Juas 2017 - v2

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Tutorial

-Particle sources-

1. Ion Source Thermal Beam divergence

An ion source is set to a potential +V=10 kV. The axis of the source is along the z axis. The source extraction electrode is placed at z=0. An electrode set to the ground potential is placed at z=d=1 cm. the ion is an argon 1+ with atomic mass 40. 1 uma=931.5 MeV/c².

1.1 Assuming a 1D problem, what is the electric field intensity in the accelerating gap. What is its direction? (we assume V=0 for z>d). plot E(z) for z=-d to 10d.

1.2 A motionless ion with a charge q is located at z=0. It is accelerated by the electric field. Express the ion velocity v as a function of z for z=0 to z=10d. Plot it. calculate the ion final velocity.

1.3 At what time td does the particle reach z=d?

1.4 At what time T does the particle reach z=10d?

1.5 Now, the plasma temperature gives the ion a transverse thermal energy Ex=1/2*kT, T=1000K. Express the transverse energy in eV. What is the transverse ion velocity vx? k=1,38*10^{-23} J/°K.

1.6 At what position x does the particle reach z=10d?

1.7 The beam pipe is 100 mm diameter, at which distance z will the particle touch the wall?

2. Space charge effect on an ion beam

An infinitely long cylinder composed of uniformly distributed electric charge density ρ is considered. The cylinder radius is R.

2.1 helped with the Gauss law, show that :

$$\begin{cases} for\ r > R, & \vec{E} = \dfrac{\rho R^2}{2\epsilon_0 r}\overrightarrow{e_r} \\ for\ r \le R, & \vec{E} = \dfrac{\rho r}{2\epsilon_0}\overrightarrow{e_r} \end{cases}$$

2.2 plot E(r)

2.3 A beam current with an intensity I, a velocity v is propagating in a beam pipe. Show that $\rho = \frac{I}{v\pi R^2}$. We assume that the beam is perfectly cylindrical and that the Electric field found in 2.1 can be used to model the space charge effect. Deduce:

$$\begin{cases} for\ r > R, & \vec{E} = \frac{I}{2\pi v \epsilon_0 r}\overrightarrow{e_r} \\ for\ r \le R, & \vec{E} = \frac{Ir}{2\pi v \epsilon_0 R^2}\overrightarrow{e_r} \end{cases}$$

2.4 The beam is composed of particle with charge q. The radial space charge induced velocity is $\vec{u} = u\overrightarrow{e_r}$, it is supposed to be null at t=0. Using Newton's 2nd law, show that the beam envelope for $r > R$ follows:

$$\frac{1}{2}mu^2 = \ln(\frac{r}{R})\ \frac{qI}{2\pi v \epsilon_0}$$

This equation cannot be treated analytically.

3. Particle trajectory in a magnetic field

A charged particle with mass m and charge q moves in a uniform magnetic field $\vec{B} = B\vec{z}$. We define $\omega = \frac{qB}{m}$. At t=0, the particle is at the point O(0,0,0) with a velocity $\vec{v} = v\vec{x}$.

3.1 From the second law of Newton, show that:

$$\begin{cases} \frac{dv_x}{dt} = \omega v_y \\ \frac{dv_y}{dt} = -\omega v_x \end{cases}$$

3.2 Deduce that:

$$\begin{cases} \frac{d^2v_x}{dt^2} = -\omega^2 v_x \\ \frac{d^2v_y}{dt^2} = -\omega^2 v_y \end{cases}$$

3.3 Look for a solution of type $v_x(t) = a\cos\omega t + b\sin\omega t$ and find :

$$\begin{cases} v_x = v\cos\omega t \\ v_y = v\sin\omega t \end{cases}$$

3.4 Deduce the trajectory coordinates:

$$\begin{cases} x(t) = \frac{v}{\omega}\sin\omega t \\ y(t) = \frac{v}{\omega}(1-\cos\omega t) \end{cases}$$

3.5 Show that the particle is making a circle with a radius $\rho = \frac{v}{\omega}$ around the center $C(0, \rho, 0)$

3.6 An electron is rotating in a 2T magnetic field. What is the electron cyclotron frequency (in Hertz)?

3.7 What is its Larmor radius if its kinetic energy is 10 keV?

4. Mass Spectrometer

An ion beam with charge state Q and mass $M = Am_A$ is extracted from an ion source set to a high voltage U. A *is the mass number*, $m_A = 1\ amu$ (931.5 MeV/c²)

4.1 show that the ion velocity is $v = \sqrt{\frac{2QeU}{M}}$

4.2 The ions are passing through a 90° bending magnet with a magnetic field B and a bending radius ρ. Knowing that $v = \rho\omega$, show :

$$B\rho = \sqrt{\frac{2m_A}{e}}\sqrt{\frac{AU}{Q}}$$

4.3 The source is ionizing H_2 gas (we assume the mass of protons to be 1 amu). The high voltage is set to 10 kV. The beam intensity is measured in a Faraday cup after the dipole. When the dipole magnetic field is ramped, three signals are obtained in the detector for the magnetic field intensities written in the table below:

Peak identified	1	2	3
Magnetic field	0,01438 T	0,02034 T	0,02492 T

What are the M/Q of those beams? Can you identify them?

5 – Beam loss by charge exchange

A 14+ ion beam is propagating into a beam pipe with a residual pressure of 10^{-6} mbar. The gas is supposed to be at 300K. What is the lost fraction of the beam after 10m? 100m?

Nitrogen ionization potential: 14.5 eV