



Saturation and geometrical scaling

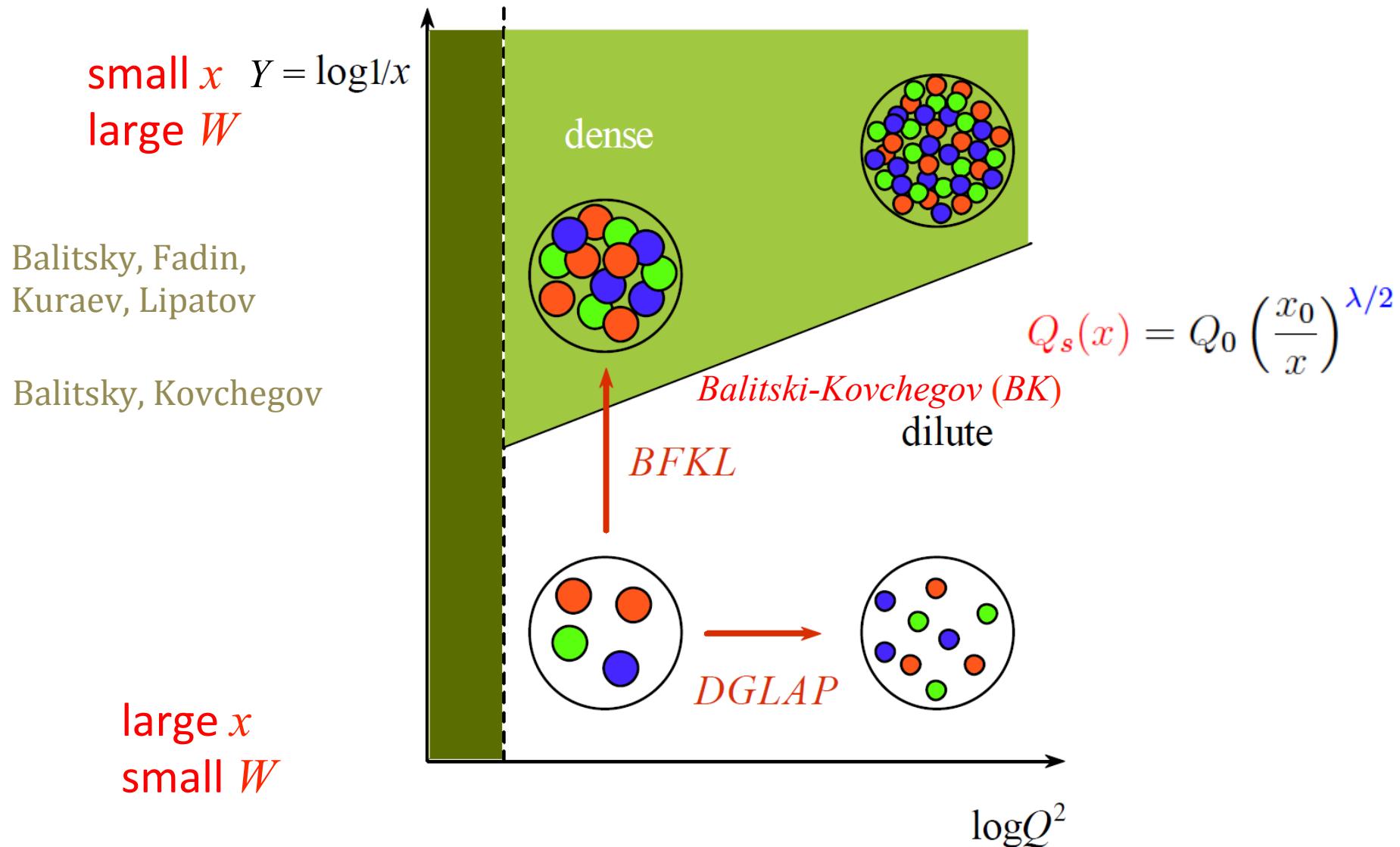
Michał Praszałowicz

M. Smoluchowski Inst. of Physics
Jagiellonian University, Kraków, Poland

Trento 27.9.2016.



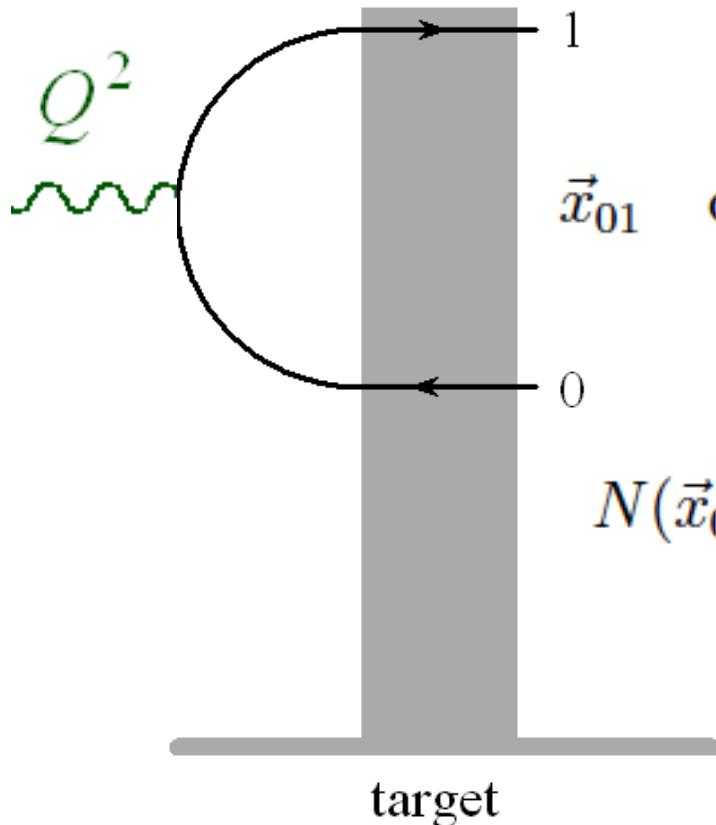
DGLAP vs BFKL Evolution





Dipole Picture

BFKL equation has very simple form and interpretation in the dipole picture of A. Mueller



A.H. Mueller and J.-w. Qiu,
Nucl. Phys. B 268 (1986) 427

\vec{x}_{01} dipole transverse size $Y = \log 1/x$

$N(\vec{x}_{01}, Y)$ dipole-target forward amplitude



BK Equation

in terms of a Fourier transform:

$$N(x, Y) = x^2 \int \frac{d^2 \vec{k}}{2\pi} e^{i\vec{k}\cdot\vec{x}} \tilde{N}(k, Y)$$

$$\frac{\partial}{\partial Y} \tilde{N}(k, Y) = \bar{\alpha}_s \chi(-\partial/\partial \ln k^2) \tilde{N}(k, Y) - \bar{\alpha}_s \tilde{N}^2(k, Y)$$

here χ is a BFKL characteristic function

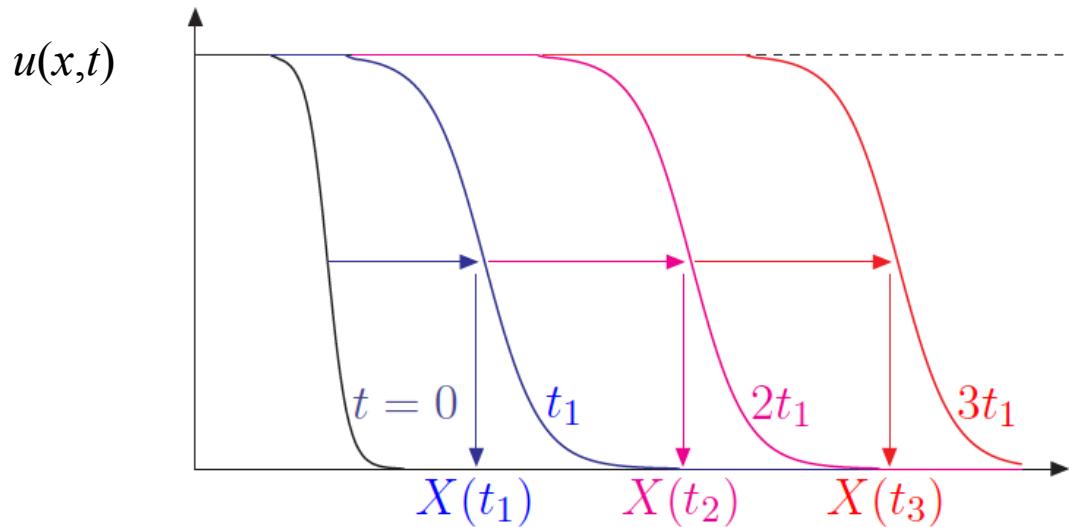
$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)$$

there exists a theorem from the '30 (Fisher, Kolomogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions



Travelling waves

identify time : $t = Y$, position : $x = \ln k^2$



Asymptotic solution:
travelling wave

$$u(x, t) = u(x - v_c t)$$

Position: $X(t) = X_0 + v_c t$ © G. Soyez



Travelling waves

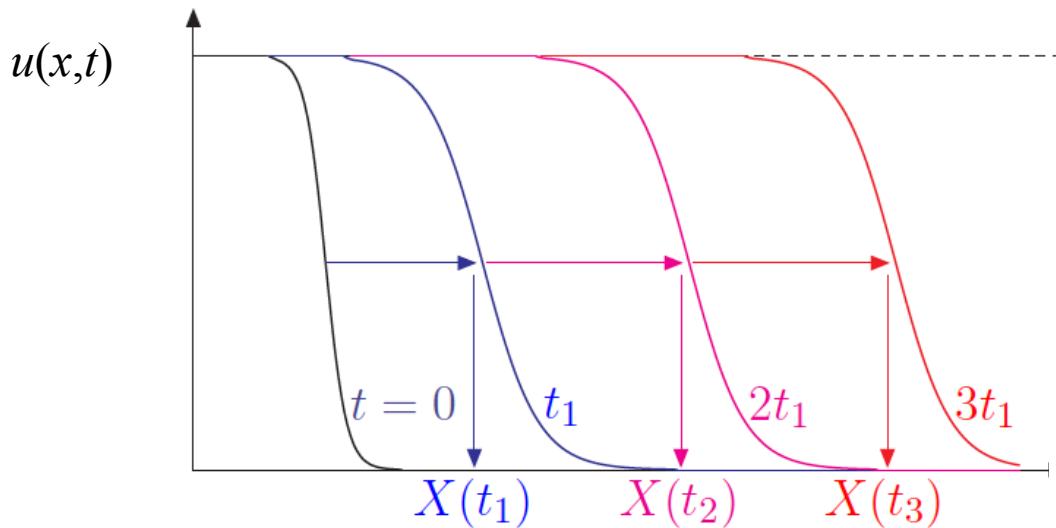
identify

time :

$$t = Y,$$

position :

$$x = \ln k^2$$



Asymptotic solution:
 travelling wave

$$u(x, t) = u(x - v_c t)$$

$$x - v_c t = \log\left(\frac{k^2}{k_0^2}\right) - v_c \log\left(\frac{1}{x}\right)$$

Position: $X(t) = X_0 + v_c t$ © G. Soyez

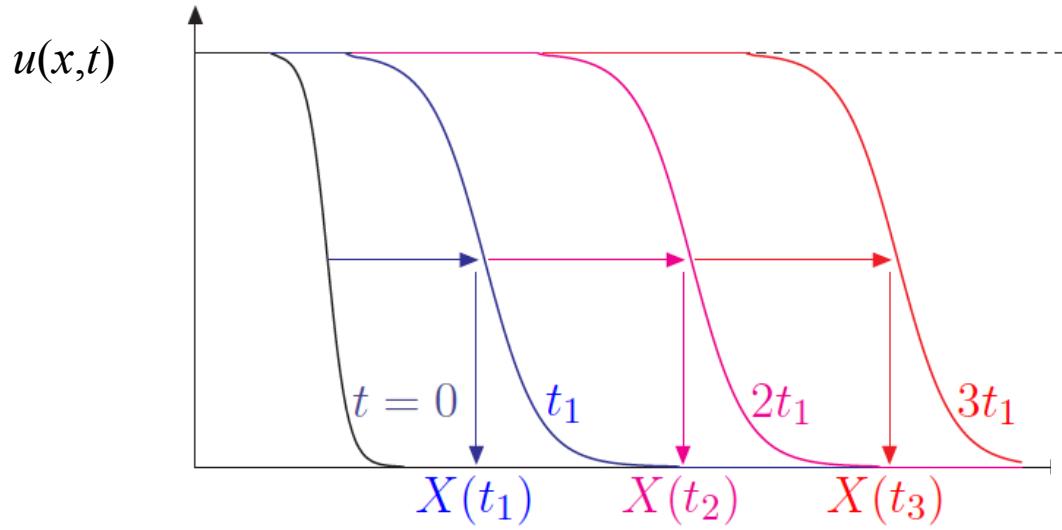
$$= \log\left[k^2 \times \frac{1}{k_0^2} \left(\frac{1}{x}\right)^{-v_c}\right]$$

$$= \log\left(\frac{k^2}{Q_{\text{sat}}^2(x)}\right)$$



Travelling waves

identify time : $t = Y$, position : $x = \ln k^2$



Asymptotic solution:
 travelling wave

$$u(x, t) = u(x - v_c t)$$

$$x - v_c t = \log\left(\frac{k^2}{k_0^2}\right) - v_c \log\left(\frac{1}{x}\right)$$

Position: $X(t) = X_0 + v_c t$ © G. Soyez

$$Q_{\text{sat}}^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$$

$$\begin{aligned} &= \log\left[k^2 \times \frac{1}{k_0^2} \left(\frac{1}{x}\right)^{-v_c}\right] \\ &= \log\left(\frac{k^2}{Q_{\text{sat}}^2(x)}\right) \end{aligned}$$



Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$



Travelling waves in QCD imply Geometrical Scaling

$$f(x, k^2) = \mathcal{F} \left(\frac{k^2}{Q_s^2(x)} \right)$$

$$Q_s(x) = Q_0 \left(\frac{x_0}{x} \right)^{\lambda/2}$$



Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges



Deep Inelastic Scattering



Model: Golec-Biernat Wüsthoff

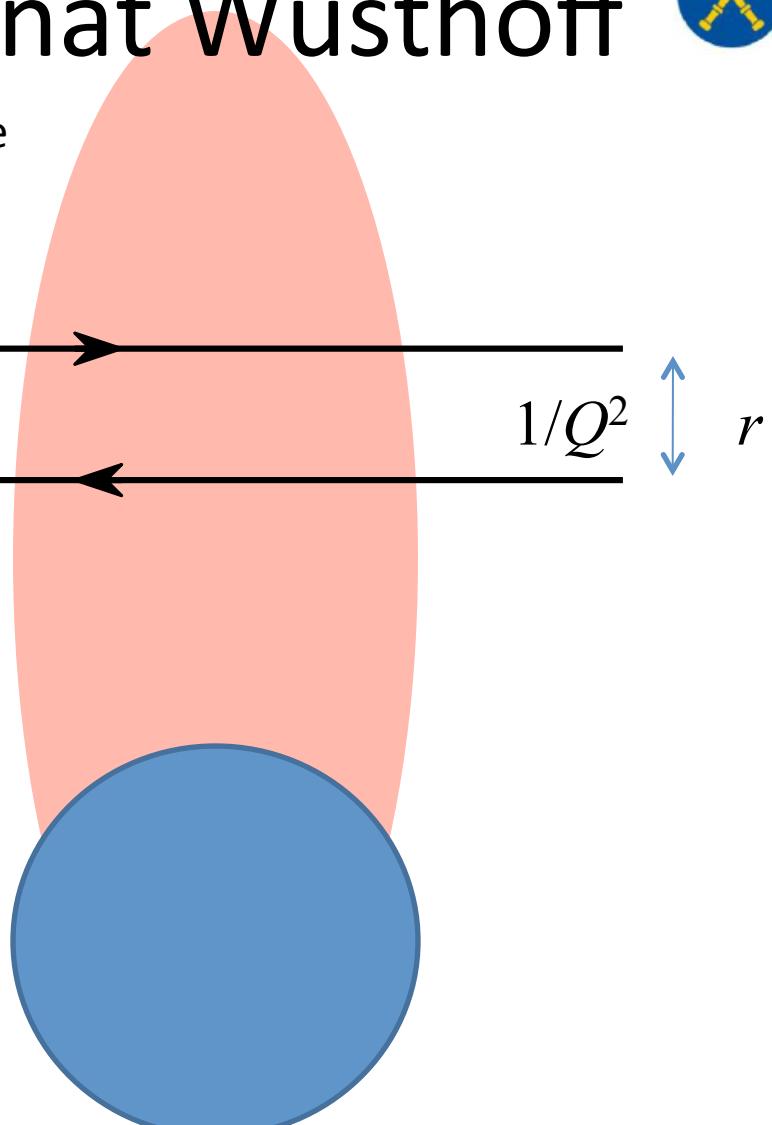
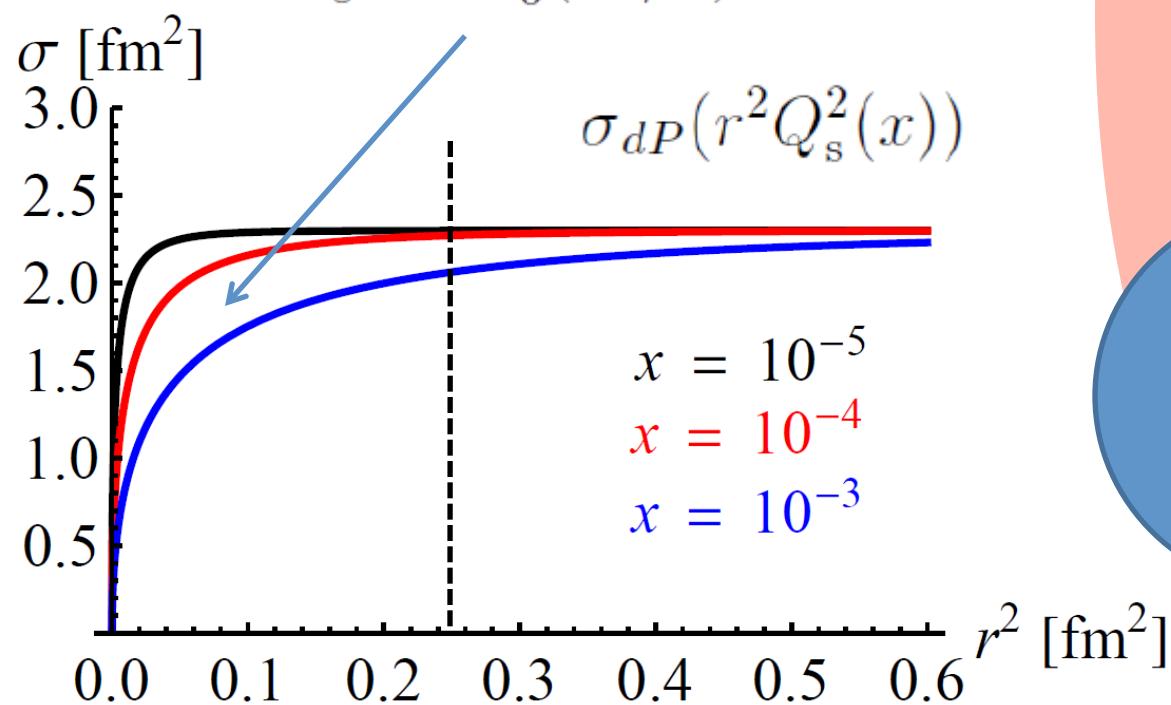
Changing the reference frame

K.J. Golec-Biernat, M. Wusthoff

PRD 59 (1998) 014017

PRD 60 (1999) 114023

$$Q_s^2 = Q_0^2 (x_0/x)^\lambda$$





Model: Golec-Biernat Wüsthoff

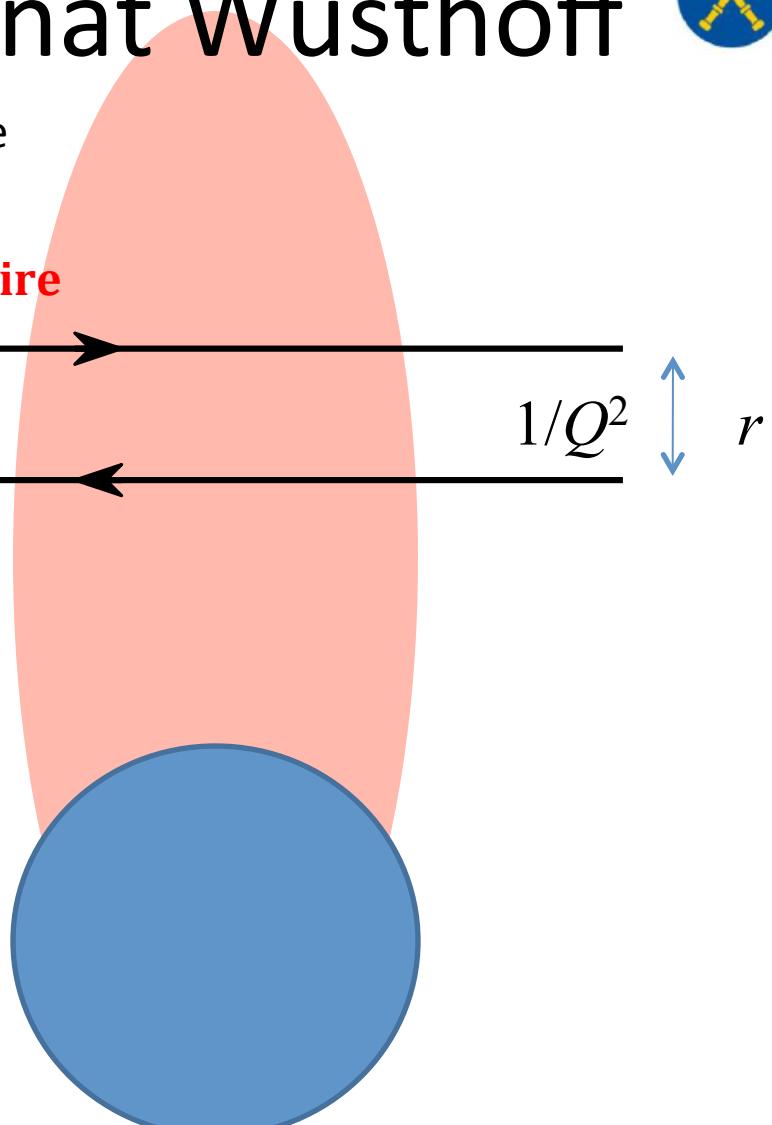
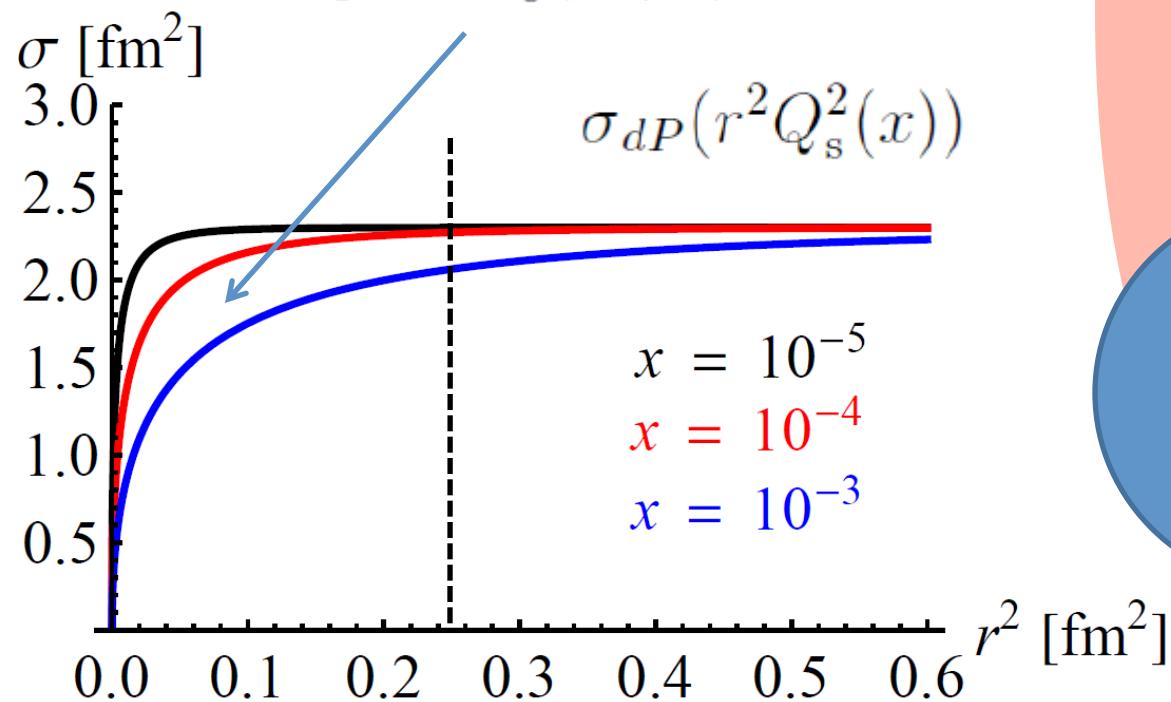
Changing the reference frame

K.J. Golec-Biernat, M. Wusthoff

PRD 59 (1998) 014017 **1036 quotations in InSpire**

PRD 60 (1999) 114023

$$Q_s^2 = Q_0^2 (x_0/x)^\lambda$$



Workshop on QCD and Diffraction

saturation 1000+

5-7 December 2016
Cracow, Poland

Organising Committee:

Wojciech Broniowski
Janusz Chwastowski
Krzysztof Kutak
Michał Praszałowicz

Christophe Royon
Anna Staśto
Rafał Staszewski



>>>KRK>2B
Krakow to business





Geometrical Scaling

$$\sigma_{\gamma^* p} = \int dr^2 |\psi(r, Q^2)|^2 \sigma_{dP}(r^2 Q_s^2(x))$$

$$\sigma_{\gamma^* p} = \sigma_{\gamma^* p} \left(\frac{Q_s(x)}{Q} \right)$$

GS does not depend on the particular form of the dipole cross-section

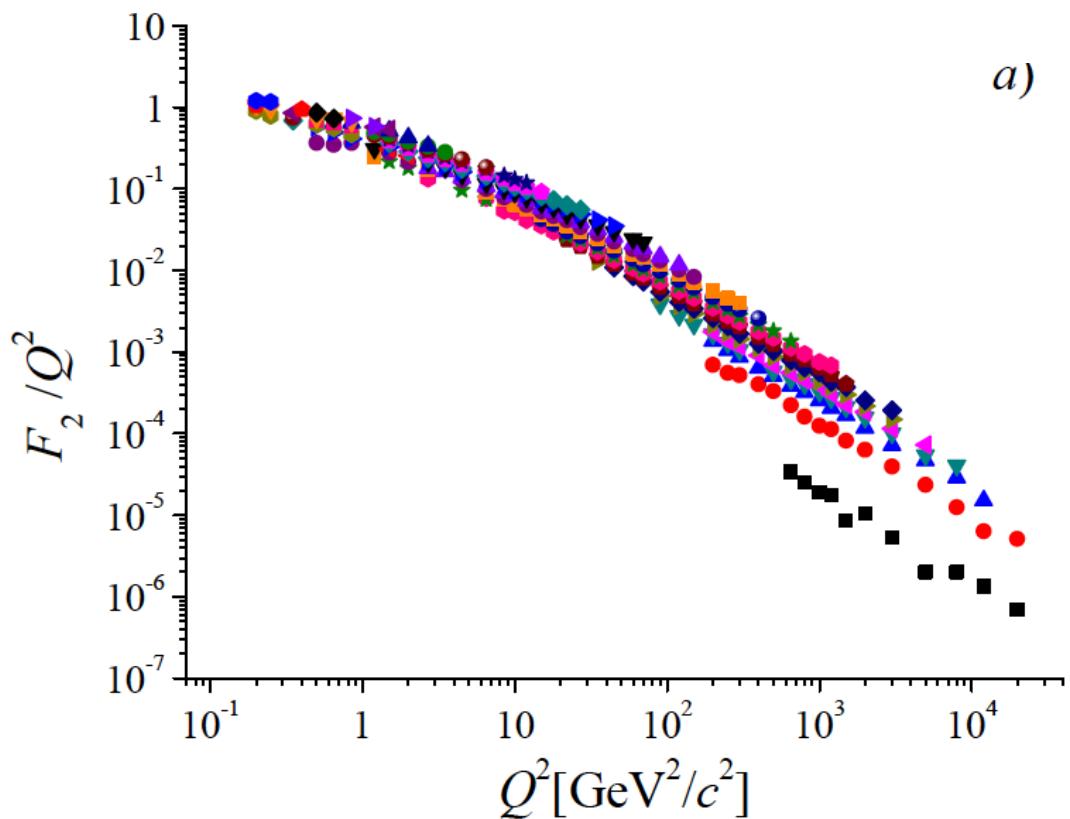


Saturation scale: energy and x dependence

$$Q_{\text{sat}}^2(x) = Q_0^2 \left(\frac{x}{x_0} \right)^{-\lambda}$$

a)

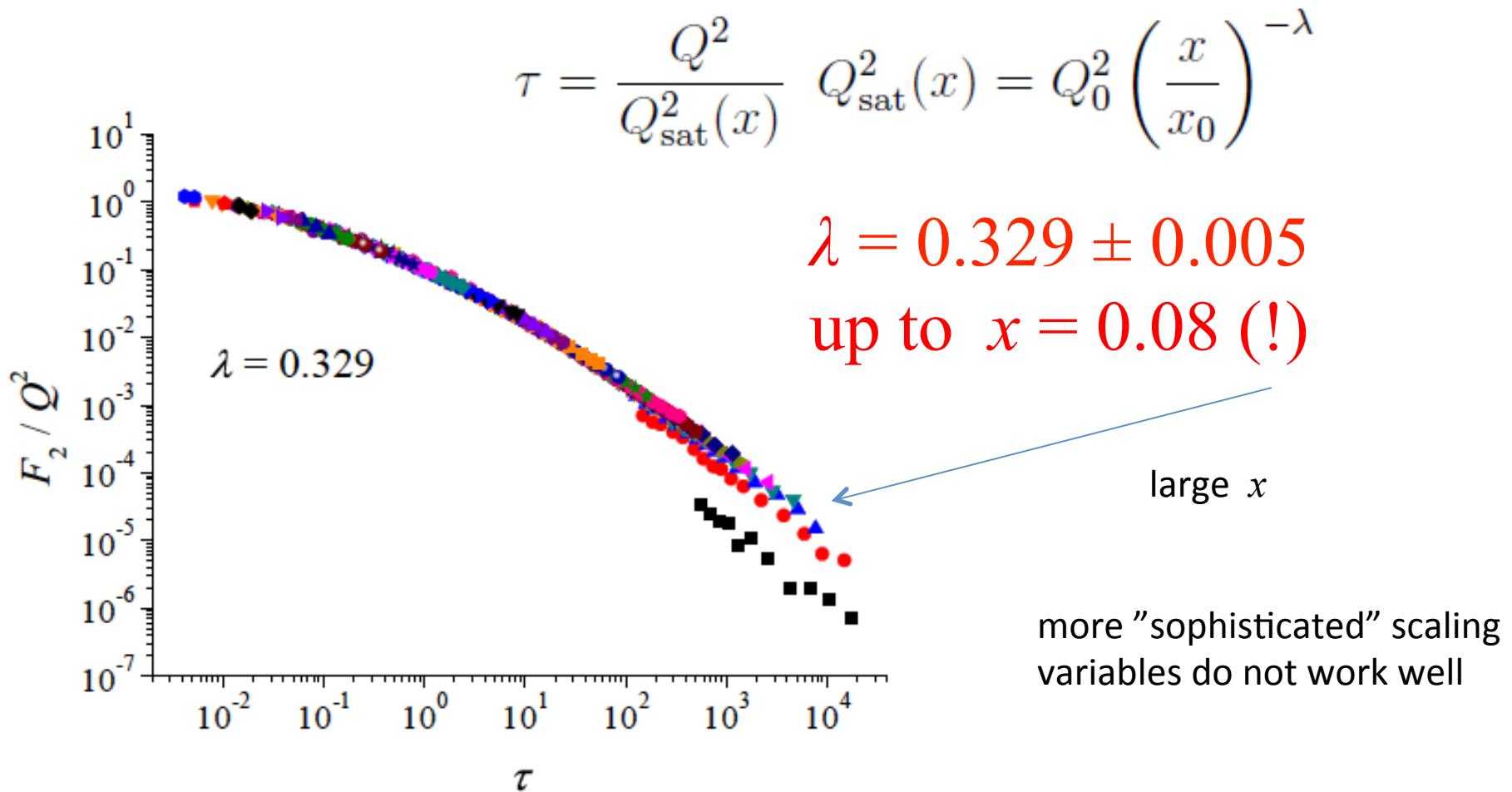
A.M. Stasto, K. J. Golec-Biernat,
J. Kwiecinski
PRL 86 (2001) 596-599



M.Praszalowicz and T.Stebel
JHEP 1303, 090 (2013)
arXiv:1211.5305 [hep-ph]
and
JHEP 1304, 169 (2013)
arXiv:1302.4227 [hep-ph]



Saturation scale: energy and x dependence





Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$



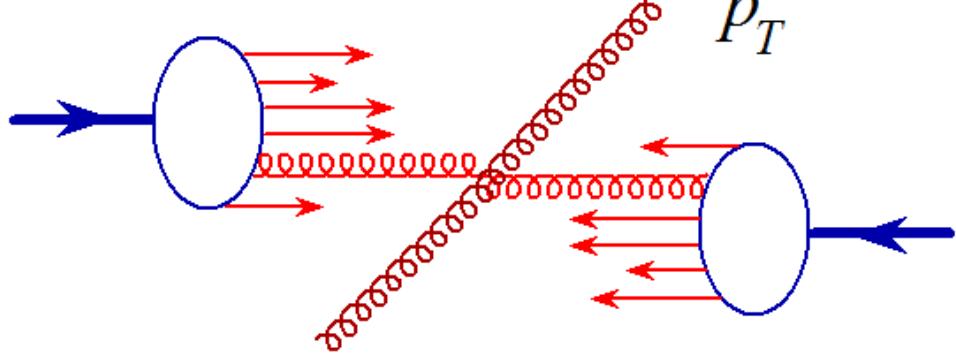
proton-proton @ LHC



Basics of geometrical scaling

Gribov, Levin Ryskin, *High p_T Hadrons In The Pionization Region In QCD.*
Phys.Lett.B100:173-176,1981.

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi \alpha_s}{2p_T^2} \int d^2 \vec{k}_T \varphi_1(x_1, \vec{k}_T^2) \varphi_2(x_2, (\vec{k} - \vec{p})_T^2)$$



$$x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}$$

gluon distribution

$$xG(x, Q^2) = \int dk_T^2 \varphi(x, k_T^2)$$

Kharzeev, Levin
Phys.Lett.B523:79-87,2001.



Basics of geometrical scaling

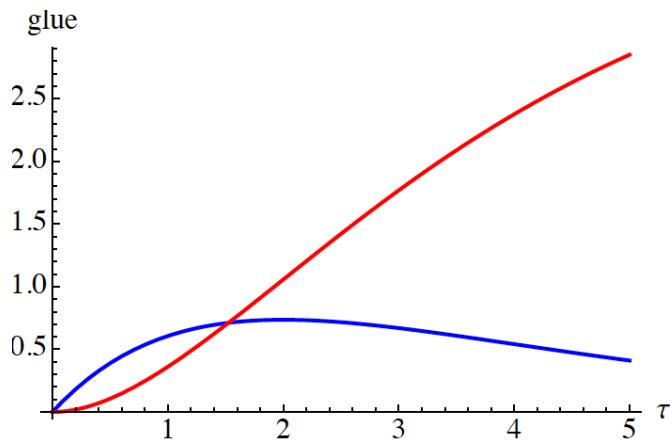
gluon distribution

$$xG(x, Q^2) = \int dk_T^2 \varphi(x, k_T^2)$$

Golec-Biernat – Wuesthoff (DIS)

$$\varphi(x, k_T^2) = S_\perp \frac{3}{4\pi^2} \frac{k_T^2}{Q_s(x)^2} \exp(-k_T^2/Q_s(x)^2)$$

$$S_\perp = \sigma_0$$



scaling variable

$$\tau = \frac{p_T^2}{Q_s^2(x)}$$

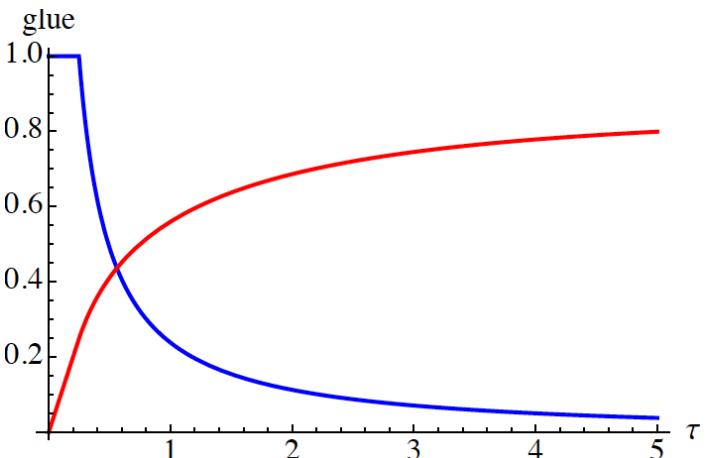
Michał Praszalowicz

unintegrated glue

Kharzeev – Levin (AA)

$$\varphi(x, k_T^2) = S_\perp \begin{cases} 1 & \text{for } k_T^2 < Q_s(x)^2 \\ Q_s(x)^2/k_T^2 & \text{for } Q_s(x)^2 < k_T^2 \end{cases}$$

S_\perp is the transverse size given by geometry





Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$



Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad Q_s(x) = Q_0 \left(\frac{x_0}{x} \right)^{\lambda/2}$$



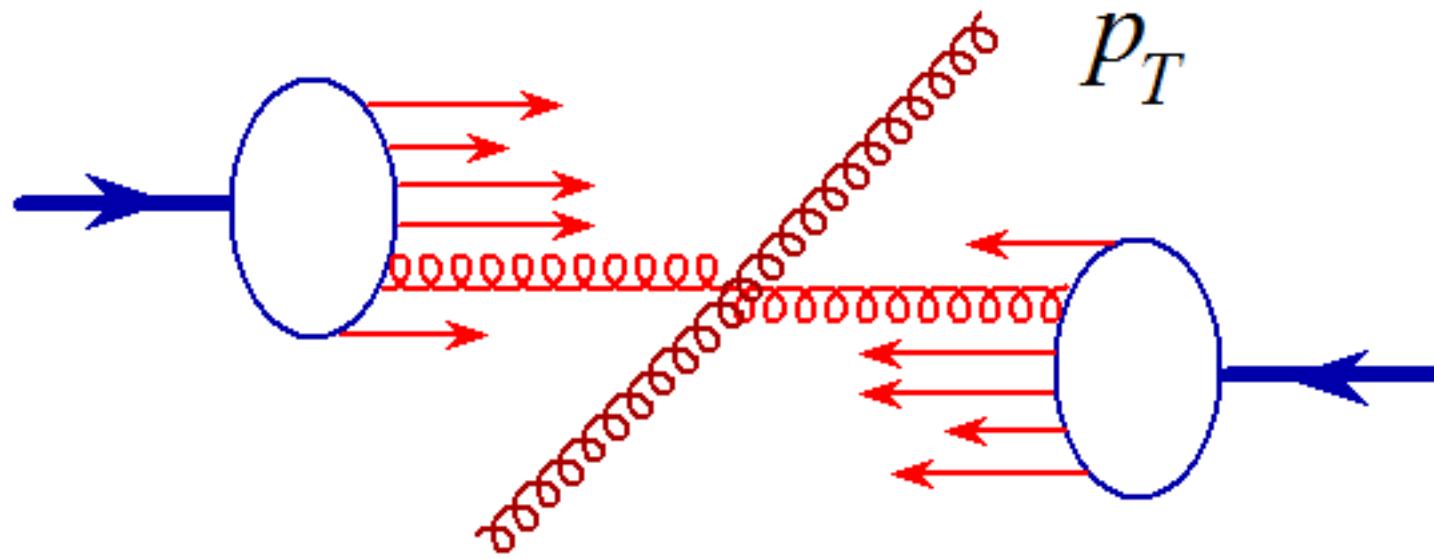
Basics of geometrical scaling

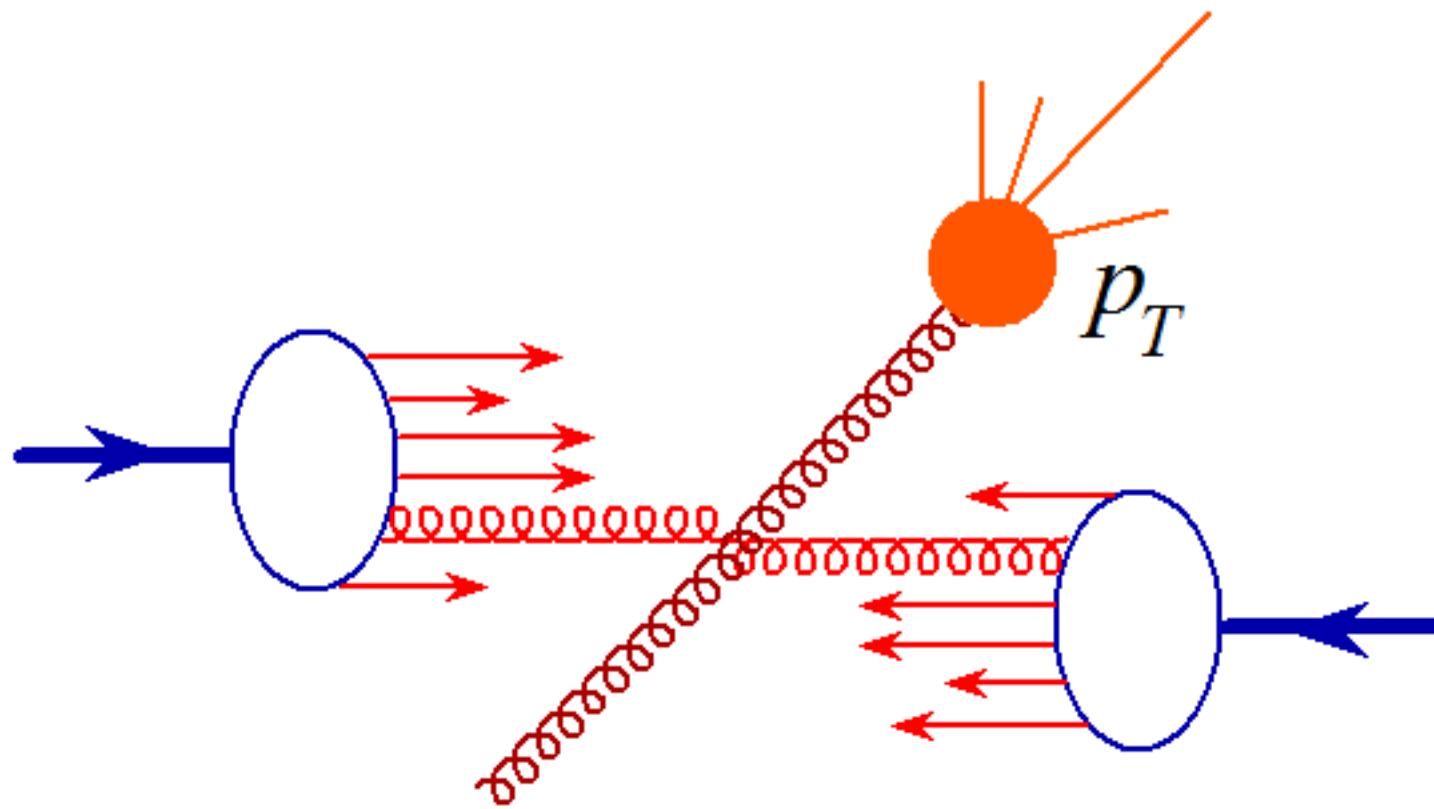
for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

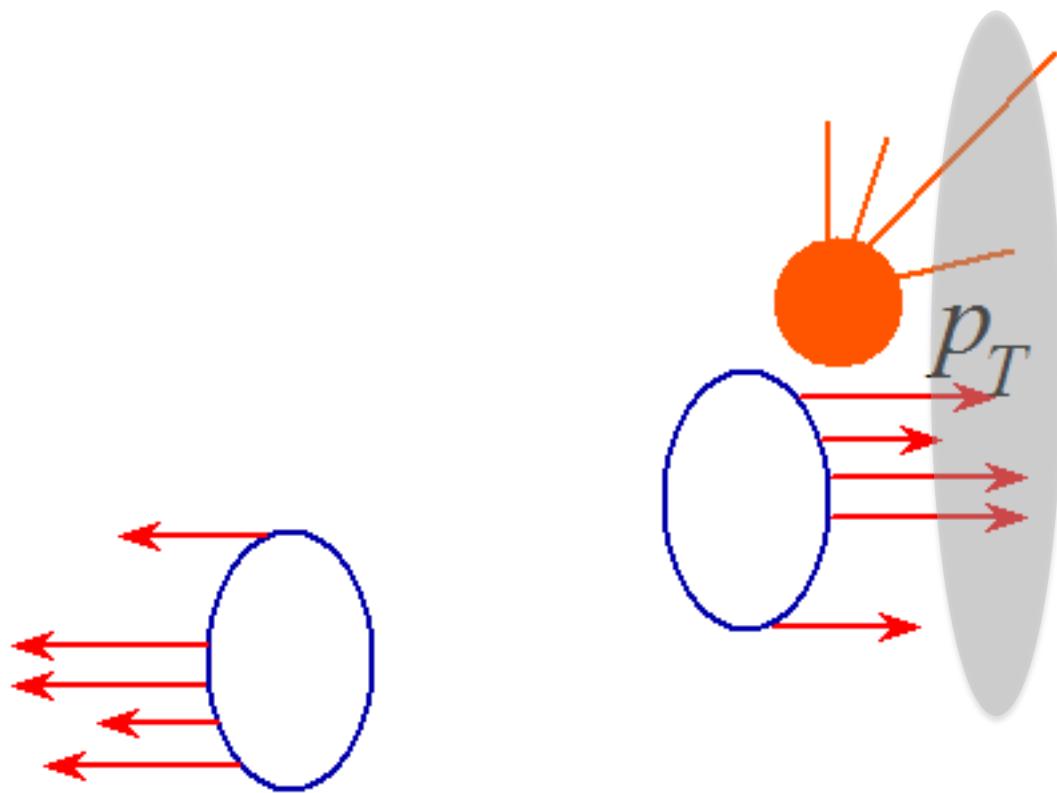
$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad Q_s(x) = Q_0 \left(\frac{x_0}{x} \right)^{\lambda/2}$$

parton – hadron duality:
power-like growth of
particle multiplicity









Geometrical scaling of p_T distributions

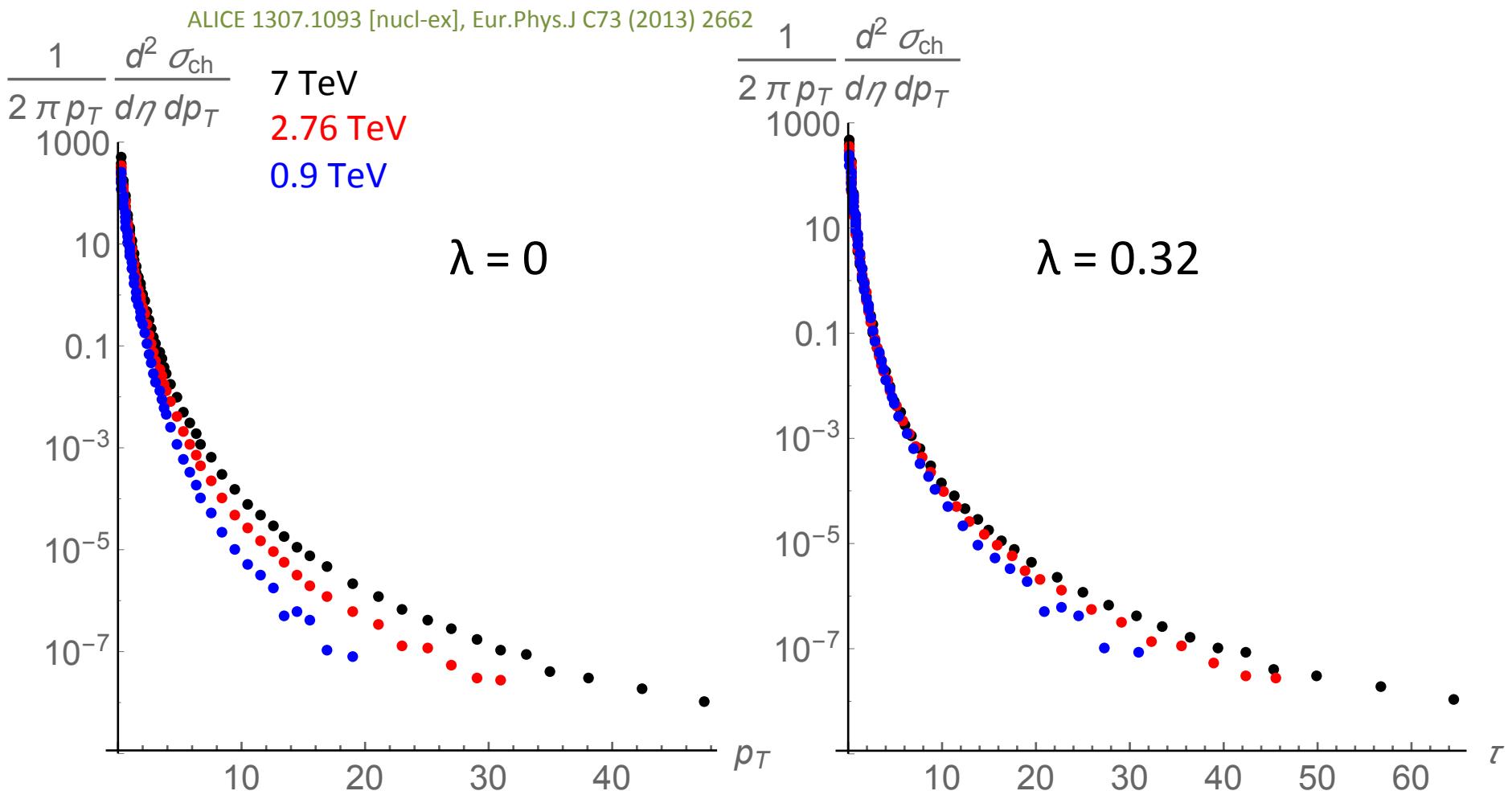
L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011

M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566
Phys.Rev. D87 (2013) 071502(R)

$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left(\frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$



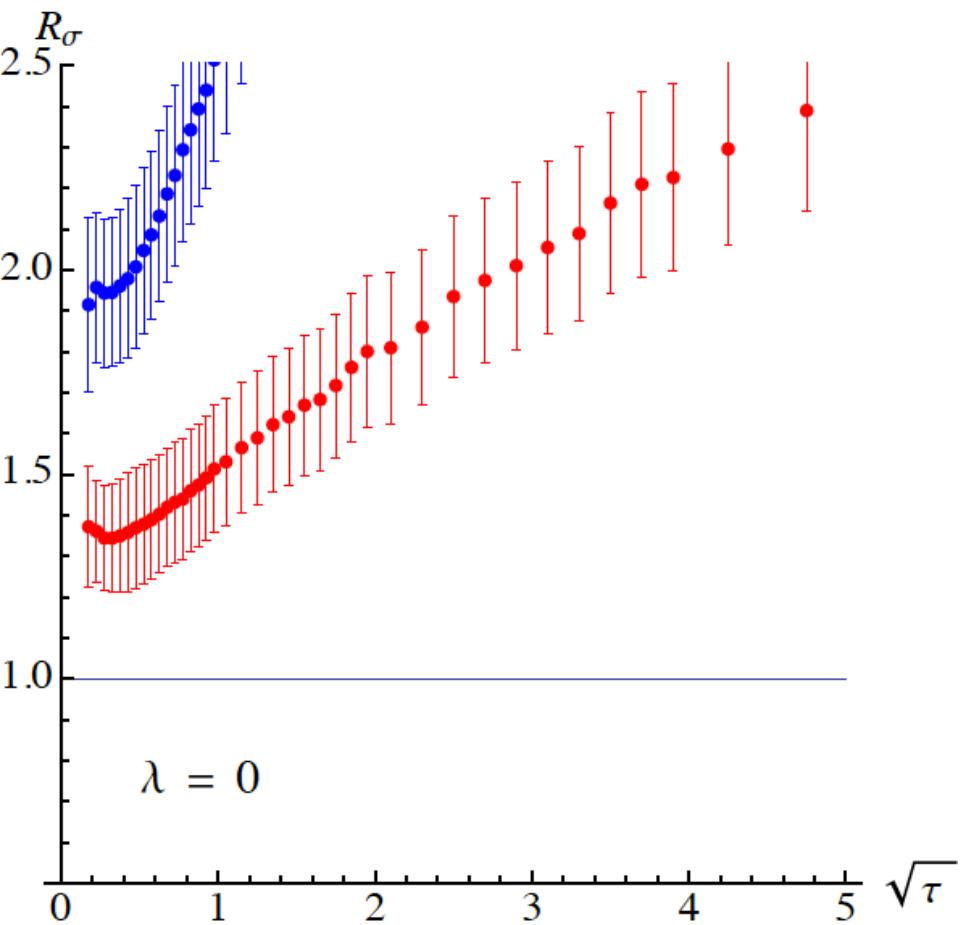
Cross-section scaling in pp





Determination of lambda

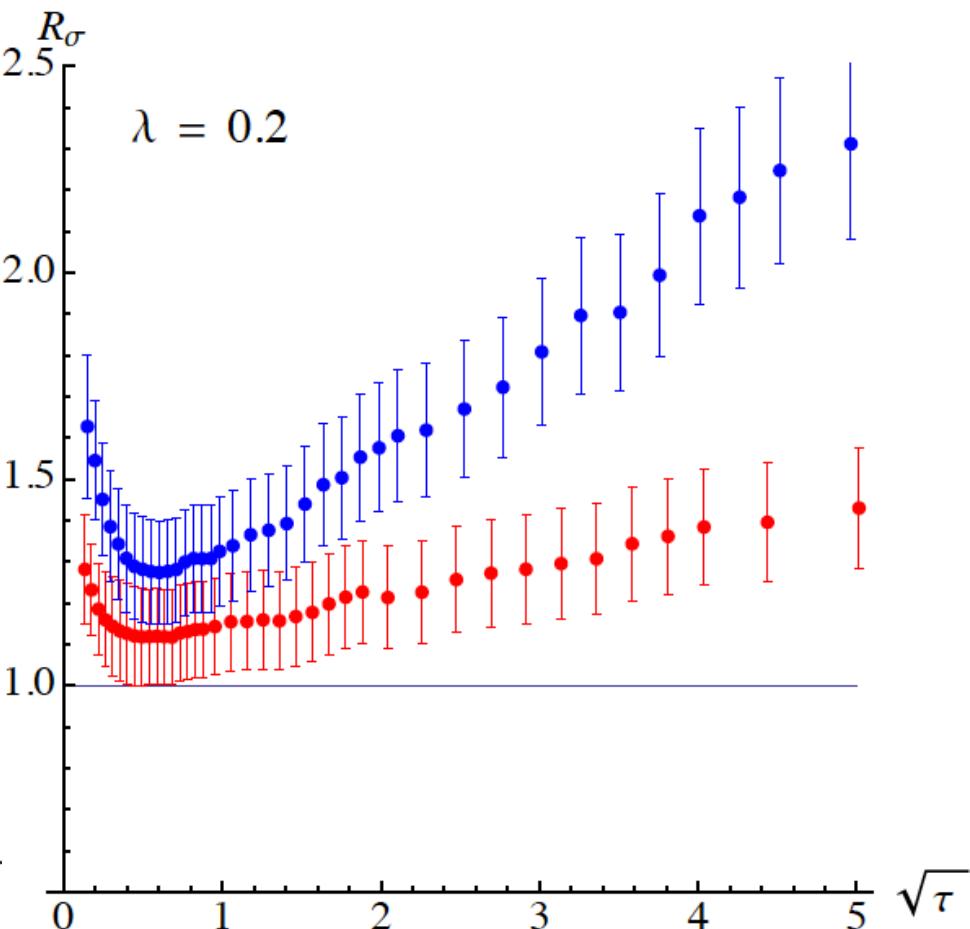
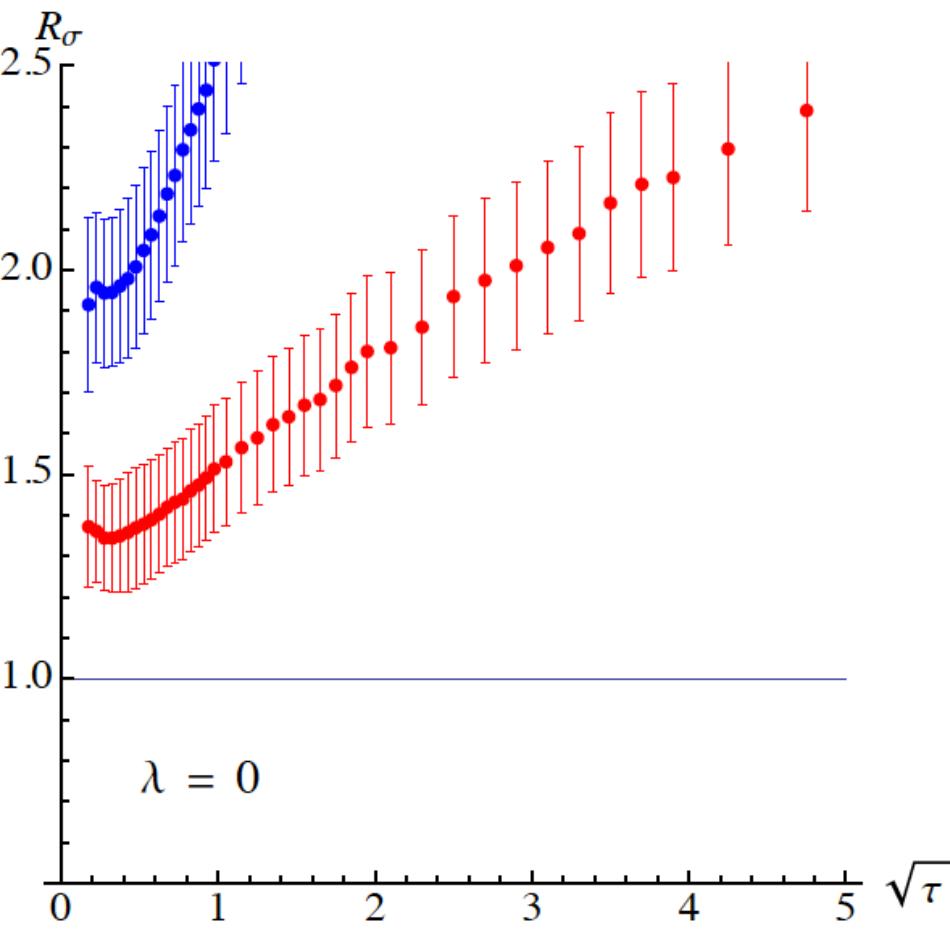
$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left(\frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$





Determination of lambda

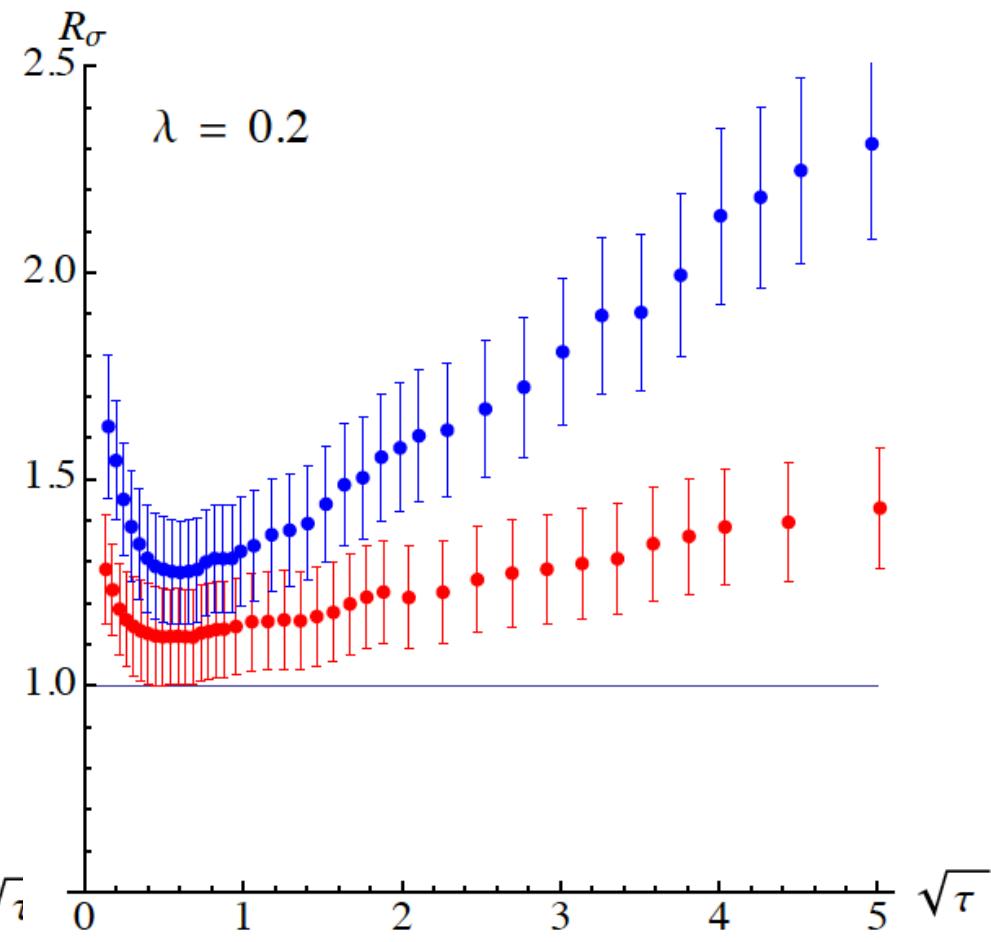
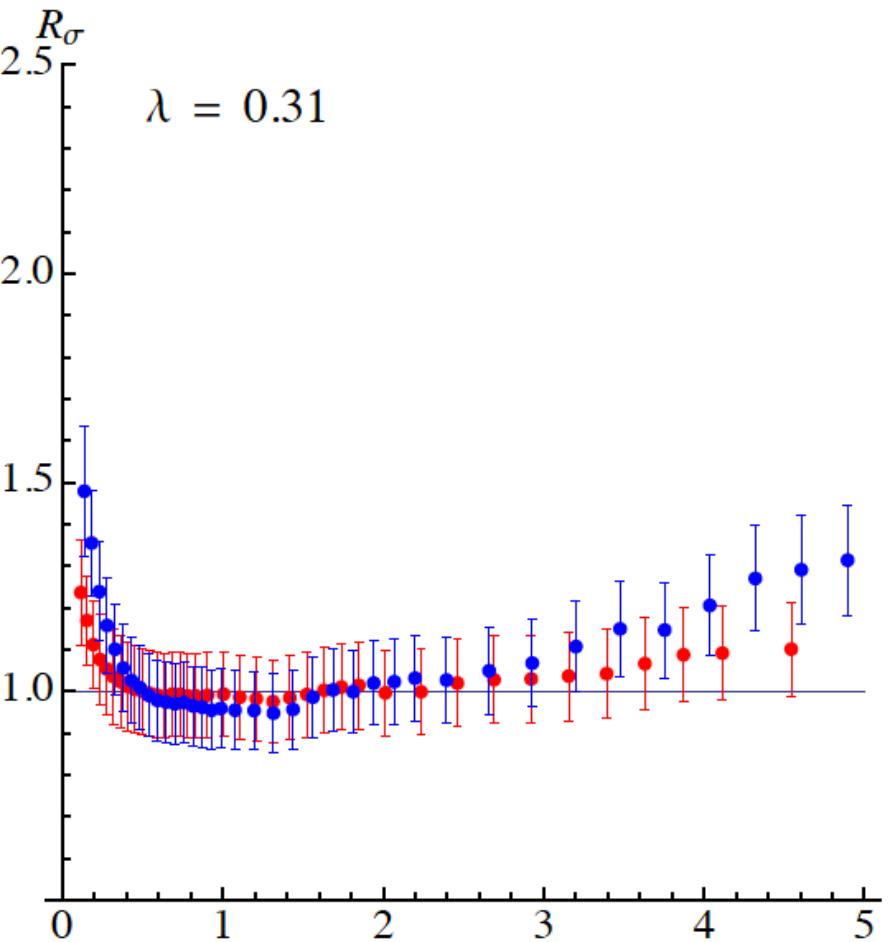
$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left(\frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$





Determination of lambda

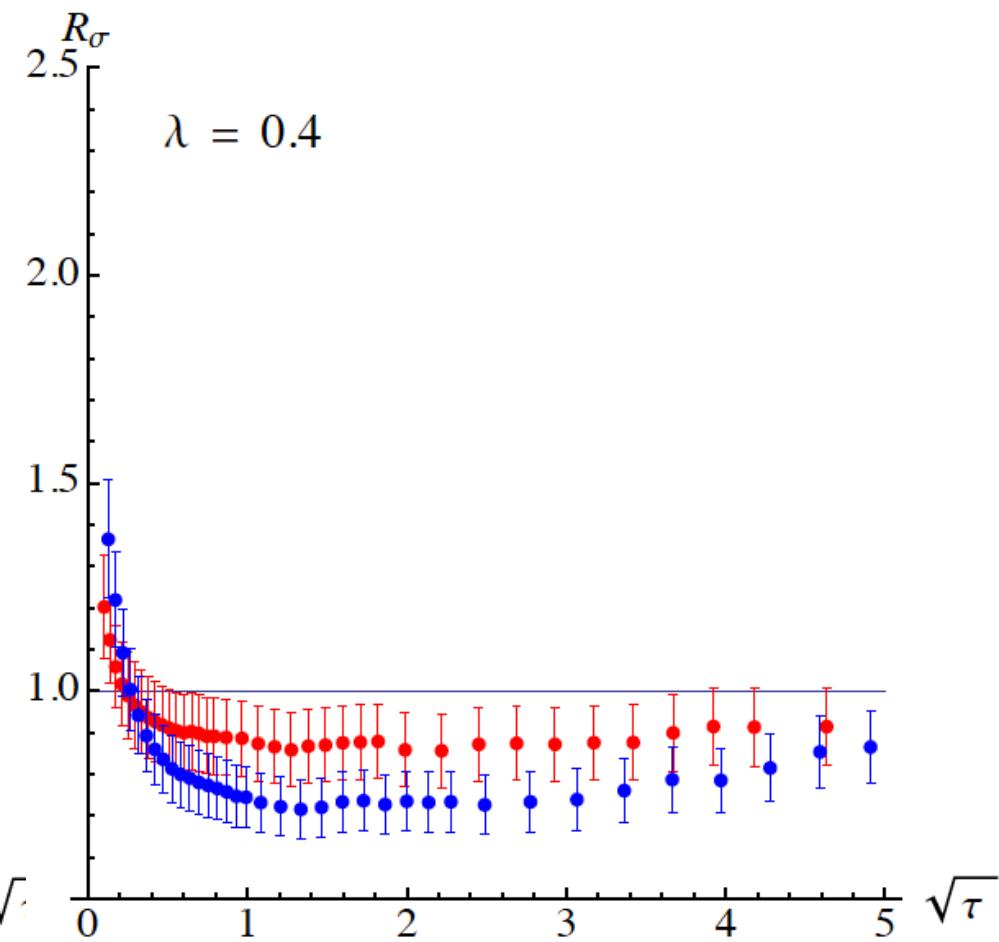
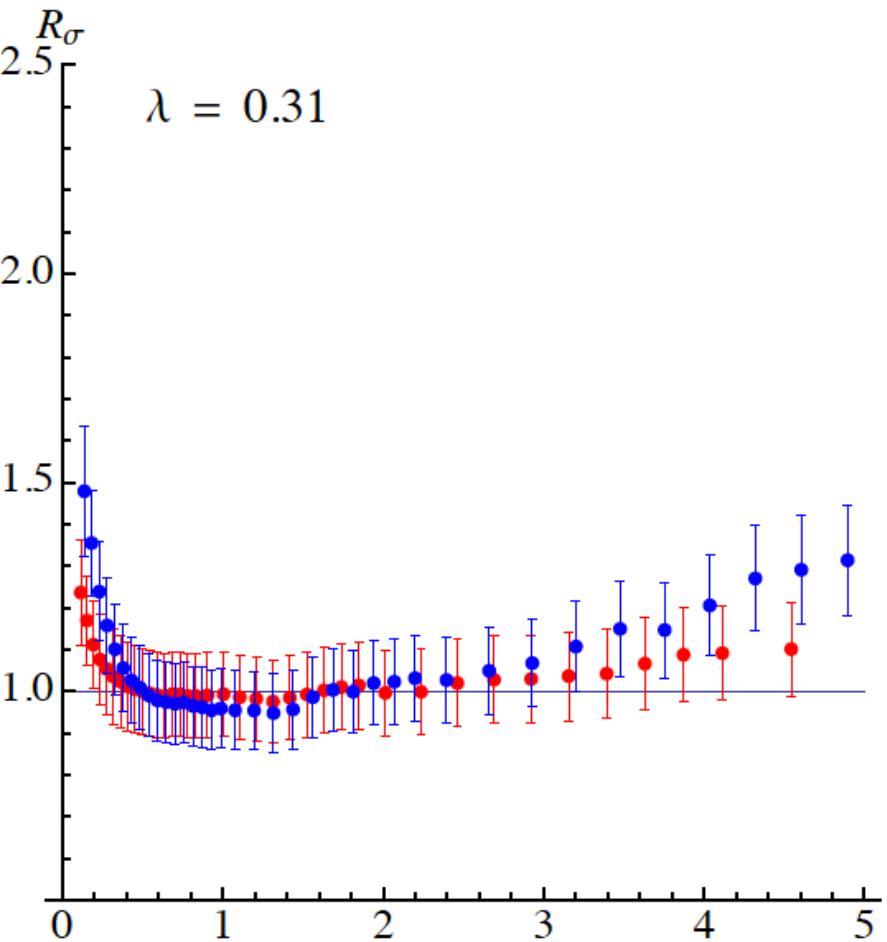
$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left(\frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$





Determination of lambda

$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left(\frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$





Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS



Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad dp_T^2 = \frac{2}{2+\lambda} \bar{Q}_s^2(W) \tau^{-\lambda/(2+\lambda)} d\tau$$

$$\bar{Q}_s(W) = Q_0 \left(\frac{W}{Q_0} \right)^{\lambda/(2+\lambda)}$$



Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad dp_T^2 = \frac{2}{2+\lambda} \bar{Q}_s^2(W) \tau^{-\lambda/(2+\lambda)} d\tau$$

$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_T = S_\perp^2 \bar{Q}_s^2(W) \int \mathcal{F}(\tau) \dots d\tau = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W)$$



Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad dp_T^2 = \frac{2}{2+\lambda} \bar{Q}_s^2(W) \tau^{-\lambda/(2+\lambda)} d\tau$$

$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_T = S_\perp^2 \bar{Q}_s^2(W) \int \mathcal{F}(\tau) \dots d\tau = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W)$$

$$\frac{d\sigma}{dy} = S_\perp \frac{dN}{dy} = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W) \rightarrow \bar{Q}_s^2(W) = \frac{\kappa}{S_\perp} \frac{dN}{dy}$$





Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2 \vec{k}_T}{Q_s^2(x)} \varphi_1 \left(\vec{k}_T^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_T^2 / Q_s^2(x) \right)$$

$$\frac{d\sigma}{dy d^2 p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \tau = \frac{p_T^2}{Q_s^2(x)} \quad dp_T^2 = \frac{2}{2+\lambda} \bar{Q}_s^2(W) \tau^{-\lambda/(2+\lambda)} d\tau$$

$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_T = S_\perp^2 \bar{Q}_s^2(W) \int \mathcal{F}(\tau) \dots d\tau = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W)$$

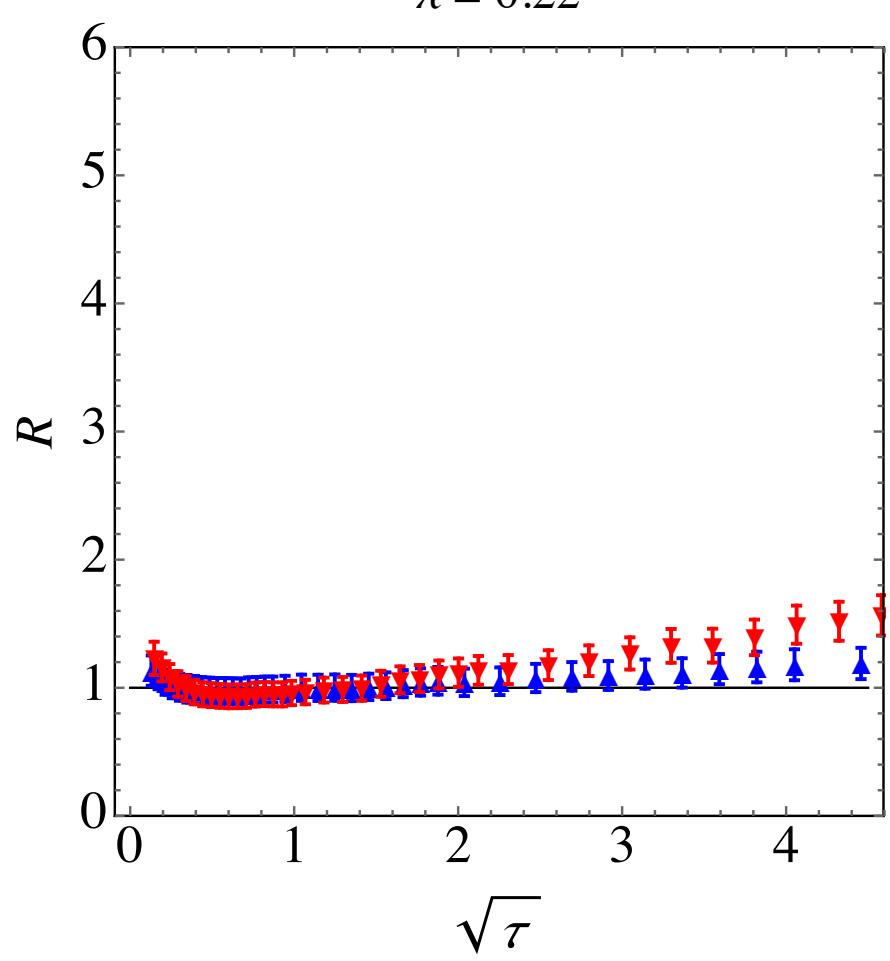
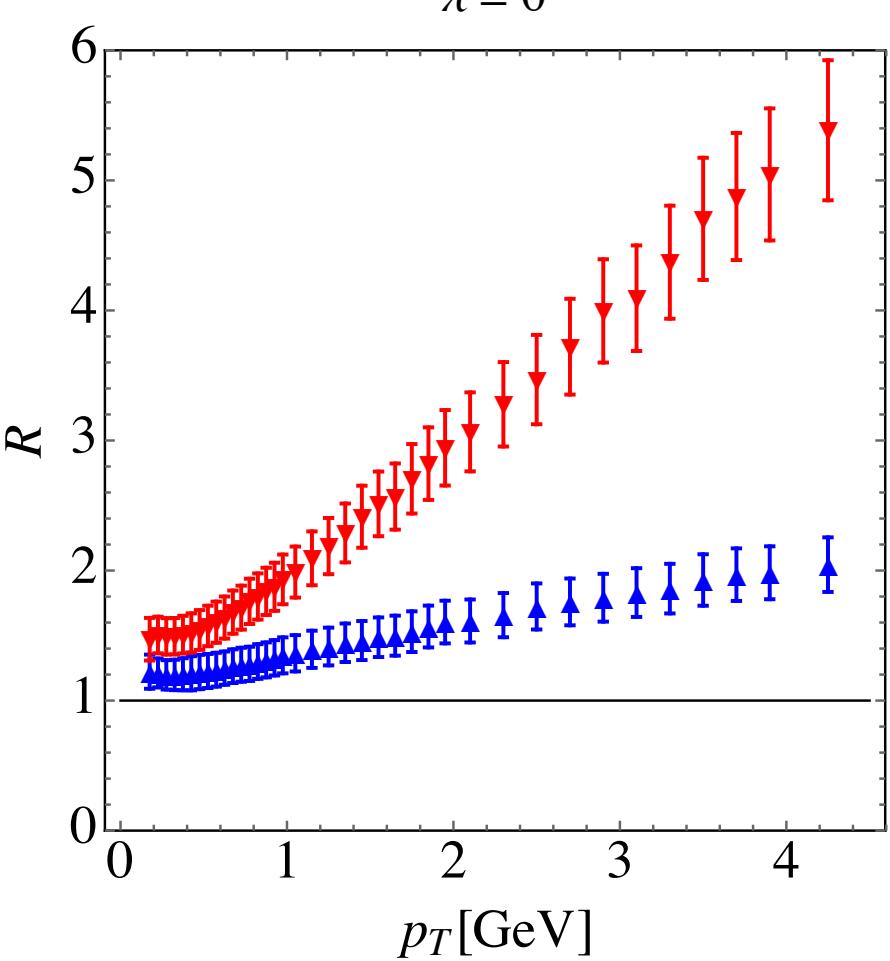
$$\frac{d\sigma}{dy} = \sigma^{\text{MB}}(W) \frac{dN}{dy} = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W) \rightarrow \boxed{S_\perp \frac{\bar{Q}_s^2(W)}{\sigma^{\text{MB}}(W)}} = \frac{\kappa}{S_\perp} \frac{dN}{dy}$$





Determination of lambda

$$\frac{dN_{\text{ch}}}{dy d^2 p_{\text{T}}} = S_{\perp} \mathcal{F}(\tau) \quad \tau = \frac{p_{\text{T}}^2}{Q_{\text{sat}}^2(p_{\text{T}}/\sqrt{s})} = \frac{p_{\text{T}}^2}{1 \text{ GeV}^2} \left(\frac{p_{\text{T}}}{\sqrt{s} \times 10^{-3}} \right)^{\lambda}$$





Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with $\lambda \sim 0.22$ (!)

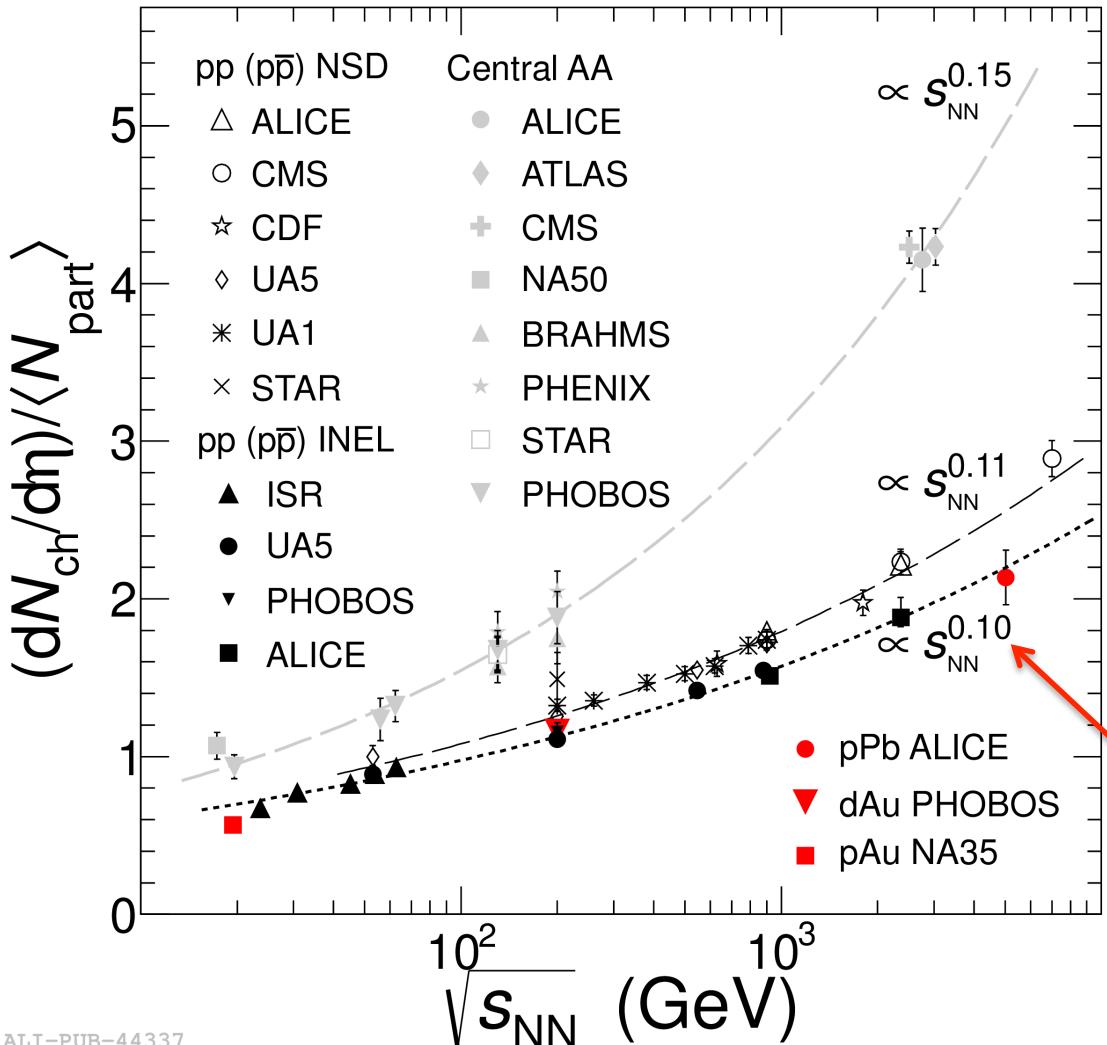


continue
with multiplicity scaling...



Power-like growth of multiplicity

http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf



plot: P. Braun-Munzinger,
54 Cracow School of
Theoretical Physics
(from ALICE-PUB-44337)

$$\frac{dN_{\text{ch}}}{dy} \sim S_{\perp} \bar{Q}_s^2(W)$$
$$\sim S_{\perp} Q_0^2 \left(\frac{W^2}{Q_0^2} \right)^{\lambda/(2+\lambda)}$$

transverse area is energy independent

$\lambda/(2 + \lambda) \simeq 0.099$



Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with $\lambda \sim 0.22$ (!)
- As a consequence total multiplicity grows with energy as $s^{0.1}$



Application to pA scattering at the LHC



Color Glass Condensate in pPB

stolen from Bozek, Bzdak, Skokov

$$\frac{dN}{dy} = S_\perp Q_p^2 \left(2 + \ln \frac{Q_A^2}{Q_p^2} \right)$$

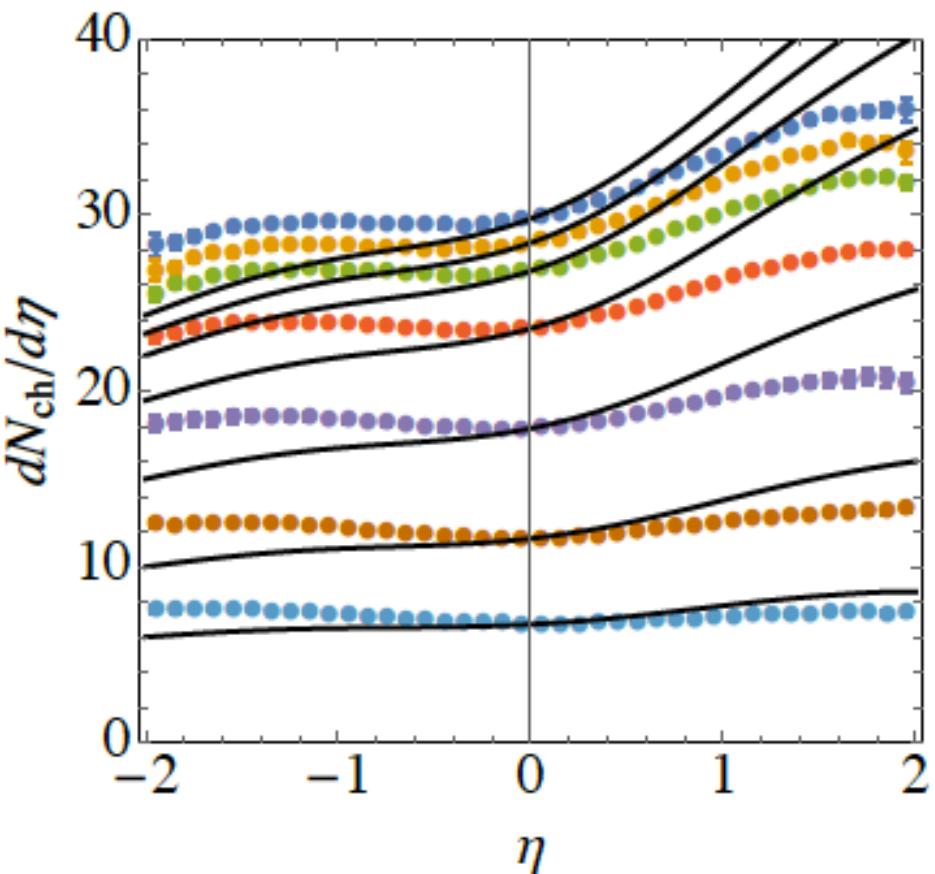
$$Q_p^2(W, y) = Q_0^2 \left(\frac{W}{W_0} \right)^\lambda \exp(\lambda y),$$

$$Q_A^2(W, y) = Q_0^2 N_{\text{part}} \left(\frac{W}{W_0} \right)^\lambda \exp(-\lambda y)$$

$$\lambda = 0.32$$



Multiplicity for pPb



$$\frac{dN_{\text{ch}}}{dy} = S_\perp Q_p^2 \left(2 + \ln \frac{Q_A^2}{Q_p^2} \right)$$

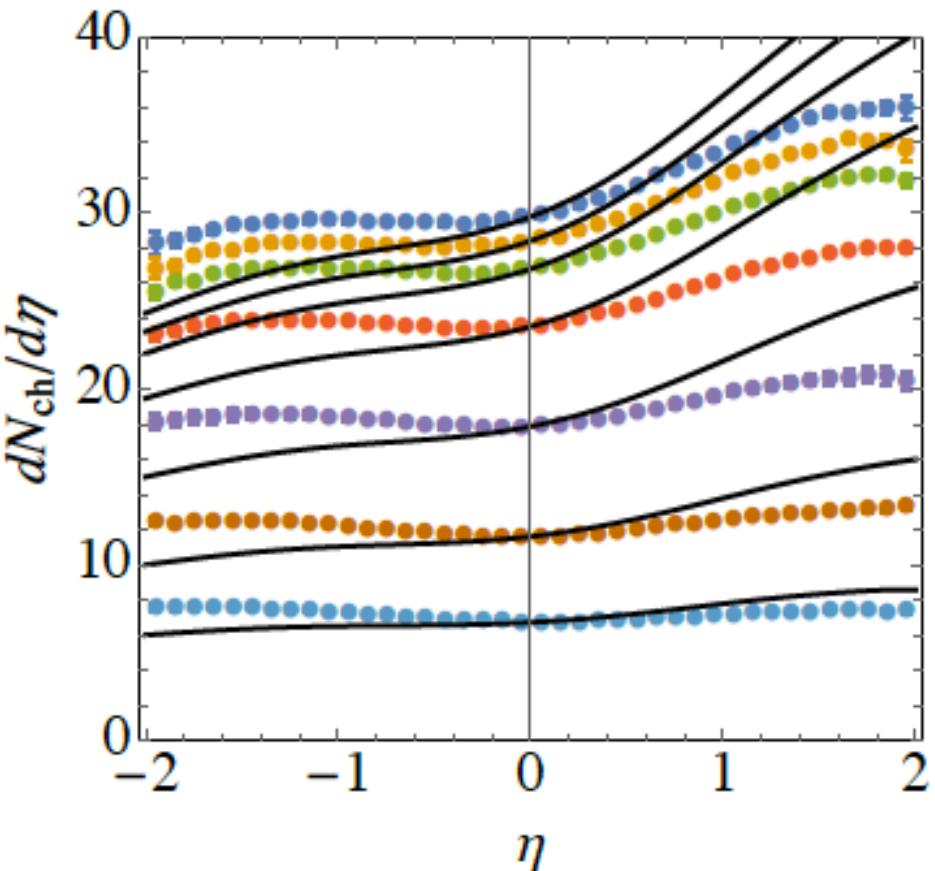
ZNA method

wounded nucleon model
does not work fot pA at the LHC

J. Adam *et al.* [ALICE Collaboration], Phys. Rev. C 91 (2015) 064905.



Multiplicity for pPb



$$\frac{dN_{\text{ch}}}{dy} = S_{\perp} Q_p^2 \left(2 + \ln \frac{Q_A^2}{Q_p^2} \right)$$

Allow Q_p to fluctuate:

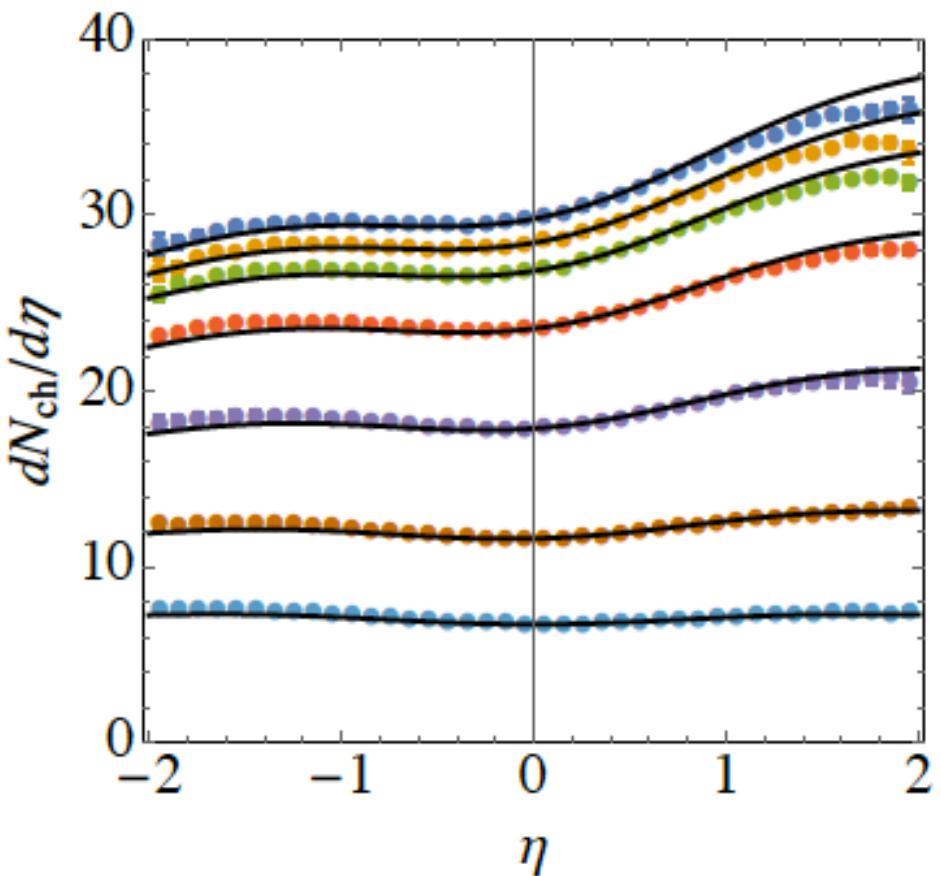
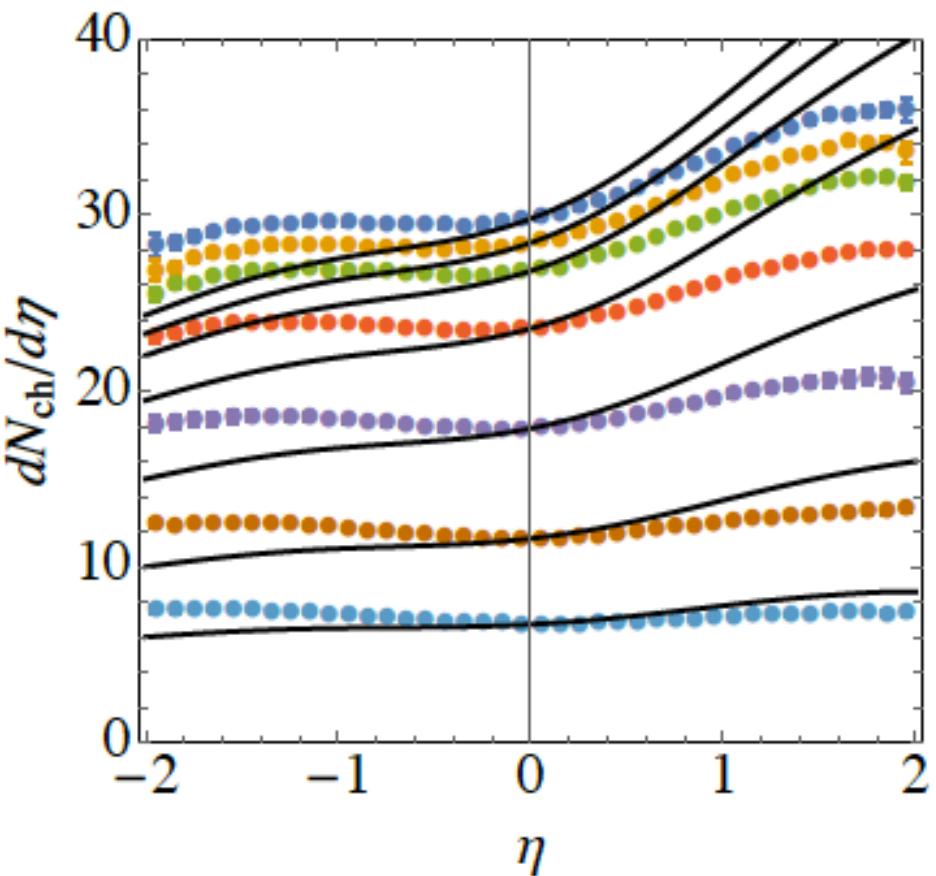
$$\frac{1}{\sqrt{2\pi}\sigma} \exp \left(-\frac{(\ln Q_p^2/Q_0^2 - \ln \bar{Q}_p^2/Q_0^2)^2}{2\sigma^2} \right)$$

E. Iancu, A. H. Mueller, S. Munier
Phys. Lett. B 606 (2005) 342

J. Adam *et al.* [ALICE Collaboration], Phys. Rev. C 91 (2015) 064905.



Fluctuations of Q_{sat} in pPb



rather large $\sigma \sim 1.55$



Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with $\lambda \sim 0.22$ (!)
- As a consequence total multiplicity grows with energy as $s^{0.1}$
- Fluctuations of the saturation scale may explain dN/dy



Conclusions

- Nonlinear BK equation generates saturation scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with $\lambda \sim 0.22$ (!)
- As a consequence total multiplicity grows with energy as $s^{0.1}$
- Fluctuations of the saturation scale may explain dN/dy



Not discussed

- Consequences of GS for F_L
- Scaling violations in pp due to $y \neq 0$
- Scaling violations in pp due to $\lambda(Q^2)$
- Scaling in pp for identified particles
- Connection with Tsallis distribution
- $\langle p_T \rangle(N)$ for identified particles
- geometrical scaling predicts energy dependence of $\langle p_T \rangle$
- $\langle p_T \rangle(N_{\text{ch}})$ difficult to describe by untuned MonteCarlos
- scaling of $\langle p_T \rangle(N_{\text{ch}})$ induced by energy dependence of Q_{sat} + CGC
- GS in heavy ion collisions – scaling with energy and N_{part}

Workshop on QCD and Diffraction

saturation 1000+

5-7 December 2016
Cracow, Poland

Organising Committee:

Wojciech Broniowski
Janusz Chwastowski
Krzysztof Kutak
Michał Praszałowicz

Christophe Royon
Anna Staśto
Rafał Staszewski



»»>KRK>2B
Krakow to business

