SARAH

http://sarah.hepforge.org/

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Overview

- A little motivation.
- What is SARAH?
- Latest developments

Motivation

Why do we need to go beyond theoretical standard candles?

The old ways are in trouble:

- For years, the dominant paradigm was SUSY, and it was a good idea to find the most minimal models possible $\rightarrow$ CMSSM, mSUGRA, MGM.
- Lots of work was put into hardcoding results for the MSSM: `SPheno, SoftSUSY, SuSpect, FeynHiggs, SuSHi` etc.
- But now the "demise" of the CMSSM/mSUGRA in terms of interpretations of LHC results is however already old news: in that model the $m_{\tilde{t}} >$ TeV, $m_\chi \gtrsim 450$ GeV and $m_{\tilde{g}} \gtrsim 2.2$ TeV gives little chance of finding new physics.
- The MSSM in general is looking less natural, why not explore e.g. λ in NMSSM?

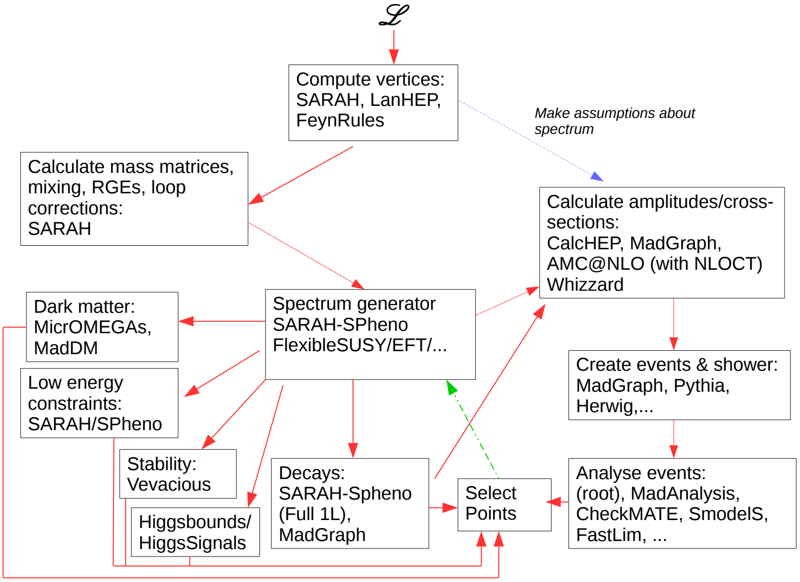
Going off the beaten track is now much easier to do:

- There has been much theoretical study of other alternatives, e.g. Dirac gauginos, Technicolour/composite Higgs/PNGB/other strong coupling; neutal naturalness, etc.
- The rise of generic tools makes it much easier to study other cases.
- E.g. `CalcHEP` and `MicrOmegas` as forerunners.
- `MadGraph/Whizard` with `UFO` from Feynrules, `LanHEP`, `SARAH`
- `SModelS, FastLim, MadAnalysis, CheckMATE, ...`

SARAH: a tool for BSM model builders

So what is SARAH ?

- Mathematica package created by F. Staub, with now several contributions from MDG.
- Takes an input model file for any SUSY or non-SUSY model: any renormalisable lagrangian.
- Specify: gauge groups, matter content, superpotential/couplings in Lagrangian.
- Spectrum generation with SPheno. Produces fortran code which compiles against the SPheno library to generate spectrum and precision observables etc for the model.
- Will calculate two-loop RGEs, one-loop masses for all particles in $\overline{DR}'$ (SUSY) or $\overline{MS}$ (non-SUSY) models.
- Calculate two-loop neutral scalar masses, in either fixed-order or SMEFT approach.
- Produces interfaces for CalcHEP, MicrOMEGAS, UFO, Whizard, Vevacious, HiggsBounds/HiggsSignals, FeynArts
- Calculation of low energy/flavour constraints (up to one loop).
- Production and decay of Higgs bosons including higher-order contributions.
- New feature in [MDG, Liebler and Staub, 1703.09237]: one-loop two-body decays (plus real corrections) for all particles.


$\mathscr{L}$
Compute vertices: SARAH, LanHEP, FeynRules
Make assumptions about spectrum
Calculate mass matrices, mixing, RGEs, loop corrections: SARAH
Calculate amplitudes/cross-sections: CalcHEP, MadGraph, AMC@NLO (with NLOCT) Whizzard
Dark matter: MicrOMEGAs, MadDM
Spectrum generator SARAH-SPheno FlexibleSUSY/EFT/...
Create events & shower: MadGraph, Pythia, Herwig,...
Low energy constraints: SARAH/SPheno
Stability: Vevacious
Higgsbounds/ HiggsSignals
Decays: SARAH-Spheno (Full 1L), MadGraph
Select Points
Analyse events: (root), MadAnalysis, CheckMATE, SmodelS, FastLim, ...

Implementing models in SARAH

SARAH requires as input:

- Gauge and global symmetries.
- Gauge eigenstates before EWSB: in principle gauge group/reps are arbitrary (e.g. for RGEs) but some features untested for $SU(N > 3)$ and reps other than (anti)fundamental/adjoint.
- Superpotential/Lagrangian. Here the notation is very compact, suppressing indices (the gauge contractions are inferred), e.g. SM lagrangian:

$$\mathtt{LagNoHC} = \mathtt{mu2}\ \ \mathtt{conj[H].H}\ \ -1/2\,\lambda\,\mathtt{conj[H].H.conj[H].H};$$
$$\mathtt{LagHC} = -(\mathtt{Yd}\,\mathtt{conj[H].d.q} + \mathtt{Ye}\,\mathtt{conj[H].e.l} + \mathtt{Yu}\,\mathtt{H.u.q});$$

- Set of vevs for gauge symmetry breaking
- Which fields mix together, name of the combined fields, name of the mixing matrices (e.g. `{{SdL, SdR}, {Sd, ZD}}`).

SARAH will then

- Calculate all gauge interactions automatically, add gauge fixing in R_ξ gauge
- Add soft SUSY breaking terms
- Calculate tadpole equations, mass matrices, vertices
- Check consistency of model: anomalies, charge conseration, missing terms, ...

Loop corrections

SARAH calculates two loop RGEs by the command `CalcRGEs[...]`:

- General renormalisable field theories: [Machacek, Vaughn, '83] $\times 3$, [Luo, Xiao hep-ph/0211440]
- For SUSY theories with simple gauge groups: [Martin, Vaughn hep-ph/9311340]
- Kinetic mixing: [Fonseca, Malinsky, (Porod), Staub 1107.2670, 1308.1674]
- Dirac gauginos: [MDG, 1206.6697]
- Running vevs in R_ξ gauge: [Sperling, Stöckinger, Voigt 1305.1548,1310.7629]

Masses/tadpoles:

1. One-loop tadpoles and self-energies are computed for all states using `CalcOneLoopCorrections[...]`, stored in mathematica output
2. Two-loop tadpoles and self-energies are computed for neutral scalars (Higgs/pseudoscalars) using `Calc2LoopCorrections[states_]`; currently this is only possble during the `SPheno` code generation (or with a patch available on request); in future it will be possible to call independently.

Decays:

- Higgs decays to gluons/photons have been supported for some time including many higher-order contributions
- With [MDG, Liebler and Staub, 1703.09237] we now generate two-body loop decays for all scalars and fermions as part of `SPheno` code output using `MakeSPheno[IncludeLoopDecays -> True]`.

Other codes in the SARAH family

As listed above SARAH interfaces with everything to allow models to be studied in all detail. But the closely related codes are

- SPheno: an MSSM spectrum generator. However, SARAH generates code that links to the SPheno library for spectrum generation etc, which has a similar structure to SPheno, but the two codes should not be confused!
- SSP (SARAH Scan and Plot), by F. Staub: sets up and analyses scans.
- Vevacious [Camargo-Molina, O'Leary, Porod, Staub, 1307.1477]: investigates vacuum stability.
- FlexibleSUSY [Athron, Park, Stöckinger, Voigt, 1406.2319]: uses SARAH output to create a SoftSUSY-like spectrum generator.

Bleeding edge

- The set of codes is now quite mature: the interfaces are well established and the flow from Lagrangian to analysis can be performed efficiently.
- The challenge is now to obtain accuracy as close as possible as can be obtained for the standard examples (SM & MSSM).
- In `MadGraph` the push is to get to NLO accuracy in all couplings (nearly there).
- In `SARAH` we now have one-loop two-body decays for all particles (plus real corrections), but this can be improved (and we could add two-loop NLO corrections to loop-induced processes, NLO three body decays ...)
- In `SARAH` the main frontier is the calculation of Higgs properties (mass, production, decays).
- Our Higgs mass calculation has already reached what can be done for the MSSM, and in some aspects exceeded it.
- $\rightarrow$ we find new technical challenges that weren't present there (e.g. Goldstone Boson Catastrophe even in gaugeless limit).

Higher-order corrections to Higgs decays

The leading order contributions to neutral scalar$\rightarrow \gamma\gamma, gg$ have been included for some time:

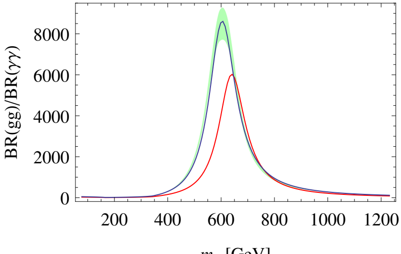
$$\Gamma(\Phi \rightarrow \gamma\gamma)_{LO} = \frac{G_F \alpha^2(0) m_\Phi^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c^f Q_f^2 r_f^\Phi A_f(\tau_f) + \sum_s N_c^s r_s^\Phi Q_s^2 A_s(\tau_s) + \sum_v N_c^v r_v^\Phi Q_v^2 A_v(\tau_v) \right|^2,$$

$$\Gamma(\Phi \rightarrow gg)_{LO} = \frac{G_F \alpha_s^2(\mu) m_\Phi^3}{36\sqrt{2}\pi^3} \left| \sum_f \frac{3}{2} D_2^f r_f^\Phi A_f(\tau_f) + \sum_s \frac{3}{2} D_2^s r_s^\Phi A_s(\tau_s) + \sum_v \frac{3}{2} D_2^v r_v^\Phi A_v(\tau_v) \right|^2.$$

However, strong corrections are known to be important. For $m_{f,s} > m_\Phi$, we include for colour triplets:

$$r_f^\Phi \rightarrow r_f \left(1 - \frac{\alpha_s}{\pi}\right), r_s^\Phi \rightarrow r_s \left(1 + \frac{8}{3}\frac{\alpha_s}{\pi}\right).$$

For $m_\Phi > 20 m_f$ we have NLO light top corrections; for $20 m_f > m_\Phi > 2 m_f$ we have up to NNNLO strong corrections fit from HDECAY.

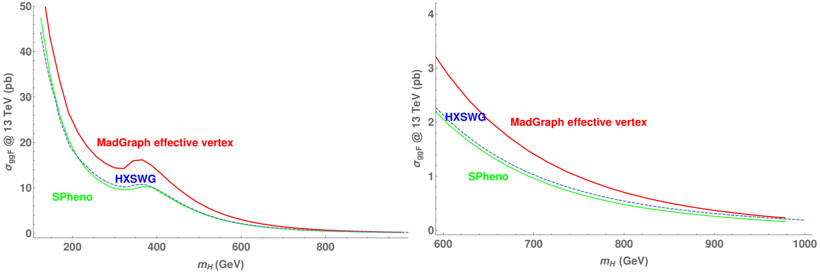


Production

- We also include the effective $\Phi F_{\mu\nu} F^{\mu\nu}$, $\Phi G_{\mu\nu} G^{\mu\nu}$ vertices in the UFO file, and the coefficients are provided in the SLHA `SPheno` output.
- ... This corresponds to the NLO-corrected values for the decays.
- However, the corrections for production are very different; need to take into account k-factor

$$k = c_{\Phi gg} \cdot \frac{\sigma_{SM}(pp \to H(M_\Phi) + \text{jet})}{\sigma_{MC}(pp \to \Phi)} \sim 2$$

- To obtain Higgs production rates, we fit to `SusHi` production rates from the effective vertex, includes many NLO strong corrections to the production; so even if we compute $pp \to \Phi + j$ in `MadGraph` we are still out (but closer):



Loop Decays

With [MDG, Liebler and Staub, 1703.09237] we now generate two-body loop decays for all scalars and fermions and include three-body tree-level Bremsstrahlung emission:

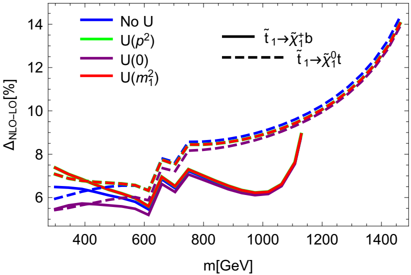
- To do this, we computed all of the generic amplitudes, and wrote routines to populate these with intermediate states, compute all of the group theory factors etc.
- The generic amplitudes are decomposed into Lorentz structures, e.g. $S \to VV$ decomposes as $\mathcal{M} \equiv \epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)\left(M_1 \eta^{\mu\nu} + M_2 p_0^\mu p_0^\nu\right)$, and we then have routines to square the amplitude by squaring the sums of these components.

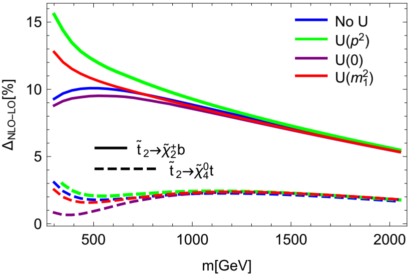
Why do we need this? Could we not just use `MadGraph` or `FormCalc`?!

- To use `MadGraph` would require a renormalised model file, in an on-shell scheme but with $\overline{\text{MS}}$ couplings. This is not always possible!
- In particular, we can use $\overline{\text{DR}}$ masses for SUSY models. We actually also allow the user to provide counterterms for their own scheme (e.g. we give those to put the W/Z on-shell).
- By automatising the process we can rapidly obtain numerical results. They can then be fed into `MadGraph` as updated widths; or it can be useful for e.g. `SmodelS`.

Z-factors

- To calculate in a momentum independent scheme means we need the LSZ reduction formula to relate the external states to the loop ones.
- We do this by introducing Z-factors, e.g. $S_i^0 \to Z_{S_i S_j} S_j = (\delta_{ij} + \frac{1}{2}\delta Z_{S_i S_j})S_j$; we give different options for these. Note that this means our results are the most accurate for even the MSSM; `SFOLD, HFOLD, FVSFOLD` do not include them. E.g. for an example MSSM point with bino LSP $M_1 = 0.3 M_{SUSY}, \mu = 0.5 M_{SUSY}, M_2 = 0.75 M_{SUSY}$:





To preserve Ward IDs we also need to enforce relationships between would-be Goldstone boson couplings and gauge boson couplings once we include Z-factors.

Output

The new SPheno output gives both the old (leading order) and new (loop-corrected) decays, e.g. contains:

```
DECAY    1000001      5.03001929E+01    # Sd_1
#   BR                NDA      ID1       ID2
   2.91393772E-01  2        6   -1000024    # BR(Sd_1 -> Fu_3 Cha_1)
   1.70527978E-01  2        6   -1000037    # BR(Sd_1 -> Fu_3 Cha_2)
...
DECAY1L    1000001      5.07518318E+01    # Sd_1
#   BR                NDA      ID1       ID2
   2.86487000E-01  2        6   -1000024    # BR(Sd_1 -> Fu_3 Cha_1)
   1.63886304E-01  2        6   -1000037    # BR(Sd_1 -> Fu_3 Cha_2)
...
```

This is because:

- The original DECAY block also allows IR-safe three-body decays
- The neutral scalar decays include our higher-order fit, and also $h \to V^{\mathrm{virt}} V^{\mathrm{virt}}$, so are more accurate – except that the new routines include $h \to Z\gamma$!

The Higgs mass as a precision electroweak observable

The current challenge is the Higgs mass calculation.
Consider the current experimental accuracy of the Higgs mass measurement:

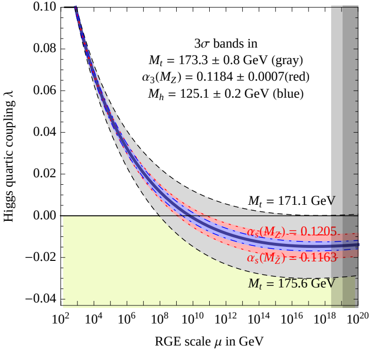
$$\text{ATLAS} + \text{CMS (Moriond 2015)}: \quad m_H = 125.09 \pm 0.21\,(\text{stat}) \pm 0.11\,(\text{syst.})$$

The uncertainty is tiny!
In the Standard Model:

- The Higgs mass is used to calculate the Higgs quartic coupling $\mathcal{L} \supset -\lambda|H|^2$ (from [Buttazzo *et al*, 1307.3536]):

$$\lambda(\mu = m_t) = 0.12604 + 0.00206\left(\frac{m_h}{\text{GeV}} - 125.15\right) - 0.00004\left(\frac{m_t}{\text{GeV}} - 173.34\right) \pm 0.00030$$

- Vital for stability analysis (also needed in principle for future triple/quartic Higgs coupling measurements):
- State-of-the-art computation includes most two-loop effects.



For many years the standard example has been the MSSM:

- Quartic predicted to be determined entirely by gauge couplings at tree level: $\lambda = \frac{1}{8}(g_Y^2 + g_2^2)\cos^2 2\beta$ in heavy M_H limit.
- Hence $\rightarrow m_h(\text{tree}) \leqslant M_Z$
- $\delta m_h^2(\text{loops}) \geqslant (86\text{GeV})^2 \gtrsim m_h^2(\text{tree})$
- Can have $\delta m_h(\text{two loops}) \lesssim 10$ GeV $\rightarrow \delta m_h^2(\text{two loops}) \sim 15\% m_h^2!$
- While at three-loop order, have $\delta m_h \sim$ few hundred MeV, $\rightarrow \delta m_h^2(\text{three loops}) \lesssim 1\% m_h^2$

Much work has led to: full one-loop calculation, two loops full diagrammatic calculation for $\alpha_s \alpha_t$ only; effective potential approximation and gaugeless limit for (Yukawa coupling)^4 diagrams, and three-loop $\alpha_s^2 \alpha_t$.

Higgs mass BSM

Two ways of performing the calculation:

1. Calculate the Higgs mass and extract all parameters (top mass, α_s, α, M_Z, ...) in the full theory, do a fixed-order calculation: good if there are other light Higgses!
2. Match the BSM theory to the SMEFT at some matching scale and use RGEs: much more accurate if BSM states are heavy!

Some recent studies in the MSSM [Bagnaschi, Pardo Vega, Slavich, '17] have shown that we only need consider renormalisable operators in method 2 if the scale is $\gtrsim$ TeV.

SARAH is mainly built for option 1, but it now implements method 2 as well, via

```
BLOCK SPHENOINPUTS
 66 1              # Two-Scale Matching
 67 1              # effective Higgs mass calculation
```

The two-loop corrections in the two-scale approach are included by matching pole masses, so in principle include some subleading log effects and should be used with caution: new technical results are needed to improve this!

Two-loop Higgs mass computation in general theories

- Expressions exist for the two-loop effective potential [Martin, '01], tadpoles [MDG, Nickel, Staub, '15], and scalar self-energies [Martin, '03] in Landau gauge.
- The scalar self-energies are known up to $\mathcal{O}(g^2) \to$ appropriate for neutral Higgs "gaugeless limit" where broken gauge couplings are set to zero.
- A set of loop functions is known, and available in `TSIL` $\to$ evaluated for general momenta by solving differential equations.
- However, evaluation of these is slow, in particular for general theories. So a useful approximation is the "effective potential limit" of $p^2 = 0 \to$ loop functions become orders of magnitude faster to evaluate.
- A general solution to the technical problem of the "Goldstone Boson Catastrophe" due to massless Goldstones in the Landau gauge was presented in [Braathen, MDG, '16] $\to$ implementation to appear soon!
- ... this will also make it possible to study two-loop Higgs mass corrections in non-SUSY models.

The Goldstone Boson Catastrophe

The Goldstone Boson Catastrophe was noticed in the MSSM electroweak corrections, the THDM – and the Standard Model, where it was studied by [Martin, '14], [Elias-Miro, Espinosa, Konstandin, '14]!

- Consider for simplicity the Abelian Goldstone Model of one complex scalar $\Phi = \frac{1}{\sqrt{2}}(v + h + iG)$ and tree-level potential

$$V = \mu^2|\Phi|^2 + \lambda|\Phi|^4.$$

- At tree level, the tadpole equation gives $\mu^2 + \lambda v^2 = 0$, and the masses are $m_G^2 = \mu^2 + \lambda v^2, M_h^2 = \mu^2 + 3\lambda v^2$.
- But we use $m_G^2 \equiv \mu^2 + \lambda v^2$ to calculate loops, and once we include loop corrections we have

$$0 = \mu^2 + \lambda v^2 + \frac{\partial \Delta V}{\partial v}$$

- ... hence $m_G^2 = \mathcal{O}(1-\text{loop})$ and is of indefinite sign!
- In fact, at two loops we find (with $A(x) \equiv x(\log x/Q^2 - 1))$

$$0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log\frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v A(m_G^2) + \frac{4\lambda^3 v^3}{M_h^2} A(M_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}}$$

On-shell scheme

When we want to generalise the approach, it turns out that resummation is very cumbersome. Instead, in [Braathen, MDG '16] we saw that we can cure the IR divergences by putting the Goldstone boson on shell:

$$(m_G^2)^{\text{run.}} \equiv (m_G^2)^{OS} - \Pi_{GG}((m_G^2)^{OS})$$

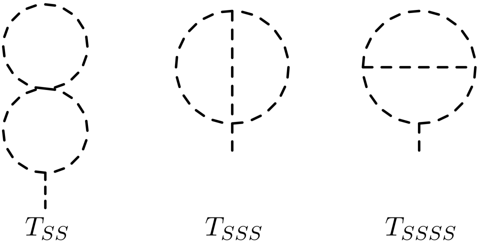
We can do this directly in the tadpole equations – and also the self-energies! So then there should be no need to take derivatives of couplings ... exactly what we want.
For example, applying the above shift to the one loop tadpole gives a two-loop correction:

$$\frac{\partial V}{\partial v} \supset \frac{\lambda v}{16\pi^2} A(m_G^2) = \frac{\lambda v}{16\pi^2}\left[\underbrace{A((m_G^2)^{OS}}_{\to 0} - \underbrace{\Pi_{GG}((m_G^2)^{OS}) \log \frac{(m_G^2)^{OS}}{Q^2}}_{\text{cancels divergent part}} + \underbrace{...}_{3-\text{loop}}\right]$$

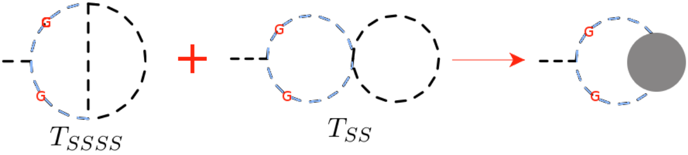
We also see that $\Pi_{GG}((m_G^2)^{OS}) = \Pi_g(0)$ (at least at this loop order) automatically!

Illustration

To see why this works, let us look at the scalar-only case. There are three classes of tadpole diagrams:

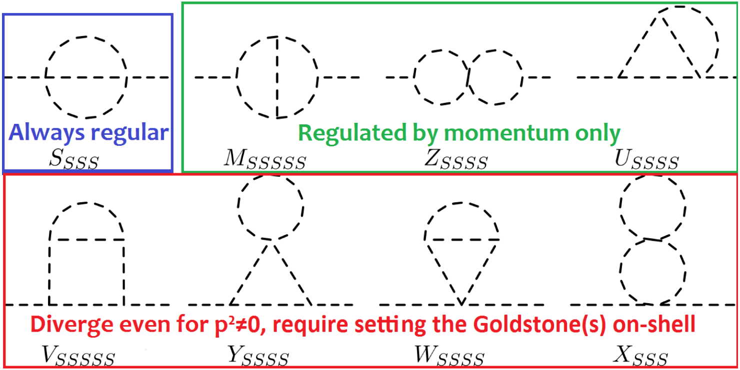


We find that the divergences only come from the T_{SS} and T_{SSSS} topologies, and they correspond to a Goldstone self-energy as a subdiagram and exactly cancel out against the on-shell shift:



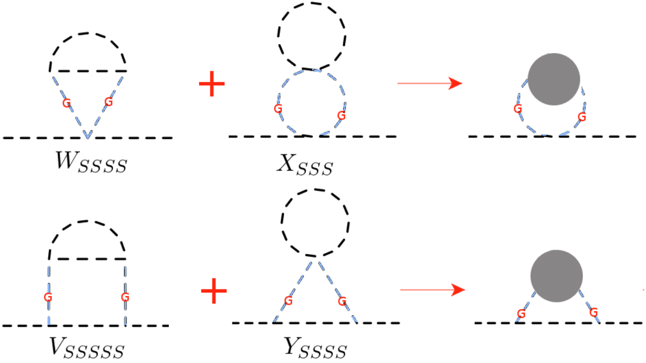
Mass diagrams

We also find that we can apply our on-shell scheme to the cancellation of divergences in self-energies! This seemed hopeless in the former approaches ... We can divide the topologies into three categories:



Mass diagram divergences

Again we find that the divergences in m_G^2 arise from Goldstone boson propagator subdiagrams:



... and once more the one-loop shifts from our on-shell scheme exactly cancel the divergences.

Generalised effective potential limit

Since we see that there are classes of diagrams that are divergent when the $p^2 \equiv s \neq 0$ and the Goldstone bosons are on-shell, the obvious response is that we cannot avoid using momentum dependence – but this is computationally expensive.
Instead, we can expand the self-energies as:

$$\Pi_{ij}^{(2)}(s) = \frac{\overline{\log}(-s)}{s} \Pi^{(2)}_{-1\,l,ij} + \frac{1}{s}\Pi^{(2)}_{-1,ij} + \Pi^{(2)}_{l^2,ij}\,\overline{\log}^2(-s) + \Pi^{(2)}_{l,ij}\,\overline{\log}(-s) + \Pi^{(2)}_{0,ij} + \sum_{k=1}^{\infty} \Pi^{(2)}_{k,ij} \frac{s^k}{k!}$$

If we discard all terms $\mathcal{O}(s)$ and higher, we have a generalised effective potential approximation! We can find closed forms for the singular terms, e.g.

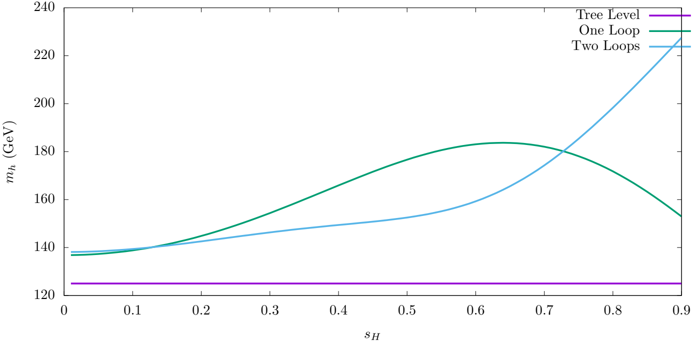
$$U(0,0,0,u) = (\overline{\log}\,u - 1)\,\overline{\log}(-s) - \frac{\pi^2}{6} + \frac{5}{2} - 2\,\overline{\log}\,u - \frac{1}{2}\,\overline{\log}^2\,u + \mathcal{O}(s).$$

This turns out to be a very good approximation, since relevant new physics should be somewhat heavier than the Higgs mass!

Impact of Higgs mass corrections

It is usual to take the Higgs "pole" masses and $\overline{\text{MS}}$ couplings as inputs in non-SUSY models (in e.g. `MadGraph` etc).

But this ignores that some sets of parameters might not make sense. E.g. consider Georgi-Machacek model with $m_h = 125$ GeV, $m_5 = 750$ GeV, $\lambda_3 = -0.2, \lambda_4 = 0.21$:



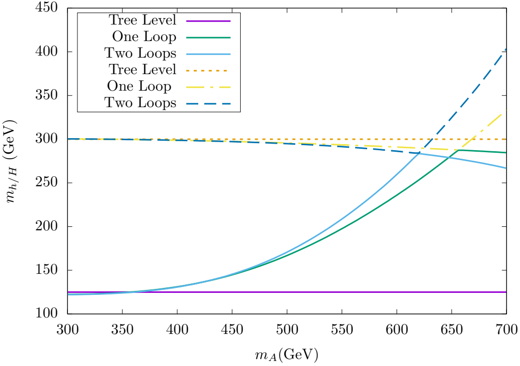
See similar issues with the THDM ...

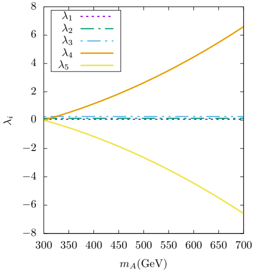
THDM

For example, suppose we take as inputs in the Two-Higgs-Doublet-Model

$$m_h, m_H, m_{H^\pm}, m_A^2, m_{12}^2, \tan\alpha, \tan\beta$$

from which we determine $\lambda_i, i = 1..5$. If we enforce the alignment limit of $\tan\alpha = -1/\tan\beta$, we can scan over the other parameters. If we take all of the Heavy Higgs masses to be 300 GeV and scan only over e.g. m_A we find:





Using the new features

From SARAH 4.12.0 we will have three new features:

```
Block SPhenoInput #
...
  7   0   # Skip two loop masses: True/False
  8   3   # Choose two-loop method
150   1   # Use consistent tadpole solution: True/False
151   1   # Generalised effective potential calculations: ↪
      True/False
410   0   # Regulator mass
```

Conclusions

- We now have the tools to study general models with high precision.
- Lots of theoretical activity to improve this.
- Challenge will also be to better integrate the flow from $\mathcal{L}$ to experiment and back again!