NLO+PS matching and perspectives for NLO PS

Sebastian Sapeta

IFJ PAN Kraków

in collaboration with:
S. Jadach, G. Nail, W. Płaczek, A. Siódmok and M. Skrzypek

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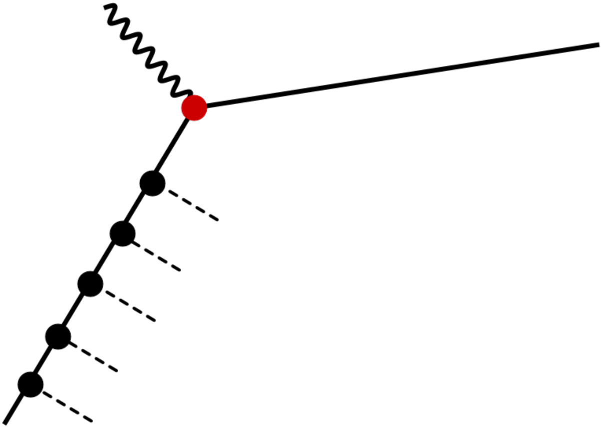
MC4BSM, SLAC, 13 May 2017

Big goal



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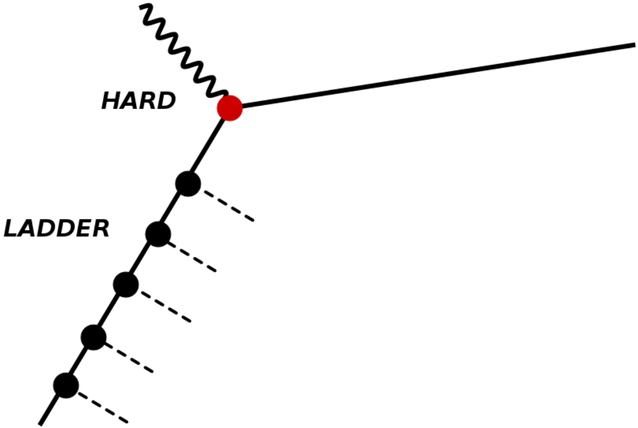

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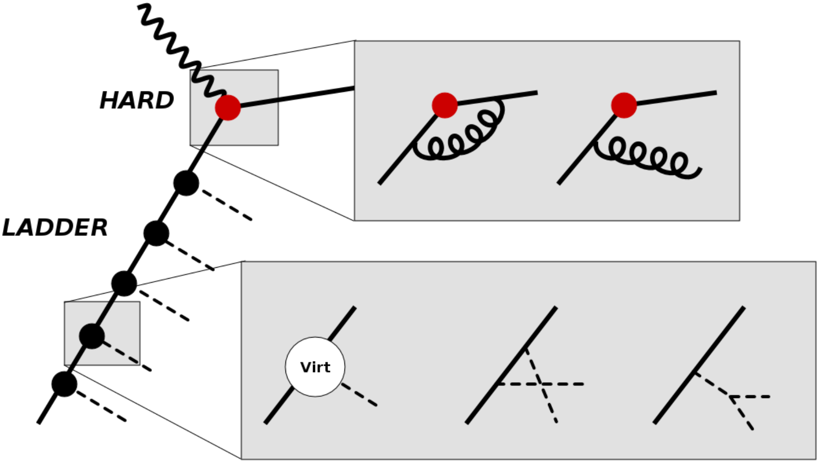

Outline and motivation

1. **A method of NLO+PS matching applied to Drell-Yan and Higgs production**

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   Key ingredients:

   - new factorization scheme leading to new PDFs
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   Why do we develop a new method?

   - By departing from $\overline{\text{MS}}$, the NLO+PS matching becomes very simple $\rightarrow$ just multiplying by a positive MC weight.
   - If it is so simple at NLO+LO PS, there is a hope that pushing it to NNLO+NLO PS will be possible.

2. **Status of the NLO parton shower development**

Benefits of matching fixed order results with parton shower

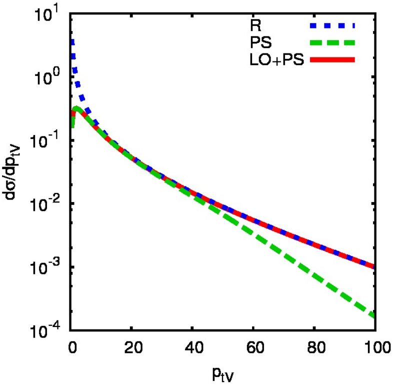
PS gives correct behaviour at low p_T and only approximate at high p_T.
The production of a gluon with p_{Tg} is given by

$$d\sigma_1^{\mathrm{PS}} = B \cdot K(p_{Tg})\, \Delta(Q, p_{Tg})\, d\phi_B d\phi_1$$

where $B \cdot K \simeq R$ and the Sudakov $\Delta(Q, p_{Tg})$ suppresses emissions between scales Q and p_{Tg}.

LO+PS can be achieved by upgrading $B \cdot K$ to the exact R

$$d\sigma_1^{\mathrm{LO+PS}} = R(p_{Tg}) \Delta(Q, p_{Tg})\, d\phi_B d\phi_1$$



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Naive addition of PS on top of a NLO event leads to double counting since PS will generate contributions already present at NLO!

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- MC@NLO [Frixione & Webber '02] and POWHEG [Nason '04]
  - Generate the hardest radiation based on the NLO cross section adjusted for subsequent shower emissions.
  - Pass the event to parton shower and let it produce further emissions.

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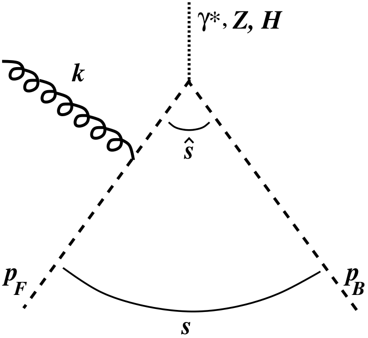
- KrkNLO [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13; Jadach, Płaczek, Sapeta, Siódmok & Skrzypek '15 – '17]
  - Run PS in a standard way.
  - Reweight the event with real×virtual NLO correction.
  - Redefine PDFs to account for "collinear" part of the NLO contribution.

The KrkNLO method

Production of a colour-neutral object



$$s = (p_F + p_B)^2$$

$$z = \frac{\hat{s}}{s}$$

Sudakov variables:

$$\alpha = \frac{2k \cdot p_B}{s} = \frac{2k^+}{\sqrt{s}}$$

$$\beta = \frac{2k \cdot p_F}{s} = \frac{2k^-}{\sqrt{s}}$$

$$z = 1 - \alpha - \beta$$

$$k_T^2 = s\alpha\beta$$

$$y = \frac{1}{2} \ln \frac{\alpha}{\beta}$$

Important subtlety

NLO cross section in $\overline{\text{MS}}$ factorization scheme (DY in $q\bar{q}$ channel)

$$d\sigma_{\text{DY}}^{\alpha_s} \quad = \quad \sigma_{\text{DY}}^{B}\, f_q^{\overline{\text{MS}}}(x_1, \hat{s}) \otimes \frac{\alpha_s}{2\pi} C_{q\bar{q}}^{\overline{\text{MS}}}(z) \otimes f_{\bar{q}}^{\overline{\text{MS}}}(x_2, \hat{s})\,,$$

where

$$C_{q\bar{q}}^{\overline{\text{MS}}}(z) = C_F \left[4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3}\pi^2 - 8 \right) \right].$$

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- We want to reproduce this with Monte Carlo, in a fully exclusive way.

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- We want to reproduce this with Monte Carlo, in a fully exclusive way.

If we use $\overline{\text{MS}}$ PDFs, we need to generate terms like $\sim \left(\frac{\ln(1-z)}{1-z}\right)_+$ which are technical artefacts of $\overline{\text{MS}}$ scheme (coming from ϵ/ϵ contributions).

- If we think of a parton shower as a procedure that unfolds PDFs, then, obviously, these are not $\overline{\text{MS}}$ PDFs!

The KrkNLO method

Two essential elements

1. **Change the factorization scheme from $\overline{\text{MS}}$ to MC**
   - produce new MC PDFs
   - differences at LO
   - universality: recovering $\overline{\text{MS}}$ NLO result
2. **Reweight parton shower**
   - correct hardest emission by "real" weight
   - upgrade the cross section/distributions to NLO by multiplicative, constant "virtual" weight

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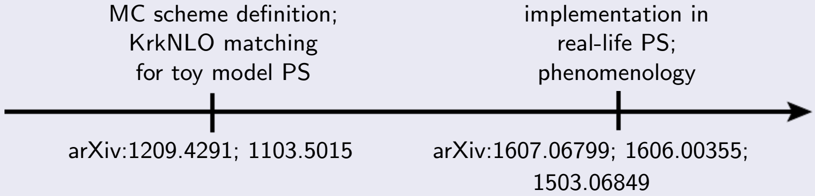
Timeline



Practical implementation for the CS shower

Implemented in Sherpa/Herwig 7 with the Catani-Seymour (CS) shower.

$\hookrightarrow$ available in the new H7 release; `http://krknlo.hepforge.org`

- The CS shower covers the entire space of (α, β).
- The evolution variable is: $q^2 \simeq \alpha\, \beta\, s = k_T^2$.
- Transformation of PDFs (example for $q\bar{q}$ channel)

$$\begin{aligned} C_{q\bar{q}}^{\overline{\mathrm{MS}}} &= \rho_{q\bar{q}}^{\mathrm{NLO}} - \Gamma_{q\bar{q}}^{\overline{\mathrm{MS}}} \\ C_{q\bar{q}}^{\mathrm{MC}} &= \rho_{q\bar{q}}^{\mathrm{NLO}} - \Gamma_{q\bar{q}}^{\mathrm{MC}} \end{aligned} \quad \Rightarrow \quad K_{q\bar{q}}^{\mathrm{MC}} = C_{q\bar{q}}^{\overline{\mathrm{MS}}} - C_{q\bar{q}}^{\mathrm{MC}}$$

which gives

$$q_{\mathrm{MC}}(x, Q^2) = q_{\overline{\mathrm{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, q_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) K_{qq}^{\mathrm{MC}}(z)$$

- Virtual correction applied multiplicatively.
- The hardest real emission is upgraded to ME by reweighting.

PDFs in MC scheme

Definition of LO PDFs in MC factorization scheme

Rotation in flavour space:

$$\begin{bmatrix} q(x,Q^2) \\ \bar{q}(x,Q^2) \\ g(x,Q^2) \end{bmatrix}_{\text{MC}} = \begin{bmatrix} q \\ \bar{q} \\ g \end{bmatrix}_{\overline{\text{MS}}} + \frac{\alpha_s}{2\pi}\int \frac{dz}{z} \begin{bmatrix} K^{\text{MC}}_{qq}(z) & 0 & K^{\text{MC}}_{qg}(z) \\ 0 & K^{\text{MC}}_{\bar{q}\bar{q}}(z) & K^{\text{MC}}_{\bar{q}g}(z) \\ K^{\text{MC}}_{gq}(z) & K^{\text{MC}}_{g\bar{q}}(z) & K^{\text{MC}}_{gg}(z) \end{bmatrix} \begin{bmatrix} q(\frac{x}{z},Q^2) \\ \bar{q}(\frac{x}{z},Q^2) \\ g(\frac{x}{z},Q^2) \end{bmatrix}_{\overline{\text{MS}}}$$

where

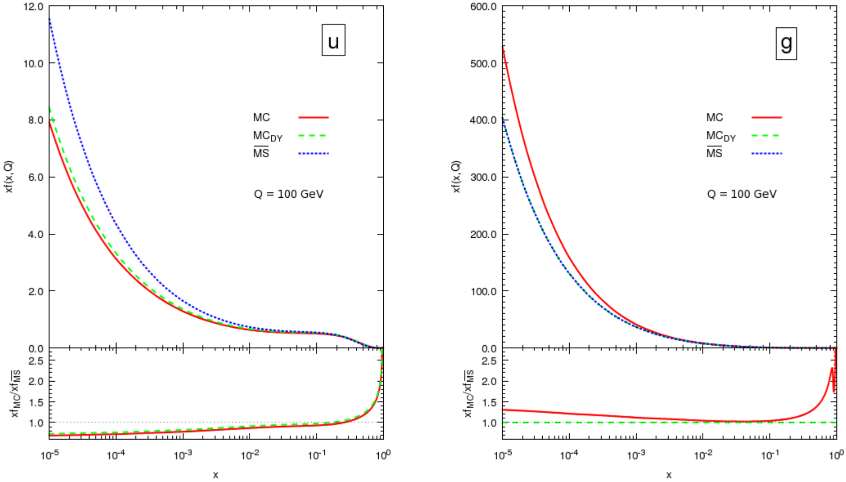
$$K^{\text{MC}}_{gq}(z) = C_F \left\{ \frac{1+(1-z)^2}{z} \ln\frac{(1-z)^2}{z} + z \right\}$$

$$K^{\text{MC}}_{gg}(z) = C_A \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ + 2\left[\frac{1}{z} - 2 + z(1-z)\right] \ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36}\frac{T_f}{C_A}\right) \right\}$$

$$K^{\text{MC}}_{qq}(z) = C_F \left\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ - (1+z)\ln\frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} + 1 - z - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{17}{4}\right) \right\}$$

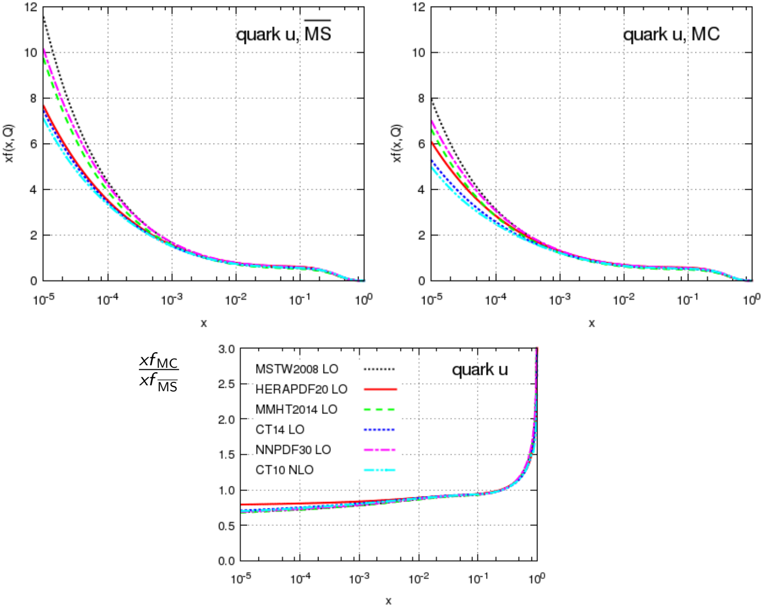
$$K^{\text{MC}}_{qg}(z) = T_R \left\{ \left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z) \right\}$$

MC PDFs



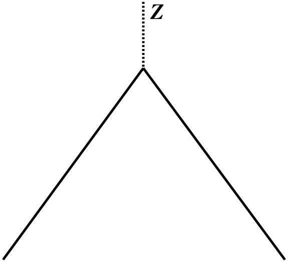
- More gluons and less quarks at low x: momentum sum rules preserved!
- We checked directly the scheme independence of NLO cross sections!

Differences between $\overline{\text{MS}}$ PDF sets carry on to MC PDFs



Reweighting the parton shower

Upgrading to NLO: the hardest emission



$$\sigma^{\mathrm{LO}} = \sigma_B \otimes f_{\oplus}(Q^2, x_{\oplus}) \otimes f_{\ominus}(Q^2, x_{\ominus})$$

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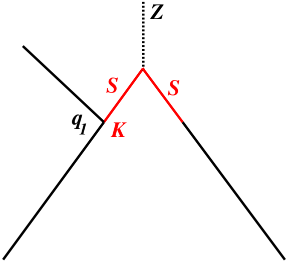
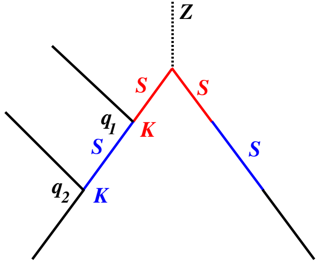


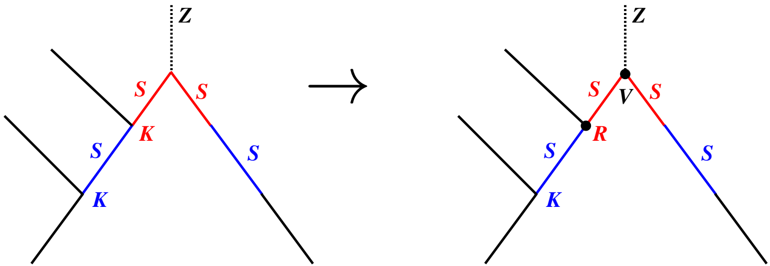
$$\sigma_{1+}^{\mathrm{PS}} = \sigma_B \otimes f_{\oplus}(Q^2, x_{\oplus}) \otimes f_{\ominus}(Q^2, x_{\ominus})$$
$$\otimes \Big\{ S_{\oplus}(q_1^2, Q^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(q_1^2, Q^2) + S_{\ominus}(q_1^2, Q^2) K_{\ominus}(q_1^2, z_1) S_{\oplus}(q_1^2, Q^2) \Big\}$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{PS}} &= \sigma_B \otimes f_{\oplus}(Q^2, x_{\oplus}) \otimes f_{\ominus}(Q^2, x_{\ominus}) \\
&\otimes \Big\{ S_{\oplus}(q_1^2, Q^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(q_1^2, Q^2) \\
&\quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \\
&\quad + S_{\ominus}(q_1^2, Q^2) \otimes K_{\ominus}(q_1^2, z_1) \otimes S_{\oplus}(q_1^2, Q^2) \\
&\quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \Big\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission



$$\sigma_{2+}^{\mathrm{NLO+PS}} = \sigma_B\,(1+V) \otimes f_\oplus(Q^2, x_\oplus) \otimes f_\ominus(Q^2, x_\ominus)$$
$$\otimes \Big\{ S_\oplus(q_1^2, Q^2) K_\oplus(q_1^2, z_1) S_\ominus(q_1^2, Q^2)\, R_\oplus(q_1^2, z_1)/K_\oplus(q_1^2, z_1)$$
$$\otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\}$$
$$+\; S_\ominus(q_1^2, Q^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(q_1^2, Q^2)\, R_\ominus(q_1^2, z_1)/K_\ominus(q_1^2, z_1)$$
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The MC weights

Real:

$$W_R^{q\bar{q}} = 1 - \frac{2\alpha\beta}{1+z^2} \qquad W_R^{qg} = 1 + \frac{\beta(\beta+2z)}{(1-z)^2+z^2}$$

$$W_R^{gg} = \frac{1+z^4+\alpha^4+\beta^4}{1+z^4+(1-z)^4} \qquad W_R^{gq} = \frac{1+\beta^2}{1+(1-z)^2}$$

Virtual:

$$W_V^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 + \frac{1}{2}\right] \qquad W_V^{qg} = 0$$

$$W_V^{gg} = \frac{\alpha_s}{2\pi} C_A \left[\frac{4}{3}\pi^2 + \frac{473}{36} + \frac{59}{18}\frac{T_f}{C_A}\right] \qquad W_V^{gq} = 0$$

- **Real weights are simple functions of kinematic variables**
  One can compute them on the fly, inside MC, or outside, using information from event record.
- **Virtual+soft weights are constant**

Results: Drell-Yan

Drell-Yan: calculational setup

KrkNLO

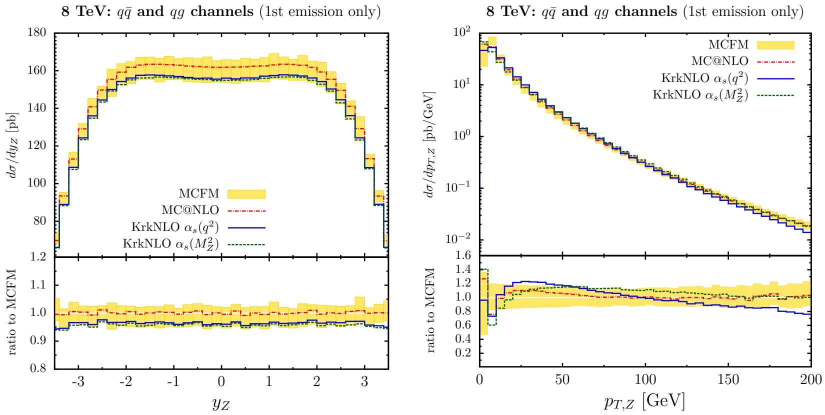
- $\mu_F^2 = m_Z^2$
- Virtual: $\mu_R^2 = m_Z^2$
- Real: two choices
  - $\mu_R^2 = m_Z^2$
  - $\mu_R^2 = q^2$, where $q \simeq k_T$ is the PS evolution variable

$\hookrightarrow$ differences formally beyond NLO, indicative of missing higher orders

Compared to:

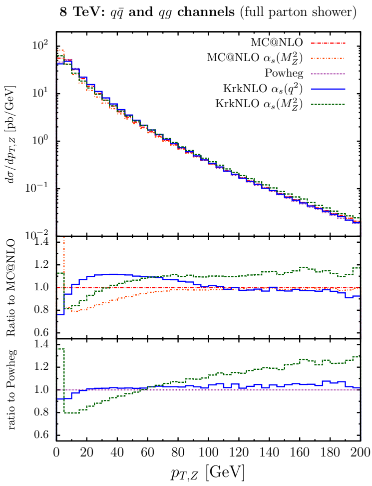
- **MCFM**: pure NLO, $\mu_F^2 = \mu_R^2 = m_Z^2$
- **MC@NLO**: from Sherpa/Herwig 7, with the evolution var. $q^2 \simeq k_T^2$
- **POWHEG**: from Herwig 7 with the evolution variable k_T^2
- **DYNNLO**: pure NNLO, $\mu_F^2 = \mu_R^2 = m_Z^2$

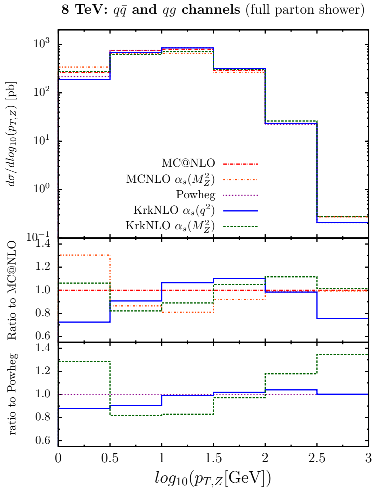
Drell-Yan: Matched results, botch channels, 1st emission



- Moderate differences between KrkNLO $\alpha_s(q^2)$ and MC@NLO in the region below M_Z and between KrkNLO $\alpha_s(M_Z^2)$ and MC@NLO in the region above M_Z

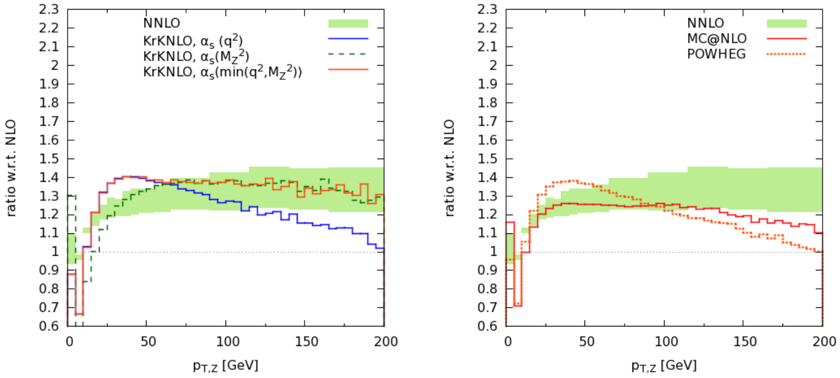
Drell-Yan: Matched results, both channels, full PS





- KrkNLO $\alpha_s(q^2)$ stays overall very close to MC@NLO
- KrkNLO $\alpha_s(q^2)$ almost coincides with POWHEG $p_{T,Z}$ distributions

Drell-Yan: Comparison to NNLO

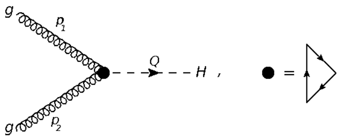


- KrkNLO with $\alpha_s(\min(q^2, M_z^2))$ nicely follows full NNLO at high $p_{T,Z}$
- the fact that the KrkNLO result is higher than NLO comes from partial accounting for $\mathcal{O}\left(\alpha_s^2\right)$ terms, those introduced by the multiplicative correction to the parton shower $R \otimes V$

Results: Higgs from gluon fusion

Higgs from gluon fusion: calculational setup

- heavy top effective vertex, $m_t \to \infty$
- $\sqrt{s} = 8$ TeV
- fully inclusive
- stable Higgs
- virtual part: $\mu_R^2 = m_H^2$
- real part: $\mu_R = \min(q^2, m_H^2)$

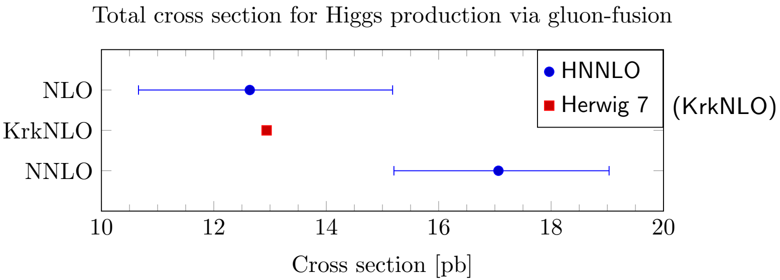


Comparisons to:

- MC@NLO
- POWHEG
  - default: p_T of PS emissions $< \mu_F$
  - original: no restriction on p_T of PS emissions
- HNNLO: fixed order result

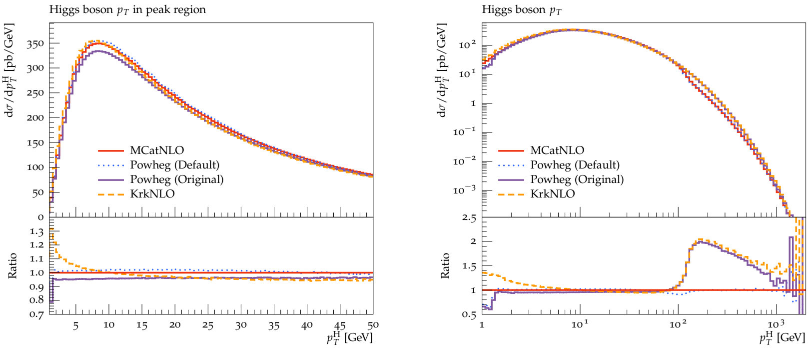
Total cross section

	MC@NLO	POWHEG		KrkNLO	HNNLO (NLO)
		Default	Original		
σ_{tot} [pb]	12.700 (2)	12.699 (2)	12.697 (2)	12.939 (2)	12.640 (1)



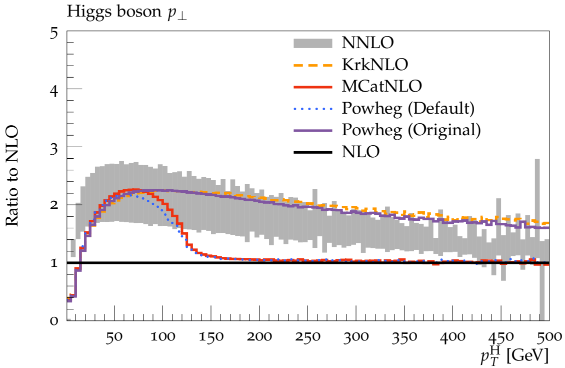
↪ 2% differences w.r.t. MC@NLO/POWHEG: $R \otimes V$ terms and $\overline{\mathrm{MS}} \to$ MC PDF transformation

Higgs transverse momentum distributions



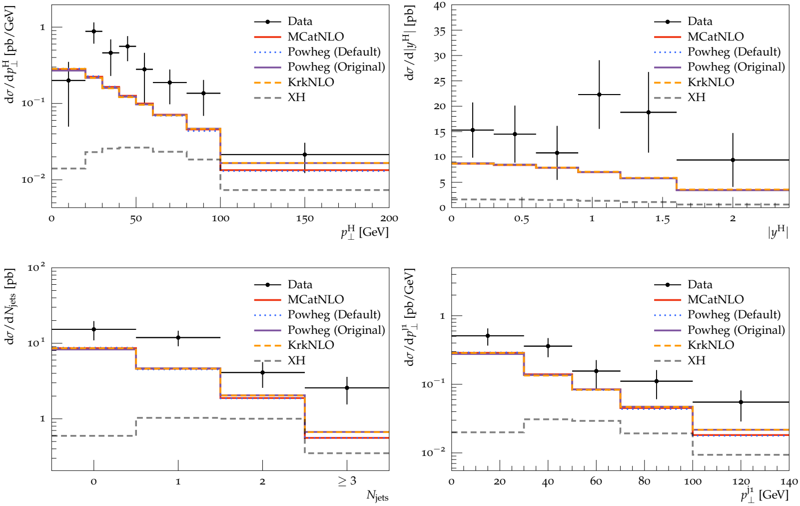
- similar results for all predictions in the range $5\,\text{GeV} < p_T < 100\,\text{GeV}$
- for $p_T > 100\,\text{GeV}$: KrkNLO differs from MC@NLO/Powheg (default), as, in the latter, PS emissions are restricted by $p_T < m_H$
- no such restriction in Powheg (orig), hence, this result is close to KrkNLO

Comparison to NNLO



- KrkNLO nicely follows full NNLO at high p_T^H; partial accounting for NNLO terms through $R \otimes V$ corrections applied to the parton shower

Comparison to data [8 TeV, ATLAS]

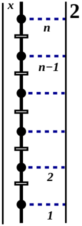


- contributions from other channels, XH, added; account for $\sim 12\%$ of the x-sec.
- all predictions compatible and undershoot the data (PDF uncert. negligible)
- experimental uncertainties still very large

Perspectives for NLO PS

LO ladder

$$\bar{D}^{LO}(x, Q) = \sum_{n=0}^{\infty}$$



$$= e^{-S} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0} \theta_{Q>a_i>a_{i-1}} \rho_1^{(0)}(k_i)$$

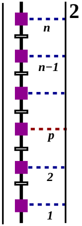
where

$$\rho_1^{(0)}(k_i) = \frac{2 C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}$$

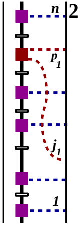
NLO ladder

$$\bar{D}^{NLO}(x, Q) =$$

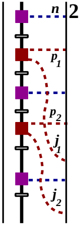
$$= e^{-S} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$



$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1 \neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2 \neq p_1, j_2}}^{p_2-1}$$



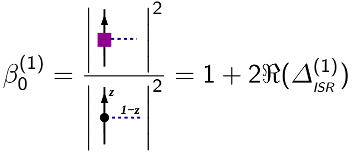
$$+ \ldots \Bigg\}$$

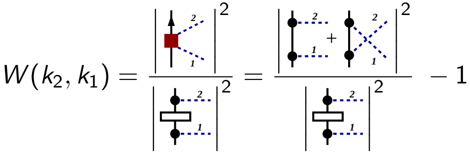
$$= e^{-S} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int \frac{d^3 k_i}{k_i^0} \, \rho_1^{(0)}(k_i) \beta_0^{(1)}(z_i) \right) \delta_{x=\prod_{j=1}^{n} x_j} \Bigg[1 +$$

$$+ \sum_{p_1, j_1} W(\tilde{k}_p, \tilde{k}_j) + \sum_{p_1, p_2, j_1, j_2} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \ldots \Bigg] \Bigg\}$$

NLO ladder

The exclusive, NLO weights read



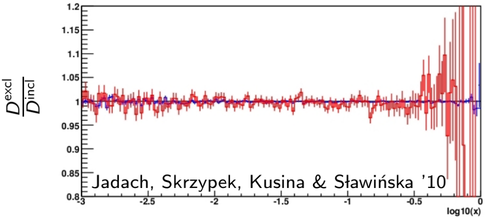


Status of NLO PS

- NLO kernels recalculated in exclusive form and cross-checked against the inclusive kernels of Curci, Furmanski, Petronzio [arXiv:1102.5083, 1401.1587, 1606.01238]

Proof of concept

- numerical comparison of toy PDFs from inclusive and exclusive DGLAP evolution



Next step

- apply the same strategy as for NLO+PS matching

existing solution for toy PS $\longrightarrow$ realistic PS

$\hookrightarrow$ Recent Development in the framework of DIRE shower [Höche, Krauss, Prestel '17]

Conclusions

- KrkNLO: a method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
  - Virtual correction is just a constant and does not depend on Born kinematics.
- The method has been implemented for Drell-Yan and Higgs production on top of Catani-Seymour shower in Sherpa 2.0 and Herwig 7 event generators.
- Comparisons to MCFM, DYNNLO, HNNLO, MC@NLO, POWHEG.
- The results of KrkNLO matching procedure at NLO+LL level come out consistent with fixed order NLO and other matching methods.
- Tails of distributions of $p_{T,Z}$ and $p_{T,H}$ close to NNLO.
- Proof of concept of NLO PS exists. Transferring that to a real-life PS is our next objective.

BACKUP

Fixed order calculations in QCD

General structure of NLO cross sections:

$$d\sigma = \Big[B + V(\alpha_s) + C(\alpha_s)\Big] d\phi_B + R(\alpha_s)\, d\phi_B d\phi_1$$

- B, R, V - Born, real and virtual part
- C - collinear subtraction counterterm (for initial state radiation case)

Calculation possible e.g. by means of subtraction procedure

$$\begin{aligned} d\sigma &= \Big[B + V(\alpha_s) + \int_1 A(\alpha_s) d\phi_1 + C(\alpha_s)\Big] d\phi_B + \\ & \int_1 \Big[R(\alpha_s) - A(\alpha_s)\Big]\, d\phi_1 d\phi_B\,, \end{aligned}$$

where $A \simeq R$, such that it reproduces collinear and soft singularities.

- Good for inclusive observables or distributions at high-p_T.

Parton shower

In the collinear region, fixed order calculation becomes unreliable because each α_s^n is accompanied by a large, logarithmic coefficient, $\ln^n$, and

$$(\alpha_s \ln)^n \sim 1 \text{ for all } n\,.$$

These terms must be summed to all orders and this is what the Parton Shower (PS) is aiming at. In the collinear limit

$$d\sigma_{n+1} \simeq d\sigma_n \frac{\alpha_s(q^2)}{2\pi} \frac{dq^2}{q^2} P(z) dz\,.$$

This can be iterated and used to resum all leading log contributions. In particular, non-emission probability (Sudakov form factor) is given by

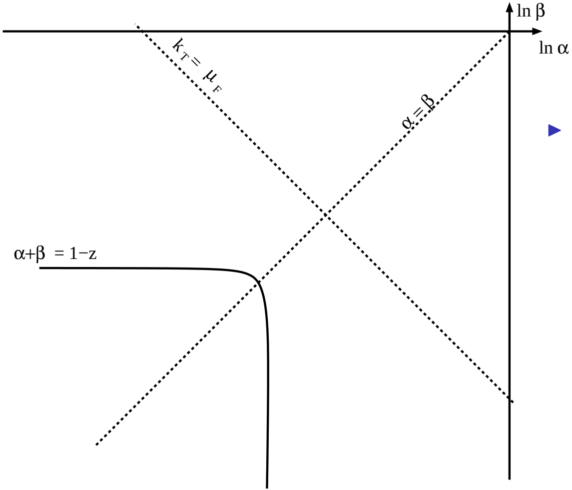
$$\Delta(q_1, q_2) = \exp\left[-\int_{q_1}^{q_2} \frac{\alpha_s(q^2)}{2\pi} \frac{dq^2}{q^2} \int_{z_0}^{1} P(z)\, dz \right].$$

In Monte Carlo event generators, the scale of i$^{\text{th}}$ emission, q_i, is found by solving

$$\Delta(q_{i-1}, q_i) = R_i\,,$$

where $R_i \in [0, 1]$ is a random number and q_{i-1} is a scale of previous emission.

Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.

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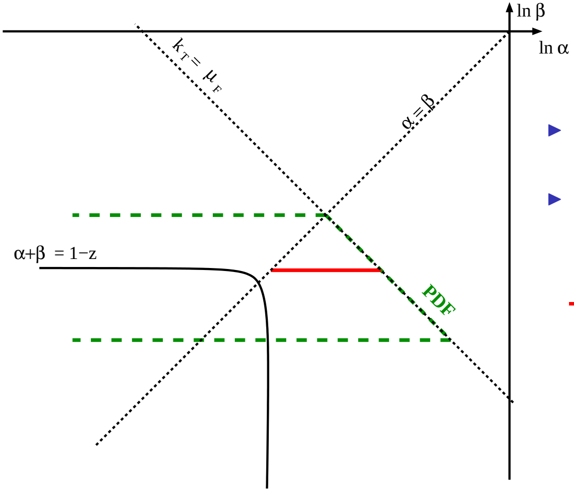

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- For one PDF we get

$$\int \frac{1}{1-z}\frac{d\beta}{\beta} = 2\frac{\ln(1-z)}{1-z}$$

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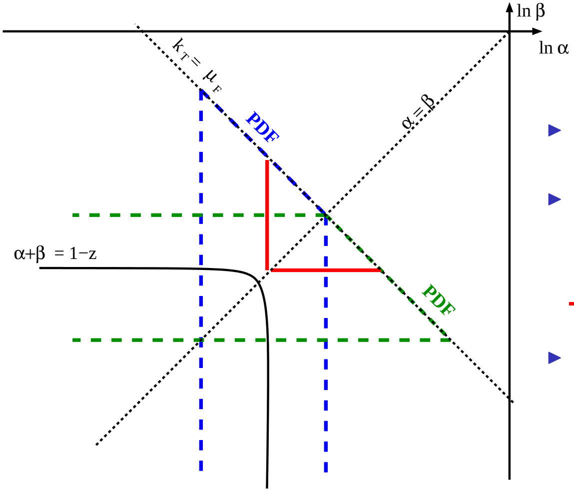

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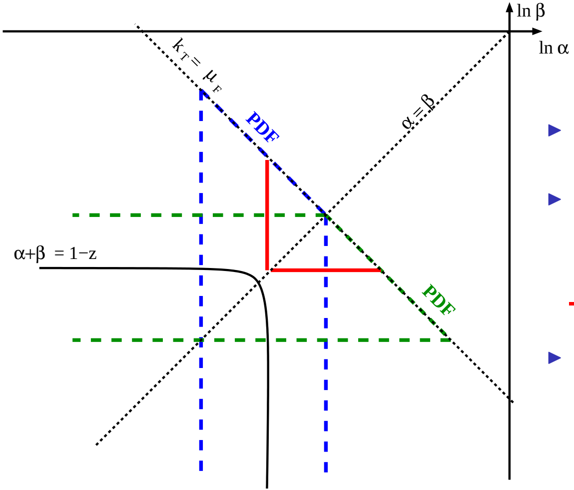

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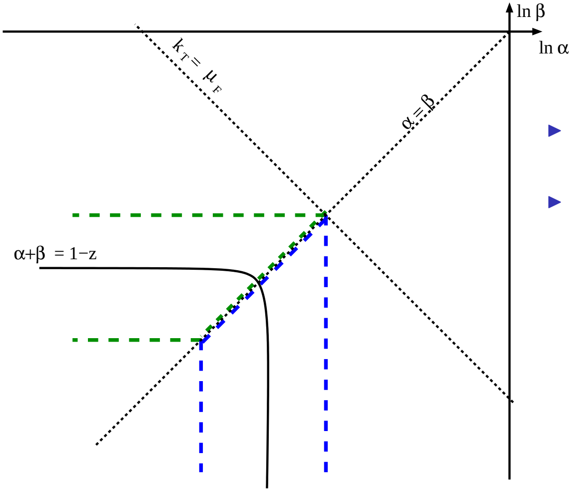
Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.

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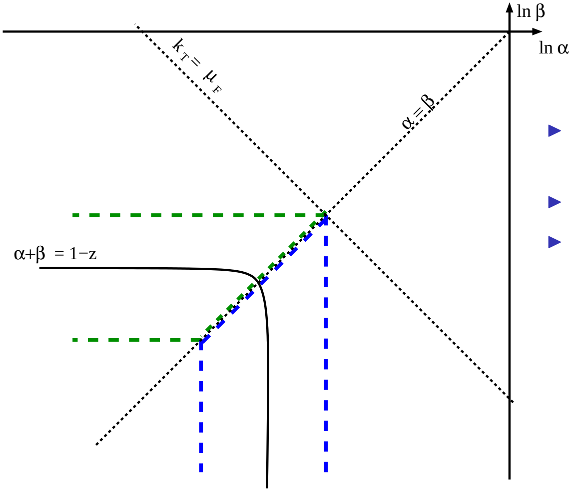

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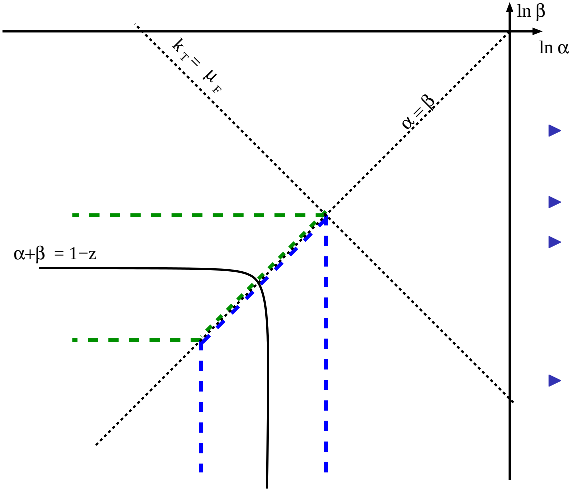

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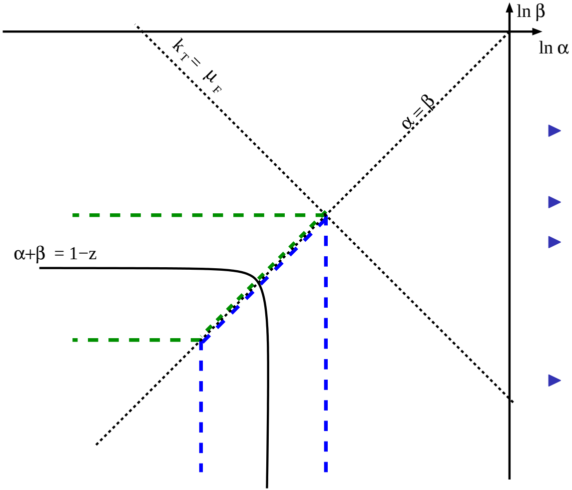

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Could the change of factorization scheme help us to simplify NLO+PS matching?

Implementation on top of the Catani-Seymour shower

$\hookrightarrow$ We used Sherpa 2.0.0 implementation of the CS shower.

Phase space measure of emitted gluon

$$\frac{d\alpha}{\alpha}\frac{d\beta}{\beta} = \frac{d\alpha d\beta}{\beta(\alpha+\beta)} + \frac{d\alpha d\beta}{\alpha(\alpha+\beta)}$$

- The evolution variable:

$$q_F^2 = s(\alpha+\beta)\beta, \qquad q_B^2 = s(\alpha+\beta)\alpha\,,$$

hence

$$\frac{d\alpha d\beta}{\alpha\beta} = \frac{dq_F^2}{q_F^2}\frac{dz}{1-z} + \frac{dq_B^2}{q_B^2}\frac{dz}{1-z}.$$

- The CS shower covers all space of (α, β).

$$\alpha+\beta \leq 1 \quad \Rightarrow \quad z \geq 0 \quad \text{and} \quad q_{F,B}^2 \leq s$$

$$\alpha, \beta > 0 \quad \Rightarrow \quad (1-z)^2 > q_F^2/s \quad \text{or} \quad (1-z)^2 > q_B^2/s$$

Implementation on top of the Catani-Seymour shower

↪ It turns out that coefficient functions of the CS shower equal to those from the MC scheme of Jadach et al. arXiv:1103.5015. Hence, CS $\equiv$ MC.

The $C_2(z)$ function:

$$C_2^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \int (R - K)$$

- For the $q\bar{q}$ channel:

$$C_{2q}^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z)\right]$$

- For the qg channel:

$$C_{2g}^{\mathrm{MC}}(z)\Big|_{\mathrm{real}} = \frac{\alpha_s}{2\pi}\, T_R\, \frac{1}{2}(1-z)(1+3z)$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $C_{2q}^{\mathrm{MC}}(z)$ and $C_{2g}^{\mathrm{MC}}(z)$ from $\overline{\mathrm{MS}}$ PDFs
  - absorbing all z-dependent terms from $\overline{\mathrm{MS}}$ coefficient functions
- The virtual correction:

$$C_{2q}\Big|_{\mathrm{virt}} = \delta(1-z)\left(\frac{4}{3}\pi^2 - \frac{5}{2}\right)$$

is applied multiplicatively.

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Simple form of the coefficient functions with no singular terms!

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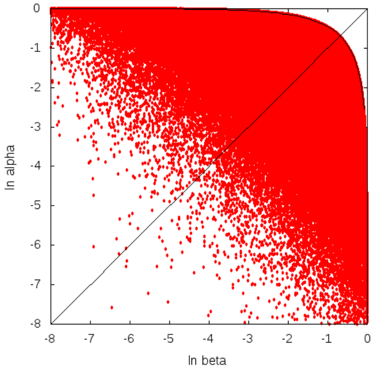
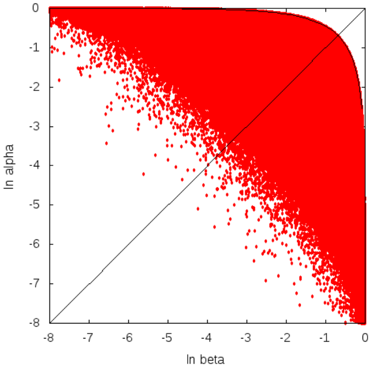
Complete coverage of phase space

Sherpa 2.0



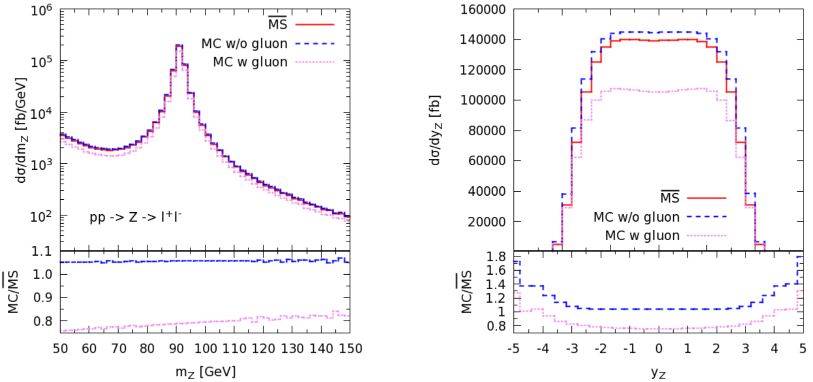
The evolution variable:
$$q^2 = (\alpha + \beta)\,\beta\, s.$$

Herwig++ (Dipole Shower)



The evolution variable:
$$q^2 = k_T^2 = \alpha\,\beta\, s.$$

$\overline{\text{MS}}$ vs MC at LO



- $+5\%$ effect at central rapidities in $q\bar{q}$ and -20% for both channels
- pronounced difference at large y coming from the $x \sim 1$ region

$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

Reweighting procedure

The "Sudakov" form factor for he CS shower

$$S(Q^2, \Lambda^2, x) = \int_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int_{z_{\text{min}}(q^2)}^{z_{\text{max}}(q^2)} dz \;\; K(q^2, z, x)\,,$$

where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1+z^2}{1-z} \; \frac{D(q^2, x/z)/z}{D(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $D(q^2, x)$ - parton distribution functions

The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Validation: $\overline{\text{MS}}$ scheme vs MC scheme at NLO

Cross section, truncated at $\mathcal{O}(\alpha_s)$, cannot depend on fact. scheme

$$\sigma_{\text{tot}}^{\overline{\text{MS}}} \stackrel{!}{=} \sigma_{\text{tot}}^{\text{MC}}$$

We have

$$\begin{aligned} \sigma_{\text{tot}}^{\overline{\text{MS}}} &= f_q \otimes (1 + \alpha_s\, C_q^{\overline{\text{MS}}}) \otimes f_{\bar{q}} \\ \sigma_{\text{tot}}^{\text{MC}} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\text{MC}}) \otimes (f_{\bar{q}} + \alpha_s \Delta f_{\bar{q}}) \end{aligned}$$

At $\mathcal{O}(\alpha_s)$:

$$C_q^{\overline{\text{MS}}} f_q f_{\bar{q}} = \Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}}$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}$$

- Final result is scheme-independent up to $\mathcal{O}(\alpha_s)$.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

$\hookrightarrow$ Identical validation performed with both $q\bar{q}$ and qg channels.

Drell-Yan: Matched results, total cross section

$q\bar{q}$ channel

	$\sigma_{\text{tot}}^{q\bar{q}}$ [pb]
MCFM	1273.4 ± 0.1
MC@NLO	1273.4 ± 0.1
POWHEG	1272.1 ± 0.7
KrkNLO $\alpha_s(q^2)$	1282.6 ± 0.2
KrkNLO $\alpha_s(M_Z^2)$	1285.3 ± 0.2

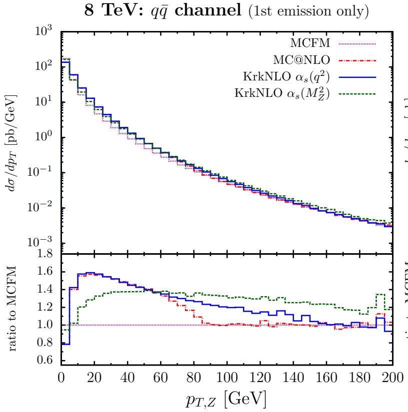
- sub-percent differences from beyond-NLO terms in the KrkNLO result (MC PDFs, mixed real-virtual)
- negligible difference between fixed and running coupling

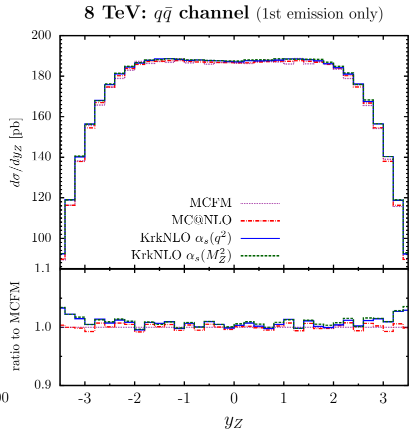
$q\bar{q} + qg$ channels

	$\sigma_{\text{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_s(q^2)$	1045.4 ± 0.1
KrkNLO $\alpha_s(M_Z^2)$	1039.0 ± 0.1

- beyond-NLO terms reach up to 4% in the KrkNLO result
  $\hookrightarrow$ resulting from large gluon luminosity leading to $f^{\text{MC}}/f^{\overline{\text{MS}}} < 1$
- small differences between fixed and running coupling choices

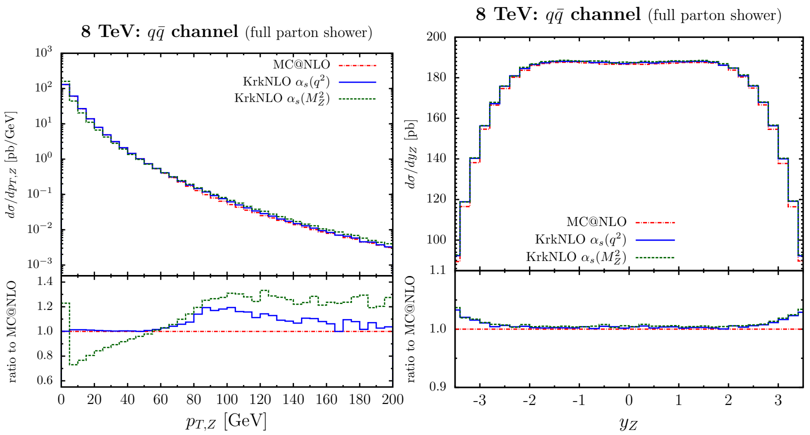
Drell-Yan: Matched results, $q\bar{q}$, 1st emission





- Reproduction of y_Z distribution at NLO.
- Agreement of KrkNLO $\alpha_s(q^2)$ with MC@NLO at low $p_{T,Z}$: PS domination
- KrkNLO results above MC@NLO and MCFM at higher $p_{T,Z}$: $\mathcal{O}(\alpha_s^2)$ terms

Drell-Yan: Matched results, $q\bar{q}$, full PS



- Low $p_{T,Z}$ part of the spectrum changes but KrkNLO $\alpha_s(q^2)$ with MC@NLO agree there because of shower domination
- KrkNLO results above pure NLO at high $p_{T,Z}$: admixture of NNLO terms
- Diffs between two KrkNLO result at high $p_{T,Z}$: running coupling effects