

# Telescope Precision Studies

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## Status report

- Introduction
- Analysis method
- Main Results
- Conclusions and Plans

# Introduction

## Motivation

The main aims of this study

- understand the position measurement in the telescope
- optimize the performance by suggesting the best plane setup

## Approach

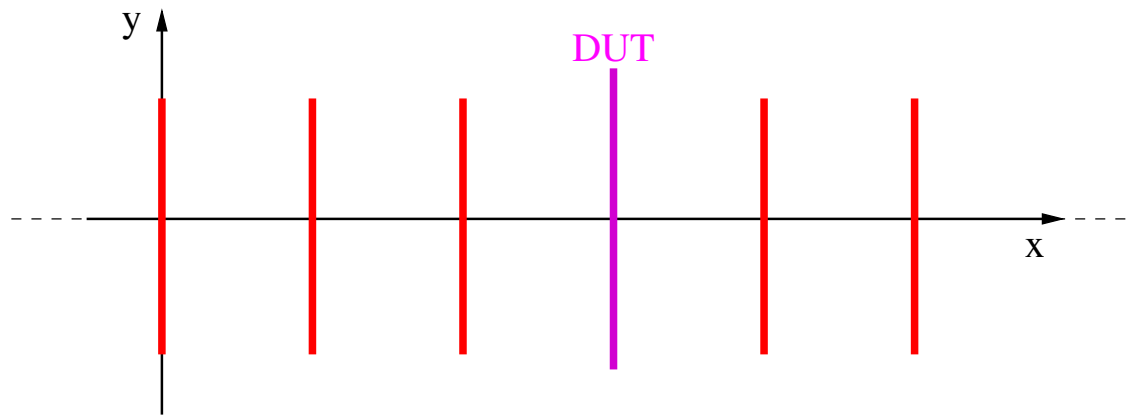
Use analytical method for track fitting including multiple scattering (!!!)

Simplifying assumptions:

- small scattering angles (Gaussian approximation)
- Gaussian position measurement errors
- perfect alignment (could be taken into account !)
- no additional material (windows, etc.) (could be taken into account as well !)

# Analysis method

## Geometry description



Geometry can be specified by giving:

- $N$  - number of detector planes (including DUT)
- $x_i$  - position of each plane ( $i = 1 \dots N$ )
- $\sigma_i$  - position resolution in each plane ( $i \neq i_{DUT}$ )
- $\Delta\theta_i$  - average scattering angle in each plane

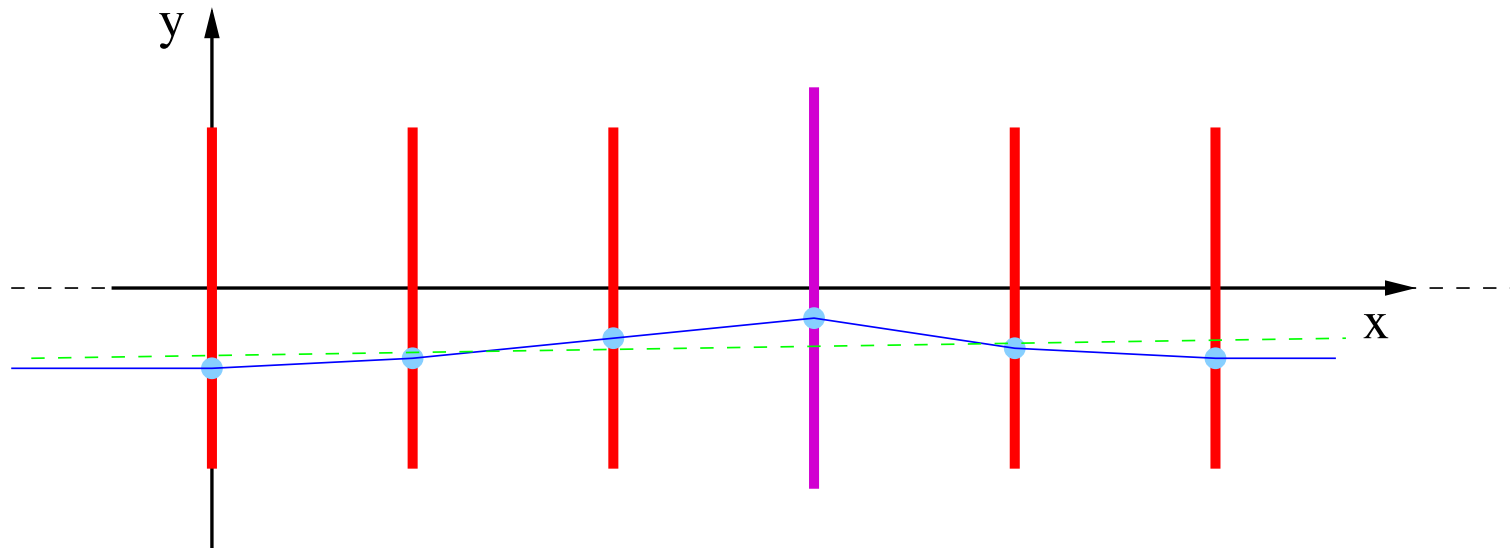
For given telescope parameters ( $N, \sigma_i, \Delta\theta_i$ ) we can look for configuration (plane ordering, values of  $x_i$ ) resulting in best determination of particle position at DUT

# Analysis method

## Multiple scattering

Distances between planes  $\sim 0(10 \text{ mm})$  + scattering angles  $\sim 0(0.1 \text{ mrad})$

$\Rightarrow$  track displacement due to scattering  $\sim 0(1 \mu\text{m})$  (for beam energy of few GeV)



Displacement comparable with position resolution ( $1 - 2 \mu\text{m}$ ) !

$\Rightarrow$  significantly influences the measurement, can not be neglected !

Straight line fit is not sufficient...

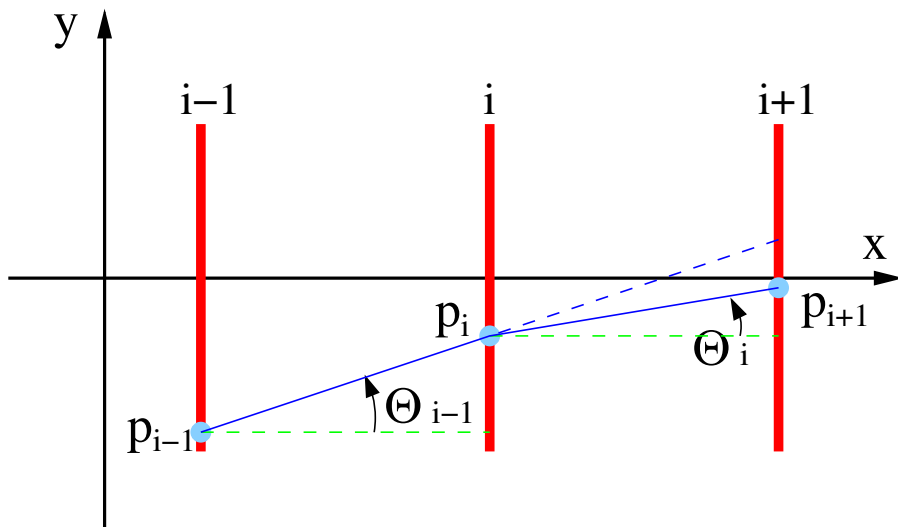
# Analysis method

## Track fitting

We want to determine track positions in each plane (including DUT), i.e.  $N$  parameters ( $p_i, i = 1 \dots N$ ), from  $N - 1$  measured positions in telescope planes ( $y_i, i \neq i_{DUT}$ ).

However, we can use constraints on multiple scattering!

Contribution of plane  $i$  to  $\chi^2$  of the fit



$$\Delta\chi_i^2 = \left( \frac{y_i - p_i}{\sigma_i} \right)^2 + \left( \frac{\Theta_i - \Theta_{i-1}}{\Delta\Theta_i} \right)^2$$

position measurement      multiple scattering

where:  $\Theta_i = \frac{p_{i+1} - p_i}{x_{i+1} - x_i}$

Both terms present for planes  $i \neq 1, i_{DUT}, N$ ,  
first term missing for DUT, second for first and last plane

$\chi^2$  minimum can be found by solving the matrix equation.

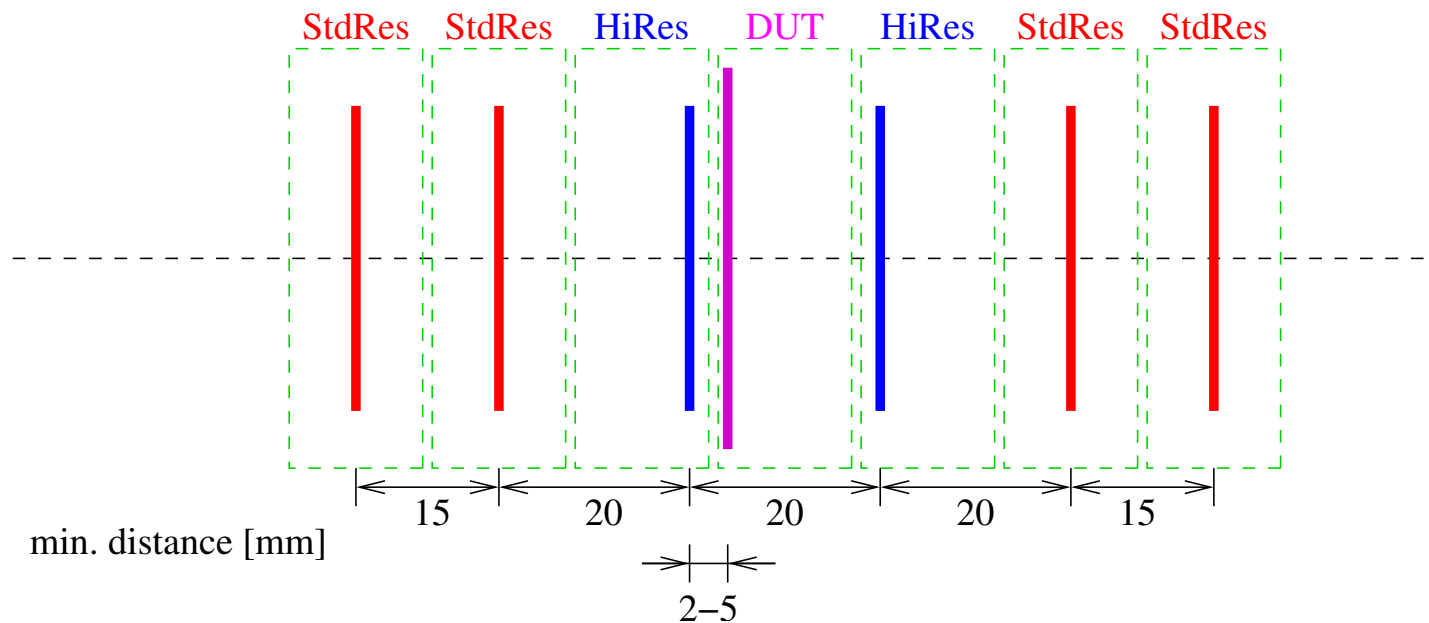
As a by-product we get also an **error** on the position reconstructed at **DUT**.

# Analysis method

## Realistic telescope geometry thanks to W.Dulinski

The minimum distance between **DUT** and **one** of the telescope planes,  $d_{min}$ , is **5 mm** (easy, realistic) or even **2 mm** (hard, optimistic).

However, other distances can not be smaller than 15 or 20 mm:



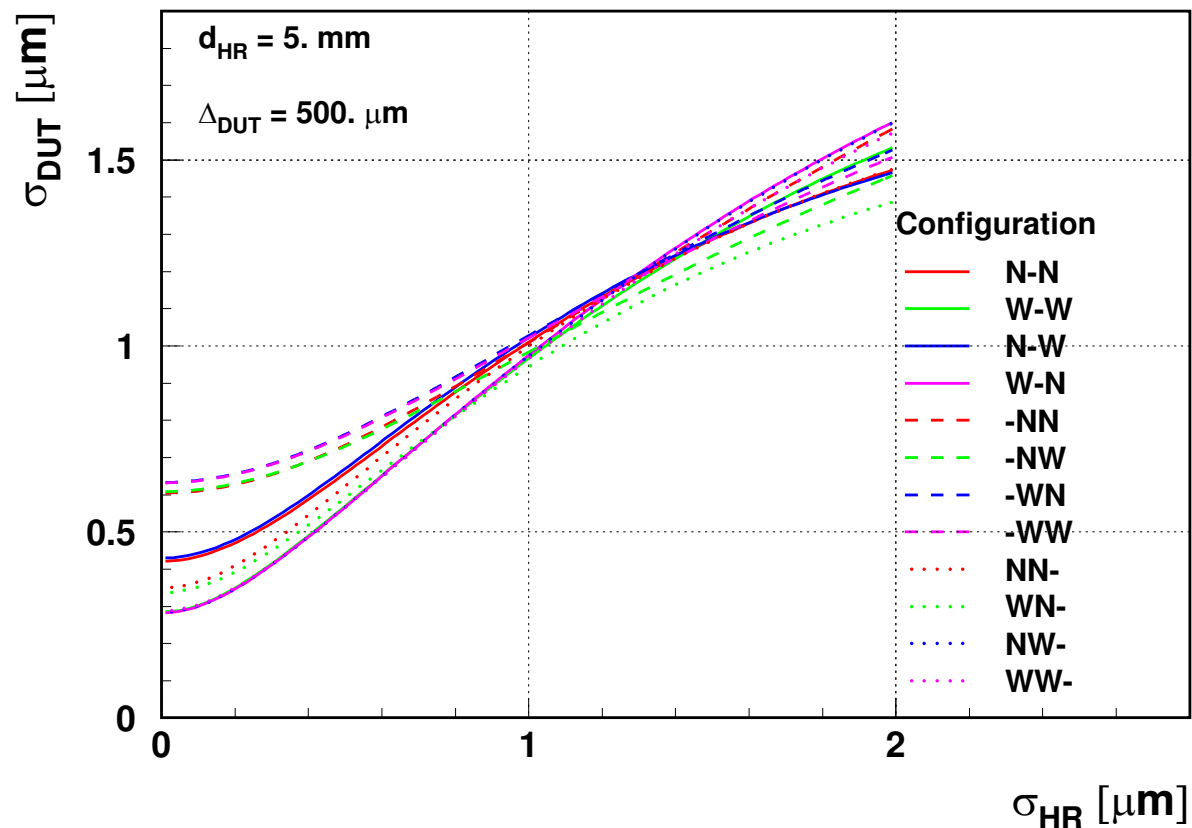
In addition to **standard sensor planes** with  $2 \mu m$  resolution we can consider adding one or **two high resolution planes** ( $\sigma_{HR} \sim 1 \mu m$ )

# Results

## 4 (1+3) telescope planes

Simplest case: 1 high resolution (HR) and 3 standard sensor planes ( $120 \mu\text{m}$  each)

Expected position error at DUT,  $\sigma_{DUT}$ , as a function of the HR plane resolution,  $\sigma_{HR}$ , for different telescope configurations: 6 GeV  $e^-$  beam, DUT thickness of  $500 \mu\text{m}$



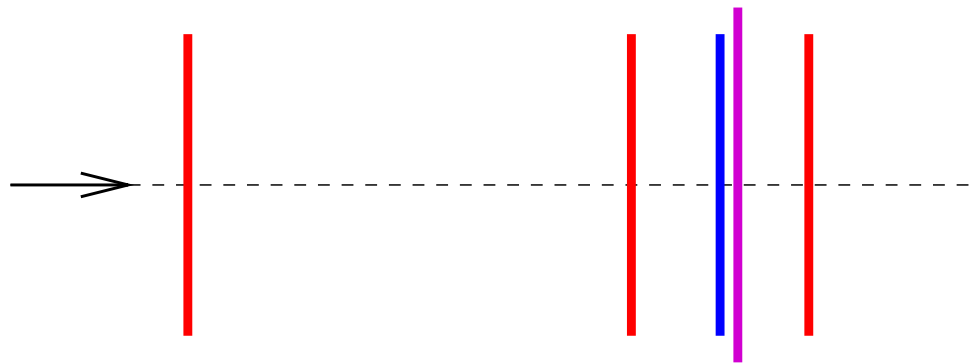
# Results

## 4 (1+3) telescope planes

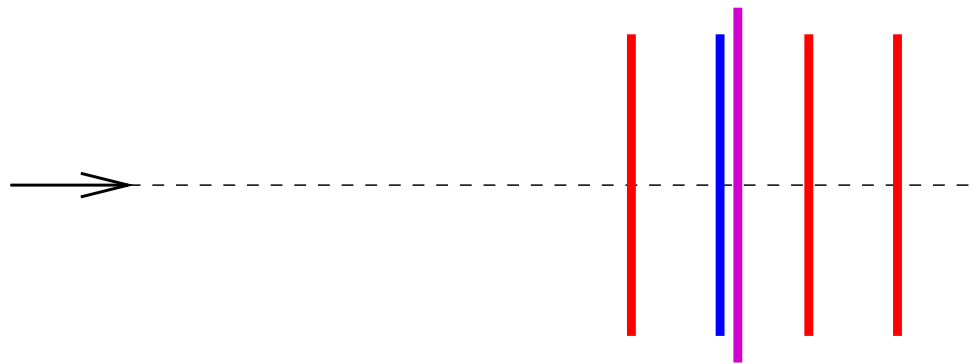
6 GeV  $e^-$  beam

Assuming HR plane resolution is not better than  $1 \mu m$  and DUT is thinner than  $1 mm$ :

Best precision for **thick DUT**,  $\Delta_{DUT} \geq 200 \mu m$ , is obtained for **WN-** configuration



**N-N** configuration gives best precision for **very thin DUT**,  $\Delta_{DUT} \leq 200 \mu m$



not to scale !



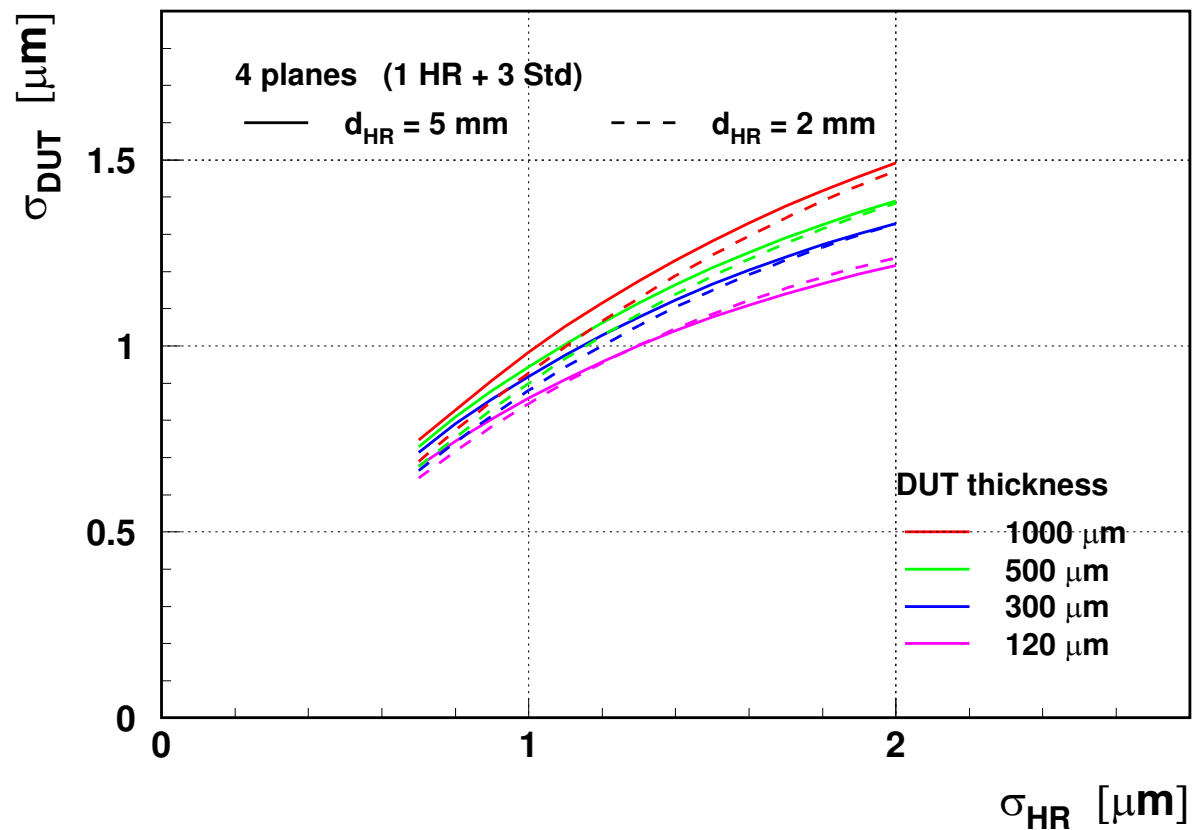
# Results

## 4 (1+3) telescope planes

6 GeV  $e^-$  beam

High resolution plane should be put as close to DUT as possible. (for  $\sigma_{HR} \sim 1 \mu\text{m}$ )

Expected position error at DUT,  $\sigma_{DUT}$ , as a function of the HR plane resolution,  $\sigma_{HR}$ , for optimum telescope configuration:



# Results

## 4 (2+2) telescope planes

Two high resolution + two standard planes: more possibilities!

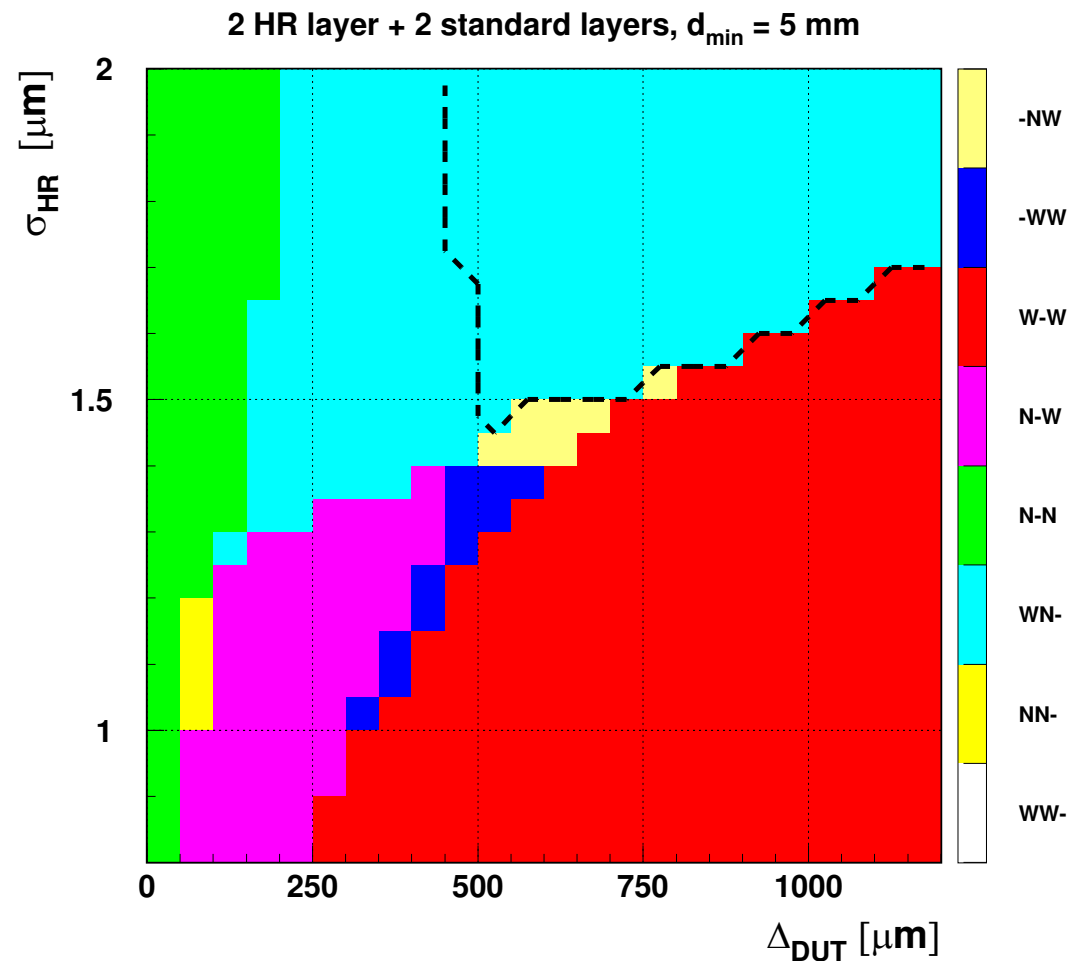
Configuration choice as a function of DUT thickness and HR plane resolution:

$$d_{min} = 5 \text{ mm}$$

Above dashed line:  
**better performance**  
if both HR planes in  
front of DUT

large  $\sigma_{HR}$  &  $\Delta_{DUT}$

6 GeV  $e^-$  beam



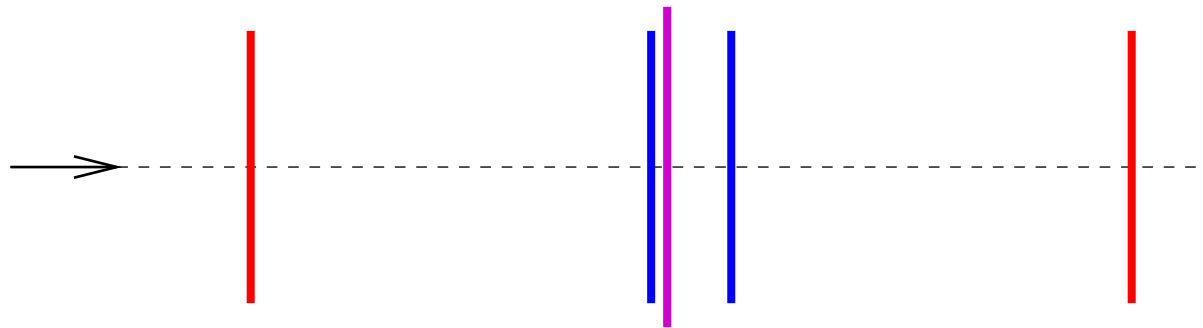
# Results

4 (2+2) telescope planes

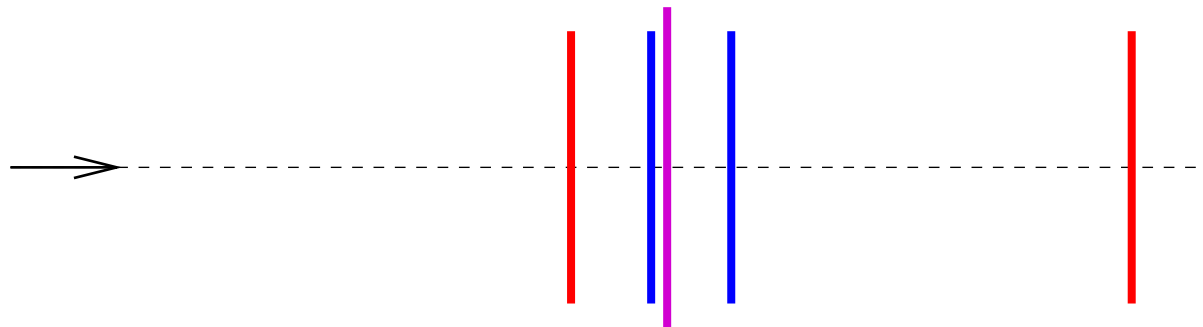
6 GeV  $e^-$  beam

Assuming HR plane resolution is of the order of  $1 \mu m$ :

**W-W** configuration gives best precision for **thick DUT**



**N-W** configuration gives best precision for **thin DUT**, or smaller  $d_{HR}$



# Results

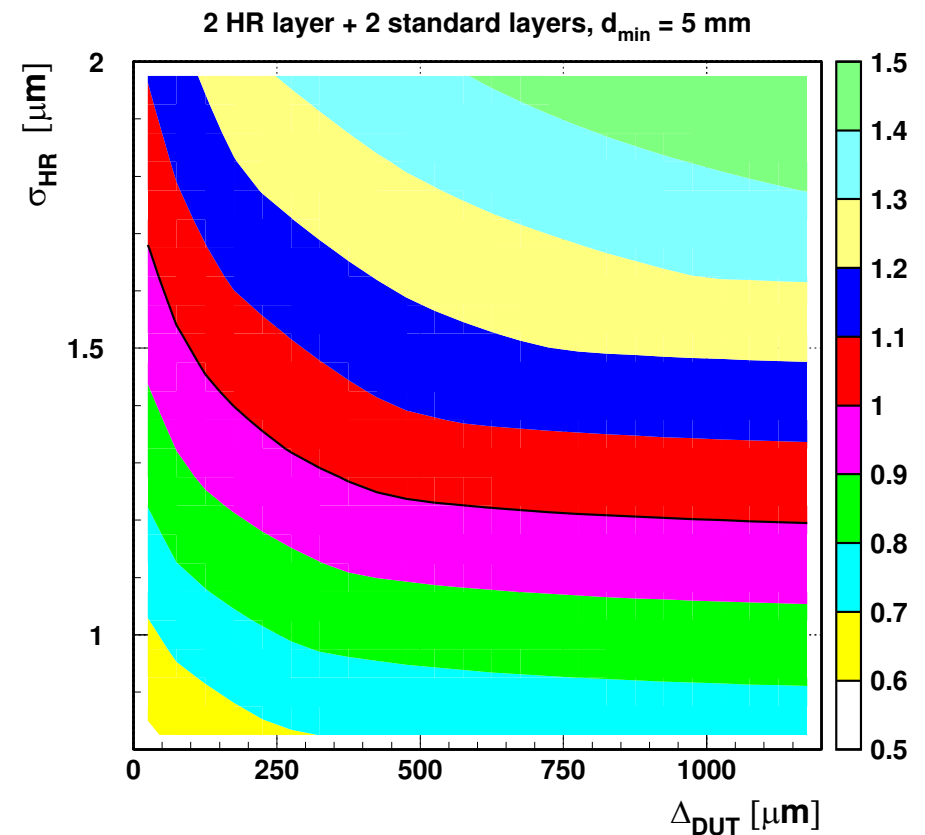
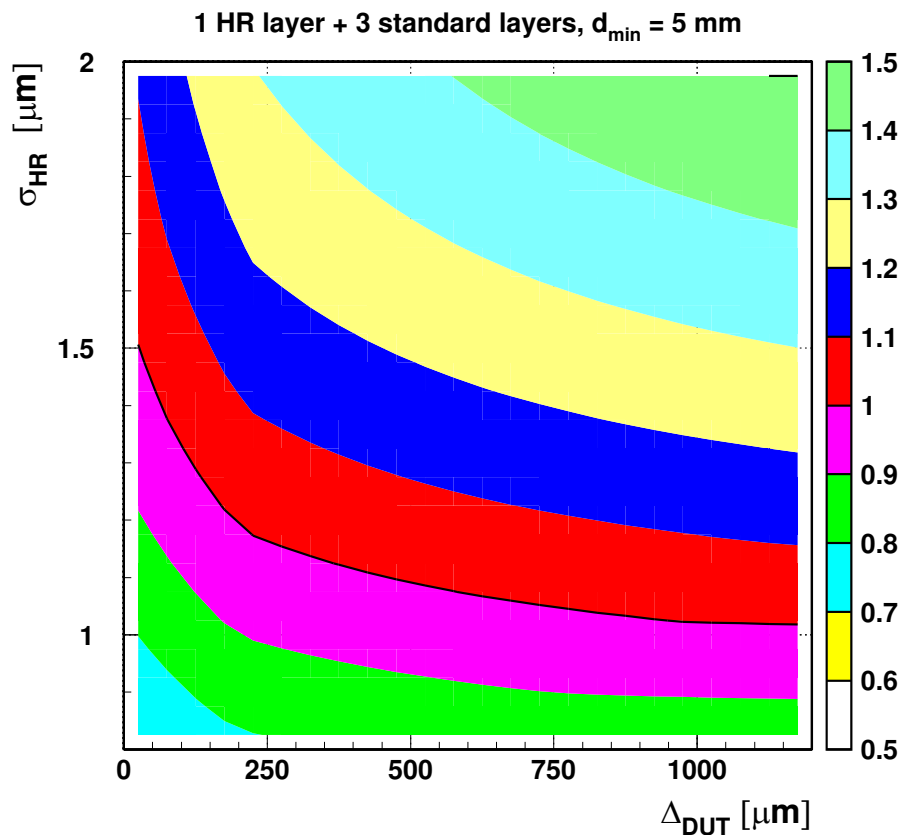
## 4 telescope planes

Configuration with two HR planes always gives better precision than with one HR plane.

Expected statistical precision of position reconstruction at DUT [ $\mu\text{m}$ ]:

1 HR plane      6 GeV  $e^-$  beam

2 HR planes       $d_{\min} = 5 \text{ mm}$



# Results

4 telescope planes

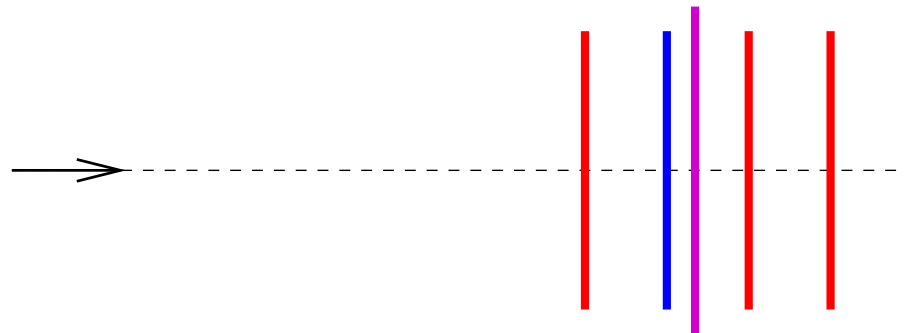
100 GeV  $\pi^-$  beam

Multiple scattering much smaller, much less important!

Best precision obtained for detector planes put close to each other (N–N configuration)

With **one HR plane** DUT should be placed in  $\sim \frac{1}{3}$  of the distance between sensors

assuming  $\sigma_{HR} \sim 1\mu m$



With **two HR planes** DUT should be placed **in the middle** between HR planes.



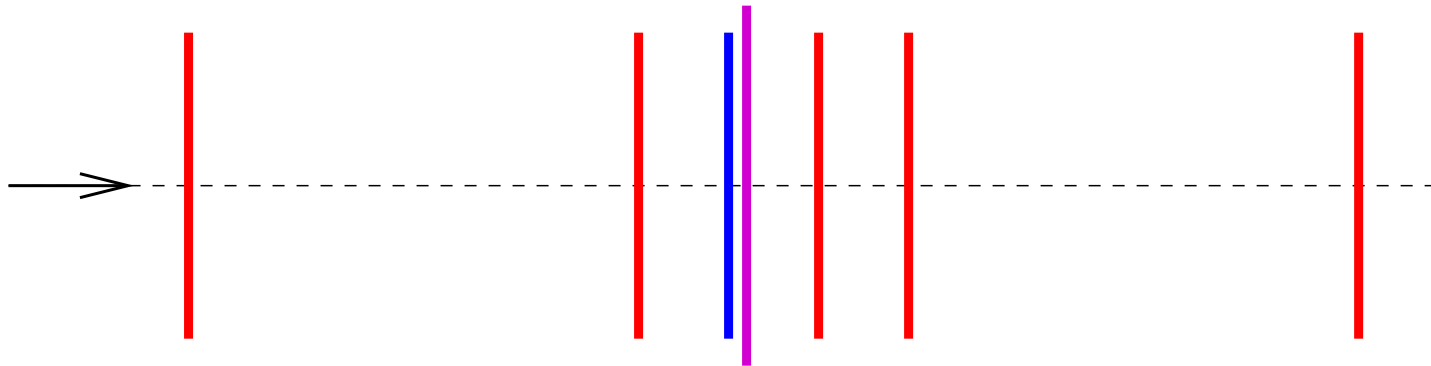
No need to minimize HR–DUT distance !

# Results

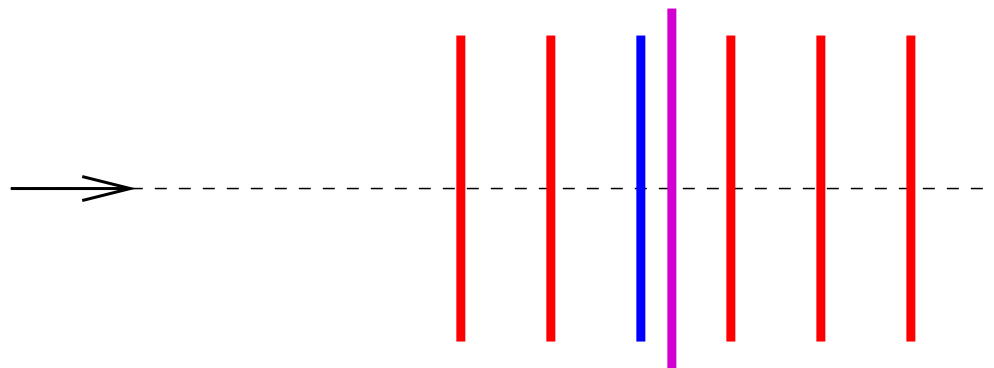
## 6 (1+5) telescope planes

One high resolution and 5 standard telescope planes

For low energy beam (eg. 6 GeV  $e^-$ ) best measurement in **WN-NW** configuration



For high energy beam (eg. 100 GeV  $\pi^-$ ) best measurement in **NN-NN** configuration



# Results

## 6 (2+4) telescope planes

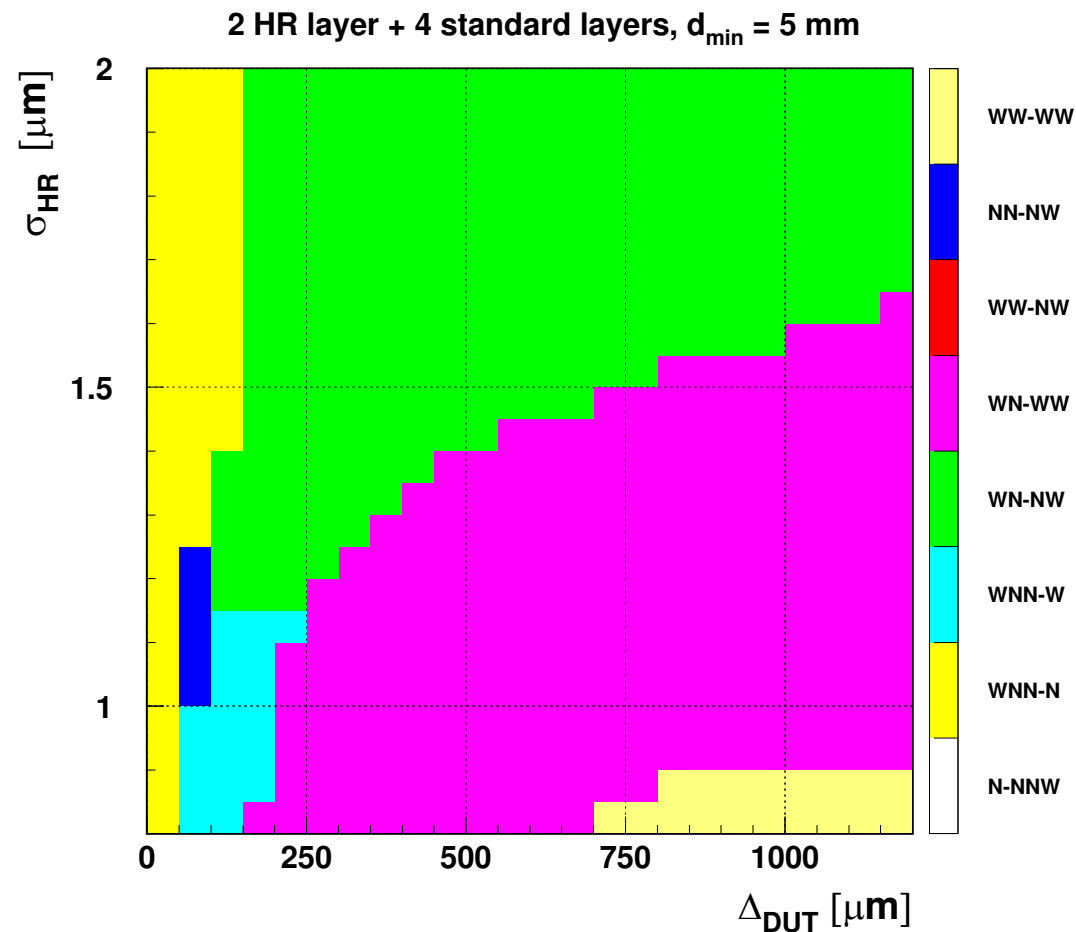
Two high resolution + four standard planes: even more possibilities!

Configuration choice as a function of DUT thickness and HR plane resolution:

$$d_{min} = 5 \text{ mm}$$

Best performance with second HR plane always placed behind DUT

6 GeV  $e^-$  beam

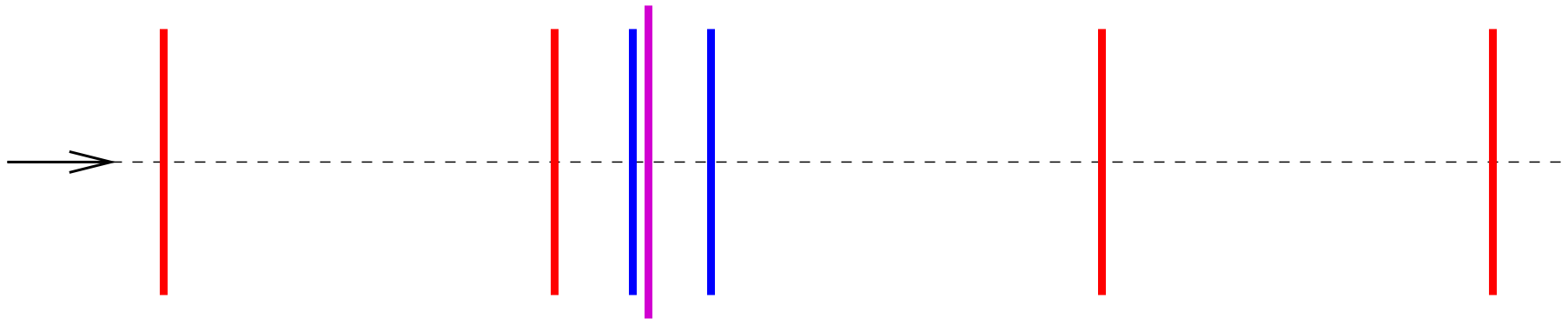


# Results

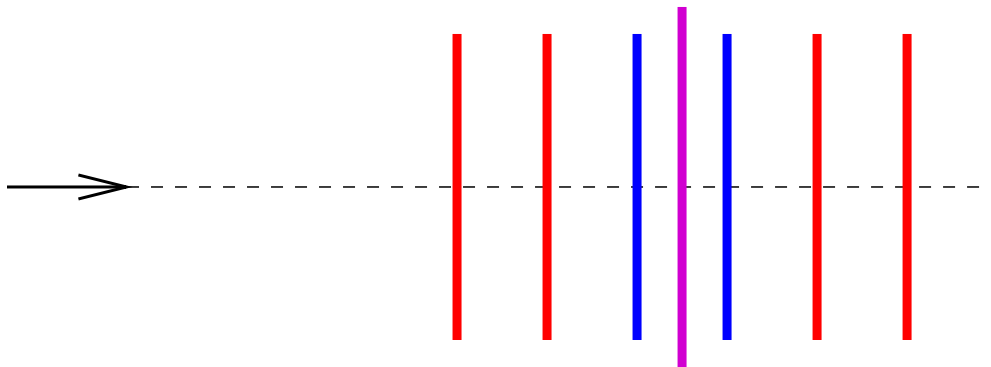
## 6 (2+4) telescope planes

Assuming HR plane resolution is of the order of  $1 \mu m$ ,  $d_{min} = 5mm$

For low energy beam (eg.  $6 \text{ GeV } e^-$ ) best measurement in **WN-WW** configuration (except for very thin DUT)



For high energy beam (eg.  $100 \text{ GeV } \pi^-$ ) best measurement in **NN-NN** configuration





# Results

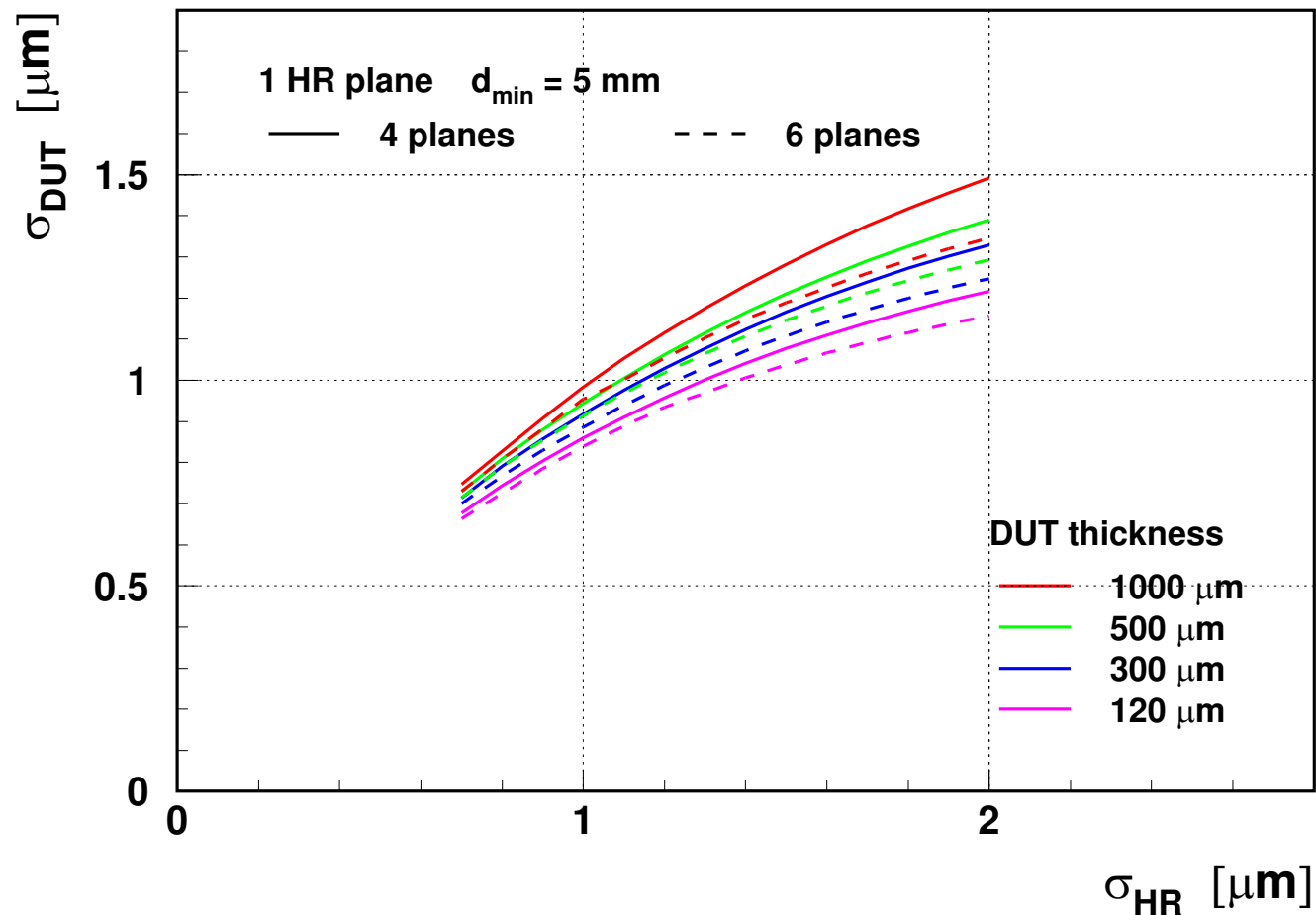
## 6 vs 4 telescope planes

Configuration with 6 planes always gives better precision than 4 planes.

Expected position error at DUT,  $\sigma_{DUT}$ , as a function  $\sigma_{HR}$

1 HR plane

$d_{min} = 5 \text{ mm}$



# Results

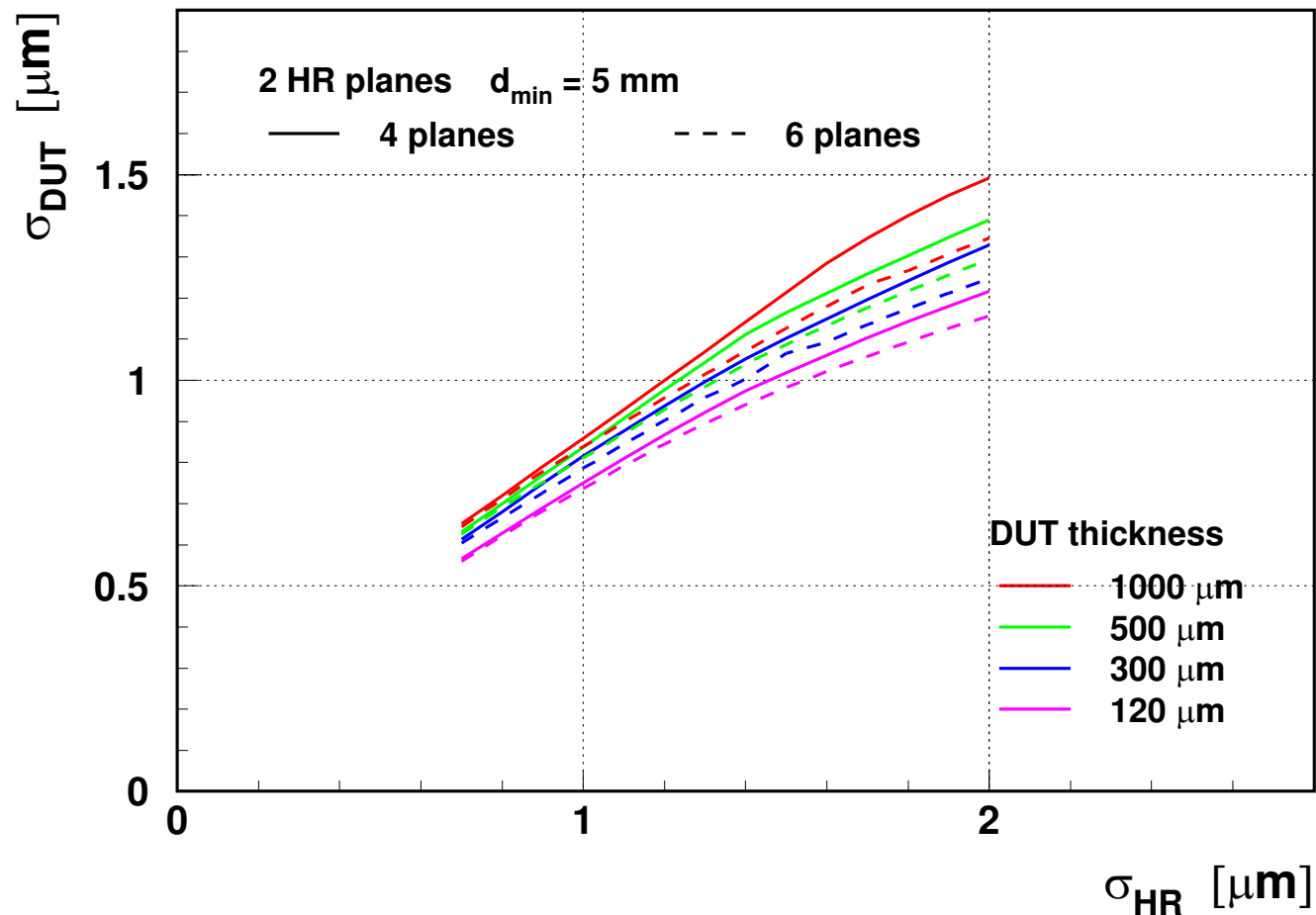
## 6 vs 4 telescope planes

Configuration with 6 planes always gives better precision than 4 planes.

Expected position error at DUT,  $\sigma_{DUT}$ , as a function  $\sigma_{HR}$

2 HR planes

$d_{min} = 5 \text{ mm}$



## Conclusions and Plans

- Analytical method used to describe the performance of the telescope with realistic geometry constraints.
- The optimum telescope setup is not uniquely defined  
⇒ few configurations, for different telescope parameters, suggested.
- To achieve error on the reconstructed particle position at DUT of  $1\ \mu m$  at least one high resolution plane is needed
- Significant improvement expected from second HR plane.
- 6 sensor planes always give better position resolution than 4 planes

Our current aim is to confirm obtained results with GEANT 4 simulation, we hope to have first results for EUDET annual meeting.

For detailed description of the analysis and current results see:

[http://hep.fuw.edu.pl/u/zarnecki/talks/afz\\_jra1\\_apr06.pdf](http://hep.fuw.edu.pl/u/zarnecki/talks/afz_jra1_apr06.pdf)

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