



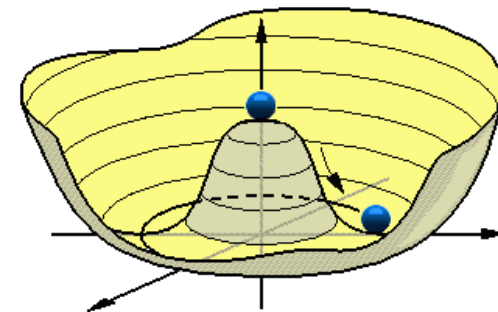
*CMB ANOMALY AND INFLATION
WITH SPONTANEOUS SYMMETRY BREAKDOWN*

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Kobe University

Particle Physics and Symmetry breakdown

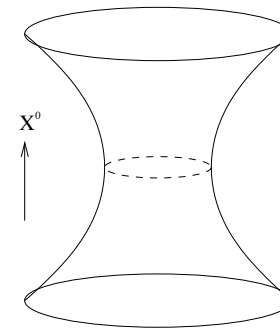
- Particle physics is governed by symmetries
a highly symmetric state is boring
- Spontaneous symmetry breakdown (SSB) is essential for describing a realistic world



- VEVs of Higgs fields control symmetry breaking patterns

Cosmology, Symmetry breakdown, and CMB

- Symmetry of de Sitter universe is maximal
- Inflation is well approximated by de Sitter but not exactly de Sitter



- SSB is essential for describing a realistic universe
- VEV of fields control symmetry breaking patterns

The main message of this talk is that

SSB gives a useful way for understanding of CMB physics

Plan of my talk



1. COBE level understanding of CMB
2. Planck level understanding of CMB
3. Understanding CMB Anomaly
4. Summary

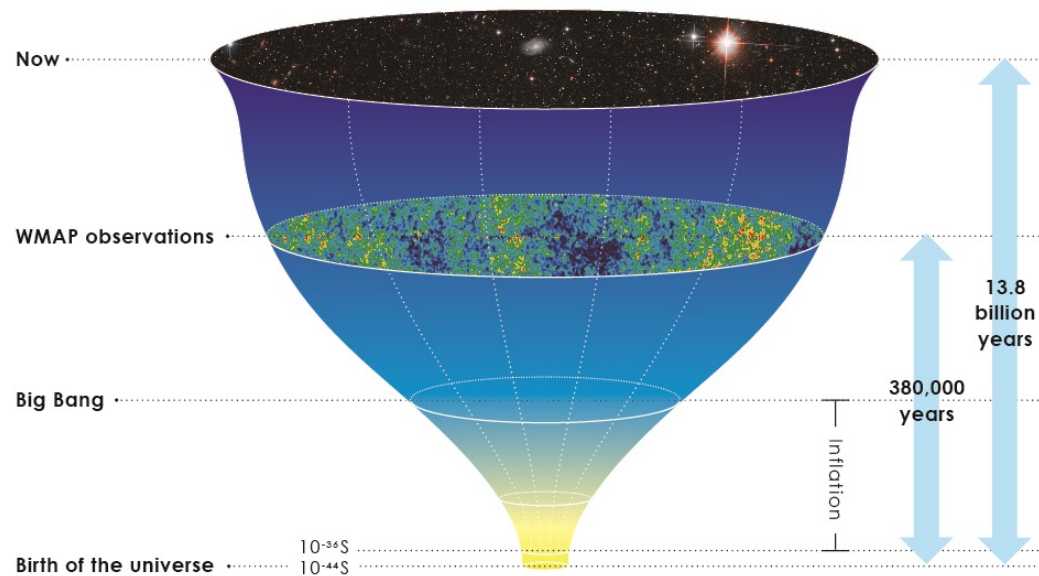


COBE level understanding of CMB

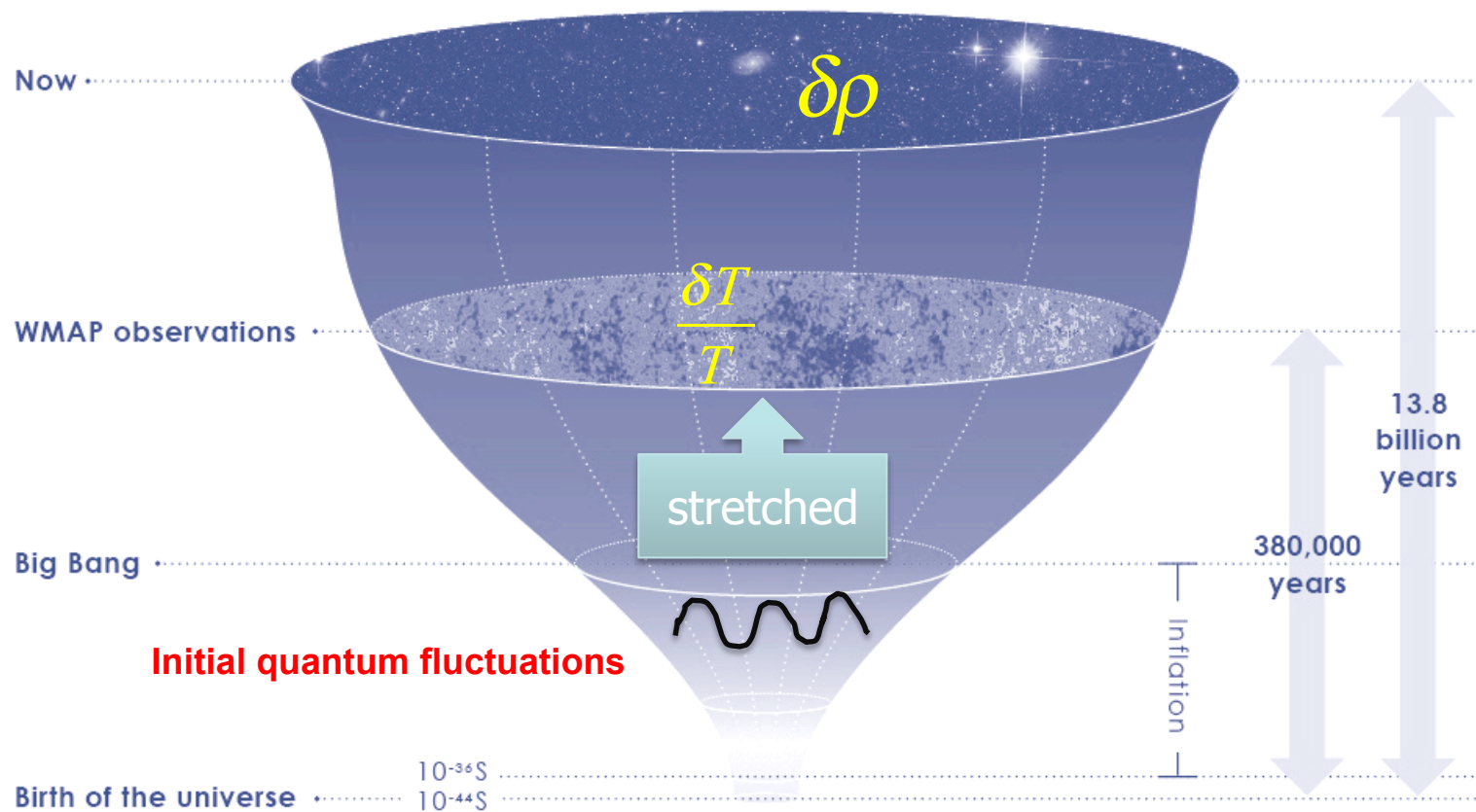
Inflation: as a part of standard cosmology

Inflation solves

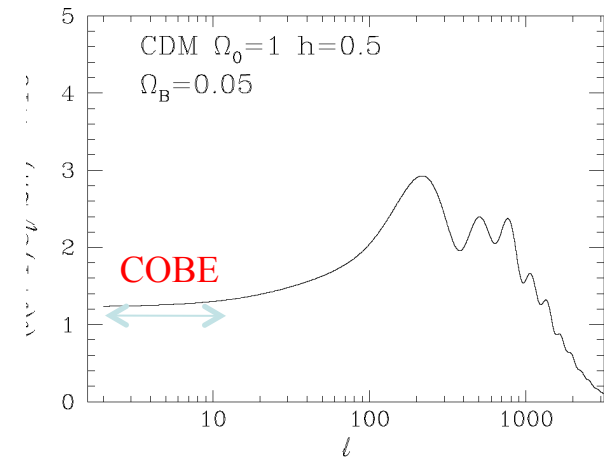
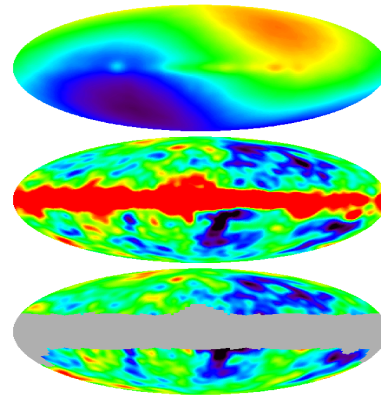
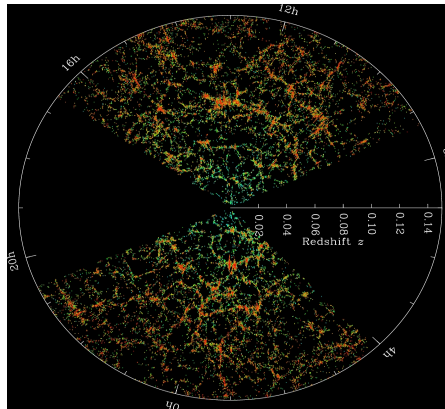
- ✓ Flatness problem
- ✓ Horizon problem
- ✓ Monopole problem
- ✓



The origin of LSS and CMB fluctuations is quantum!!



Primordial fluctuations inferred from observations



$$\frac{\delta T}{T} = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$\langle a_{\ell m} a_{\ell' m'} \rangle = C_{\ell} \delta_{\ell \ell'} \delta_{m m'}$$

Nature of primordial fluctuations

- ◆ statistically homogeneous
- ◆ statistically isotropic
- ◆ Gaussian
- ◆ scale invariant

Inflation: the highest symmetric universe

Shift symmetry in field space leads to quasi - de Sitter inflation

de Sitter spacetime $ds^2 = -dt^2 + e^{2Ht} (dx^2 + dy^2 + dz^2)$

$O(4,1)$: de Sitter symmetry

➤ **Spatial translation symmetry**

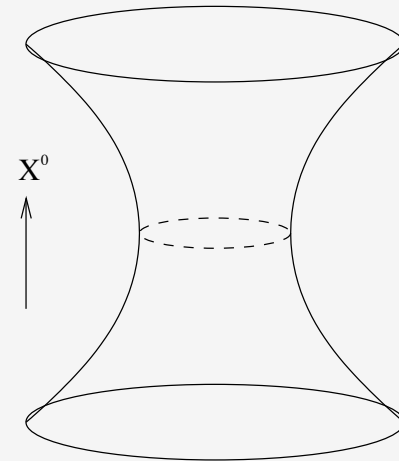
$$x^i \rightarrow x^i + c^i, \quad c^i \text{ is a constant vector}$$

➤ **Rotation symmetry**

$$x^i \rightarrow R^i_j x^j, \quad R^T R = I$$

➤ **Temporal de Sitter symmetry**

$$t \rightarrow t + c, \quad x^i \rightarrow e^{-Hc} x^i$$



Nature of primordial fluctuations

Suppose there exist primordial fluctuations ζ

Shift symmetry implies **Gaussianity** $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = P_\zeta(\mathbf{k}_1, \mathbf{k}_2)$

Spatial translation symmetry implies **statistical homogeneity**

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2)P_\zeta(\mathbf{k}_1)$$

Rotation symmetry implies **statistical isotropy**

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2)P_\zeta(k_1 = |\mathbf{k}_1|)$$

Temporal de Sitter symmetry implies **scale invariance**

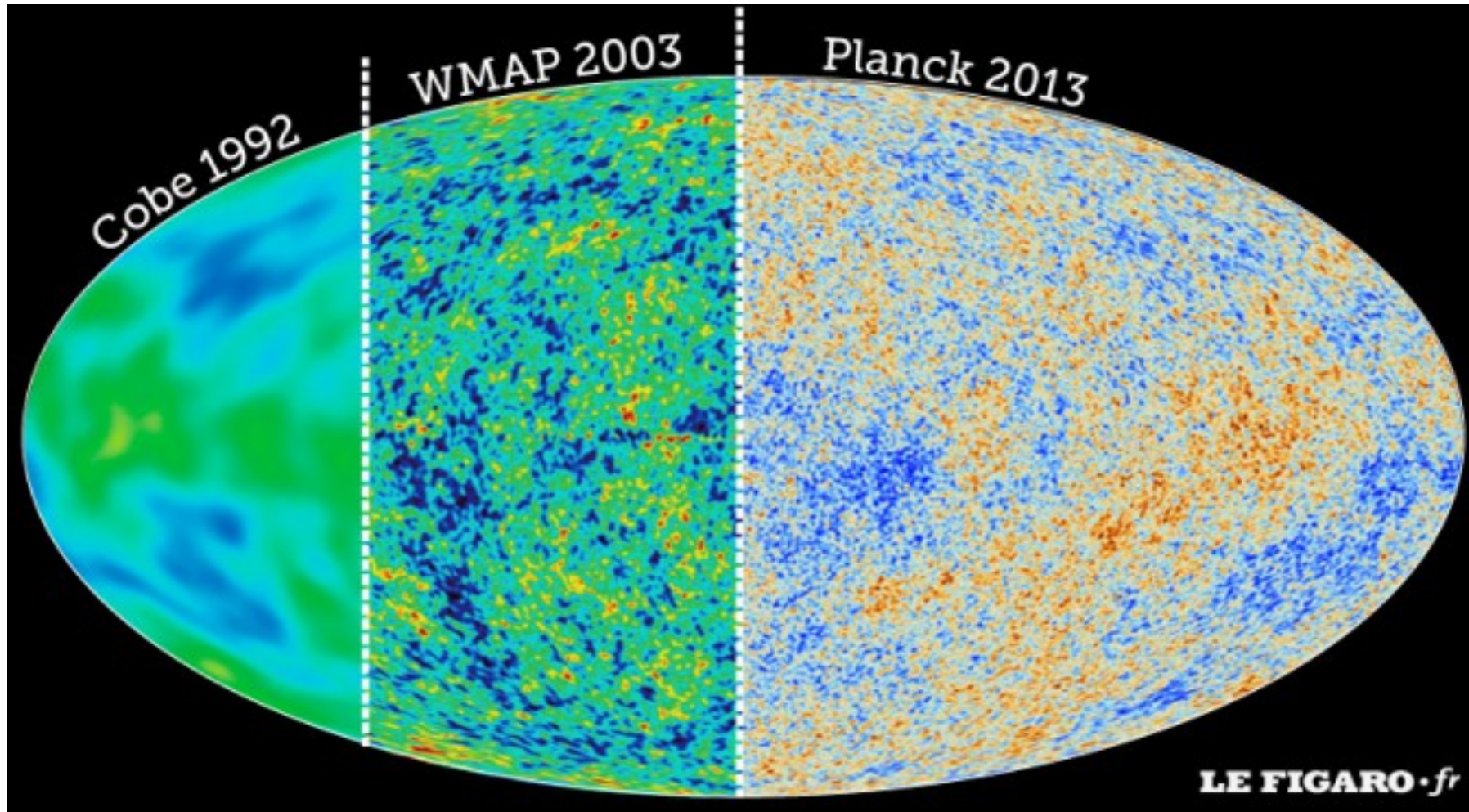
$$P_\zeta(k) \approx \text{const.}$$

Thus, we have succeeded in understanding COBE results.



Planck level understanding of CMB

From COBE to WMAP, Planck



Planck requires precision theory

Precision cosmology

Almost isotropic

Almost scale free

$$P_\zeta(k) \approx \Delta_\zeta^2 k^{n_s - 1}$$

$$n_s = 0.968 \pm 0.006$$

$$\Delta_\zeta^2 = 2.457^{+0.092}_{-0.093} \times 10^{-9}$$

Almost Gaussian

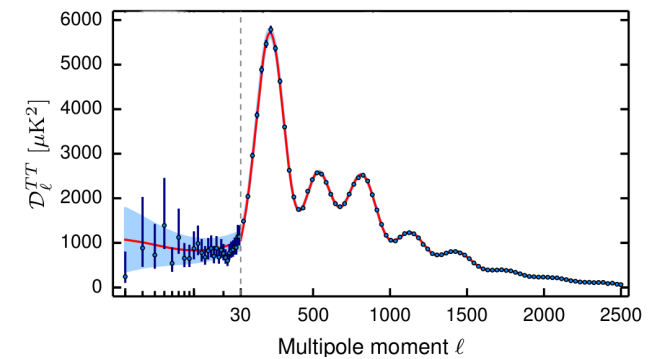
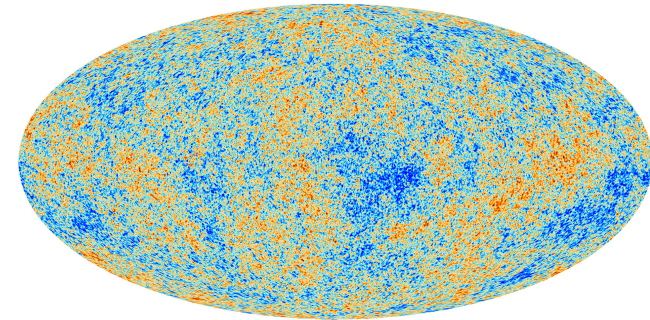
$$f_{NL}^{local} = 2.5 \pm 5.7$$

$$\zeta = \zeta_G + \frac{3}{5} f_{NL}^{local} \zeta_G^2$$

PGWs

$$r = \frac{\Delta_h^2}{\Delta_\zeta^2} < 0.11$$

Planck data



Spontaneous symmetry breakdown of O(4,1)

In the highly symmetric state, nothing can fluctuate.

Time dependent VEV of inflaton: $\phi(t) \equiv \langle \hat{\phi}(x^i, t) \rangle$

→ Spontaneous symmetry breakdown of temporal de Sitter symmetry

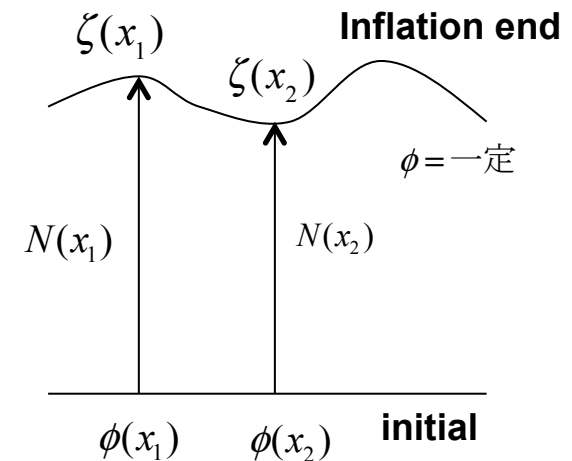
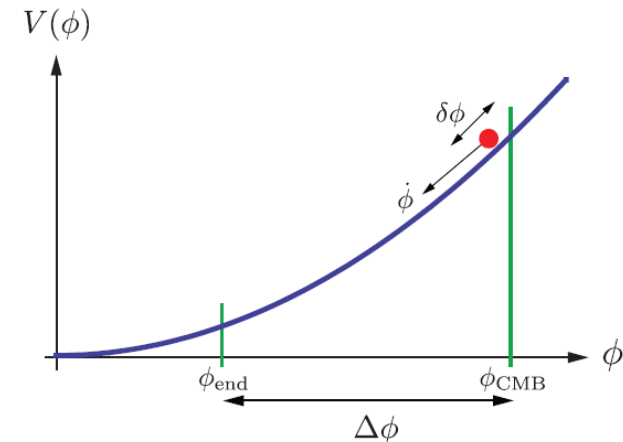
Nambu-Goldstone mode $t \rightarrow t + \pi(t, x^i)$

Cheung et al. 2008

$$\delta\phi(t, x^i) \equiv \phi(t + \pi(t, x^i)) - \phi(t) \simeq \dot{\phi}(t) \pi(t, x^i)$$

$$\zeta(t) = -\delta N = -H \delta t = -H \frac{\delta\phi}{\dot{\phi}} = -H \pi$$

This allows us to perform explicit calculations.



Symmetry breakdown gives rise to variety!

Precision cosmology forces us to look at fine structures of fluctuations!

Violation of temporal de Sitter symmetry \rightarrow spectral tilt

There should be a slight tilt because the expansion is not exactly deSitter. The deviation from deSitter can be characterized by the slow roll parameter. Hence, the tilt should be of the order of the slow roll parameter.

$$n_s = 1 - 2\varepsilon - \delta \quad \varepsilon = -\frac{\dot{H}}{H^2} \quad \delta = \frac{\dot{\varepsilon}}{H\varepsilon}$$

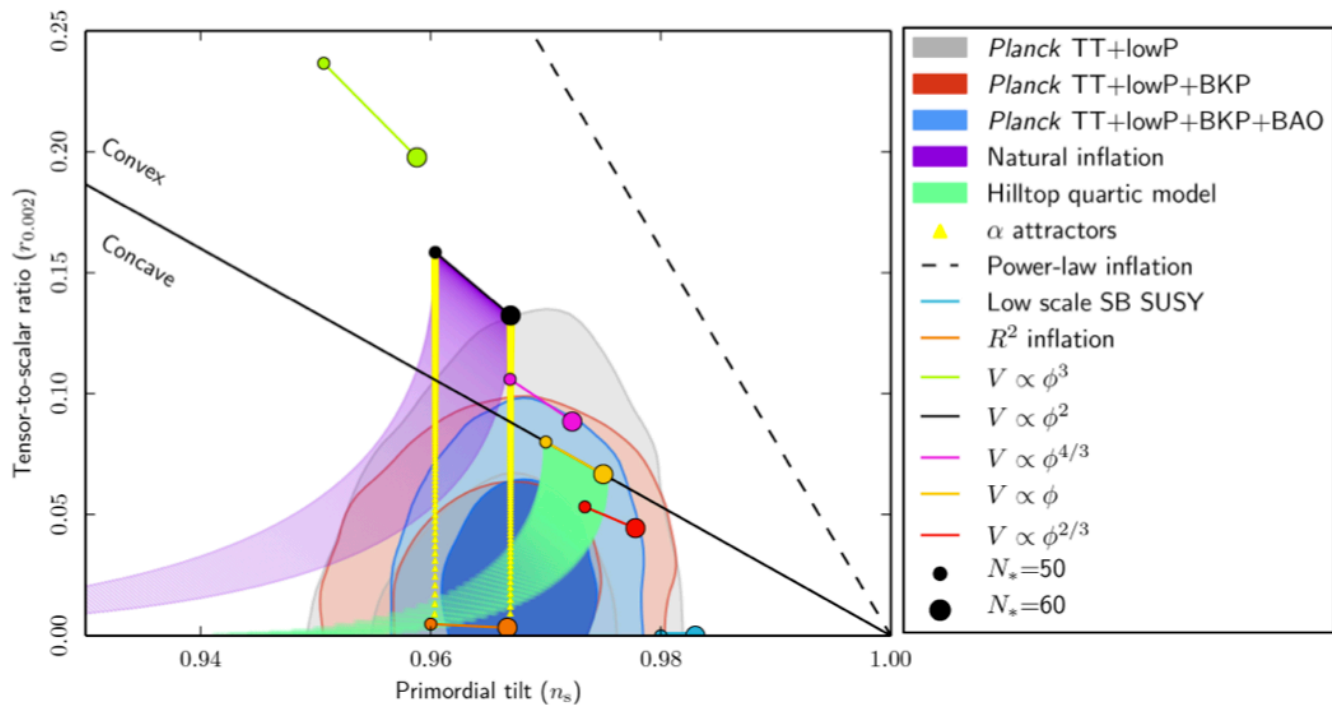
Violation of shift symmetry \rightarrow non-Gaussianity

There should be small non-gaussianity of the order of the slow roll parameter because the shift symmetry is not exact.

$$f_{NL}^{local} = O(\varepsilon) \quad \text{Maldacena 2003}$$

PGWs $r = \frac{\Delta_h^2}{\Delta_\zeta^2} = 16\varepsilon$

Standard way to constrain inflation





Understanding CMB Anomaly

CMB anomaly found by WMAP and Planck

Hemisphere Asymmetry

Eriksen et al. 2004 Hansen et al. 2009

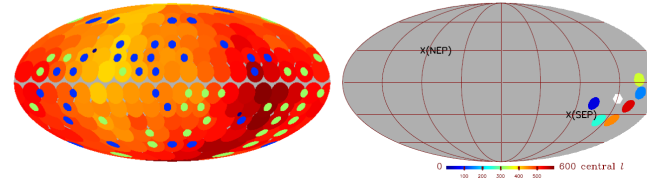


Figure 4: The left panel (from Eriksen et al.¹²) illustrates the hemisphere asymmetry seen in WMAP. The color of each large disk indicates the ratio of power in a hemisphere centered on the disk to power in the opposite

Multipole alignments

Tegmark et al. (2003),
de Oliveira-Costa et al. (2004),
Schwarz et al. (2004)

Preferred direction?

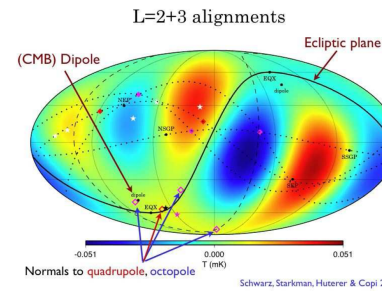


Figure 5: From Schwarz et al.⁹ The quadrupole plus octopole of the WMAP data. Several directions that can be computed from these multipoles are indicated, along with the orientations of the ecliptic and dipole.

Power spectrum anisotropy

Groeneboom & Eriksen (2008)

$$P(k) = P_0(k) \left[1 + g(k) (\vec{k} \cdot \vec{n})^2 \right]$$

\vec{n} : preferred direction

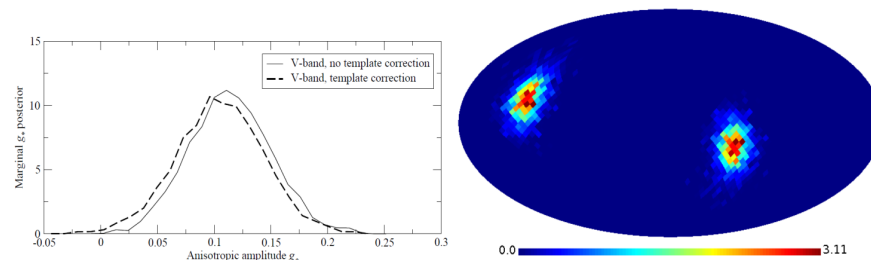


FIG. 10.— Marginal ACW posteriors obtained from the non-template corrected V-band, $P(\hat{n}|\mathbf{d})$ (right) and $P(g_*|\mathbf{d})$ (left). Notice how $P(g_*|\mathbf{d})$ is shifted insignificantly with respect to the template-corrected V-band posterior.

Symmetry breakdown gives variety!

Time dependent VEV of inflaton: $\phi(t) \equiv \langle \hat{\phi}(x^i, t) \rangle$

- ◆ statistically homogeneous
- ◆ statistically isotropic
- ◆ ~~Gaussian~~
- ◆ ~~scale invariant~~

Time dependent VEVs of inflaton and a gauge field:

$$\phi(t) \equiv \langle \hat{\phi}(x^i, t) \rangle \quad A_i(t) \equiv \langle \hat{A}_i(x^i, t) \rangle$$

gives rise to

a physical mechanism for the statistical anisotropy.

Watanabe, Kanno, Soda, arXiv:0902.2833; PRL 102, 191302, 2009

Gauge field and inflation

power-law inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]$$

$$V = V_0 e^{\lambda \frac{\phi}{M_p}}$$

In this case, it is well known that there exists an isotropic power law inflation

$$ds^2 = -dt^2 + t^{4/\lambda^2} (dx^2 + dy^2 + dz^2)$$

$$\frac{\phi}{M_p} = -\frac{2}{\lambda} \log t$$

What happens if a gauge field exists with a non-trivial kinetic term?

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right]$$

gauge kinetic function

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$f = f_0 e^{\rho \frac{\phi}{M_p}}$$

Let the direction of the gauge field to be x - axis.

$$A_\mu = (0, A_x(t), 0, 0)$$

Then, the metric should be Bianchi Type-I

$$ds^2 = -dt^2 + e^{2\alpha(t)} \left[e^{-4\sigma(t)} dx^2 + e^{2\sigma(t)} (dy^2 + dz^2) \right]$$

Exact Anisotropic inflation

For the parameter region $\lambda^2 + 2\rho\lambda - 4 > 0$, we found the following new solution

$$ds^2 = -dt^2 + t^{2\omega} \left[t^{-4\zeta} dx^2 + t^{2\zeta} (dy^2 + dz^2) \right]$$

$$\omega = \frac{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}{6\lambda(\lambda + 2\rho)} \quad \zeta = \frac{\lambda^2 + 2\rho\lambda - 4}{3\lambda(\lambda + 2\rho)}$$

$$\frac{\phi}{M_p} = -\frac{2}{\lambda} \log t$$

$$\dot{A}_x(t) = C t^\gamma$$

$$\gamma = 4\frac{\rho}{\lambda} - \omega - 4\zeta$$

Apparently, the expansion is anisotropic and its degree of anisotropy is given by

$$\frac{\Sigma}{H} = \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{1}{3} I \varepsilon \approx \frac{\rho_A}{V}$$

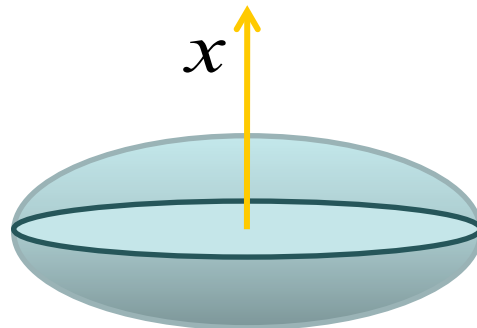
$$I = \frac{\lambda^2 + 2\rho\lambda - 4}{\lambda^2 + 2\rho\lambda}$$

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{6\lambda(\lambda + 2\rho)}{\lambda^2 + 8\rho\lambda + 12\rho^2 + 8}$$

$$0 \leq I < 1$$

slow roll parameter

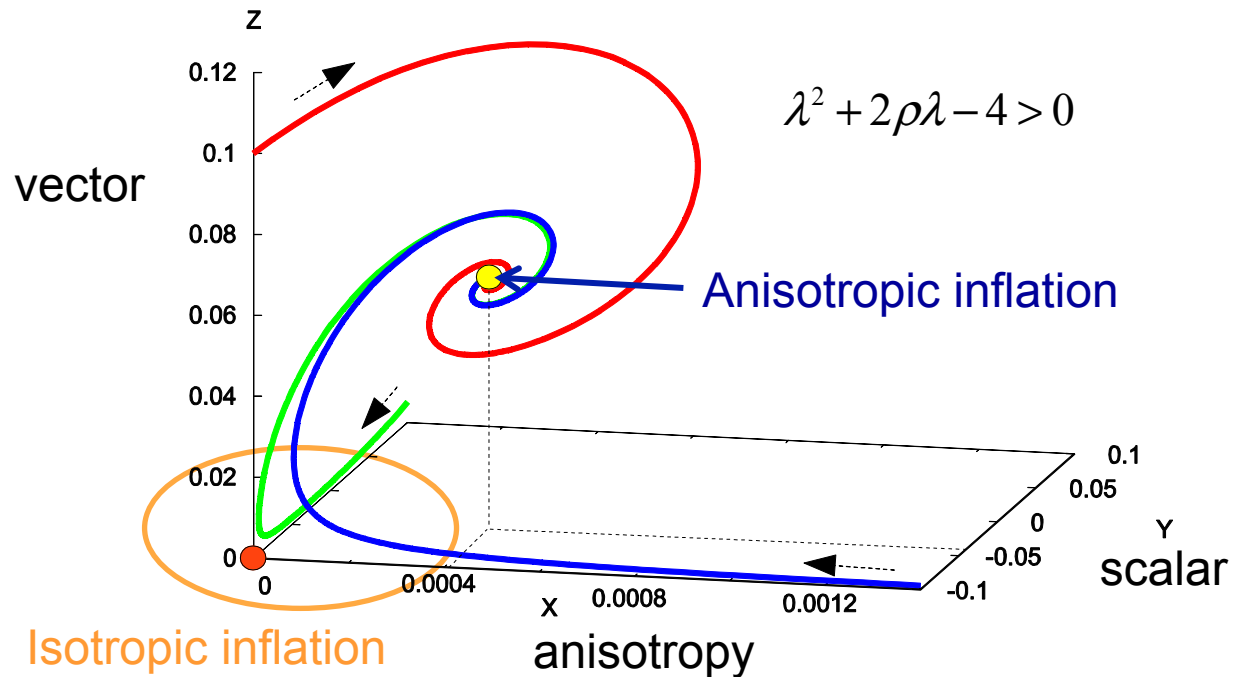
oblate



The phase space structure

Kanno, Watanabe, Soda, JCAP, 2010

We have shown there are two fixed points in the phase space.



Quantum fluctuations generate seeds of coherent gauge fields.

This can be regarded as **the spontaneous breakdown of rotational symmetry.**

The result universally holds for other set of potential and gauge kinetic functions.

Universality of Anisotropic inflation

Supergravity action

$$S = \int d^4x \left[\sqrt{-g} R + g_{i\bar{j}} \partial^\mu \phi^i \partial_\mu \phi^{\bar{j}} - e^{\kappa^2 K(\phi)} g^{i\bar{j}} \left(D_i W(\phi) D_{\bar{j}} \bar{W}(\phi) - 3\kappa^2 |W(\phi)|^2 \right) \right. \\ \left. - \frac{1}{4} \text{Re} f_{ab}(\phi^i) F^{a\mu\nu} F_{\mu\nu}^b - \frac{1}{4} \text{Im} f_{ab}(\phi^i) F^{a\mu\nu} \tilde{F}_{\mu\nu}^b + \dots \right]$$

$$ds^2 = -dt^2 + e^{2Ht} \left[e^{-4\Sigma t} dx^2 + e^{2\Sigma t} (dy^2 + dz^2) \right]$$

$$\frac{\Sigma}{H} \approx \frac{\rho_A}{V} = \frac{\text{energy density of gauge field}}{\text{energy density of inflaton field}}$$



Statistical Symmetry Breaking in the CMB

Predictions of anisotropic inflation

Watanabe, Kanno, Soda, PTP, 2010

Dulaney, Gresham, PRD, 2010

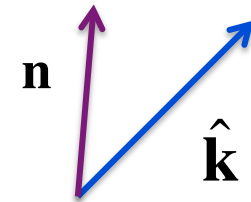
Gumrukcuoglu, Himmetoglu, Peloso PRD, 2010

Statistical anisotropy in curvature perturbations

$$P_\zeta(\mathbf{k}) = P_0(k) \left[1 - g_\zeta (\mathbf{n} \cdot \hat{\mathbf{k}})^2 \right]$$

$$g_\zeta = \frac{24}{\epsilon} \frac{\rho_A}{V} N^2(k)$$

preferred
direction



Statistical anisotropy in GW spectrum

$$P_h(\mathbf{k}) = P_0(k) \left[1 - g_h (\mathbf{n} \cdot \hat{\mathbf{k}})^2 \right]$$

$$g_h = 6 \frac{\rho_A}{V} N^2(k)$$

Cross correlation between curvature perturbations and GW

$$r_c = \frac{\langle \zeta h \rangle}{\langle \zeta \zeta \rangle} = -24 \frac{\rho_A}{V} N^2(k)$$

TB correlation in CMB

consistency relations between observables

$$4g_h = \epsilon_H g_\zeta \quad r_c = -4g_h$$

Angular power spectrum without rotation invar.

Cross correlation

$$P_{\zeta h}(\mathbf{k}) = \sqrt{2} P_{\zeta}(k) \varepsilon g_{\zeta} \sin^2 \theta$$

Angular power spectrum of X and Y reads

$$C_{\ell\ell'}^{XY} \propto \int d\Omega_k P_{XY}(\mathbf{k}) {}_{-s}Y_{\ell m}^*(\hat{\mathbf{k}}) {}_{-s'}Y_{\ell' m'}(\hat{\mathbf{k}})$$

For isotropic spectrum, $P(\mathbf{k}) = P(k)$, we have $C_{\ell\ell'}^{XY} \propto \delta_{\ell\ell'}$

For anisotropic spectrum, there are off-diagonal components.

For example,

$$C_{\ell\ell'}^{TB} \propto \int d\Omega_k P_{TB}(\mathbf{k}) Y_{\ell m}^*(\hat{\mathbf{k}}) {}_{-2}Y_{\ell' m'}(\hat{\mathbf{k}}) \propto \delta_{\ell, \ell' \pm 1}$$

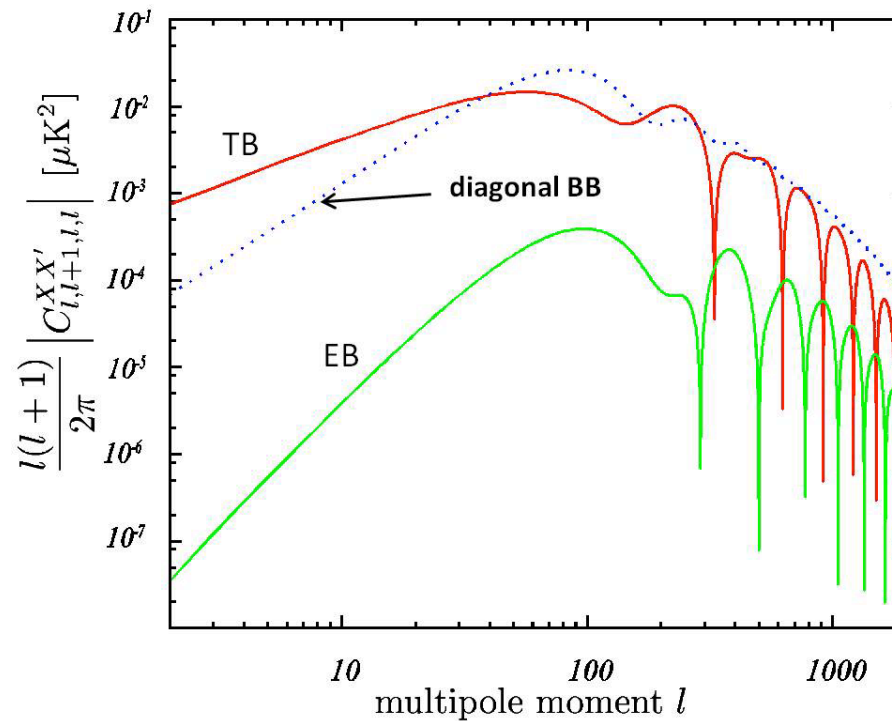
The off-diagonal part of the angular power spectrum tells us if the gauge kinetic function plays a role in inflation.

The impact of gauge fields on primordial GWs

When we assume the tensor to the scalar ratio $r = 0.3$
and scalar anisotropy $g_\zeta = 0.3$

The off-diagonal spectrum becomes

Watanabe, Kanno, Soda, MNRAS Letters, 2011



The anisotropic inflation can be tested through the CMB observation!

Observational bound on the statistical anisotropy

$$P(\mathbf{k}) = P(k) \left[1 + g_*(k) (\mathbf{k} \cdot \vec{n})^2 \right]$$

Table 16. Constraints on g_* , the quadrupolar asymmetry amplitude, determined from the SMICA, NILC, SEVEM, and Commander foreground-cleaned maps. Error bars are 68 % CL.

Ade et al. 2015

$g_* \times 10^2$	$q = 0$	$q = 1$	$q = 2$
Commander ...	$0.19^{+1.97}_{-1.40}$	$-0.09^{+1.85}_{-1.83}$	$-0.27^{+1.14}_{-1.10}$
NILC	$0.60^{+1.73}_{-1.62}$	$0.16^{+1.51}_{-0.99}$	$-0.04^{+0.73}_{-0.66}$
SEVEM	$0.13^{+1.55}_{-1.26}$	$0.03^{+1.38}_{-1.04}$	$-0.01^{+0.98}_{-0.71}$
SMICA	$0.23^{+1.70}_{-1.24}$	$0.16^{+1.47}_{-1.00}$	$0.13^{+1.01}_{-0.61}$

Cf. WMAP constraint $g_* \leq 0.01$

Kim & Komatsu 2013

Statistically anisotropic Non-gaussianity

Bispectrum

$$B = \frac{3}{10} (2\pi)^{5/2} f_{NL} P^2 \frac{\sum_{i=1}^3 k_i^3}{\prod_{i=1}^3 k_i^3} \quad r_2 = \frac{k_2}{k_1}, \quad r_3 = \frac{k_3}{k_1}$$

$$f_{NL}^{local} \approx 2.6 \left(\frac{g_\zeta}{0.01} \right) \left(\frac{N_{CMB}}{60} \right) \quad \text{Bartolo et al. 2012}$$

There was no large local type non-gaussianity in Planck data!

Planck data

Non-gaussianity

$$f_{NL}^{local} = 2.7 \pm 5.8$$

Ade et al. 2015

Further symmetry break down ...

Supergravity action

$$S = \int d^4x \left[\sqrt{-g} R + g_{i\bar{j}} \partial^\mu \phi^i \partial_\mu \phi^{\bar{j}} - e^{\kappa^2 K(\phi)} g^{i\bar{j}} \left(D_i W(\phi) D_{\bar{j}} \bar{W}(\phi) - 3\kappa^2 |W(\phi)|^2 \right) - \frac{1}{4} \text{Re} f_{ab}(\phi^i) F^{a\mu\nu} F_{\mu\nu}^b - \frac{1}{4} \text{Im} f_{ab}(\phi^i) F^{a\mu\nu} \tilde{F}_{\mu\nu}^b + \dots \right]$$

Spatial translation symmetry

dipole anisotropy

Parity symmetry

chromo-natural inflation
parity violating non-gaussianity
chiral GWs

Summary



Based on the view of spontaneous symmetry breakdown,

- We have found **Anisotropic inflation** can be realized in the presence of gauge fields.
- We have found **a mechanism for generating the statistical anisotropy in the CMB.**

More precisely, the mechanism predicts

- ✓ the statistical anisotropy in scalar and tensor fluctuations
 - ✓ the cross correlation between scalar and tensor
 - ✓ the sizable non-gaussianity
- It turned out that
inflation with spontaneous symmetry breakdown
gives a unified picture of CMB physics.