### Measurement of transverse Emittance



The emittance characterizes the whole beam quality, assuming linear behavior as described by second order differential equation.

It is defined within the phase space as:  $\varepsilon_x = \frac{1}{\pi} \int_A dx dx'$ 

#### The measurement is based on determination of:

either profile width  $\sigma_x$  and angular width  $\sigma_x'$  at one location or  $\sigma_x$  at different locations and linear transformations.

#### Different devices are used at transfer lines:

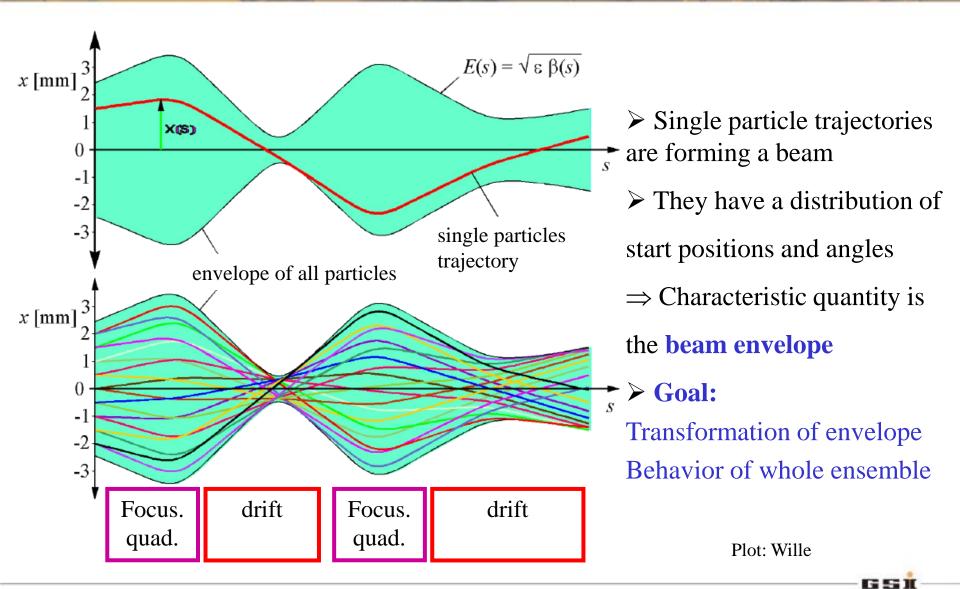
- $\triangleright$  Lower energies  $E_{kin}$  < 100 MeV/u: slit-grid device, pepper-pot (suited in case of non-linear forces).
- ➤ All beams: Quadrupole variation, 'three grid' method using linear transformations (**not** well suited in the presence of non-linear forces)

Synchrotron: lattice functions results in stability criterion

$$\Rightarrow \text{ beam width delivers emittance:} \quad \varepsilon_x = \frac{1}{\beta_x(s)} \left[ \sigma_x^2 - \left( D(s) \frac{\Delta p}{p} \right) \right] \text{ and } \quad \varepsilon_y = \frac{\sigma_y^2}{\beta_y(s)}$$

# Excurse: Particle Trajectory and Characterization of many Particles





### Definition of transverse Emittance



The emittance characterizes the whole beam quality:

 $\varepsilon_x = \frac{1}{2} \int_{A} dx dx'$ 

Ansatz:

Beam matrix at one location:  $\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \varepsilon \cdot \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} \text{ with } \boldsymbol{x} = \begin{pmatrix} x \\ x' \end{pmatrix}$ 

It describes a 2-dim probability distr.

The value of emittance is:

$$\varepsilon_x = \sqrt{\det \mathbf{\sigma}} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

For the profile and angular measurement:

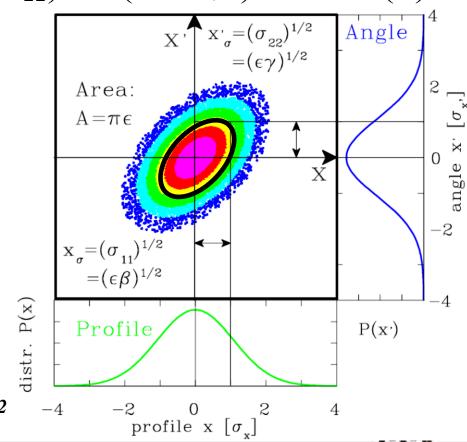
$$x_{\sigma} = \sqrt{\sigma_{11}} = \sqrt{\varepsilon \beta}$$
 and

$$x'_{\sigma} = \sqrt{\sigma_{22}} = \sqrt{\varepsilon \gamma}$$

Geometrical interpretation:

All points x fulfilling  $x^t \cdot \sigma^{-1} \cdot x = 1$ are located on a ellipse

$$\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2 = \det \sigma = \varepsilon_x^2$$



### The Emittance for Gaussian Beams



The density function for a 2-dim Gaussian distribution is:

$$\rho(x, x') = \frac{1}{2\pi\epsilon} \exp\left[-\frac{1}{2} \vec{x}^T \sigma^{-1} \vec{x}\right]$$
$$= \frac{1}{2\pi\epsilon} \exp\left[\frac{-1}{2 \det \sigma} \left(\sigma_{22} x^2 - 2\sigma_{12} x x' + \sigma_{11} x'^2\right)\right]$$

It describes an ellipse with the characteristics profile and angle Gaussian distribution of width

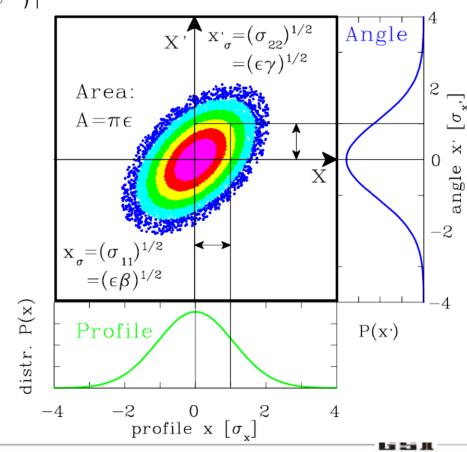
$$x_{\sigma} \equiv \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}}$$
 and

$$x'_{\sigma} \equiv \sqrt{\left\langle x'^2 \right\rangle} = \sqrt{\sigma_{22}}$$

and the correlation or covariance

$$cov \equiv \sqrt{\langle xx' \rangle} = \sqrt{\sigma_{12}}$$

For 
$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
 it is  $\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$  assuming  $\det(\mathbf{A}) = ad - bc \neq 0 \Leftrightarrow$  matrix invertible



## The Emittance for Gaussian and non-Gaussian Beams



### The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- right space charged dominated beams at LINAC & synchrotron
- > cooled beams in storage rings

**Variances** 

Covariance i.e. correlation

## General description of emittance

### using terms of 2-dim distribution:

It describes the value for 1 stand. derivation

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx \rangle}$$

$$\langle x \rangle \equiv \mu = \frac{\iint x \cdot \rho(x, x') \, dx dx'}{\iint \rho(x, x') \, dx dx'} \qquad \langle x' \rangle \equiv \mu' = \frac{\iint x' \cdot \rho(x, x') \, dx dx'}{\iint \rho(x, x') \, dx dx'} \qquad \sum_{i,j} \rho(i,j) \cdot x_i x'_j$$

$$\langle x^n \rangle = \frac{\iint (x - \mu)^n \cdot \rho(x, x') \, dx dx'}{\iint \rho(x, x') \, dx dx'} \qquad \langle x'^n \rangle = \frac{\iint (x' - \mu')^n \cdot \rho(x, x') \, dx dx'}{\iint \rho(x, x') \, dx dx'} \qquad \langle x \rangle = \frac{i,j}{\sum_{i,j} \rho(i,j)}$$

$$\text{covariance} : \langle xx' \rangle = \frac{\iint (x - \mu)(x' - \mu') \cdot \rho(x, x') \, dx dx'}{\iint \rho(x, x') \, dx dx'} \qquad \text{and correspondingly fo}$$

For discrete distribution:

$$\langle x \rangle = \frac{\sum_{i,j} \rho(i,j) \cdot x_i x'_j}{\sum_{i,j} \rho(i,j)}$$

and correspondingly for all other moments

### The Emittance for Gaussian and non-Gaussian Beams



### The beam distribution can be non-Gaussian, e.g. at:

- beams behind ion source
- > space charged dominated beams at LINAC & synchrotron

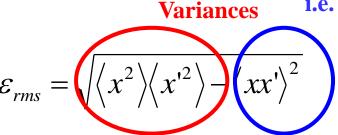
> cooled beams in storage rings

Covariance i.e. correlation

General description of emittance

using terms of 2-dim distribution:

It describes the value for 1 stand, derivation



### For Gaussian beams only:

 $\varepsilon_{rms} \leftrightarrow \text{interpreted as area containing a fraction } f \text{ of ions: } \varepsilon(f) = -2\pi\varepsilon_{rms} \cdot \ln(1-f)$ 

factor to $\epsilon_{rms}$	$1 \cdot \epsilon_{rms}$	$\pi \cdot \epsilon_{rms}$	$2\pi \cdot \epsilon_{rms}$	$4\pi \cdot \epsilon_{rms}$	$6\pi \cdot \epsilon_{rms}$	$8\pi \cdot \epsilon_{rms}$
faction of beam $f$ [%]	15	39	63	86	95	98

Care: no common definition of emittance concerning the fraction f



### **Outline:**

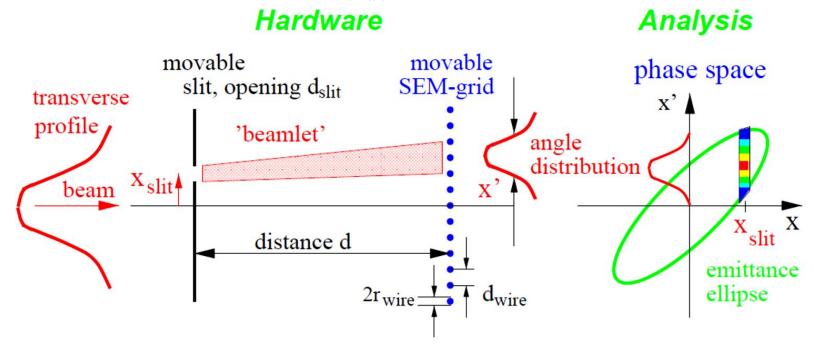
- > Definition and some properties of transverse emittance
- ➤ Slit-Grid device: scanning method
  scanning slit → beam position & grid → angular distribution
- **▶** Pepper-pot device: single shot device
- > Quadrupole strength variation and position measurement
- > Summary

### The Slit-Grid Measurement Device



Slit-Grid: Direct determination of position and angle distribution.

Used for protons/heavy ions with  $E_{kin} < 100 \text{ MeV/u} \Rightarrow \text{range } R < 1 \text{ cm}$ .



*Slit*: position P(x) with typical width: 0.1 to 0.5 mm

**Distance:** 10 cm to 1 m (depending on beam velocity)

**SEM-Grid:** angle distribution P(x')

### Slit & SEM-Grid



Slit with e.g. 0.1 mm thickness

 $\rightarrow$  Transmission only from  $\Delta x$ .

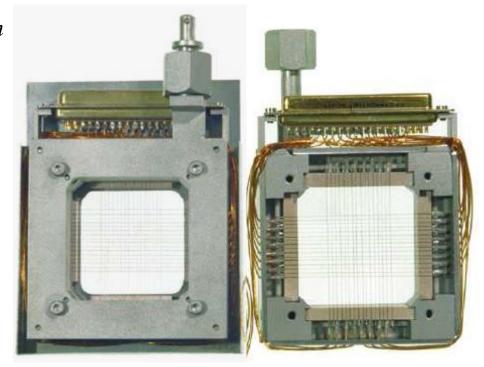
Example: Slit of width 0.1 mm (defect)
Moved by stepping motor, 0.1 mm resolution



Beam surface interaction: e<sup>-</sup> emission

→ measurement of current.

Example: 15 wire spaced by 1.5 mm:



Each wire is equipped with one I/U converter different ranges settings by  $R_i$ 

 $\rightarrow$  very large dynamic range up to  $10^6$ .

# **Display of Measurement Results**



### The distribution of the ions is depicted as a function of

- ➤ Position [mm]
- ➤ Angle [mrad]

#### The distribution can be visualized by

- ➤ Mountain plot
- ➤ Contour plot

**Calc. of 2**<sup>nd</sup> **moments**  $< x^2 >$  ,  $< x^2 > & < xx^2 >$ 

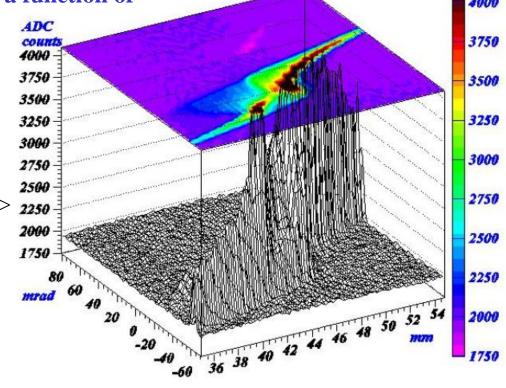
## Emittance value $\varepsilon_{rms}$ from

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

- ⇒ Problems:
- Finite **binning** results in limited resolution
- $\triangleright$  Background  $\rightarrow$  large influence on  $\langle x^2 \rangle$ ,  $\langle x'^2 \rangle$  and  $\langle xx' \rangle$

### Or fit of distribution i.e. ellipse to data

⇒ Effective emittance only



Beam: Ar<sup>4+</sup>, 60 KeV, 15 μA at Spiral2 Phoenix ECR source. Plot from P. Ausset, DIPAC 2009

### The Resolution of a Slit-Grid Device



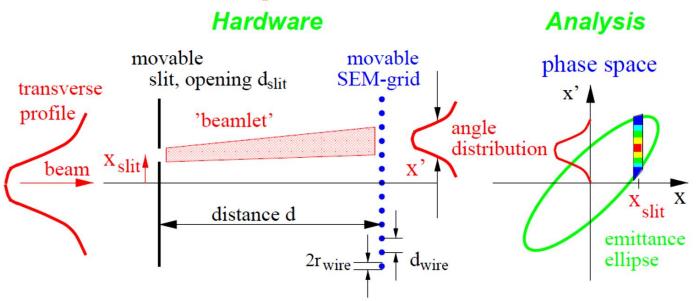
The width of the slit  $d_{slit}$  gives the resolution in space  $\Delta x = d_{slit}$ .

The angle resolution is  $\Delta x' = (d_{wire} + 2r_{wire})/d$ 

 $\Rightarrow$  discretization element  $\Delta x \cdot \Delta x'$ .

By scanning the SEM-grid the angle resolution can be improved.

Problems for small beam sizes or parallel beams.

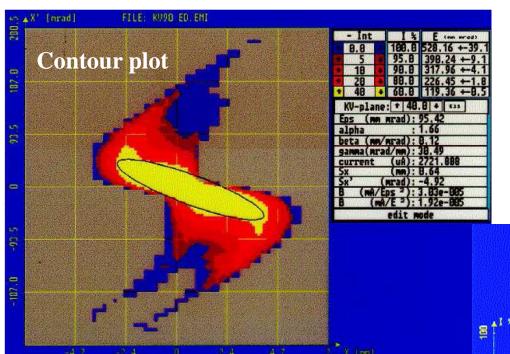


For pulsed LINACs: Only one measurement each pulse  $\rightarrow$  long measuring time required.

### Result of an Slit-Grid Emittance Measurement



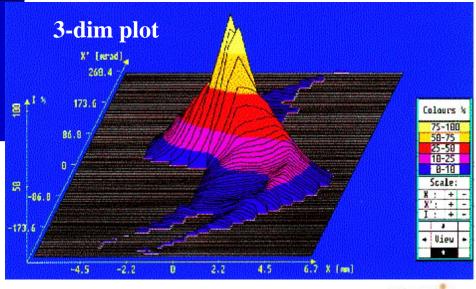
**Result for a beam behind ion source:** ➤ here aberration in quadrupoles due to large beam size



- ➤ different evaluation and plots possible
- > can monitor any distribution

### Low energy ion beam:

→ well suited for emittance showing space-charge effects or aberrations.





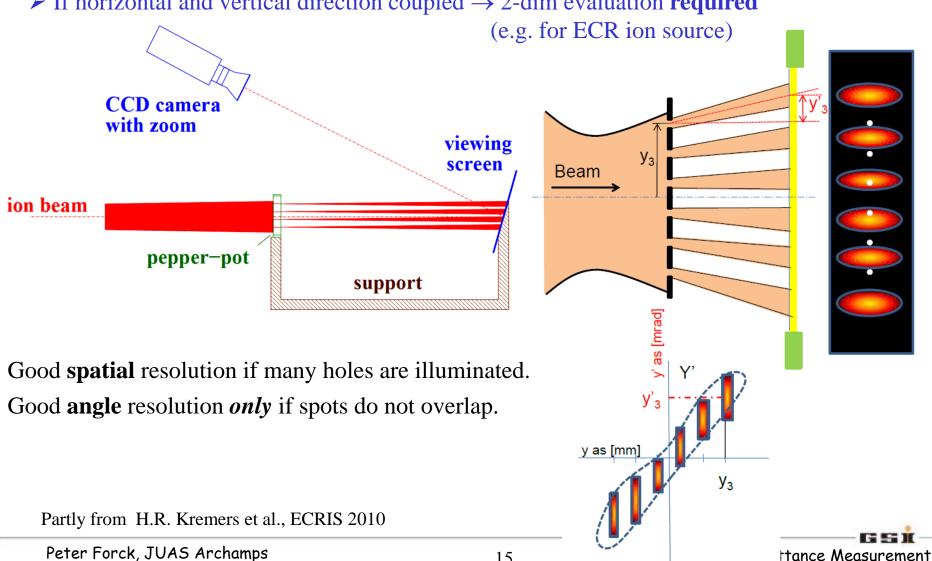
### **Outline:**

- > Definition and some properties of transverse emittance
- Slit-Grid device: scanning method
   scanning slit → beam position & grid → angular distribution
- ➤ Pepper-pot device: single shot device hole-plate → beam position & screen → angular distribution
- > Quadrupole strength variation and position measurement
- > Summary

## The Pepperpot Emittance Device



- For pulsed LINAC: Measurement within one pulse is an advantage
- $\triangleright$  If horizontal and vertical direction coupled  $\rightarrow$  2-dim evaluation **required**



Peter Forck, JUAS Archamps

# The Pepperpot Emittance Device at GSI UNILAC

viewing screen



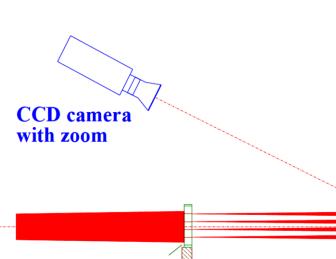
### Example GSI-LINAC 0.12 to 11 MeV/u:

*Pepper-pot*: 15 × 15 holes with Ø 0.1mm on a  $50 \times 50 \text{ mm}^2$  copper plate

*>Distance*: pepper-pot-screen: 25 cm

> Screen: Al<sub>2</sub>O<sub>3</sub>, Ø 50 mm

**▶** Data acquisition: high resolution CCD

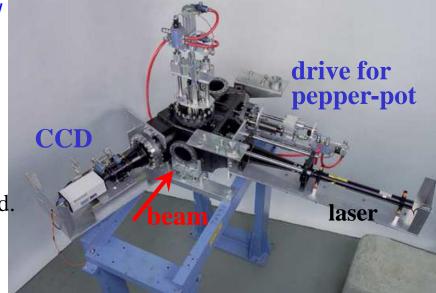


ion beam

pepper-pot

support

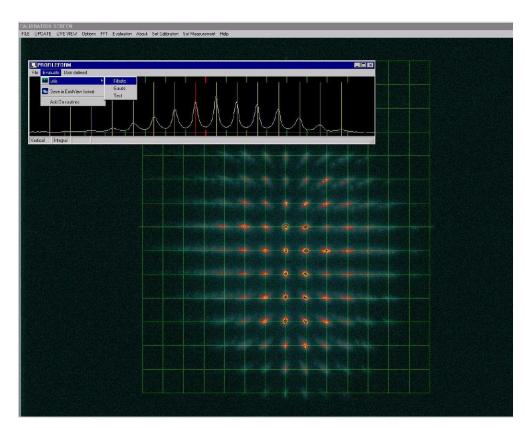
Good **spatial** resolution if many holes are illuminated. Good **angle** resolution *only* if spots do not overlap.



# Result of a Pepperpot Emittance Measurement

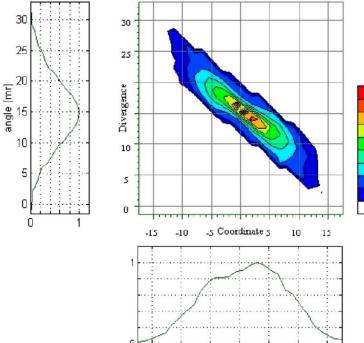


*Example*: Ar <sup>1+</sup> ion beam at 1.4 MeV/u, screen image from single shot at GSI:



### Data analysis:

Projection on horizontal and vertical plane → analog to slit-grid.



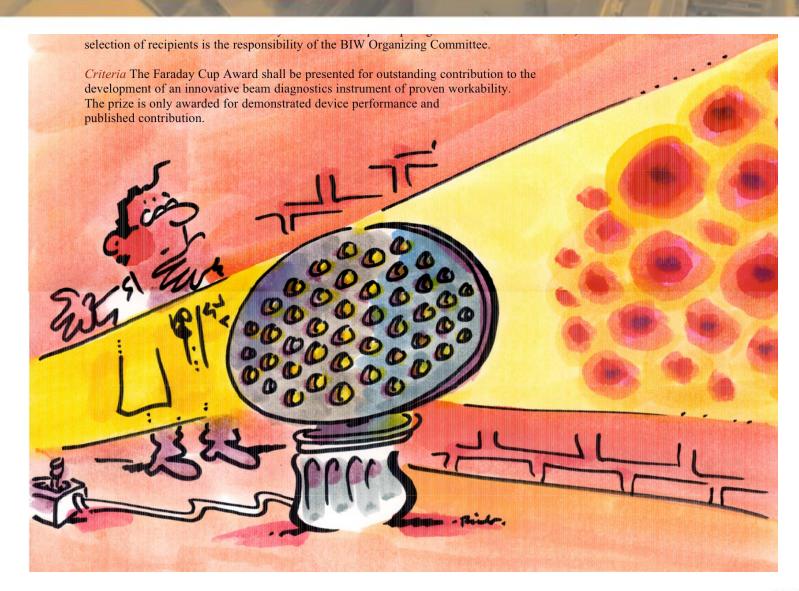
17

coordinate [mm]

10

# The Artist View of a Pepperpot Emittance Device





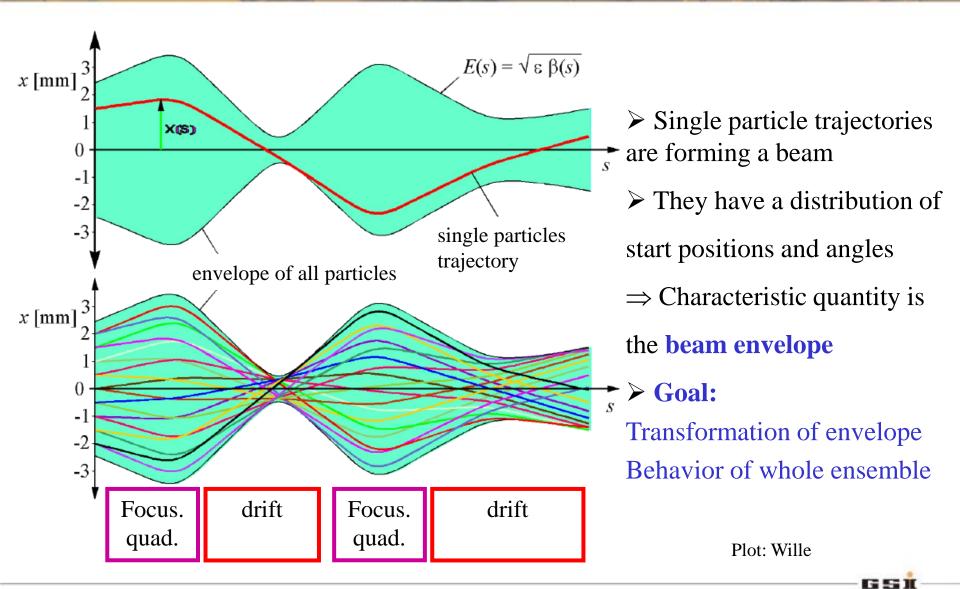


### **Outline:**

- > Definition and some properties of transverse emittance
- Slit-Grid device: scanning method
   scanning slit → beam position & grid → angular distribution
- ➤ Pepper-pot device: single shot device hole-plate → beam position & screen → angular distribution
- ➤ Quadrupole strength variation and position measurement emittance from several profile measurement and beam optical calculation
- > Summary

# Excurse: Particle Trajectory and Characterization of many Particles





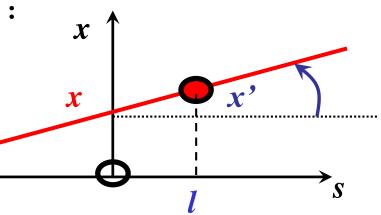
# Excurse: Definition of Offset and Divergence



## Horizontal and vertical coordinates at s = 0:

- $\triangleright x$ : Offset from reference orbit in [mm]
- $\triangleright x'$ : Angle of trajectory in unit [mrad]

$$x' = dx/ds$$



### **Assumption: par-axial beams:**

- $\triangleright x$  is small compared to  $\rho_0$
- $\triangleright$  Small angle with  $p_x/p_s << 1$

### **Longitudinal coordinate:**

- $\triangleright$  Longitudinal orbit difference:  $l = -v_0 \cdot (t t_0)$  in unit [mm]
- $\triangleright$  Momentum deviation:  $\delta = (p p_0) / p_0$  sometimes in unit [mrad] = [‰]

For **continuous** beam: l has no meaning  $\Rightarrow$  set  $l \equiv 0$ !

**Reference particle:** no horizontal and vertical offset  $x \equiv y \equiv 0$  and  $l \equiv 0$  for all s

### Excurse: Definition of Coordinates



$$\vec{x}(s) = \begin{vmatrix} y \\ y' \end{vmatrix}$$

hori. spatial deviation
horizontal divergence
vert. spatial deviation
vertical divergence
longitudinal deviation
momentum deviation

The transformation from a location  $s_{\theta}$  to  $s_{I}$  is given by the Transfer Matrix R

$$\vec{x}(s_1) = \mathbf{R}(\mathbf{s}) \cdot \vec{x}(s_0) =$$

**Remark**: At ring accelerator a comparable (i.e. a bit different) matrix is called **M** 

$$\begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_0 \\ x_0 \\ y_0 \\ y_0 \\ l_0 \\ \delta_0 \end{pmatrix}$$

### Excurse: Conservation of Emittance



#### **Liouville's Theorem:**

## The phase space density can not changes with conservative e.g. linear forces.

The beam distribution at one location  $s_{\theta}$  is described by the beam matrix  $\sigma(s_{\theta})$ 

This beam matrix is transported from location  $s_0$  to  $s_1$  via the transfer matrix

$$\sigma(s_1) = \mathbf{R} \cdot \sigma(s_0) \cdot \mathbf{R}^T$$

### 6-dim beam matrix with <u>decoupled</u> horizontal and vertical plane:

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 & 0 & \sigma_{15} & \sigma_{16} \\ \sigma_{12} & \sigma_{22} & 0 & 0 & \sigma_{25} & \sigma_{26} \\ 0 & 0 & \sigma_{33} & \sigma_{34} & 0 & 0 \\ 0 & 0 & \sigma_{34} & \sigma_{44} & 0 & 0 \\ \sigma_{15} & \sigma_{25} & 0 & 0 & \sigma_{55} & \sigma_{56} \\ \sigma_{16} & \sigma_{26} & 0 & 0 & \sigma_{56} & \sigma_{66} \end{pmatrix}$$
beam matrix: the three coordinates: 
$$\sigma_{11} = \langle x^2 \rangle \qquad x_{rms} = \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}}$$

$$\sigma_{12} = \langle x \cdot x' \rangle \qquad y_{rms} = \sqrt{\langle y^2 \rangle} = \sqrt{\sigma_{33}}$$

$$\sigma_{22} = \langle x'^2 \rangle \qquad l_{rms} = \sqrt{\langle l^2 \rangle} = \sqrt{\sigma_{55}}$$

#### Horizontal

$$\sigma_{11} = \langle x^2 \rangle$$

$$\sigma_{12} = \langle x \cdot x' \rangle$$

$$\sigma_{22} = \langle x'^2 \rangle$$

#### Beam width for

$$x_{rms} = \sqrt{\langle x^2 \rangle} = \sqrt{\sigma_{11}}$$

$$y_{rms} = \sqrt{\langle y^2 \rangle} = \sqrt{\sigma_{33}}$$

$$l_{rms} = \sqrt{\langle l^2 \rangle} = \sqrt{\sigma_{55}}$$

# Excurse: Some Examples for linear Transformations



The 2-dim sub-space (x,x') can be used in case there is coupling like dispersion  $R_{16} = (x/\delta) = 0$ 

**Important examples are:** 

> Drift with length 
$$L$$
:  $\mathbf{R}_{\text{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ 

 $\triangleright$  Horizontal **focusing** with quadrupole constant k end effective length l:

$$\mathbf{R}_{\text{focus}} = \begin{pmatrix} \cos \sqrt{k} \, l & \frac{1}{\sqrt{k}} \sin \sqrt{k} \, l \\ -\sqrt{k} \cdot \sin \sqrt{k} \, l & \cos \sqrt{k} \, l \end{pmatrix} \Rightarrow \mathbf{R}_{\text{focus}}^{\text{thin lens}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

 $\triangleright$  Horizontal <u>de-focusing</u> with quadrupole constant k end effective length l:

$$\mathbf{R}_{\text{de-focus}} = \begin{pmatrix} \cosh \sqrt{k} \, l & \frac{1}{\sqrt{k}} \sinh \sqrt{k} \, l \\ \sqrt{k} \cdot \sinh \sqrt{k} \, l & \cosh \sqrt{k} \, l \end{pmatrix} \Rightarrow \mathbf{R}_{\text{de-focus}}^{\text{thin lens}} = \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}$$

Ideal quad.: field gradient  $g = B_{pole}/a$ ,  $B_{pole}$  field at poles, a aperture

 $\rightarrow$  quadrupole constant  $k = /g //(B\rho)_0$ 

Thin lens approximation:  $l \rightarrow 0 \Rightarrow kl \rightarrow \text{const} \Rightarrow kl \equiv 1/f$ 

 $\Rightarrow$  simple transfer matrix (math. proof by 1<sup>st</sup> order Taylor expansion)

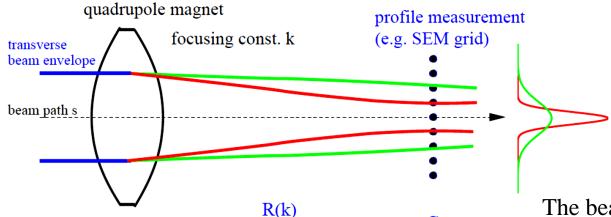
**Horizontal focusing:** 

Kick:  $\Delta x' = -x/f$ 

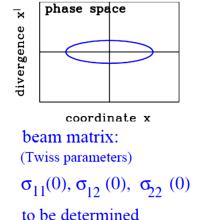
# Emittance Measurement by Quadrupole Variation



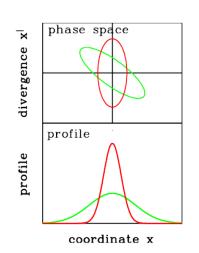
From a profile determination, the emittance can be calculated via linear transformation, if a well known and constant distribution (e.g. Gaussian) is assumed.



linear transformation



location:



 $S_1$ 

The beam width  $x_{max}$  and  $x_{max}^2 = \sigma_{11}(1, k)$  is measured, matrix  $\mathbf{R}(\mathbf{k})$  describes the focusing.

measurement:  $\mathbf{x}^{2}(\mathbf{k}) = \sigma_{11}(1, \mathbf{k})$ 

### Measurement of transverse Emittance



- The beam width  $x_{max}$  at  $s_1$  is measured, and therefore  $\sigma_{11}(1, k_i) = x_{max}^2(k_i)$ .
- Different focusing of the quadrupole  $k_1, k_2...k_n$  is used:  $\Rightarrow \mathbf{R_{focus}}(k_i)$ , including the drift, the transfer matrix is changed  $\mathbf{R}(k_i) = \mathbf{R_{drift}} \cdot \mathbf{R_{focus}}(k_i)$ .
- Task: Calculation of beam matrix  $\sigma(0)$  at entrance  $s_0$  (size and orientation of ellipse)
- The transformations of the beam matrix are:  $\sigma(1, k) = \mathbf{R}(k) \cdot \sigma(0) \cdot \mathbf{R}^{\mathbf{T}}(k)$ .  $\Longrightarrow$  Resulting in a redundant system of linear equations for  $\sigma_{ij}(0)$ :

$$\begin{array}{lcl} \sigma_{11}(1,k_1) & = & R_{11}^2(k_1) \cdot \sigma_{11}(0) + 2R_{11}(k_1)R_{12}(k_1) \cdot \sigma_{12}(0) + R_{12}^2(k_1) \cdot \sigma_{22}(0) & \text{focusing } k_1 \\ & : \\ \sigma_{11}(1,k_n) & = & R_{11}^2(k_n) \cdot \sigma_{11}(0) + 2R_{11}(k_n)R_{12}(k_n) \cdot \sigma_{12}(0) + R_{12}^2(k_n) \cdot \sigma_{22}(0) & \text{focusing } k_n \end{array}$$

• To learn something on possible errors, 
$$n > 3$$
 settings have to be performed. A setting with a focus close to the SEM-grid should be included to do a good fit.

- Assumptions:
  - Only elliptical shaped emittance can be obtained.
  - No broadening of the emittance e.g. due to space-charge forces.
  - If not valid: A self-consistent algorithm has to be used.

### Measurement of transverse Emittance



Using the 'thin lens approximation' i.e. the quadrupole has a focal length of f:

$$\mathbf{R}_{focus}(K) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \implies \mathbf{R}(L, K) = \mathbf{R}_{drift}(L) \cdot \mathbf{R}_{focus}(K) = \begin{pmatrix} 1 + LK & L \\ K & 1 \end{pmatrix}$$

ELETTRA 100 MeV e Linac, YAG:Ce:

Vertical Plane

 $\varepsilon_{\rm n}$  =11 ± 1.5 [ $\mu$ m rad]

Focusing strength K [m<sup>-1</sup>]

Example: Square of the beam width at Measurement of  $\sigma(1,K) = \mathbf{R}(K) \cdot \sigma(0) \cdot \mathbf{R}^{\mathrm{T}}(K)$ 

$$\sigma_{11}(1,K) = L^2 \sigma_{11}(0) \cdot K^2$$

$$+2\cdot (L\sigma_{11}(0) + L^{2}\sigma_{12}(0)) \cdot K$$

$$+L^{2}\sigma_{22}(0) + \sigma_{11}(0)$$

$$\equiv a\cdot K^{2} - 2ab\cdot K + ab^{2} + c$$

 $\beta = 15.9 \pm 0.1 \text{ [m]}$  $\alpha$  =29.9 ± 0.4  $\gamma = 56.1 \pm 0.5 [\text{m}^{-1}]$ 

The  $\sigma$ -matrix at quadrupole is:

$$\sigma_{11}(0) = \frac{a}{L^2}$$

$$\sigma_{12}(0) = -\frac{a}{L^2} \left(\frac{1}{L} + b\right)$$

$$\sigma_{22}(0) = \frac{1}{L^2} \left(ab^2 + c + \frac{2ab}{L} + \frac{a}{L^2}\right)$$

 $\epsilon = \sqrt{\det \sigma(0)} = \sqrt{\sigma_{11}(0)\sigma_{22}(0) - \sigma_{12}^2(0)} = \sqrt{ac/L^2}$ 

G. Penco et al., EPAC'08

0.25

0.05

### The 'Three Grid Method' for Emittance Measurement



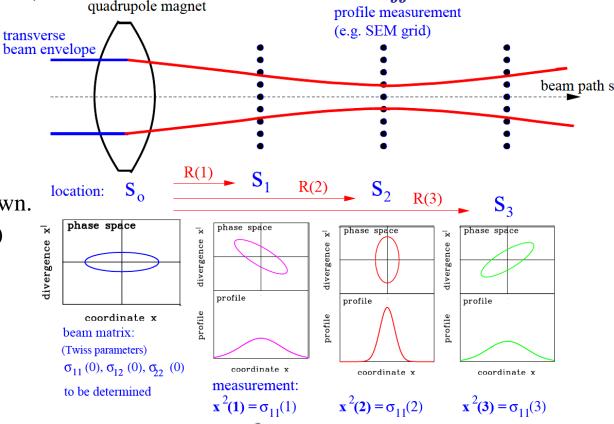
# Instead of quadrupole variation, the beam width is measured at *different* locations:

#### The procedure is:

- Beam width x(i) measured at the locations  $s_i$ 
  - $\Rightarrow$  beam matrix element  $x^2(i) = \sigma_{11}(i)$ .
- The transfer matrix  $\mathbf{R}(i)$  is known. (without dipole a  $3 \times 3$  matrix.)
- > The transformations are:

$$\sigma(i) = \mathbf{R}(i)\sigma(0)\mathbf{R}^{\mathrm{T}}(i)$$

 $\Rightarrow$  redundant equations:



$$\begin{array}{lll} \sigma_{11}(1) & = & R_{11}^2(1) \cdot \sigma_{11}(0) + 2R_{11}(1)R_{12}(1) \cdot \sigma_{12}(0) + R_{12}^2(1) \cdot \sigma_{22}(0) & \mathbf{R}(1) : s_0 \to s_1 \\ \sigma_{11}(2) & = & R_{11}^2(2) \cdot \sigma_{11}(0) + 2R_{11}(2)R_{12}(2) \cdot \sigma_{12}(0) + R_{12}^2(2) \cdot \sigma_{22}(0) & \mathbf{R}(2) : s_0 \to s_2 \\ & : & & \vdots \end{array}$$

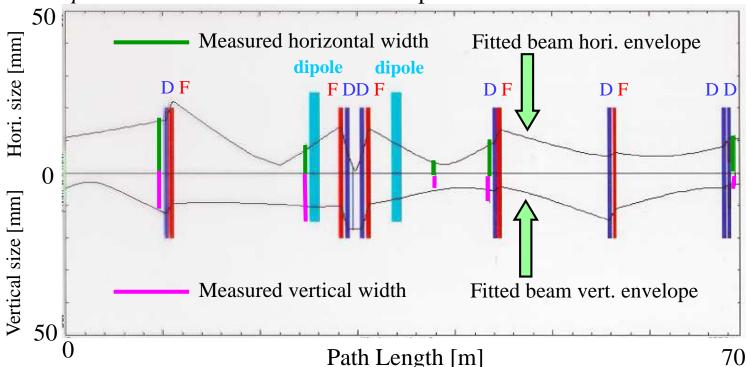
$$\sigma_{11}(n) = R_{11}^2(n) \cdot \sigma_{11}(0) + 2R_{11}(n)R_{12}(n) \cdot \sigma_{12}(0) + R_{12}^2(n) \cdot \sigma_{22}(0) \quad \mathbf{R}(n) : s_0 \to s_n$$

### Results of a 'Three Grid Method' Measurement



**Solution:** Solving the linear equations like for quadrupole variation or fitting the profiles with linear optics code (e.g. TRANSPORT, WinAgile, MadX).

*Example:* The hor. and vert. beam envelope and the beam width at a transfer line:



Assumptions: > constant emittance, in particular no space-charge broadening

- ≥100 % transmission i.e. no loss due to vacuum pipe scraping
- > no misalignment, i.e. beam center equals center of the quadrupoles.

# Summary for transverse Emittance Measurement



Emittance measurements are very important for comparison to theory.

It includes size (value of  $\varepsilon$ ) and orientation in phase space ( $\sigma_{ij}$  or  $\alpha$ ,  $\beta$  and  $\gamma$ )

(three independent values)

Techniques for transfer lines (synchrotron: width measurement sufficient):

## Low energy beams $\rightarrow$ direct measurement of x- and x'-distribution

- ightharpoonup Slit-grid: movable slit  $\to x$ -profile, grid  $\to x'$ -profile
- ightharpoonup Pepper-pot: holes  $\rightarrow x$ -profile, scintillation screen  $\rightarrow x'$ -profile

## All beams $\rightarrow$ profile measurement + linear transformation:

- ➤ Quadrupole variation: one location, different setting of a quadrupole
- > 'Three grid method': different locations
- ➤ Assumptions: ➤ well aligned beam, no steering
  - ➤ no emittance blow-up due to space charge.

# Appendix GSI Ion LINAC: Emittance Measurement Devices





Slit Grid Emittance: Standard device, total 9 device



Pepper-pot Emittance: Special device, total 1 device

Transfer to Synchrotron

