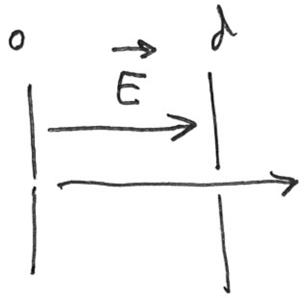
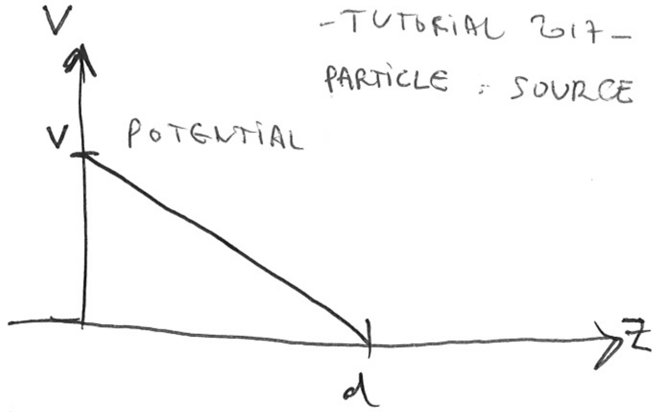
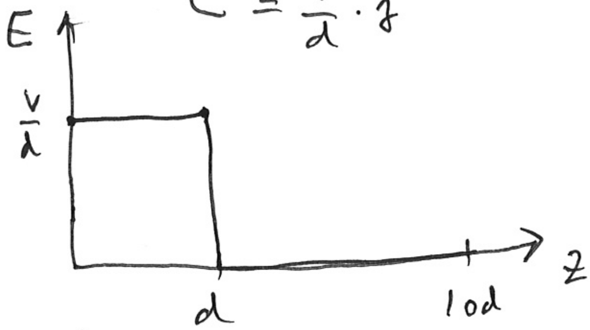
1.1 $E = \frac{V}{d}$





$$\vec{E} = \frac{V}{d}\cdot\vec{z}$$



$$E = \frac{10^4}{10^{-2}} V/m = 1 MV/m$$

1.2 $0 \leq z \leq d$

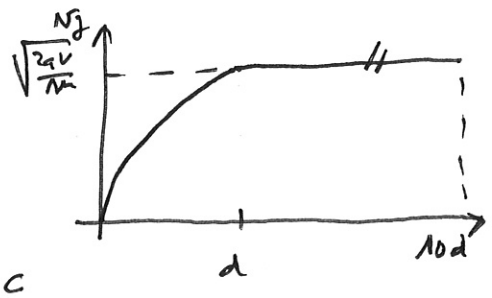
$$m\frac{dv_z}{dt} = qE \qquad v_z = \frac{dz}{dt} \Rightarrow dt = \frac{dz}{v_z}$$

$$\Rightarrow \frac{dv_z}{dz} m v_z = qE \Rightarrow m\int_0^{v_z} v\,dv = \int_0^z qE\,dz$$

$$\Rightarrow \frac{1}{2} m v_z^2 - 0 = qEz - 0$$

$$\Rightarrow v_z = \sqrt{\frac{2qE.z}{m}} \qquad z \leq d$$

$$v_z = \sqrt{\frac{2qEd}{m}} = \sqrt{\frac{2qV}{m}} \qquad z \geq d$$



$V_z = V$

$$v_z = \sqrt{\frac{2.1.10^4}{40.931,5\,10^6}}\cdot c \qquad v_z = 219793 \text{ m/s}$$

1.3

$$m\frac{dv}{dt} = qE \rightarrow v(t) - \cancel{v(0)} = \frac{qE}{m}(t-0)$$

$$\Rightarrow z(t) - \cancel{z(0)} = \frac{1}{2}\frac{qEt^2}{m} - 0 \quad \text{for } t \,/\, z(t) \leq d$$

$$z(t_d) = d \Rightarrow d = \frac{qE t_d^2}{2m} \Rightarrow t_d = \sqrt{\frac{2md}{qE}} \qquad E = \frac{V}{d}$$

$$t_d = \sqrt{\frac{2md^2}{qV}} = \sqrt{\frac{2m}{qV}}.d = \sqrt{\frac{2A.uma}{qV}}d = 9,1.10^{-8}\ s.$$

1.4 Δr time to travel from $z = d$ to $z = 10d$

$$v_z = \frac{10d - d}{\Delta r} \qquad \Delta r = \frac{v_z}{9d} = \frac{219\,793}{0{,}09} = 4{,}09.10^{-7}\,s.$$

$$T = t_d + \Delta r = 9{,}1.10^{-8} + 4{,}09.10^{-7} = 5.10^{-7}\,s$$

1.5 $E_x = \frac{1}{2} kT \qquad E_x = \frac{1{,}38.10^{-23}.1000}{1{,}6.10^{-19}} = 8{,}63.10^{-2}\,eV$

$$\frac{1}{2} m v_x^2 = \frac{1}{2} kT \Rightarrow v_x = \sqrt{\frac{kT}{m}}$$

$$v_x = \sqrt{\frac{8{,}63.10^{-2}}{40.\underbrace{amu}_{931{,}5.10^{6}}}}.c = 804\,m/s$$

1.6

$x = v_x . t_x = 804 \times \underset{5.10^{-7}}{\cancel{4{,}29.10^{-4}}} = 4.10^{-4}\,m$ very small!

1.7 $\delta x = 0{,}05\,m \Rightarrow \delta t_x = \frac{\delta x}{v_x} = \frac{0{,}05}{804} = 6{,}2.10^{-5}\,s.$

$v_z = 219793\,m/s = \frac{L}{\delta r_x} \Rightarrow L \simeq v_z . \delta t_x = 13{,}6\,m$

3. particle trajectory in $\vec{B}$

3.1 $m\frac{d\vec{v}}{dt} = q\vec{v}\times\vec{B}$ $\quad \vec{v}\begin{vmatrix} v_x \\ v_y \\ 0 \end{vmatrix} \quad \vec{B}\begin{vmatrix} 0 \\ 0 \\ B \end{vmatrix}$

$$\Rightarrow \begin{cases} \frac{dv_x}{dt} = \frac{q v_y B}{m} = \omega v_y \\ \frac{dv_y}{dt} = -\frac{qB}{m} v_x = -\omega v_x \end{cases}$$

3.2
$$\begin{cases} \frac{d^2 v_x}{dt^2} = \omega \frac{dv_y}{dt} = -\omega^2 v_x \\ \frac{d^2 v_y}{dt} = -\omega \frac{dv_x}{dt} = -\omega^2 v_y \end{cases}$$

3.3 $$\left.\begin{array}{l} v_x(t) = a\cos\omega t + b\sin\omega t \\ v_x(0) = v \end{array}\right\} \Rightarrow a = v$$

$$v_y(t) = \frac{1}{\omega}\frac{dv_x}{dt} = \frac{1}{\omega}(-\omega v\sin\omega t + b\omega\cos\omega t)$$

$$\left.\begin{array}{l} v_y(t) = -v\sin\omega t + b\cos\omega t \\ v_y(0) = 0 \end{array}\right\} \Rightarrow b = 0$$

So $\begin{cases} v_x(t) = v\cos\omega t \\ v_y(t) = v\sin\omega t \end{cases}$

3.4 $\frac{dx}{dt} = v\cos\omega t \Rightarrow \underset{\substack{\| \\ 0}}{x(t) - x(0)} = \frac{v}{\omega}\sin\omega t - 0$

$\frac{dy}{dt} = v\sin\omega t \Rightarrow \underset{\substack{\| \\ 0}}{y(t) - y(0)} = -\frac{v}{\omega}\cos\omega t + \frac{v}{\omega}$

3.5 $$\underbrace{x^2 + \left(y - \frac{v}{\omega}\right)^2}_{} = \frac{v^2}{\omega^2}\underbrace{(\cos^2 \omega t + \sin^2 \omega t)}_{=1}$$

equation of a circle with center $C\begin{pmatrix} 0 \\ \frac{v}{\omega} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ +\rho \\ 0 \end{pmatrix}$ ← mistake in the Tutorial Text

and a radius $\frac{v}{\omega} = \rho$ = Larmor radius

3.6 $\omega = 2\pi f = \frac{qB}{m}$

$m = 511 \text{keV}/c^2$ $\quad f = \frac{qB}{2\pi m} = \frac{\cancel{e}.2}{511.10^3 \cancel{e}} . c^2 = 56 \text{ GHz}$

3.7 $v = \rho\omega = \rho 2\pi f$

$\rho = \frac{v}{2\pi f}$

$\frac{1}{2} m v^2 = T = 10 \text{keV} \Rightarrow v = \sqrt{\frac{2T}{m}}$

$v = \sqrt{\frac{2.10^4.\cancel{e}}{511.10^3 \cancel{e} / c^2}} = \sqrt{\frac{20}{511}} . c = 5,94.10^7 \text{ m/s}$

$\beta = 0,2 \rightarrow$ relativistic !

So the calculation is FALSE! we need to switch to relativistic equation:

$E = T + mc^2 = \gamma mc^2$

$T = (\gamma - 1) mc^2 \Rightarrow \gamma = 1 + \frac{T}{m_e c^2} = 1 + \frac{10^4}{511.10^3} = 1,0195$

and $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0,1949$

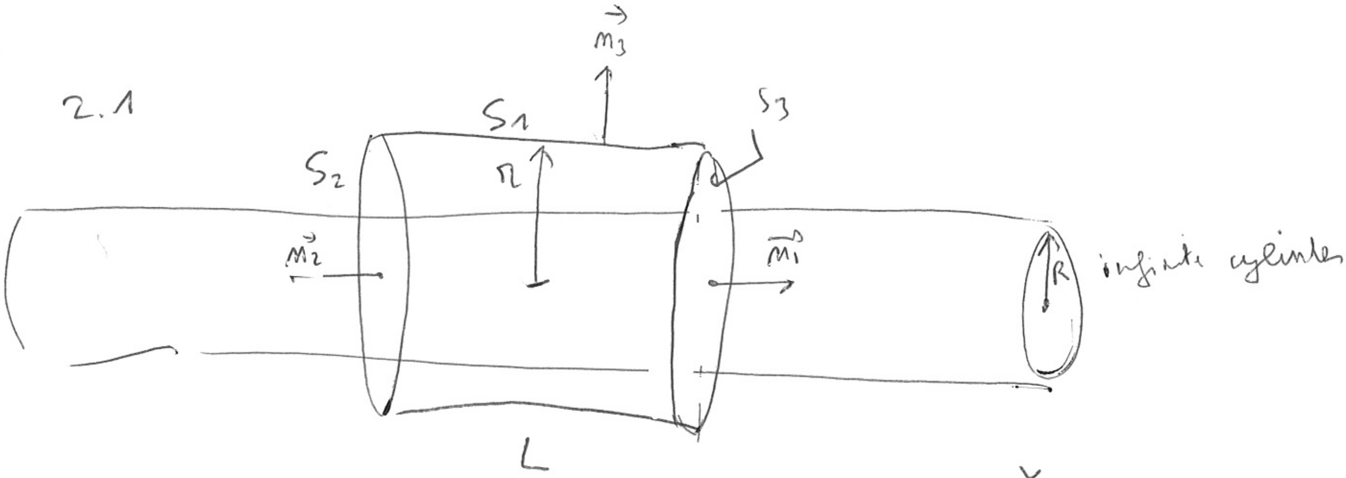
$\Rightarrow v = 5,84 . 10^7 \text{ m/s}$.

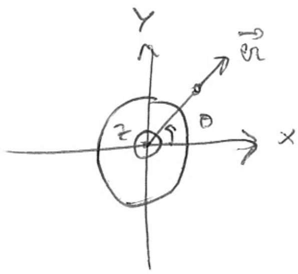
$$\rho = \frac{v}{\omega} = \frac{5,84 \cdot 10^{7}}{2\pi \cdot 56\,10^{9}} = 1,66 \cdot 10^{-4}\ m$$

VERY SMALL!

2.

2.1





gauss Law:

symmetry $\Rightarrow \vec{E} = E_r \cdot \vec{e_r}$

for $r > R$

$$\iint_{S_1+S_2+S_3} \vec{E} \cdot d\vec{S} = \frac{\rho \cdot \pi R^2 \cdot L}{\varepsilon_0}$$

$$\iint_{S_2} \vec{E} \cdot d\vec{S} = 0 \qquad \vec{E} \cdot \vec{n_2} = 0 \quad \& \quad \iint_{S_3} \vec{E} \cdot d\vec{S} = 0 \qquad \vec{E} \cdot \vec{n_3} = 0$$

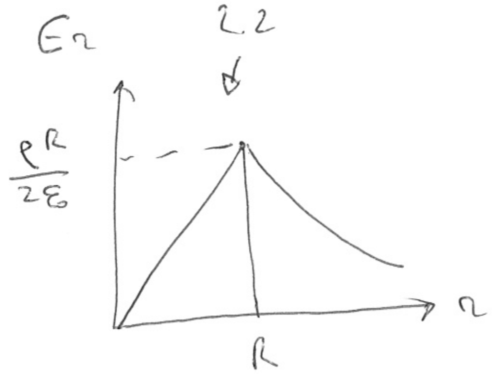
$$\iint_{S_2} \vec{E}\, d\vec{S} = \int_{z=0}^{L}\int_{\theta=0}^{2\pi} E_r\, r\, d\theta\, dz = r E_r \cdot 2\pi L$$

$$\Rightarrow 2\cancel{\pi} \cancel{L} r E_r = \frac{\rho \cancel{\pi} R^2 \cancel{L}}{\varepsilon_0} \Rightarrow E_r = \frac{\rho R^2}{2 \varepsilon_0 r} \quad r \geq R$$

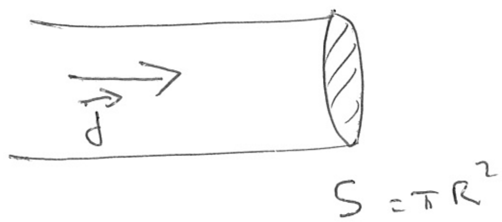
if $r < R$, we get

$$2\cancel{\pi}\cancel{l}\cancel{r}\,E_r = \frac{\rho\,\cancel{\pi}\,r^{\cancel{2}}\,\cancel{l}}{\varepsilon_0}$$

$$\Rightarrow E_r = \frac{\rho r}{2\varepsilon_0} \qquad r < R$$



2.3



$\vec{j} = \rho . \vec{v}$ ← velocity

↑

CURRENT DENSITY

$$I = \iint_S \vec{j} \cdot \vec{ds} = j \cdot \pi R^2 \quad \text{for a cylinder}$$

$$I = \pi R^2 \rho v \Rightarrow \rho = \frac{I}{v \pi R^2}$$

substitute ρ in solution 2.1 gives the answer

2.4

$$\vec{e_r} = \frac{x}{r}\vec{x} + \frac{y}{r}\vec{y}$$

$$m\frac{d\vec{v}}{dt} = qE.\vec{e_r}$$ → charge of a particle on the beam envelope.

So
$$\begin{cases} m\dfrac{dV_x}{dt} = qE\dfrac{x}{r} \\[2ex] m\dfrac{dV_y}{dt} = q\dfrac{Ey}{r} \\[2ex] m\cancel{\dfrac{dV_z}{dt}} = 0 \leftarrow \text{we don't care} \end{cases}$$

$v \quad \vec{v_r} = u.\vec{e_r} = \vec{v_x} + \vec{v_y}$

$\dot{\vec{v_r}} = \dot{\vec{v_x}} + \dot{\vec{v_y}}$

$$\Rightarrow \quad m\frac{du}{dt} = qE = \frac{qI}{2\pi N\varepsilon_0}\cdot\frac{1}{r}$$

$$\frac{dr}{dt} = u \Rightarrow dt = \frac{dr}{u}$$

$$\Rightarrow \quad m\frac{du}{dr}\cdot u = \frac{qI}{2\pi N\varepsilon_0}$$

$$\Rightarrow \quad \frac{1}{2} m\, du^2 = \frac{dr}{r}\,\frac{qI}{2\pi N\varepsilon_0}$$

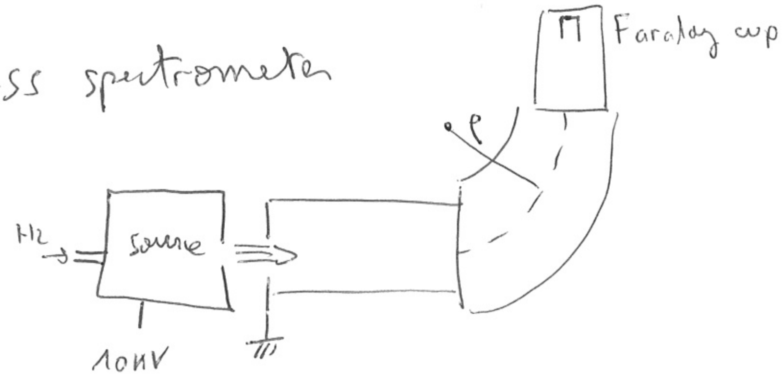
$$\Rightarrow \quad \int_{u=0}^{u} \frac{1}{2} m\, du^2 = \int_{R}^{r>R} \frac{dr}{r}\cdot\frac{qI}{2\pi N\varepsilon_0}$$

$$\Rightarrow \quad \frac{1}{2} m u^2 = qE \ln\frac{r}{R} = \ln\left(\frac{r}{R}\right)\cdot\frac{qI}{2\pi N\varepsilon_0}$$

charge of a particule on the beam envelope

we cannot go further analytically

4. Mass spectrometer



Q charge number
A mass number

4.1 $\frac{1}{2}mv^2 = QeU$ (1) $\Rightarrow v = \sqrt{\frac{2QeU}{m}}$

4.2 $v = \rho\omega = \rho\frac{QeB}{m} = \frac{\rho e}{m_A}\frac{QB}{A}$

$$v^2 = \frac{\rho^2 e^2}{m_A^2}\frac{Q^2B^2}{A^2} \quad (2)$$

$$(1)+(2) \Rightarrow \frac{\rho^2 e^{\cancel{2}}}{m_A^{\cancel{2}}}\frac{Q^2B^2}{A^{\cancel{2}}} = \frac{2\cancel{Q}\,\cancel{e}\,U}{\cancel{A}\,\cancel{m_A}}$$

$$\Rightarrow B^2 = \frac{1}{\rho^2}\cdot\frac{A.U}{Q}\cdot\frac{2m_A}{e} \quad \Rightarrow B = \frac{1}{\rho}\sqrt{\frac{2m_A}{e}}\cdot\sqrt{\frac{A.U}{Q}}$$

4.3 $\frac{A}{Q} = B^2\rho^2 \frac{e}{2m_A}\cdot\frac{1}{U}$

$\frac{A}{Q}$	1	2	3
B	0.01438	0.02034	0,02492

$\frac{A}{Q} = 1 \Rightarrow A = 1$ proton H^+

H_2 feed gas $\Rightarrow \frac{A}{Q} = 2 \Rightarrow H_2^+$

H_2 gas $\rightarrow \frac{A}{Q} = 3 \Rightarrow H_3^+$!

$$\frac{2m_A}{e} = \frac{2\cdot 931{,}5\cdot 10^6\,\cancel{e}V}{\cancel{e}\;c^2} = 1{,}863\cdot 10^9\,\frac{V}{c^2}$$

$$\frac{\rho^2 e}{2m_A}\cdot\frac{1}{U} = \frac{1}{10^4\cdot 1{,}863\cdot 10^9}\cdot c^2$$

5 Beam Loss

see p. 54

$$\sigma \sim 1{,}43 . 10^{-12} \, Q^{1{,}17} \, I_0^{-2{,}76} \; (cm^2)$$ charge exchange

N Ionization Potential $\sim 14.5\,eV$

$\sigma_{1+} \sim 8{,}911 . 10^{-16}\ cm^2 \Rightarrow \sigma_{1+} = 8{,}911 . 10^{-20}\ m^2$

$\sigma_{14+} \sim 1{,}95 . 10^{-14}\ cm^2 \Rightarrow \sigma_{14+} = 1{,}95 . 10^{-18}\ m^2$

at 300K $kT \sim \frac{1}{40}\ eV$

$P = nkT$
↳ density

$$n = \frac{P}{kT} = \frac{10^{-6} . 10^{5} . 10^{-3}}{\frac{1}{40} . e}$$

$$n = 2{,}42 . 10^{16}\ m^{-3}$$

Mean Free Path

$$\left(\lambda_{14+} = \frac{1}{\sigma_{14+} . n} = 21{,}2\ m \right)$$ not asked

$$\lambda_{1+} = 465\ m$$

$I(x) = I(0) e^{-\frac{x}{\lambda}}$ transmitted

$I_{1+}(10\ m) = 0{,}978 \Rightarrow$ Lost $= 2{,}1\%$

Lost $= 1 - e^{-\frac{x}{\lambda}}$

$I_{1+}(100\ m) = 0{,}806 \Rightarrow$ Lost $= 19{,}36\%$ | <u>not negligible</u>

$\Rightarrow$ a base vacuum $\ll 10^{-6}$ mbar is required