓ **Color dipole approach** — the universal way of inclusive/diffractive scattering treatment, accounts for all the sources of Regge factorisation breaking at the amplitude level (Kopeliovich, RP et al)

Good-Walker picture of diffractive scattering

R. J. Glauber, Phys. Rev. 100, 242 (1955).
E. Feinberg and I. Ya. Pomeranchuk, Nuovo. Cimento. Suppl. 3 (1956) 652.
M. L. Good and W. D. Walker, Phys. Rev. 120 (1960) 1857.

Projectile has a substructure!

Hadron can be excited:
not an eigenstate of interaction!

$$|h\rangle = \sum_{\alpha=1} C_\alpha^h |\alpha\rangle \qquad \hat{f}_{el}|\alpha\rangle = f_\alpha |\alpha\rangle$$

Completeness and orthogonality

$$\langle h'|h\rangle = \sum_{\alpha=1} (C_\alpha^{h'})^* C_\alpha^h = \delta_{hh'}$$

$$\langle \beta|\alpha\rangle = \sum_{h'} (C_\beta^{h'})^* C_\alpha^{h'} = \delta_{\alpha\beta}$$

fluctuations — semi-hard/ semi-soft — soft

	$\lvert C_\alpha\rvert^2$	σ_α	$\sigma_{tot} = \sum_{\alpha=soft}^{hard} \lvert C_\alpha\rvert^2 \sigma_\alpha$	$\sigma_{sd} = \sum_{\alpha=soft}^{hard} \lvert C_\alpha\rvert^2 \sigma_\alpha^2$
Hard	~ 1	$\sim \frac{1}{Q^2}$	$\sim \frac{1}{Q^2}$	$\sim \frac{1}{Q^4}$
Soft	$\sim \frac{m_q^2}{Q^2}$	$\sim \frac{1}{m_q^2}$	$\sim \frac{1}{Q^2}$	$\sim \frac{1}{m_q^2 Q^2}$

Elastic and single diffractive amplitudes

$$f_{el}^{h\to h} = \sum_{\alpha=1} |C_\alpha^h|^2 f_\alpha$$

$$f_{sd}^{h\to h'} = \sum_{\alpha=1} (C_\alpha^{h'})^* C_\alpha^h f_\alpha$$

Single diffractive cross section

$$\sum_{h'\neq h} \frac{d\sigma_{sd}^{h\to h'}}{dt}\bigg|_{t=0} = \frac{1}{4\pi}\left[\sum_{h'} |f_{sd}^{hh'}|^2 - |f_{el}^{hh}|^2\right]$$

$$= \frac{1}{4\pi}\left[\sum_\alpha |C_\alpha^h|^2 |f_\alpha|^2 - \left(\sum_\alpha |C_\alpha^h| f_\alpha\right)^2\right] = \boxed{\frac{\langle f_\alpha^2\rangle - \langle f_\alpha\rangle^2}{4\pi}}$$

Dispersion of the eigenvalues distribution

Important basis for the dipole picture!

Phenomenological dipole approach

Eigenvalue of the total cross section is
the universal dipole cross section

see e.g. B. Kopeliovich et al, since 1981

Eigenstates of interaction in QCD:
color dipoles

Dipole:
- cannot be excited
- experience only elastic scattering
- have no definite mass, but only separation
- universal – elastic amplitude can be extracted in one process and used in another

$$\sum_{h'} \frac{d\sigma_{sd}^{h\to h'}}{dt}\bigg|_{t=0} = \sum_{\alpha=1} |C_\alpha^h|^2 \frac{\sigma_\alpha^2}{16\pi} =$$

SD cross section

$$\int d^2 r_T |\Psi_h(r_T)|^2 \frac{\sigma^2(r_T)}{16\pi} = \frac{\langle \sigma^2(r_T)\rangle}{16\pi}$$

wave function of
a given Fock state

total DIS cross section

$$\sigma_{tot}^{\gamma^* p}(Q^2, x_{Bj}) = \int d^2 r_T \int_0^1 dx \left|\Psi_{\gamma^*}(r_T, Q^2)\right|^2 \sigma_{\bar{q}q}(r_T, x_{Bj})$$

partonic interpretation of a scattering does depend on frame of reference!

Theoretical calculation of
the dipole CS is a challenge

BUT! *Can be extracted from data and used in ANY process!*

Example: Naive GBW parameterization
of HERA data

$$\sigma_{\bar{q}q}(r_T, x) = \sigma_0 \left[1 - e^{-\frac{1}{4} r_T^2 Q_s^2(x)}\right]$$

saturates at
large separations

$$r_T^2 \gg 1/Q_s^2$$

color transparency $\sigma_{\bar{q}q}(r_T) \propto r_T^2 \qquad r_T \to 0$

QCD factorisation $\sigma_{q\bar{q}}(r, x) \propto r^2 x g(x)$

A point-like colorless object
does not interact with
external color field!

ANY inclusive/diffractive scattering is due to an interference of dipole scatterings!

Gluon distribution amplitudes and dipole CS

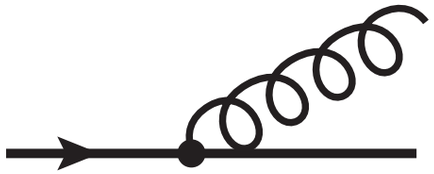
In most cases, a scattering cross section in the target rest frame can be represented in terms of three basic ingredients:

- **Gluon to quark-antiquark splitting amplitude:**

$$\Phi^T_{Q\bar{Q}} = \sqrt{\alpha_s} \int \frac{d^2\kappa}{(2\pi)^2} (\xi^\mu_Q)^\dagger \frac{m_Q(\vec{e}_{ini}\cdot\vec{\sigma}) + (1-2\beta)(\vec{\sigma}\cdot\vec{n})(\vec{e}_{ini}\cdot\vec{\kappa}) + i(\vec{e}_{ini}\times\vec{n})\cdot\vec{\kappa}}{\kappa^2+\epsilon^2} \tilde{\xi}^{\bar{\mu}}_{\bar{Q}}\, e^{-i\vec{\kappa}\vec{r}}$$

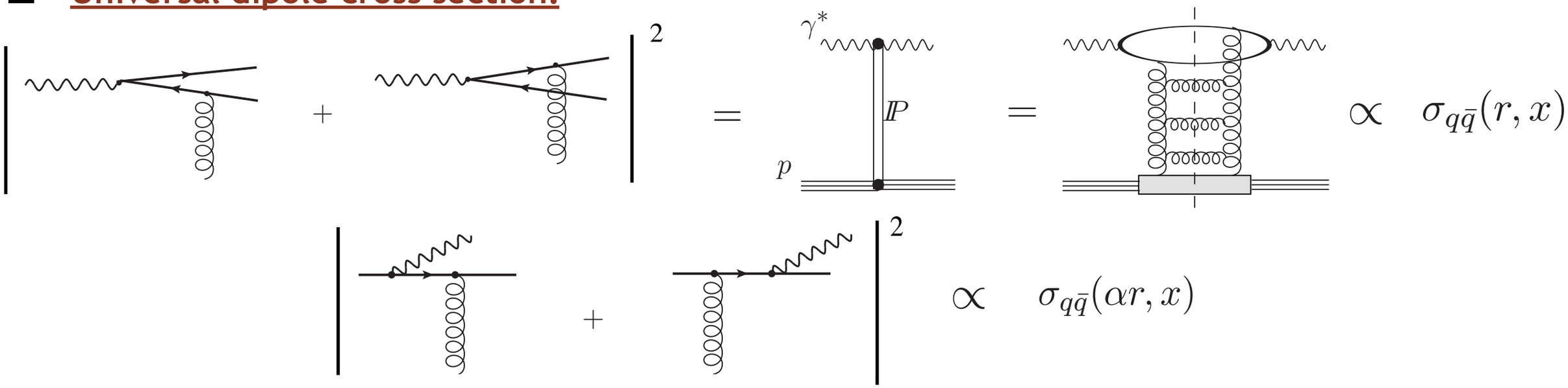
$$= \frac{\sqrt{\alpha_s}}{2\pi} (\xi^\mu_Q)^\dagger \Big\{ m_Q(\vec{e}_{ini}\cdot\vec{\sigma}) + i(1-2\beta)(\vec{\sigma}\cdot\vec{n})(\vec{e}_{ini}\cdot\vec{\nabla}_r) - (\vec{e}_{ini}\times\vec{n})\cdot\vec{\nabla}_r \Big\} \tilde{\xi}^{\bar{\mu}}_{\bar{Q}}\, K_0(\epsilon r)\,,$$

- **Gluon Bremsstrahlung off a quark:**



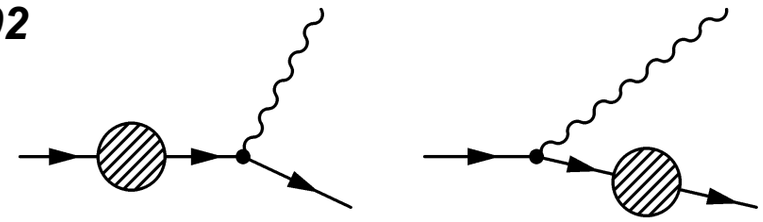
$$\Phi^T_{qG^*}(\alpha,\vec{\pi}) = \sqrt{\alpha_s}\,(\eta^s_Q)^\dagger \frac{(2-\alpha)(\vec{e}_*\cdot\vec{\pi}) + i m_q \alpha^2 (\vec{n}\times\vec{e}_*)\cdot\vec{\sigma} - i\alpha(\vec{\pi}\times\vec{e}_*)\cdot\vec{\sigma}}{\vec{\pi}^2 + \alpha^2 m_q^2}\, \eta^{s'}_Q$$

- **Universal dipole cross section:**



Dipole approach vs NLO QCD: Drell-Yan

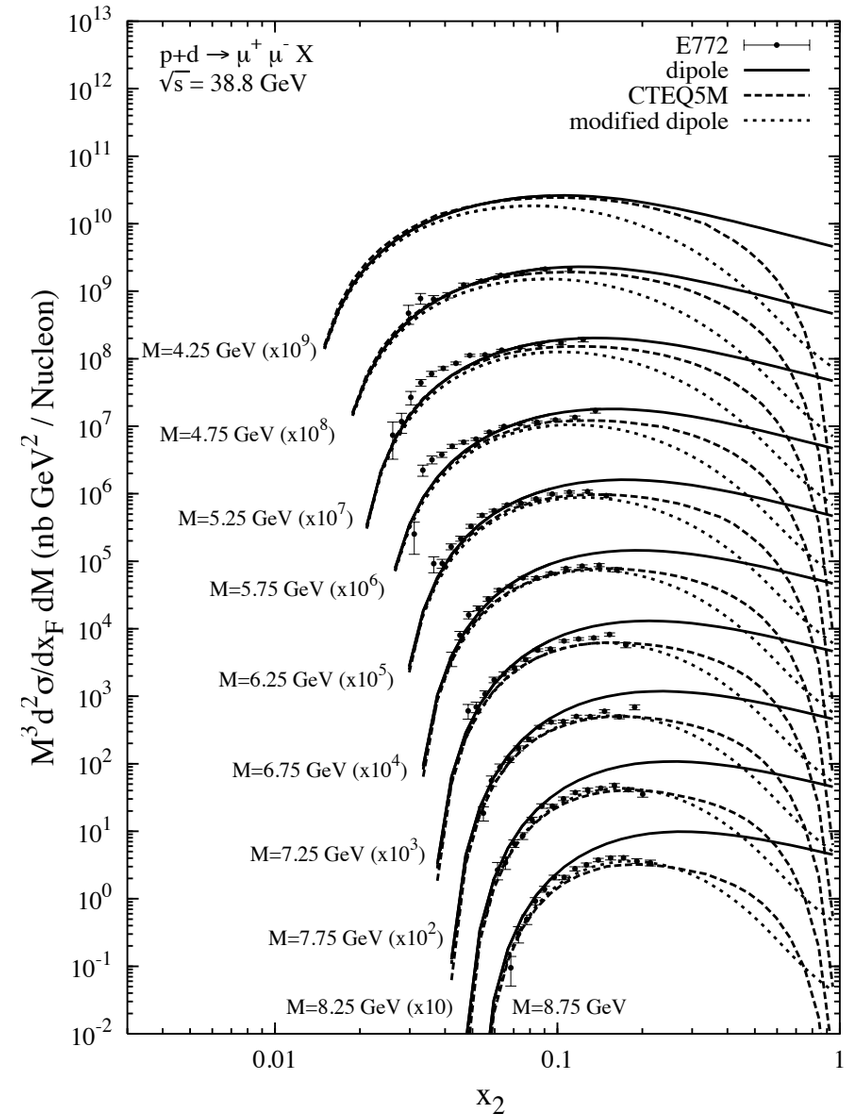
J. Raufeisen et al, PRD66 2002

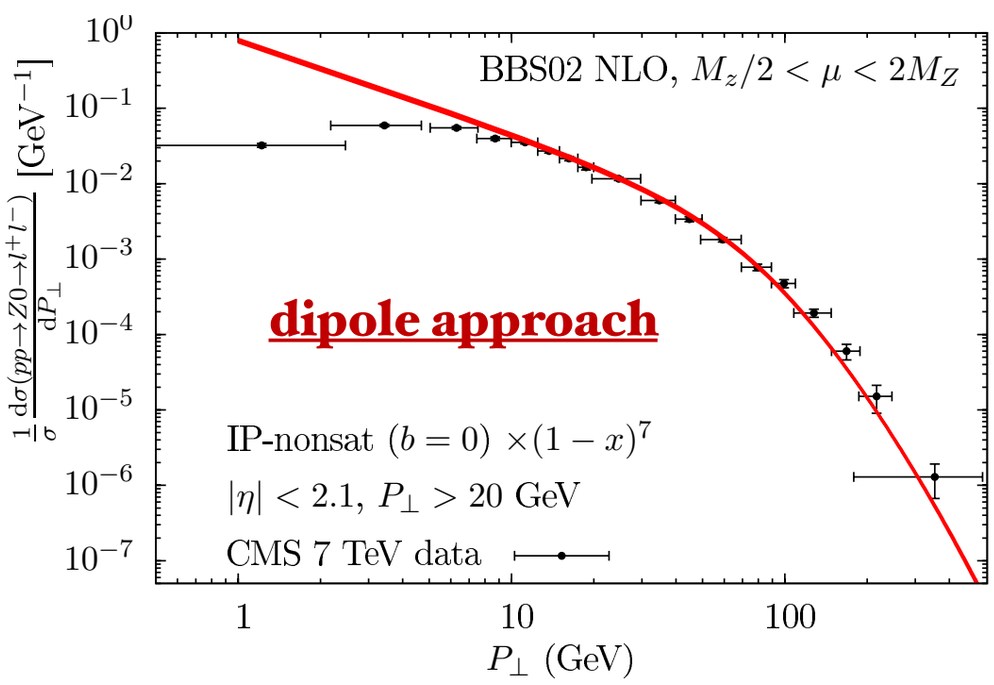


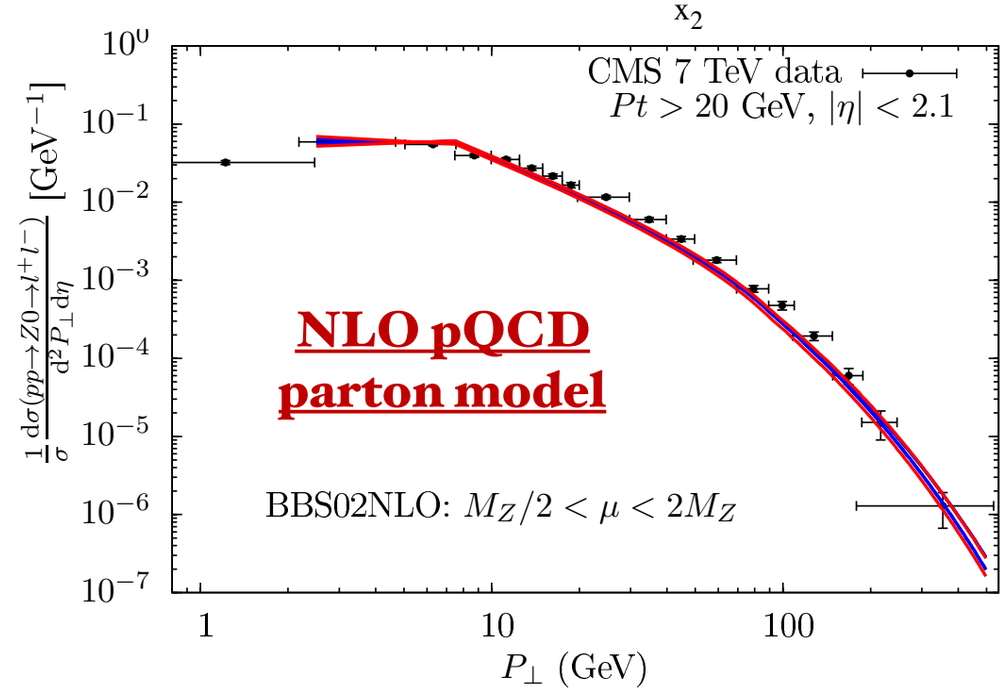
$$\frac{d\sigma(qN \to \gamma^* X)}{d\ln\alpha} = \int d^2\rho \; |\Psi_{\gamma^* q}(\alpha,\rho)|^2 \; \sigma^N_{q\bar{q}}(\alpha\rho, x)$$

$$\frac{d^2\sigma(pN \to l^+ l^- X)}{dM^2 dx_F} = \frac{\alpha_{em}}{3\pi M^2}\frac{x_1}{x_1+x_2}\int_{x_1}^{1}\frac{d\alpha}{\alpha^2}\sum_{f=1}^{N_f} Z_f^2\left[q_f\left(\frac{x_1}{\alpha},\widetilde{Q}\right)+\bar{q}_f\left(\frac{x_1}{\alpha},\widetilde{Q}\right)\right]$$
$$\times \int d^2\rho \; |\Psi_{\gamma^* q}(\alpha,\rho)|^2 \; \sigma^N_{q\bar{q}}(\alpha\rho, x) \; .$$

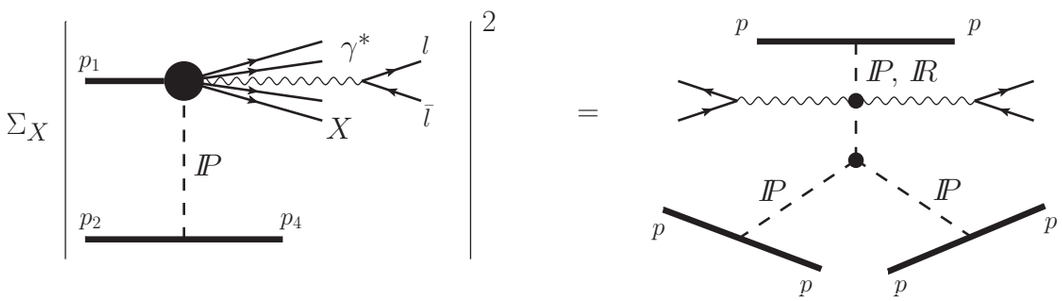
Dipole approach predictions effectively account for higher order QCD corrections!



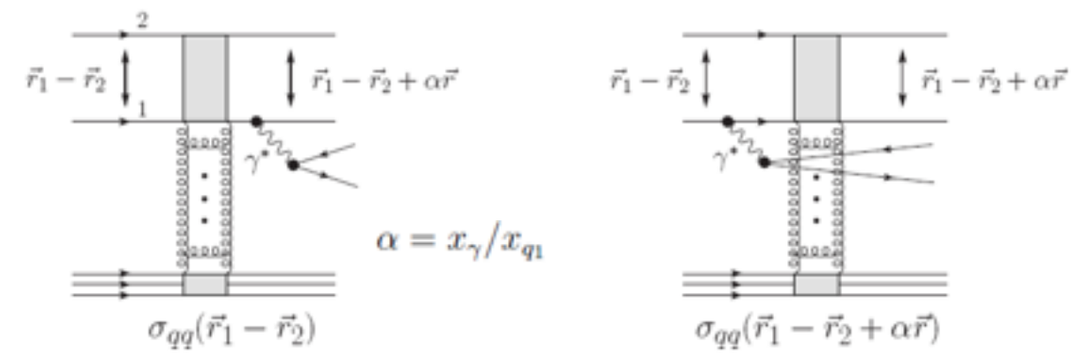




Diffractive Abelian (e.g. Drell-Yan) radiation via dipoles



Diffractive Drell Yan (semi-hard)



interplay between hard and soft fluctuations is pronounced!

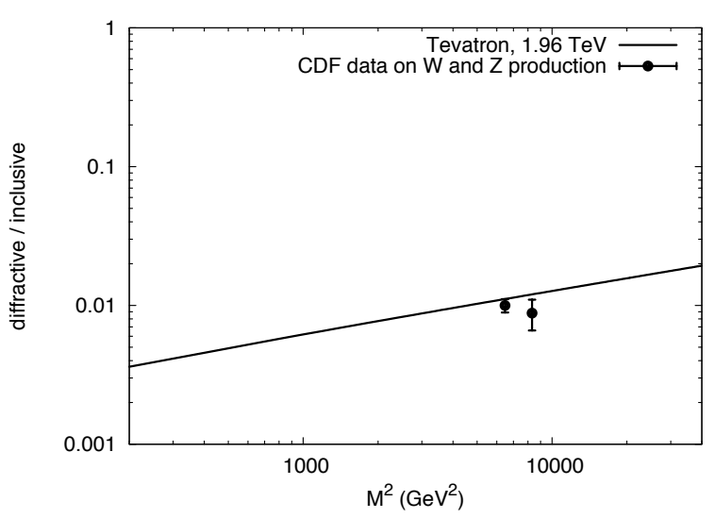
superposition has a Good-Walker structure

$$\propto \sigma(\vec{R}) - \sigma(\vec{R} - \alpha\vec{r}) = \frac{2\alpha\,\sigma_0}{R_0^2(x_2)}\, e^{-R^2/R_0^2(x_2)} \left(\vec{r}\cdot\vec{R}\right) + O(r^2)$$

Diffractive DIS $\propto r^4 \propto 1/M^4$ vs diffractive DY $\propto r^2 \propto 1/M^2$

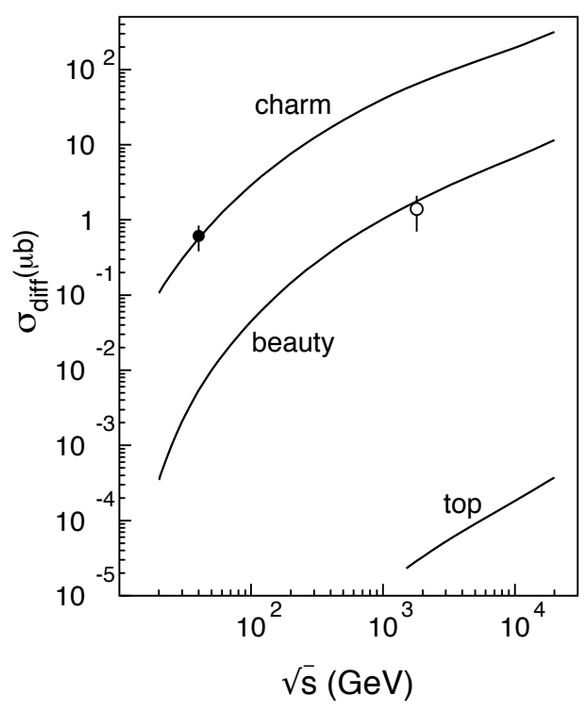
$$r \sim 1/(1-\alpha)M$$

SD DY/gauge bosons



RP et al 2011,12

SD heavy quarks



Kopeliovich et al 2006

- ★ *diffractive factorisation is automatically broken*
- ★ *any SD reaction is a superposition of dipole amplitudes*
- ★ *gap survival is automatically included at the amplitude level on the same footing as dip. CS*
- ★ *works for a variety of data in terms of universal dip. CS*

Sophisticated dipole cascades are being put into MC: Lund Dipole Chain model (DIPSY)
Ref. G. Gustafson, and L. Lönnblad

Elastic amplitude and gap survival

Dipole elastic amplitude has **eikonal form**:

$$\mathrm{Im}\, f_{el}(\vec{b}, \vec{r}_1 - \vec{r}_2) = 1 - \exp\left[i\chi(\vec{r}_1) - i\chi(\vec{r}_2)\right]$$

$$\sigma_{\bar{q}q}(r_p, x) = \int d^2b\, 2\,\mathrm{Im} f_{el}(\vec{b}, \vec{r}_p) = \sigma_0(1 - e^{-r_p^2/R_0^2(x)})$$

$$\chi(b) = -\int_{-\infty}^{\infty} dz\, V(\vec{b}, z)$$

potential is nearly imaginary at high energies!

Diffractive amplitude is proportional to

$$\mathrm{Im}\, f_{el}(\vec{b}, \vec{r}_1 - \vec{r}_2 + \alpha\vec{r}) - \mathrm{Im}\, f_{el}(\vec{b}, \vec{r}_1 - \vec{r}_2) = \underbrace{\exp\left[i\chi(\vec{r}_1) - i\chi(\vec{r}_2)\right]}\, \exp\left[i\alpha\, \vec{r}\cdot\vec{\nabla}\chi(\vec{r}_1)\right]$$

$$|\vec{r}_i - \vec{r}_j| \sim b \sim R_p,\ i \neq j$$

another source of QCD factorisation breaking

Exactly the soft survival probability amplitude

controlled by soft spectator partons

vanishes in the black disc limit!

Absorption effect is automatically included into elastic amplitude at the amplitude level

SD-to-inclusive ratio for diffractive gauge bosons production

RP et al 2011,12

$$\mathrm{Im}\, f_{el}(\vec{b}, \vec{R}_{ij} + \alpha\vec{r}) - \mathrm{Im}\, f_{el}(\vec{b}, \vec{R}_{ij}) \simeq \frac{\partial \mathrm{Im}\, f_{el}(\vec{b}, \vec{R}_{ij})}{\partial \vec{R}_{ij}}\, \alpha\vec{r}$$

$$|\Psi_i(\vec{r}_1, \vec{r}_2, \vec{r}_3; x_q, \{x_q^{2,3,\dots}\}, \{x_g^{2,3,\dots})|^2 = \frac{3a^2}{\pi^2} e^{-a(r_1^2+r_2^2+r_3^2)} \rho(x_q, \{x_q^{2,3,\dots}\}, \{x_g^{2,3,\dots}\})$$
$$\times\ \delta(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)\delta(1 - x_q - \sum_j x_{q/g}^j),$$

$$a = \langle r_{ch}^2 \rangle^{-1}$$

$$\int d^2r_1 d^2r_2 d^2r_3\, e^{-a(r_1^2+r_2^2+r_3^2)} \delta(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) = \frac{1}{9} \int d^2R_{12} d^2R_{13} e^{-\frac{2a}{3}(R_{12}^2 + R_{13}^2 + \vec{R}_{12}\vec{R}_{13})}$$

$$\frac{d\sigma^{sd}_{\lambda_G}/d^2q_\perp\, dx_1\, dM^2}{d\sigma^{incl}_{\lambda_G}/d^2q_\perp dx_1\, dM^2} = \frac{a^2}{6\pi} \frac{\bar{R}_0^2(M_\perp^2/x_1 s)}{B_{sd}(s)\,\bar{\sigma}_0} \frac{\sigma_0^2(s)}{R_0^4(s)} \frac{1}{A_2} \Big[\frac{2}{(A_2 - 4A_1)^2} + \frac{A_2^2}{(A_2^2 - 4A_3^2)^2} \Big]$$

$$A_1 = \frac{2a}{3} + \frac{2}{R_0^2(s)}, \qquad A_2 = \frac{2a}{3}, \qquad A_3 = \frac{2a}{3} + \frac{1}{R_0^2(s)}$$

Hard GBW (small dipoles)

$$\bar{\sigma}_0 = 23.03\,\mathrm{mb}, \quad \bar{R}_0(x_2) = 0.4\,\mathrm{fm} \times (x_2/x_0)^{0.144}, \quad x_0 = 3.04 \times 10^{-4}$$

diffractive (Regge) slope $\quad B_{sd}(s) \simeq \langle r_{ch}^2 \rangle/3 + 2\alpha'_{I\!P} \ln(s/s_0)$

At the leading twist, the dipole approach predicts the same angular correlation in DDY as in inclusive DY!

Soft KST (large dipoles)

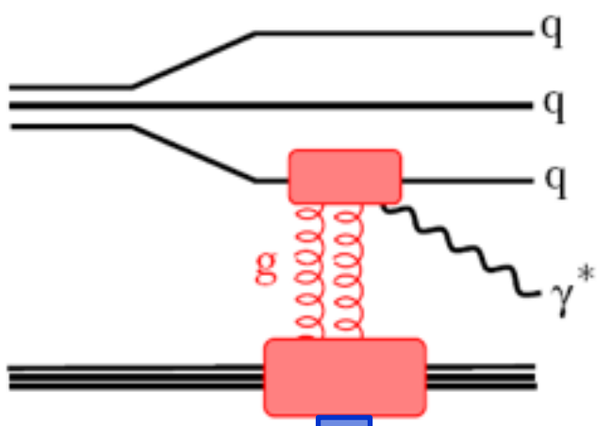
$$R_0(s) = 0.88\,\mathrm{fm}\,(s_0/s)^{0.14}$$

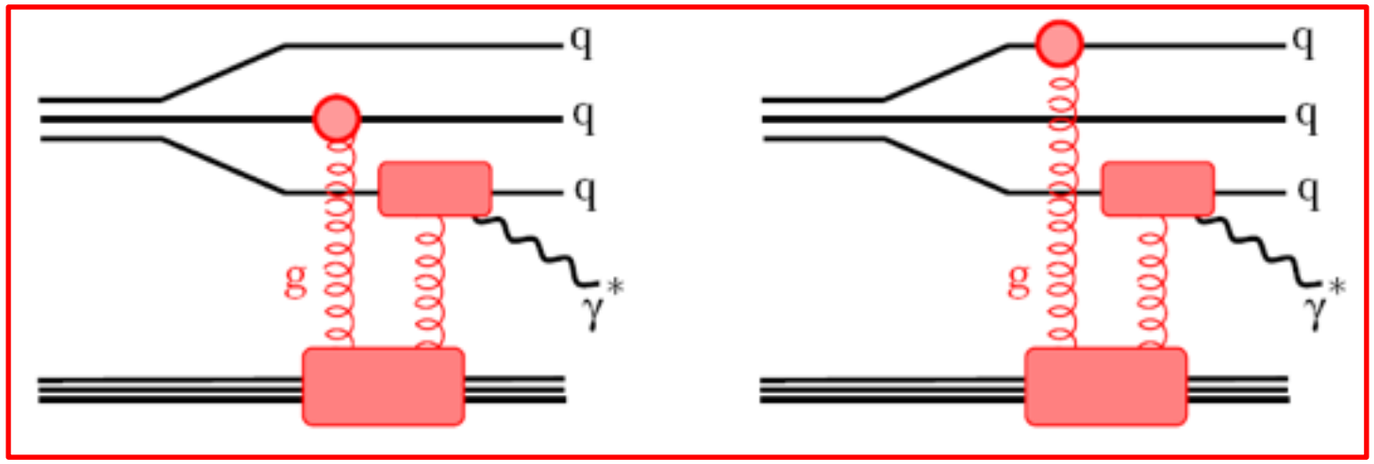
$$\sigma_0(s) = \sigma_{tot}^{\pi p}(s)\Big(1 + \frac{3R_0^2(s)}{8\langle r_{ch}^2 \rangle_\pi}\Big)$$

$$\sigma_{tot}^{\pi p}(s) = 23.6(s/s_0)^{0.08}\ \mathrm{mb}$$

$$\langle r_{ch}^2 \rangle_\pi = 0.44\ \mathrm{fm}^2$$

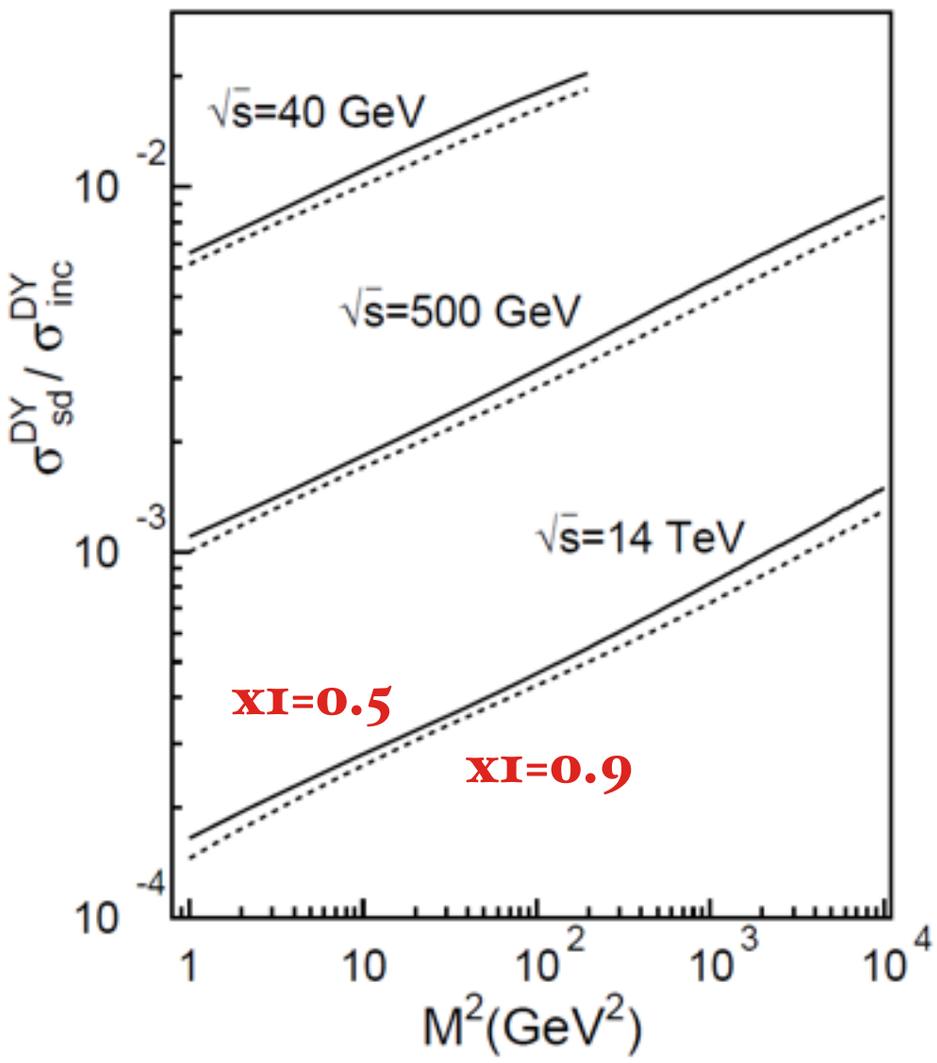
Diffractive factorisation breaking in DDY





vanishes in the forward limit,
higher twist effect!

leading twist effect!



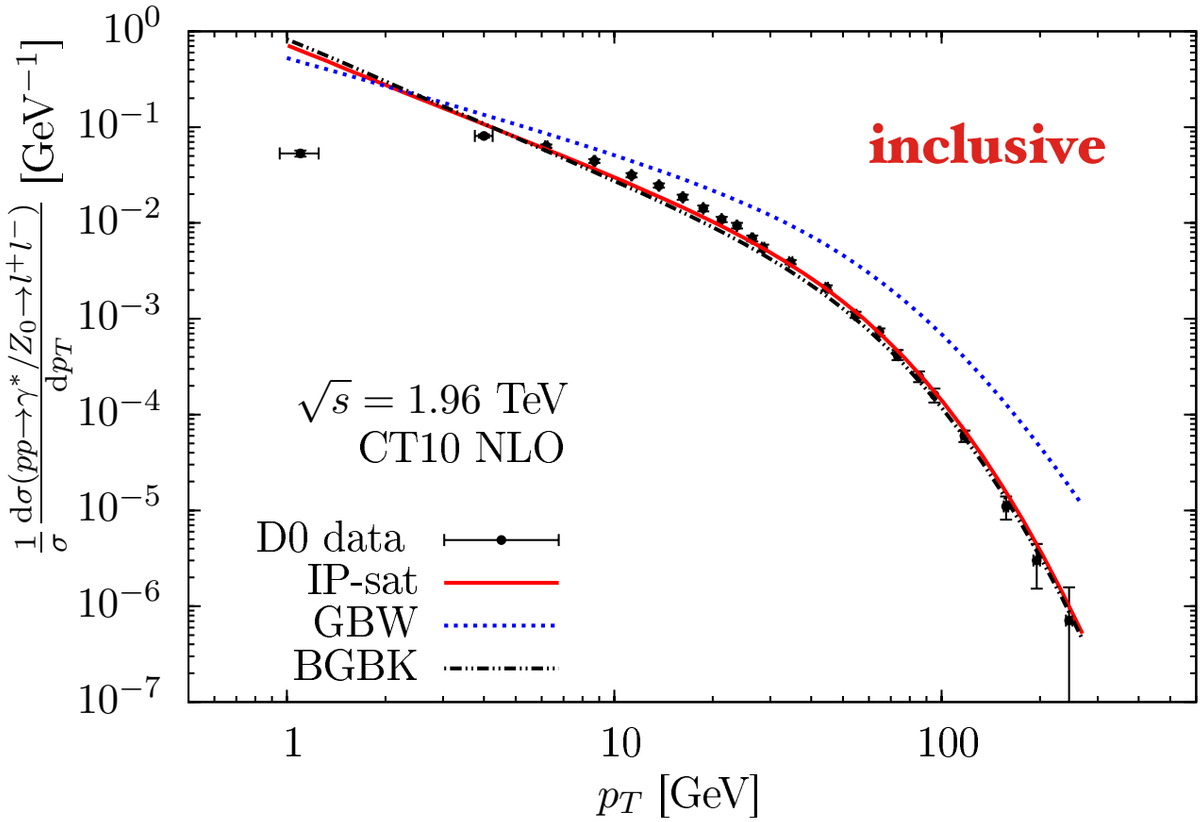
saturated shape of the dipole CS
+
unitarity corrections

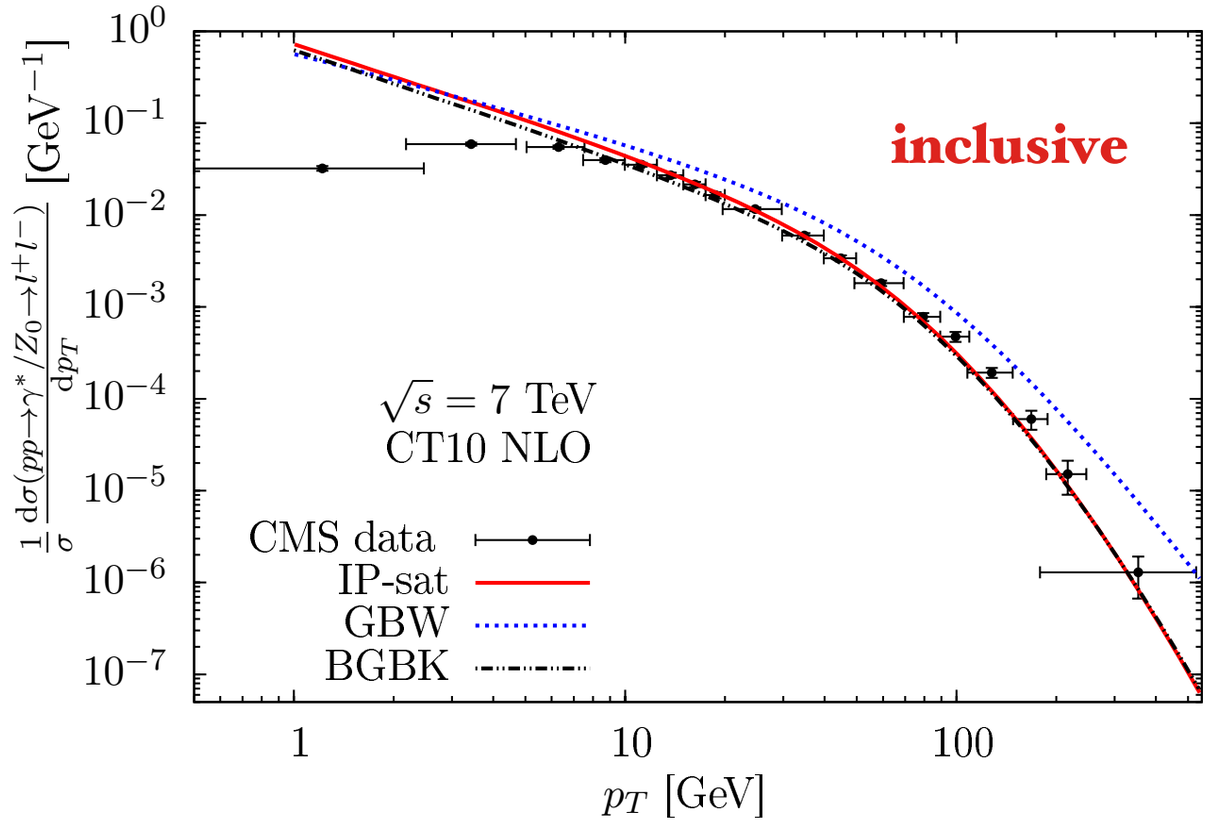
Fraction of diffractive events

- **steeply falls with energy**
- **grows with the hard scale**

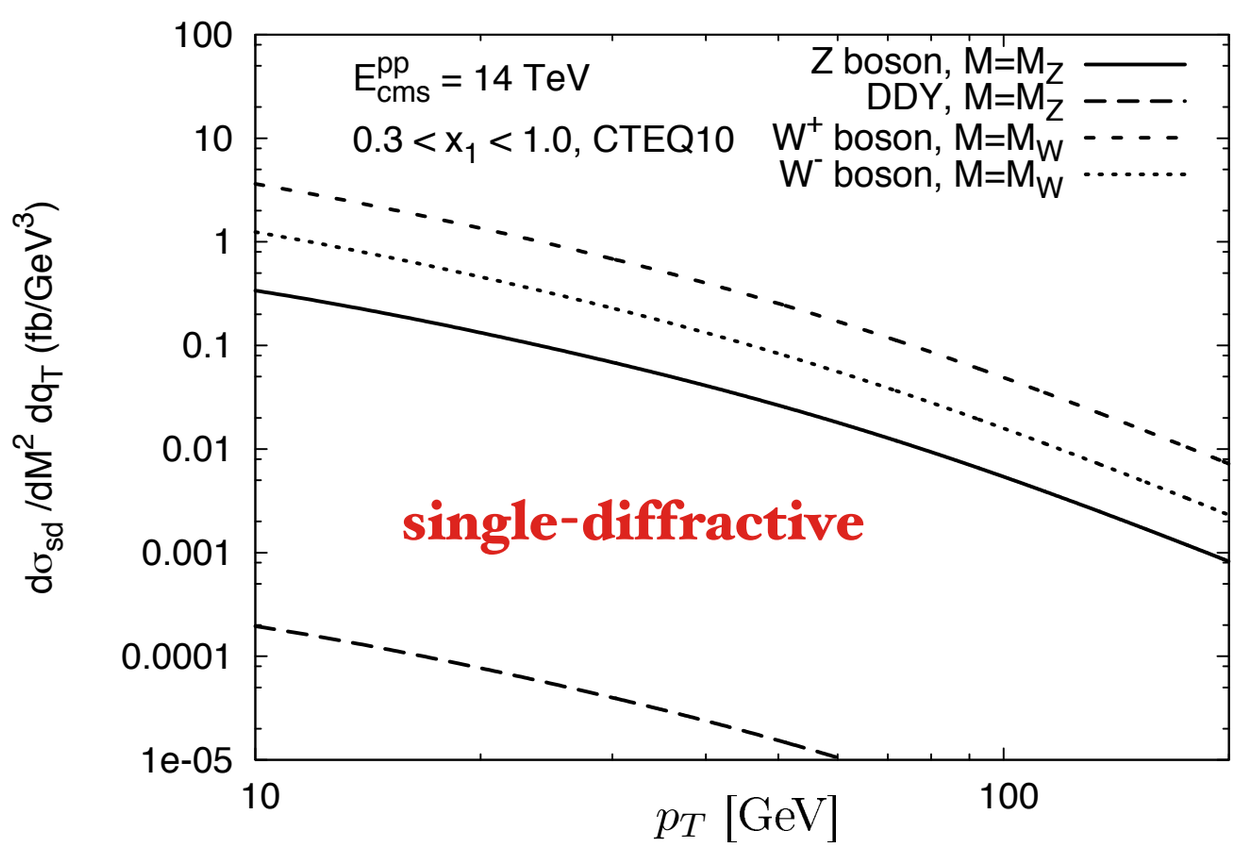
Opposite to factorization-based results (like Ingelman-Schlein)

PT correlations in inclusive and diffractive Drell-Yan



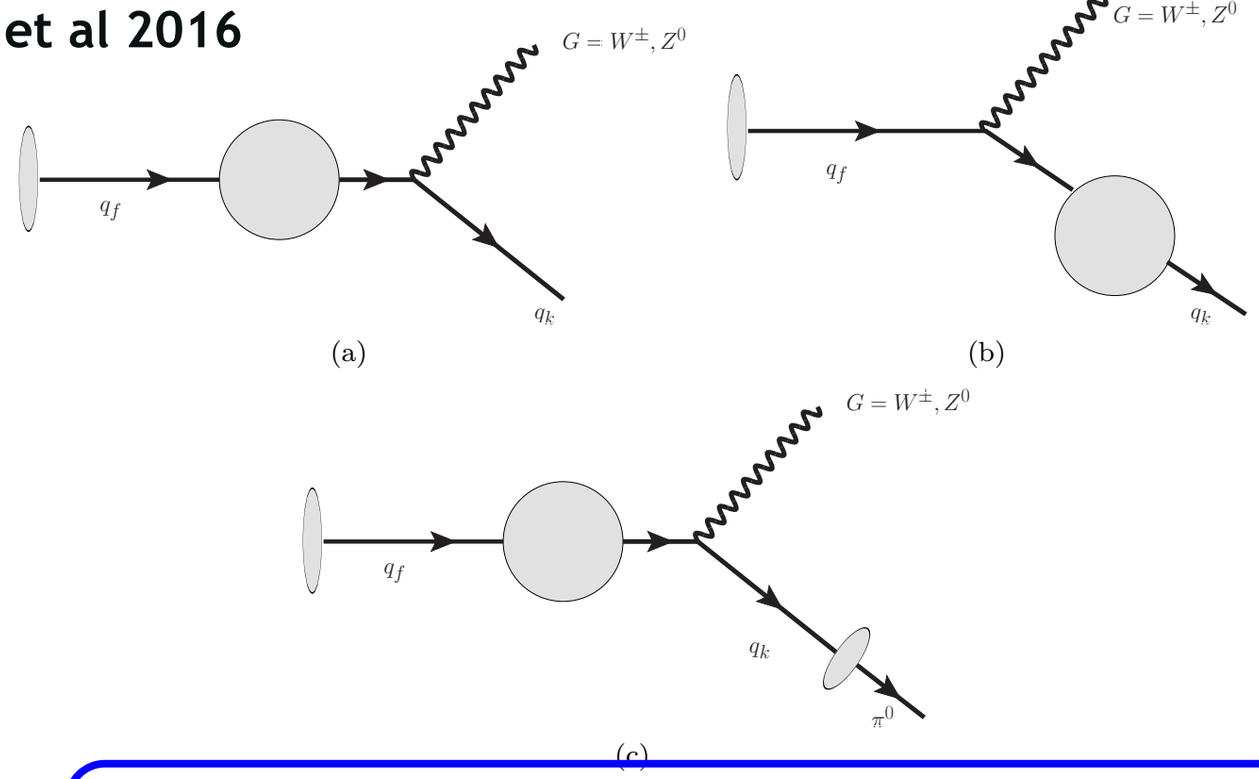


RP et al 2013, 2016

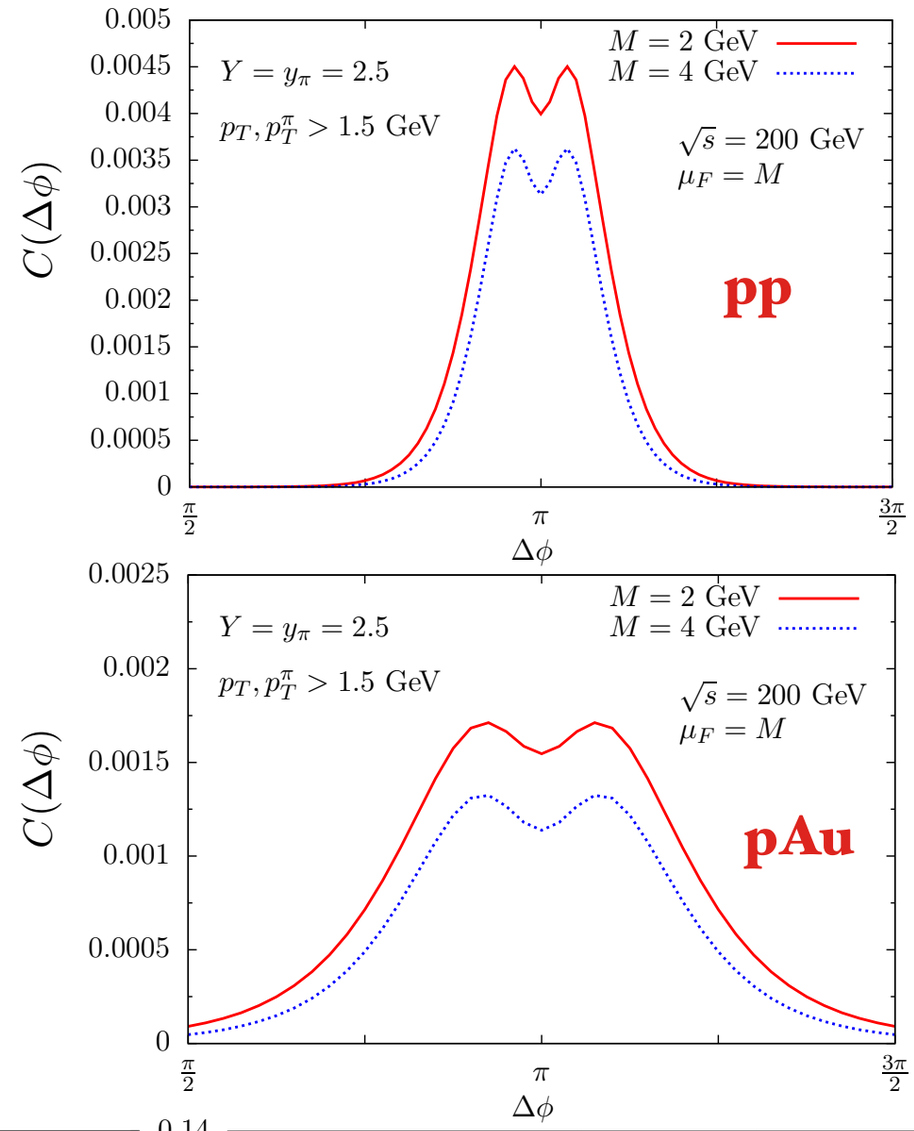


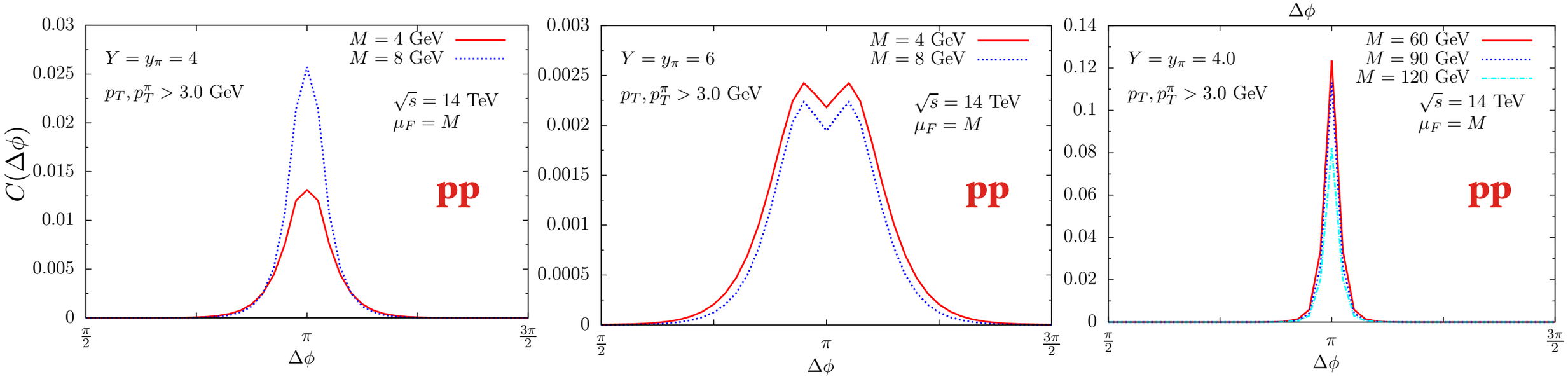
Angular correlations in Drell-Yan as a probe for saturation

A. Stasto et al, 2012
RP et al 2016



$$C(\Delta\phi) = \frac{2\pi \int_{p_T, p_T^h > p_T^{\rm cut}} dp_T p_T \, dp_T^h p_T^h \, \frac{d\sigma(pp \to hG^* X)}{dY dy_h d^2 p_T d^2 p_T^h}}{\int_{p_T > p_T^{\rm cut}} dp_T p_T \, \frac{d\sigma(pp \to G^* X)}{dY d^2 p_T}}$$





This picture does not change when turning to diffractive Drell-Yan

Heavy flavour production: Bremsstrahlung vs Fusion

B. Kopeliovich et al, PRD76 2007

Gauge-invariant sub-sets of diagrams

"Bremsstrahlung" component

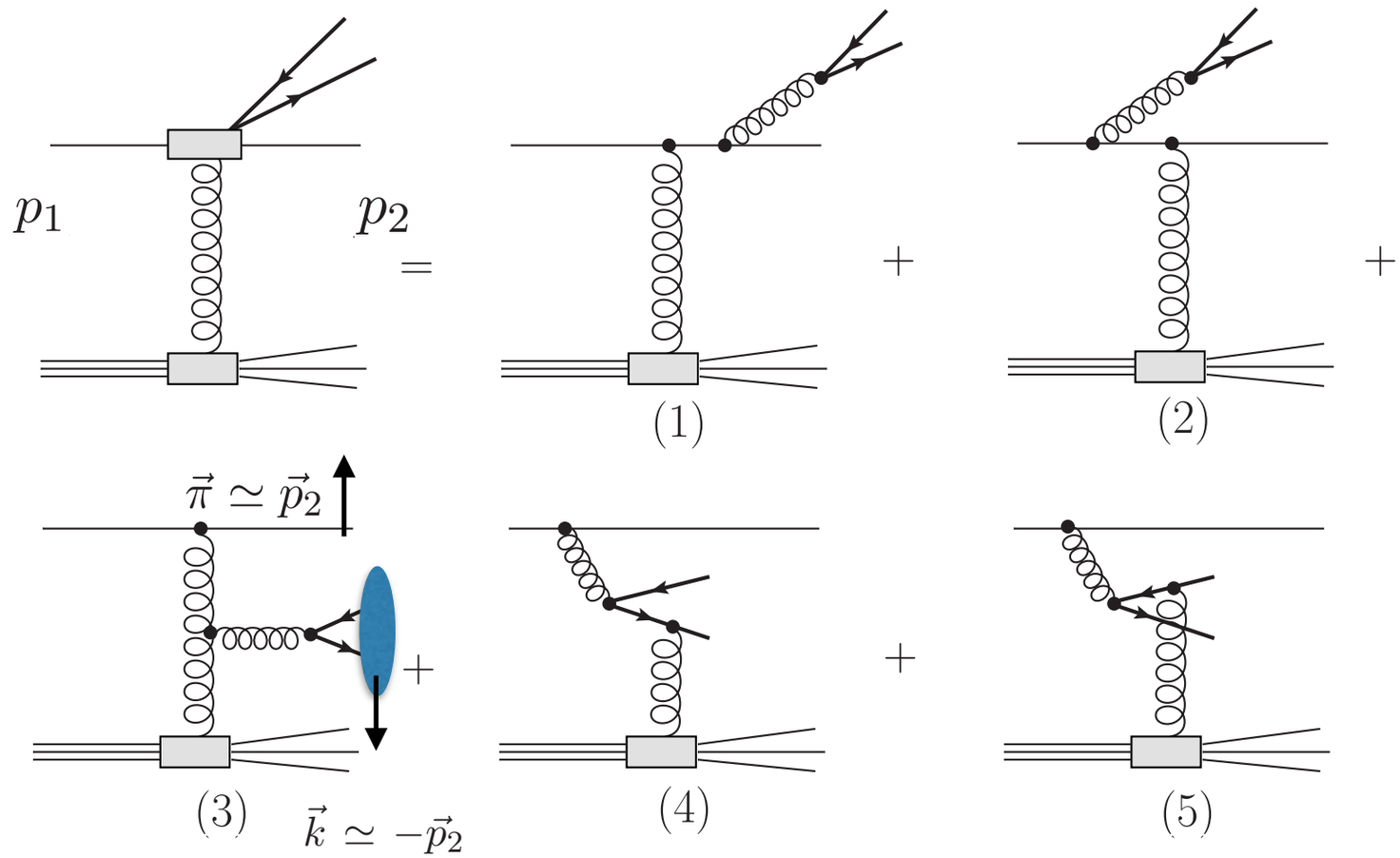
$$M^T_{\rm Br} = M^T_1 + M^T_2 + \frac{Q^2}{M^2+Q^2} M^T_3$$

suppressed by QQ mass!

"Fusion" component

$$M^T_{\rm Pr} = \frac{M^2}{M^2+Q^2} M^T_3 + M^T_4 + M^T_5$$

Dominates!



Gluon virtuality

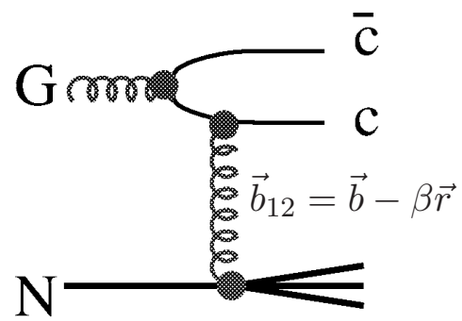
$$(p_2 - p_1)^2 \equiv -Q^2\,, \qquad Q^2 = \frac{\vec{\pi}^2 + \alpha^2 m_q^2}{\bar{\alpha}} \qquad \vec{\pi} = \alpha\vec{p}_2 - \bar{\alpha}\vec{k}\,, \qquad \vec{k} = \sum_i \vec{k}_i$$

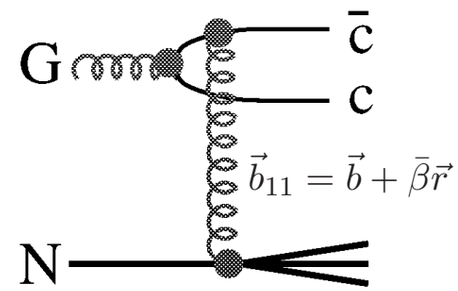
Basis for heavy flavour production in the dipole picture

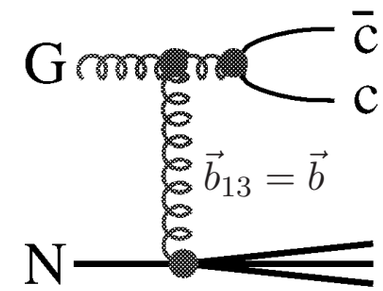
Dipole framework for heavy flavor production

"Fusion" components

$$G + N \to \bar{c}c + X$$







LC momenta

$$k_1 \simeq \bar{\beta}k - \kappa\,, \qquad k_2 \simeq \beta k + \kappa \qquad \vec{\kappa} = \bar{\beta}\vec{k}_2 - \beta\vec{k}_1$$

impact parameter representation

$$\hat{A}(\vec{s},\vec{r}) = \frac{1}{(2\pi)^4}\int d^2\vec{q}\,d^2\vec{\kappa}\;\hat{A}(\vec{q},\vec{\kappa})\,e^{-i\vec{q}\cdot\vec{s}-i\vec{\kappa}\cdot\vec{r}}$$

$$\hat{A} \simeq \frac{\sqrt{3}}{2}\sum_r \Big\{\tau_r\,\tau_a\langle f|\hat{\gamma}_r(\vec{b}_{11})|i\rangle - \tau_a\,\tau_r\langle f|\hat{\gamma}_r(\vec{b}_{12})|i\rangle$$
$$- i\sum_c f_{cra}\tau_c\langle f|\hat{\gamma}_r(\vec{b}_{13})|i\rangle\Big\}\Phi_{Q\bar{Q}}(\vec{r},\beta)\,,$$

$$|A|^2 \equiv \frac{1}{8}\frac{1}{2}\sum_{\lambda_*,\mu,\bar{\mu}}\langle\hat{A}^\dagger\hat{A}\rangle_{|3q\rangle_1}$$

$$\sum_X\langle i|\hat{\gamma}_a(\vec{b}_k)\hat{\gamma}_{a'}(\vec{b}_l)|i\rangle_{|3q\rangle_1} = \frac{3}{4}\delta_{aa'}S(\vec{b}_k,\vec{b}_l)$$

The universal dipole cross section

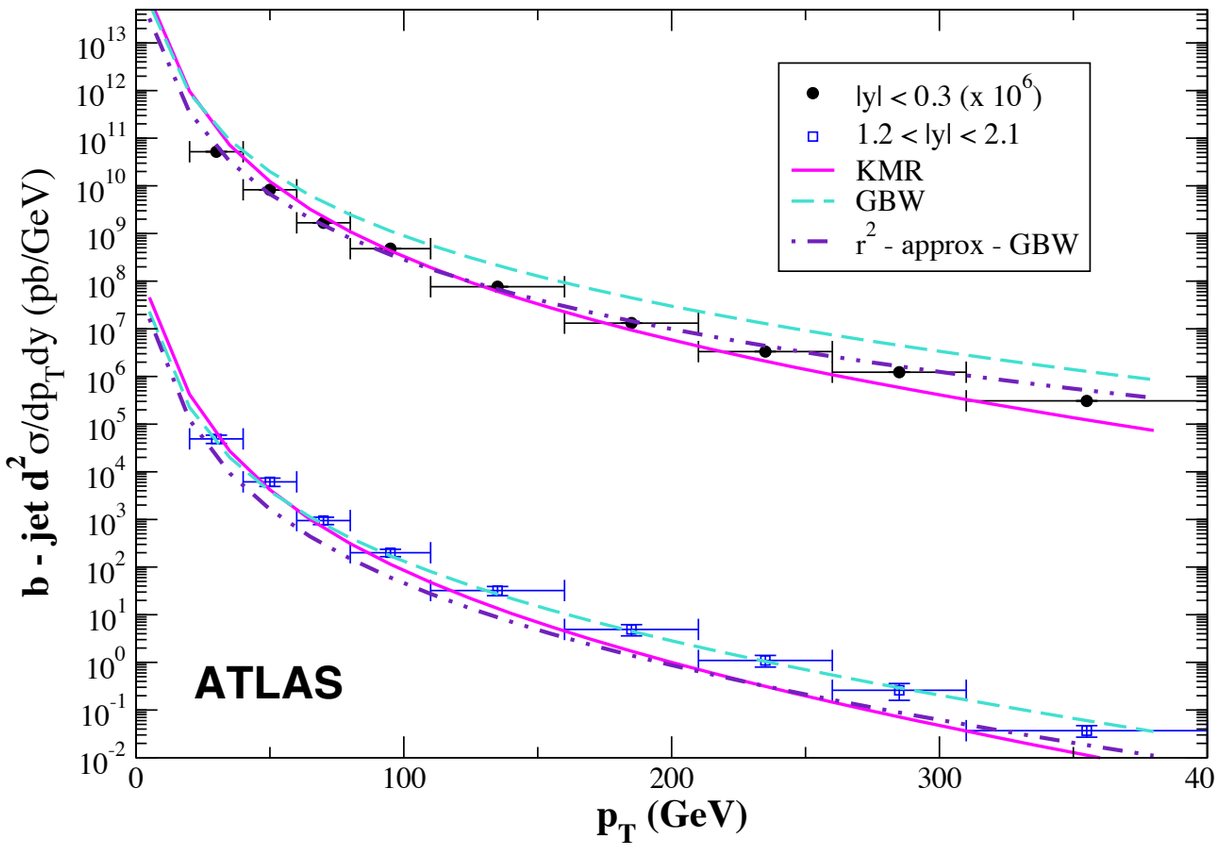
$$\sigma_{\bar{q}q}(\vec{r}_1-\vec{r}_2) \equiv \int d^2b\Big[S(\vec{b}+\vec{r}_1,\vec{b}+\vec{r}_1) + S(\vec{b}+\vec{r}_2,\vec{b}+\vec{r}_2) - 2S(\vec{b}+\vec{r}_1,\vec{b}+\vec{r}_2)\Big]$$

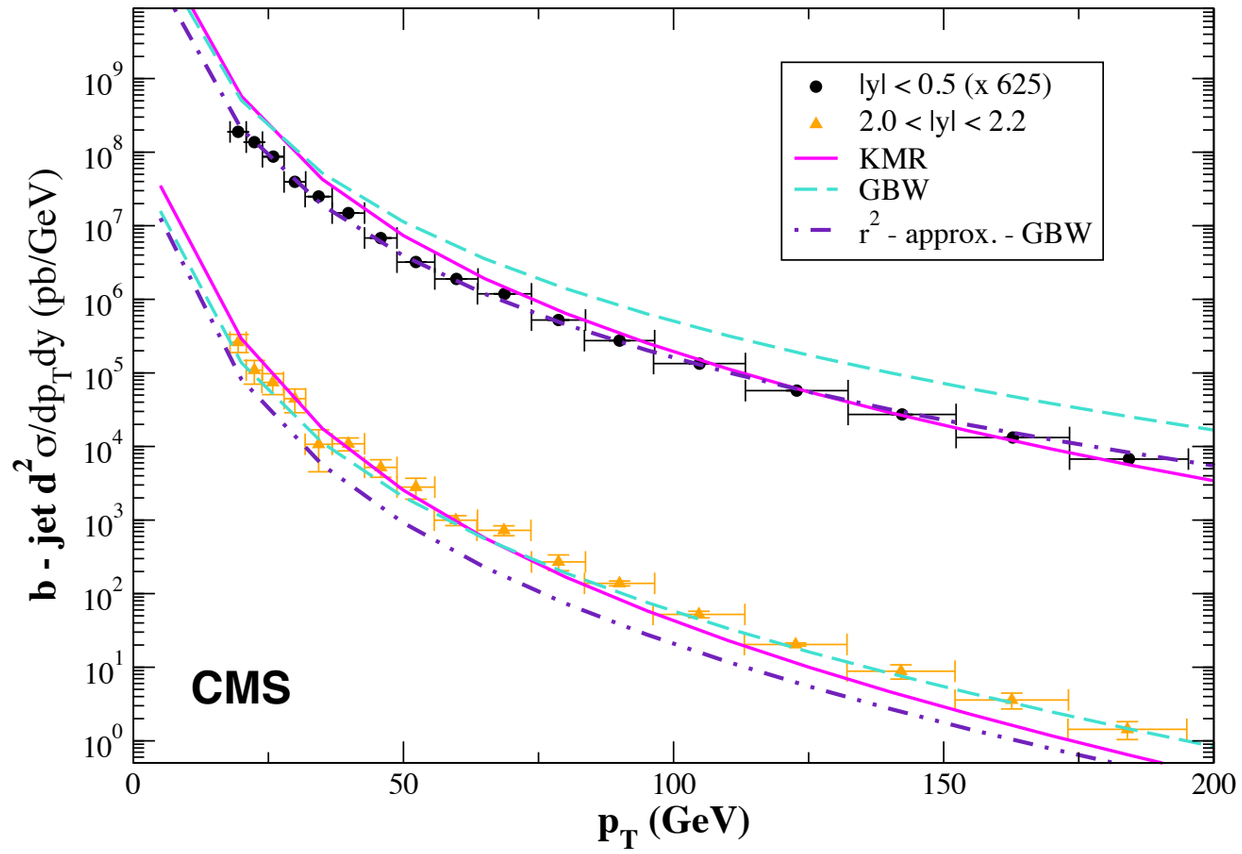
The total cross section

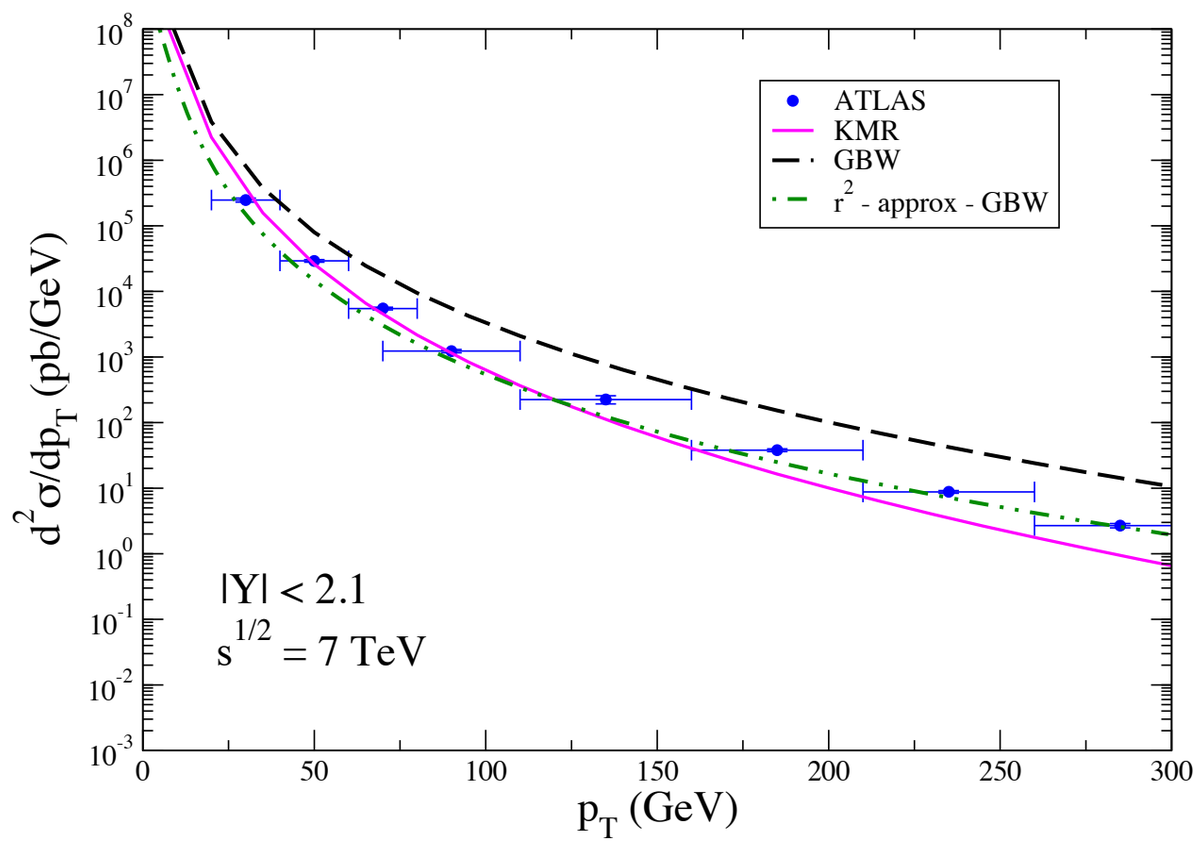
$$\sigma(G+p\to c\bar{c}+X) = \sum_{\mu\bar{\mu}}\int_0^1 d\beta\int d^2r\,\sigma_3(r,\beta,x_2)\,|\Phi_{Q\bar{Q}}(\vec{r},\beta)|^2$$

$$\sigma_3(r,\beta,x_2) = \frac{9}{8}\Big(\sigma_{\bar{q}q}(\bar{\beta}r,x_2) + \sigma_{\bar{q}q}(\beta r,x_2)\Big) - \frac{1}{8}\sigma_{\bar{q}q}(r,x_2)\,, \qquad x_2 = \frac{M^2_{c\bar{c}}}{2m_pE_G}$$

Inclusive Q-jet pT distribution in pp collisions vs LHC data

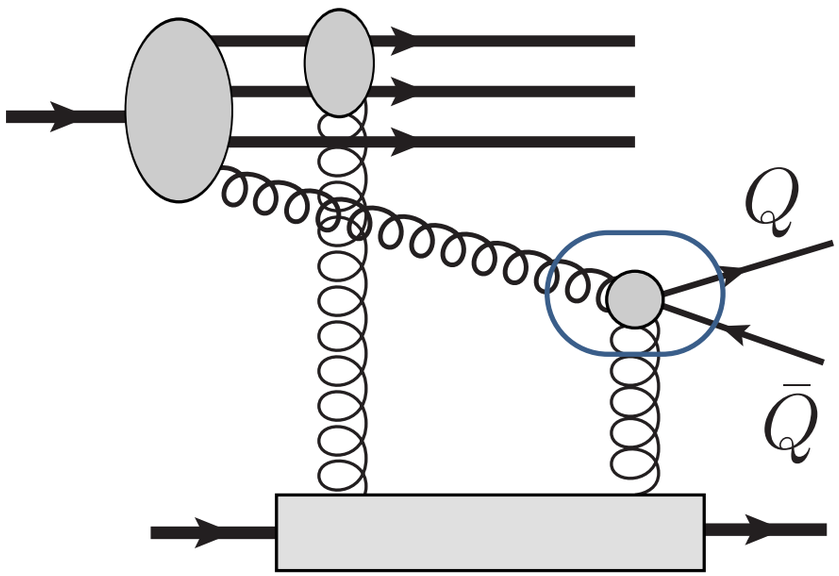


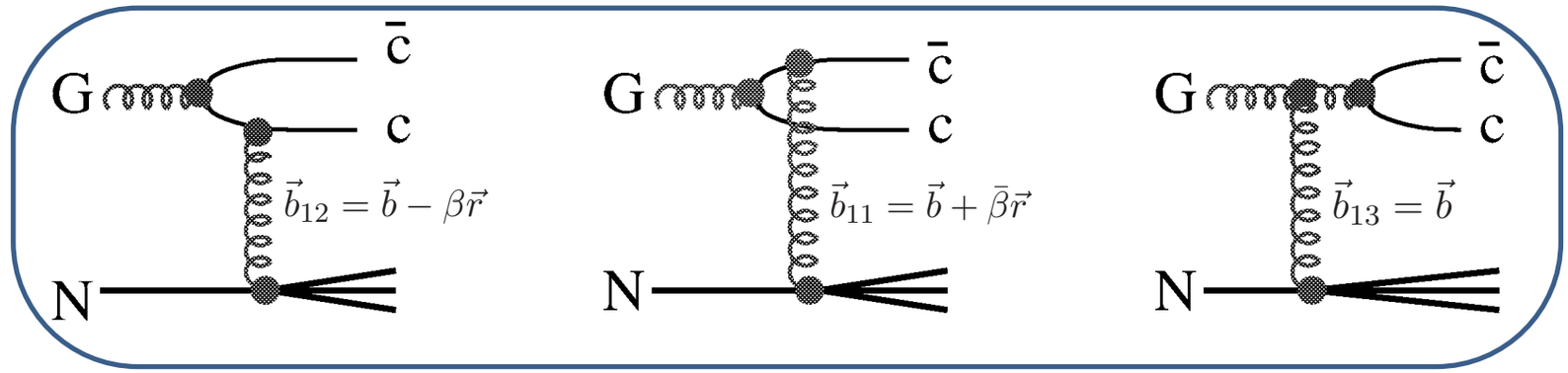




not worse than in NLO pQCD!

Diffractive non-Abelian (gluon) radiation via dipoles





"skeleton" contributions are subject for "dressing!"

leading twist effect!

B. Kopeliovich et al, 2007
RP et al, in progress

when the LO contributions get generalised to all-order results, ALL possible higher-order (perturbative+nonperturbative) corrections due to NON-RESOLVED emissions are AUTOMATICALLY resumed and accounted for by the dipole formula!

SD amplitude

$$\overline{|A_{\rm SD}|^2} \simeq \frac{3}{256}\,|\Psi_{in}|^2|\Psi_{fin}|^2 \sum_{i,j=1}^{2}\left[\nabla^i\Psi^*_{Q\bar{Q}}(\alpha,\vec{r})\,\nabla^j\Psi_{Q\bar{Q}}(\alpha,\vec{r}')\right]\Omega^{ij}_{\rm soft}$$

"soft color screening" part

$$\Omega^{ij}_{\rm soft} = \left[\nabla^i\sigma_{q\bar{q}}(\vec{r}_{12}) + \nabla^i\sigma_{q\bar{q}}(\vec{r}_{13})\right]\left[\nabla^j\sigma_{q\bar{q}}(\vec{r}_{12}) + \nabla^j\sigma_{q\bar{q}}(\vec{r}_{13})\right]$$

SD-to-inclusive ratio

$$\frac{d\sigma_{\rm SD}}{d\Omega} \simeq \frac{\bar{R}_0^2(x_2)}{\bar{\sigma}_0}\left[\alpha^2+\bar{\alpha}^2-\frac{1}{4}\alpha\bar{\alpha}\right]^{-1} F_{\rm S}(x_1,s)\,\frac{d\sigma_{\rm incl}}{d\Omega}$$

$$F_{\rm S}(x_1,s) \equiv \frac{729\,a^2\sigma_0(x_1 s)^2\,\Lambda(x_1 s)}{4096\,\pi^2\,B_{\rm SD}(s)}$$

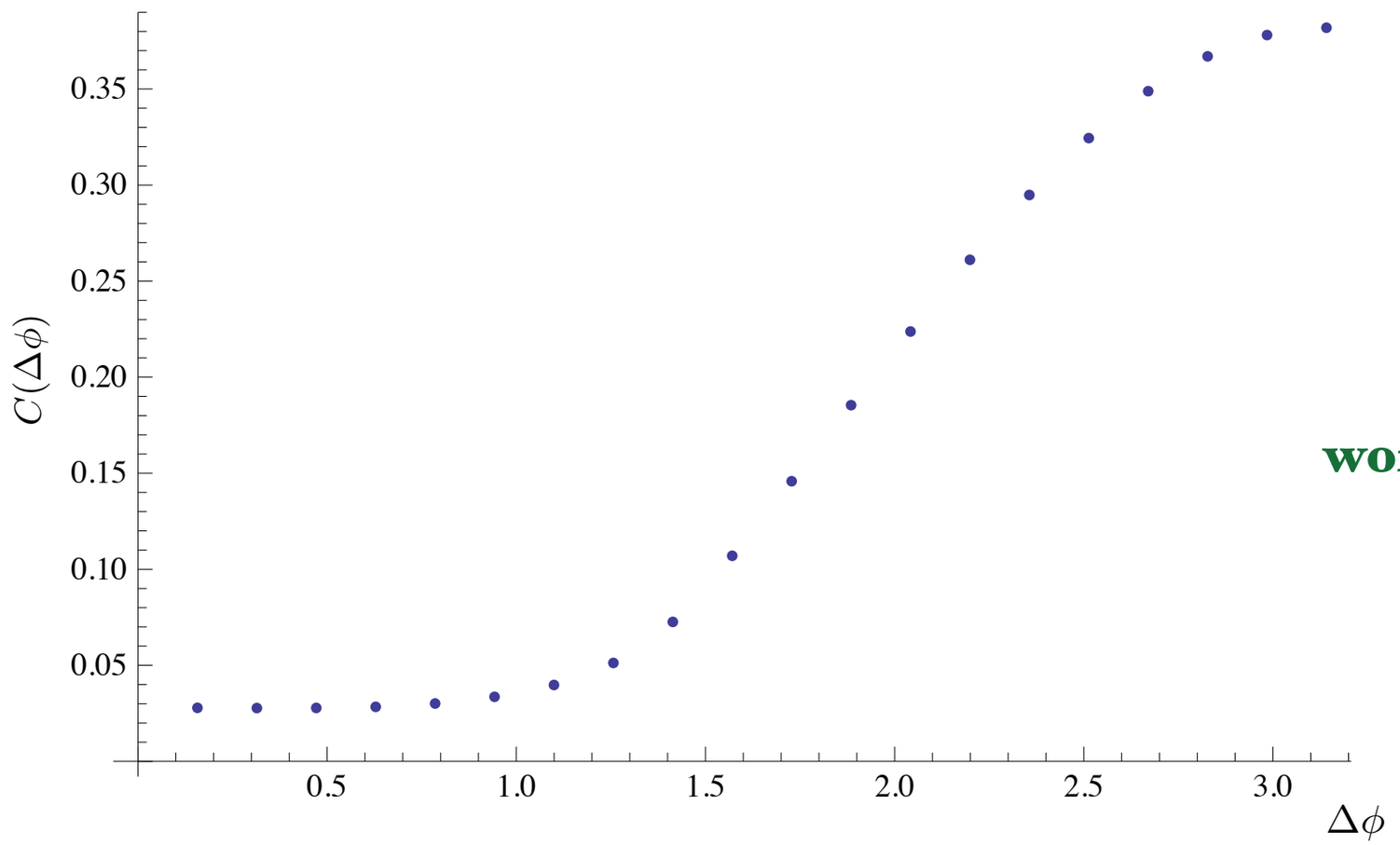
The angular correlation is affected by color-screening interaction in higher-twist diffraction (e.g. in DIS, see talk by A. Rezaeian) but not in the leading twist!

Heavy QQbar angular correlation

followed by a discussion with O. Teryaev and M. Tasevsky

$$\frac{d^3\sigma(G \to Q\bar{Q} + X)}{d(\ln\alpha)d^2p_T} = \frac{1}{6\pi}\int \frac{d^2\kappa_\perp}{\kappa_\perp^4}\alpha_s^2\mathcal{F}(x,\kappa_\perp^2) \times$$

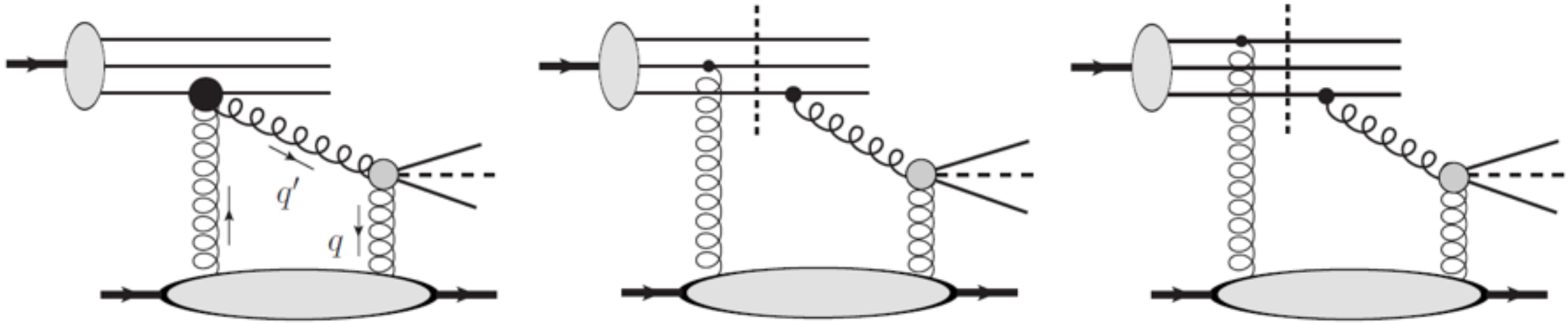
$$\left\{\left[\frac{9}{8}\mathcal{H}_0(\alpha,\bar{\alpha},p_T) - \frac{9}{4}\mathcal{H}_1(\alpha,\bar{\alpha},p_T,\kappa) + \mathcal{H}_2(\alpha,\bar{\alpha},p_T,\kappa) + \frac{1}{8}\mathcal{H}_3(\alpha,\bar{\alpha},p_T,\kappa)\right] + [\alpha \longleftrightarrow \bar{\alpha}]\right\}$$



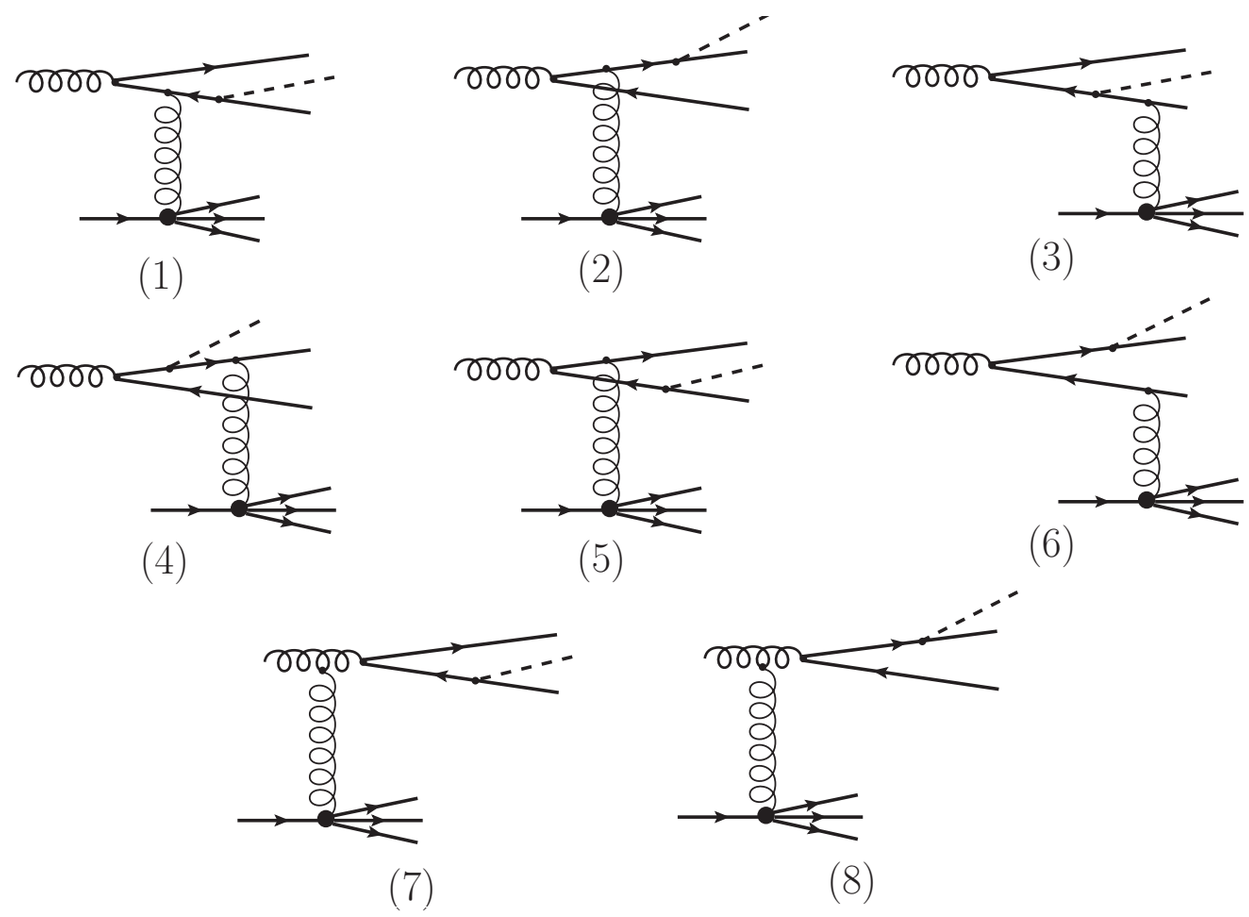
work in progress!

The same for inclusive and leading-twist single-diffractive QQbar production!

Diffractive Higgsstrahlung off heavy quarks

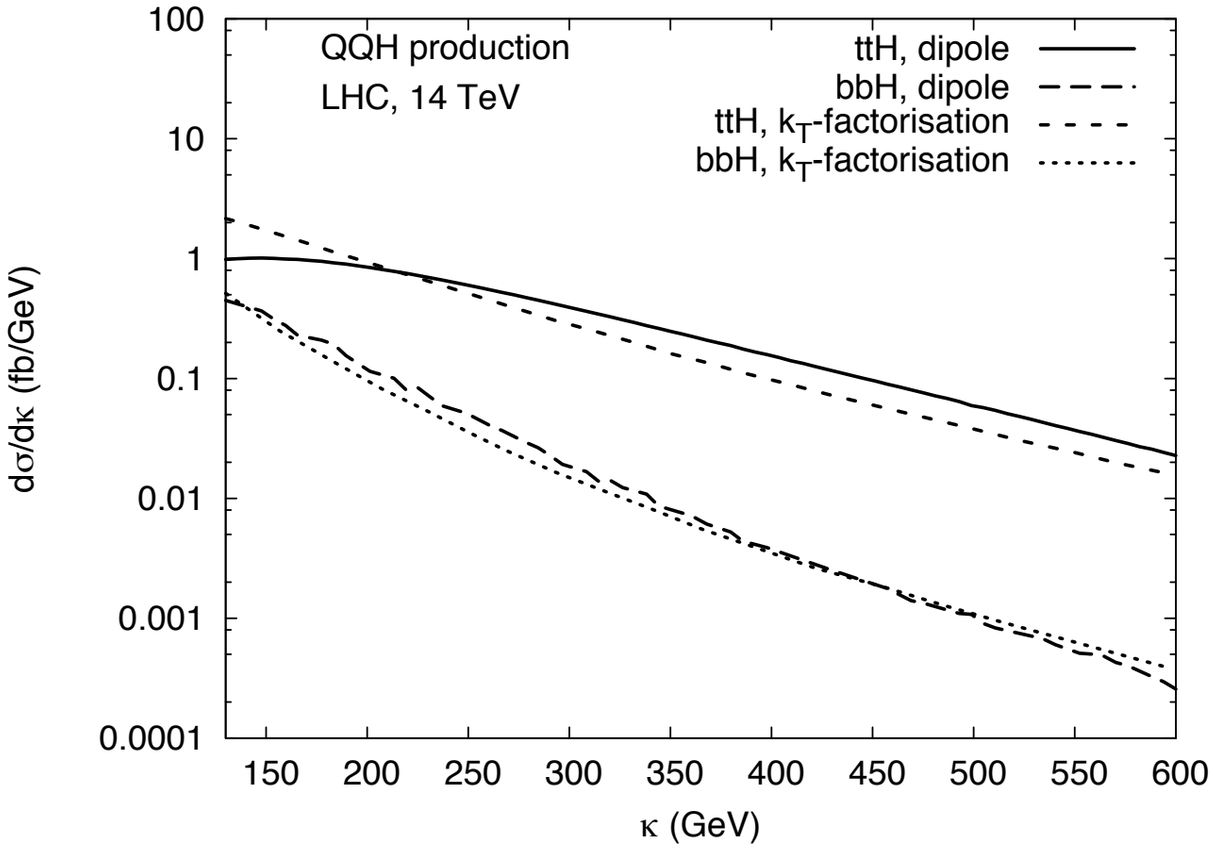


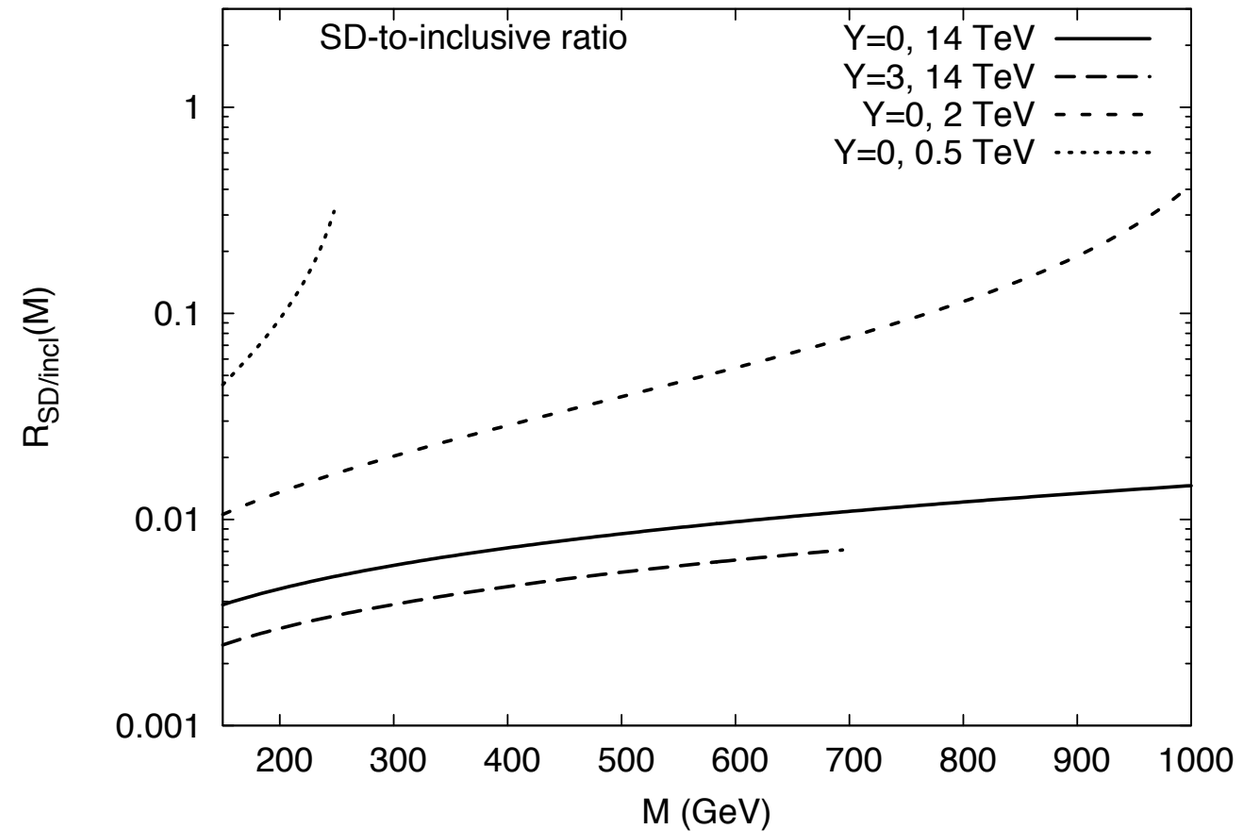
$$p + p \to \bar{Q}Qh + X + p$$

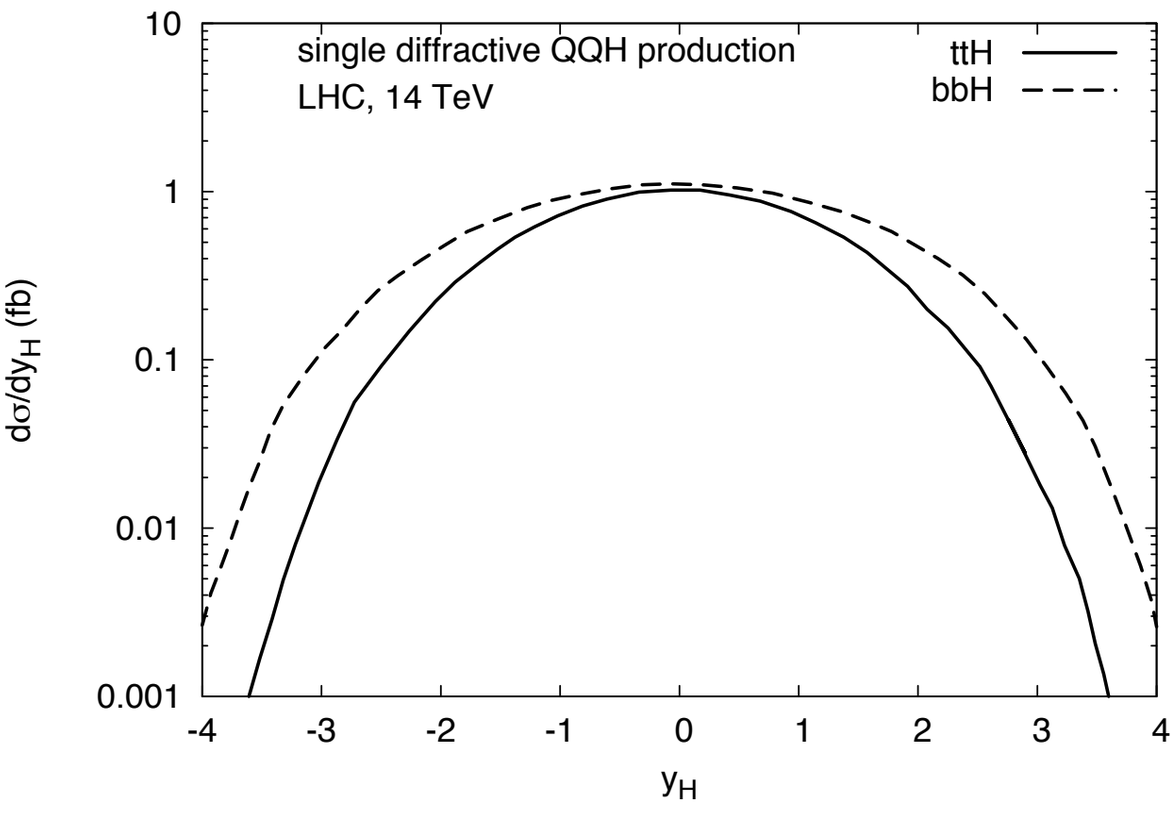


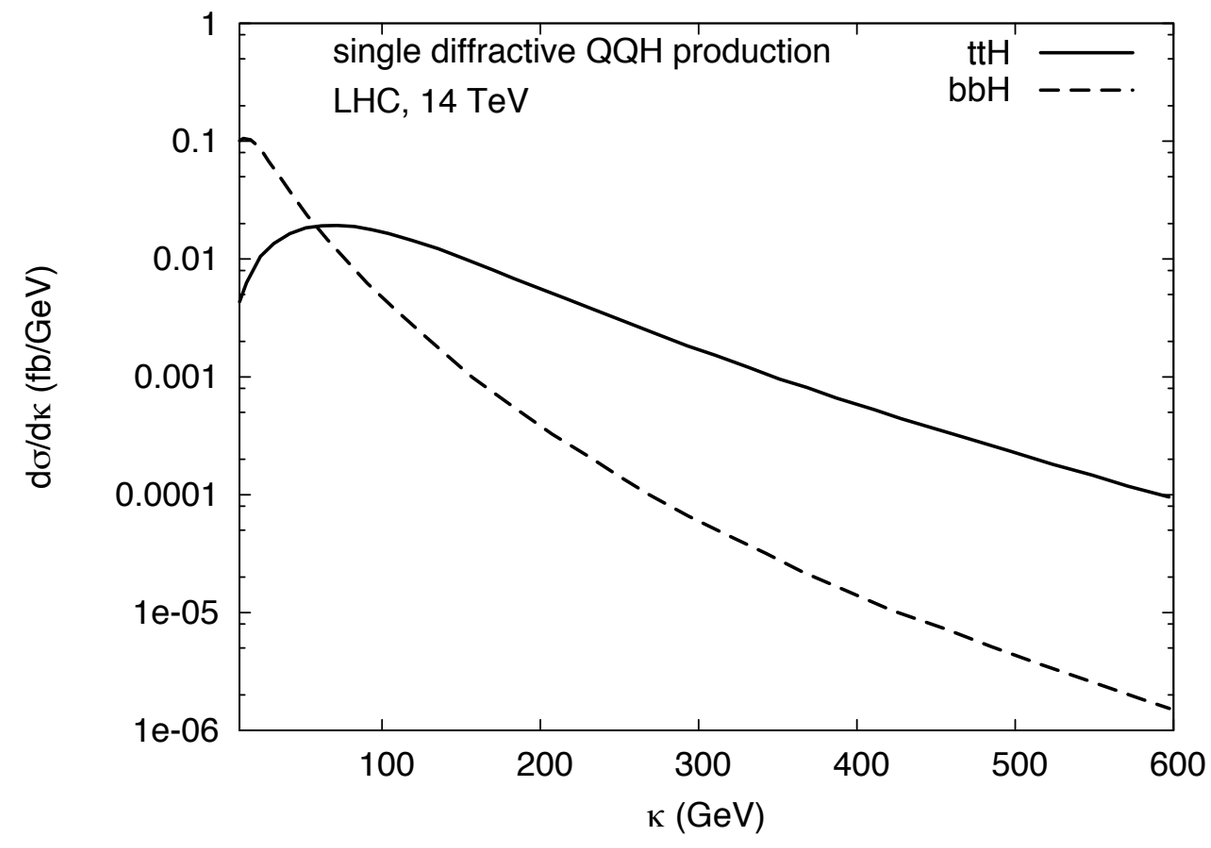
Gluon-Gluon fusion strongly dominates over gluon Bremsstrahlung!

Diffractive Higgsstrahlung off heavy quarks









Conclusions

- ✓ The dipole picture provides universal and robust means for studies the inclusive and single-diffractive processes in both pp and pA collisions at large Feynman xF beyond QCD factorisation

- ✓ Major sources of diffractive factorisation breaking in hadron-hadron collisions are (i) the absorptive corrections, and (ii) the hard-soft interplay due to transverse motion of spectators, making the hadronic diffraction of the leading-twist nature

- ✓ The universal partial dipole amplitude accounts for the absorptive corrections such that no additional probabilistic fudge factors are necessary in the dipole picture

- ✓ Single-diffractive gauge bosons' (e.g. Drell-Yan) and heavy flavour production at large Feynman xF has been studied beyond diffractive factorisation

- ✓ The SD-to-diffractive ratio affects the scale and rapidity dependence of the leading-twist hadronic diffractive observables compared to the inclusive ones, the angular correlations are the same as in the inclusive case.