Joint Universities Accelerator School

JUAS 2017

Archamps, France, 27th February – 3rd March 2017

Normal-conducting accelerator magnets
Lecture 1: Basic principles

Thomas Zickler

CERN

Scope of the lectures

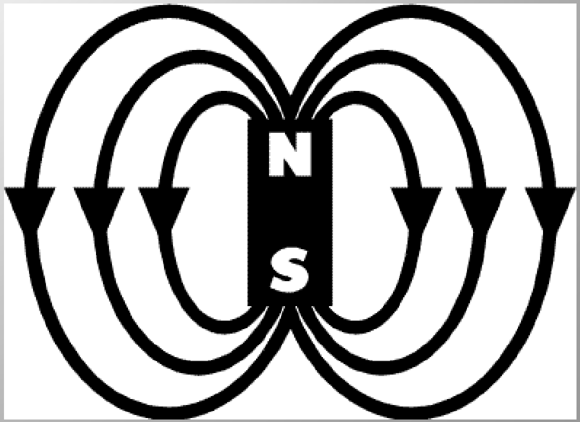
Overview of electro-magnetic technology as used in particle accelerators considering *normal-conducting, iron-dominated* electro-magnets (generally restricted to direct current situations)

Main goal is to:

- create a fundmental understanding in accelerator magnet technology
- provide a guide book with practical instructions how to start with the design of a standard accelerator magnet
- focus on applied and practical design aspects using ‘real’ examples
- introduce finite element codes for practical magnet design
- present an outlook into magnet manufacturing, testing and measurements

Not covered:

– permanent magnet technology
– superconducting technology



Literature

- Fifth General Accelerator Physics Course, CAS proceedings, University of Jyväskylä, Finland, September 1992, CERN Yellow Report 94-01
- International Conference on Magnet Technology, Conference proceedings
- Iron Dominated Electromagnets, J. T. Tanabe, World Scientific Publishing, 2005
- Magnetic Field for Transporting Charged Beams, G. Parzen, BNL publication, 1976
- Magnete, G. Schnell, Thiemig Verlag, 1973 (German)
- Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization, S. Russenschuck, Wiley-VCH, 2010
- Practical Definitions & Formulae for Normal Conducting Magnets, D. Tommasini, Sept. 2011
- CAS proceedings, Magnetic measurements and alignment, Montreux, Switzerland, March 1992, CERN Yellow Report 92-05
- CAS proceedings, Measurement and alignment of accelerator and detector magnets, Anacapri, Italy, April 1997, CERN Yellow Report 98-05
- Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen, K. Wille, Teubner Verlag, 1996
- CAS proceedings, Magnets, Bruges, Belgium, June 2009, CERN Yellow Report 2010-004

Program (1)

Lecture 1 — Monday 27.2. (10:45 – 12:15)

Introduction & Basic principles

A bit of history…
Why do we need magnets?
Basic principles and concepts
Magnet types

Lecture 2 — Monday 27.2. (16:15 – 17:15)

Analytical design

What do we need to know before starting?
Yoke design
Coil dimensioning
Cooling layout

Lecture 3 — Monday 27.2. (17:15 – 18:15)

Magnet production, tests and measurements

Magnetic materials
Manufacturing techniques
Quality assurance
Recurrent quality issues
Cost estimation and optimization

Program (2)

Lecture 4 — Tuesday 28.2. (15:00 – 16:00)

Applied numerical design

- Building a basic 2D finite-element model
- Interpretation of results
- Typical application examples / limitation of numerical design

Tutorial — Tuesday 28.2. (16:15 – 18:15)

Case study (part 1)

- Students are invited to design and specify a ‚real' magnet
- Analytical magnet design on paper

Mini-workshop — Wednesday, 1.3. (9:00 – 12:15)

Case study (part 2)

- Computer work
- Numerical magnet design

Consultation hour — Monday, 13.3. (14:00 – 16:00)

Only upon request!

Lecture 1: Basic principles

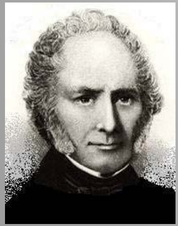
- A bit of history…
- Why do we need magnets?
- Magnet technologies
- Basic principles and concepts
- Field description
- Magnet types and applications

A bit of history...

1820: Hans Christian Oersted (1777-1851) finds that electric current affects a compass needle

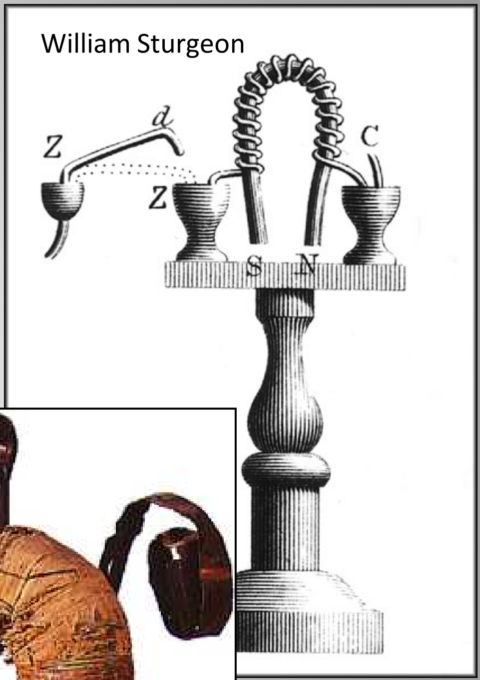
1820: Andre Marie Ampere (1775-1836) in Paris finds that wires carrying current produce forces on each other

1820: Michael Faraday (1791-1867) at Royal Society in London develops the idea of electric fields and studies the effect of currents on magnets and magnets inducing electric currents



1825: British electrician, William Sturgeon (1783-1850) invented the first electromagnet

1860: James Clerk Maxwell (1831-1879), a Scottish physicist and mathematician, puts the theory of electromagnetism on mathematical basis

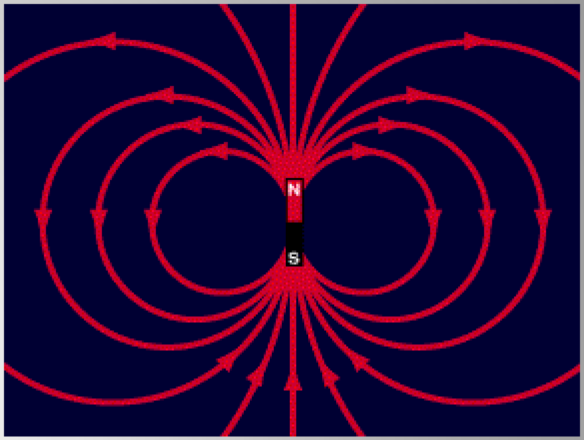


William Sturgeon


Joseph Henry

Magnetic units

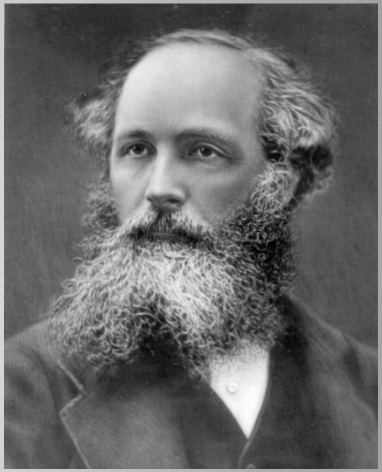
IEEE defines the following units:



- Magnetic field:
  - H (vector) [A/m]
  - the magnetizing force produced by electric currents
- Electromotive force:
  - e.m.f. or U [V or (kg m^2)/(A s^3)]
  - here: voltage generated by a time varying magnetic field
- Magnetic flux density or magnetic induction:
  - B (vector) [T or kg/(A s^2)]
  - the density of magnetic flux driven through a medium by the magnetic field
  - Note: induction is frequently referred to as "Magnetic Field“
  - H, B and μ relates by: $B = \mu H$
- Permeability:
  - $\mu = \mu_0 \mu_r$
  - permeability of free space $\mu_0 = 4\,\pi\,10^{-7}$ [V s/A m]
  - relative permeability μ_r (dimensionless): $\mu_{\text{air}} = 1$; $\mu_{\text{iron}} > 1000$ (not saturated)
- Magnetic flux:
  - ϕ [Wb or (kg m^2)/(A s^2)]
  - surface integral of the flux density component perpendicular trough a surface

Maxwell's equations

In 1873, Maxwell published "Treatise on Electricity and Magnetism" in which he summarized the discoveries of Coulomb, Oersted, Ampere, Faraday, et. al. in four mathematical equations:



Gauss' law for electricity:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \oint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss' law of flux conservation:

$$\nabla \cdot \vec{B} = 0 \qquad \oint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

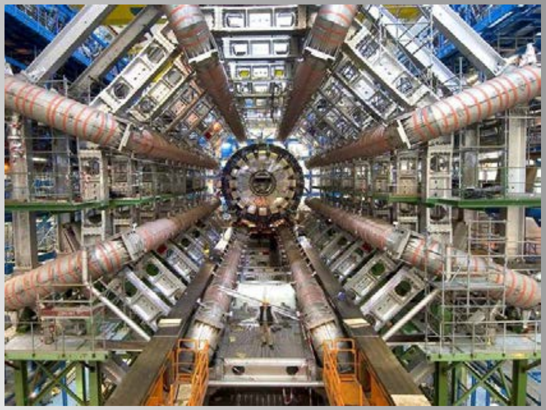
Faraday's law of induction:

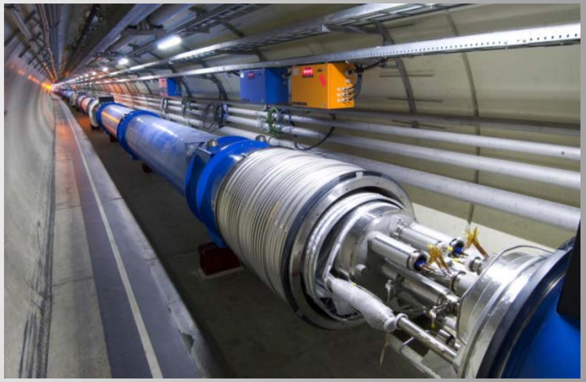
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

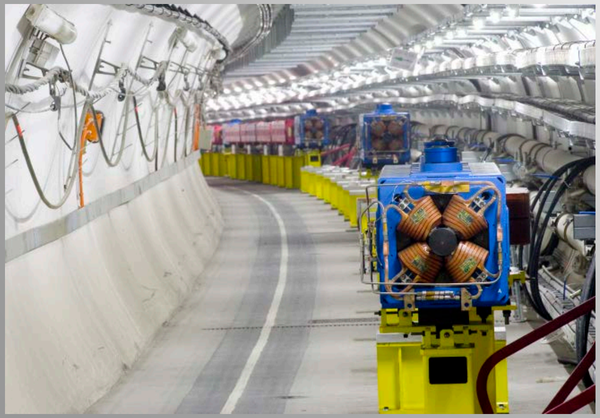
Ampere's circuital law:

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint_{\partial A} \vec{B} \cdot d\vec{s} = \int_A \mu_0 \vec{J} \cdot d\vec{A} + \frac{d}{dt}\int_A \mu_0 \varepsilon_0 \vec{E} \cdot d\vec{A}$$

Magnets at CERN



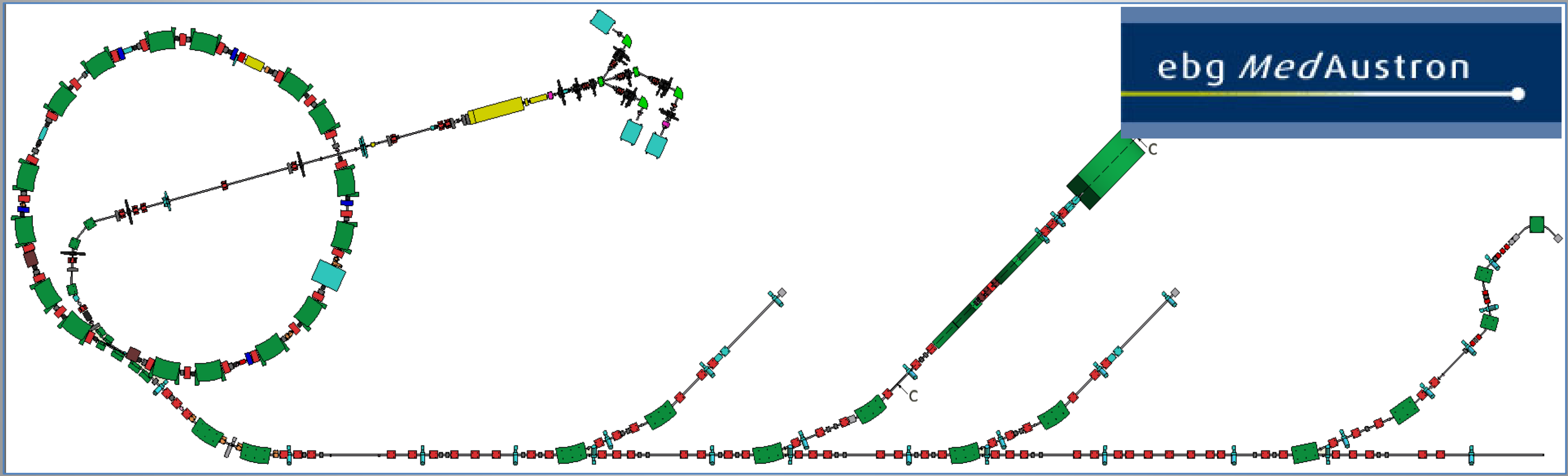




Normal-conducting magnets:
4800 magnets (50 000 tons) are installed in the CERN accelerator complex
Superconducting magnets:
10 000 magnets (50 000 tons) mainly in LHC
Permanent magnets:
150 magnets (4 tons) in Linacs & EA

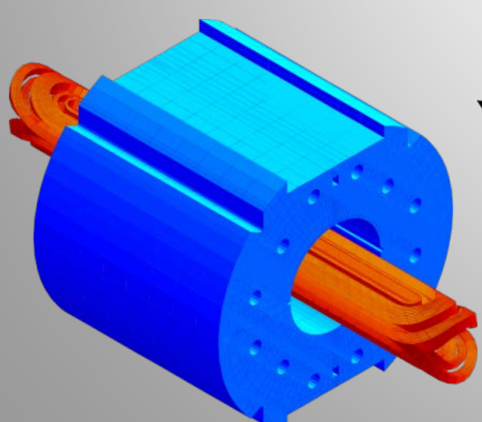
Why do we need magnets?

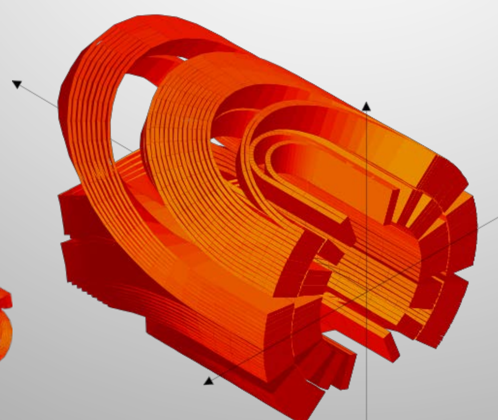
- Interaction with the beam
  - guide the beam to keep it on the orbit
  - focus and shape the beam
- Lorentz's force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
  - for relativistic particles this effect is equivalent if $\vec{E} = c\vec{B}$
  - if $B = 1$ T then $E = 3 \cdot 10^8$ V/m

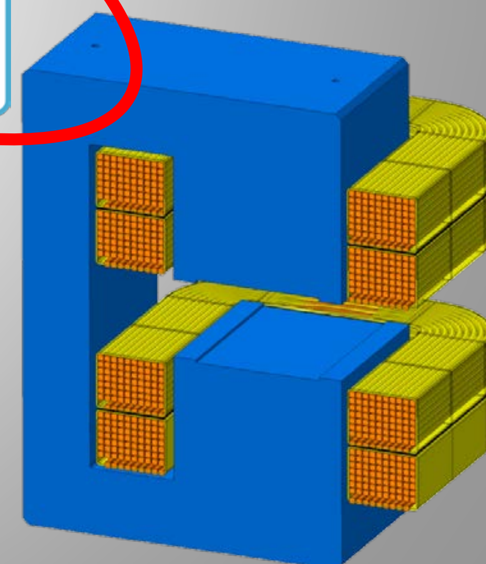


Magnet technologies

- Magnets
  - Electro-magnets
    - Superconducting
      - **Coil dominated** — B < 11T
      - Iron dominated — B < 2 T
    - Normal-conducting
      - Coil dominated — B < 1 T
      - **Iron dominated** — B < 2 T
  - Permanent magnets

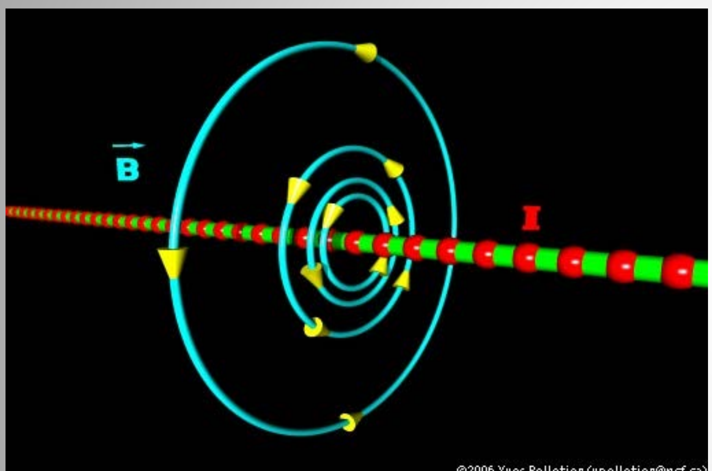






How does an electro-magnet work?

- Permanent magnets provide only constant magnetic fields
- Electro-magnets can provide adjustable magnetic fields

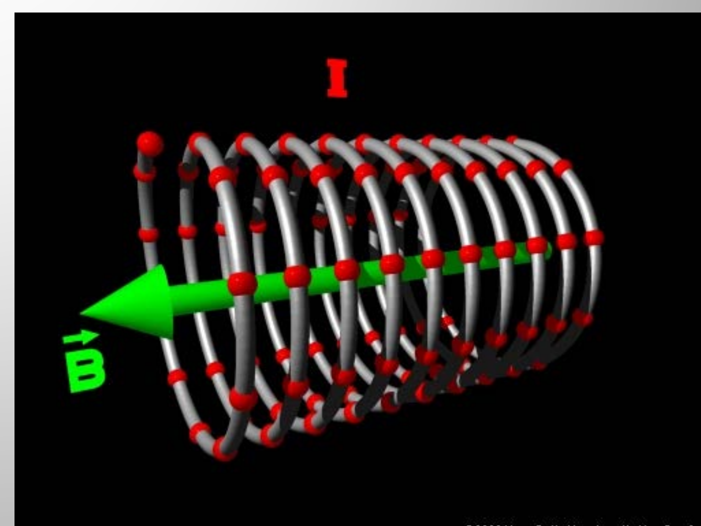


Maxwell & Ampere:

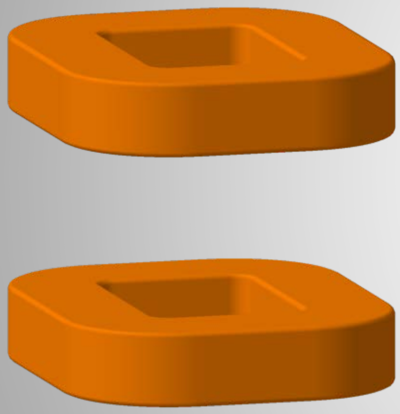
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

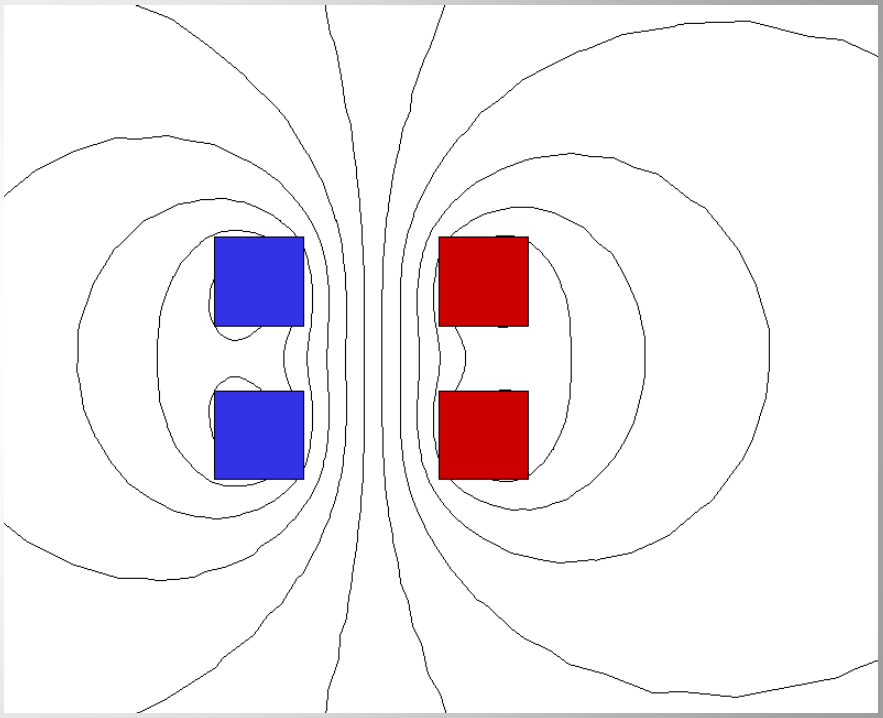
„An electrical current is surrounded by a magnetic field“

„Right hand rule“ applies



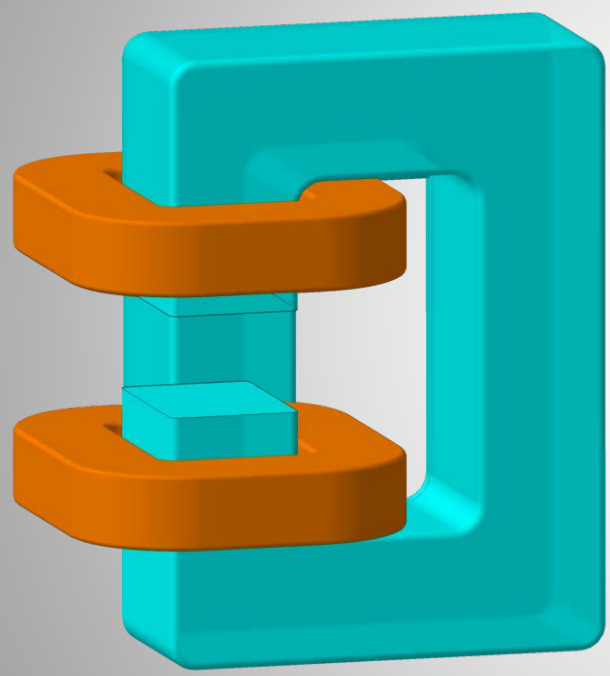
Magnetic circuit

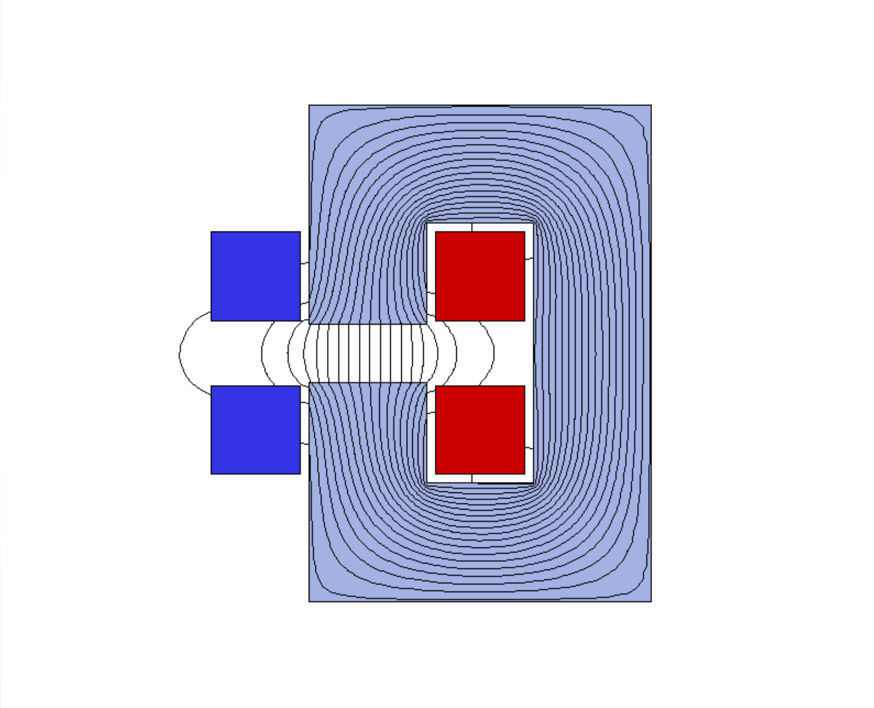




Flux lines represent the magnetic field
Coil colors indicate the current direction

Magnetic circuit





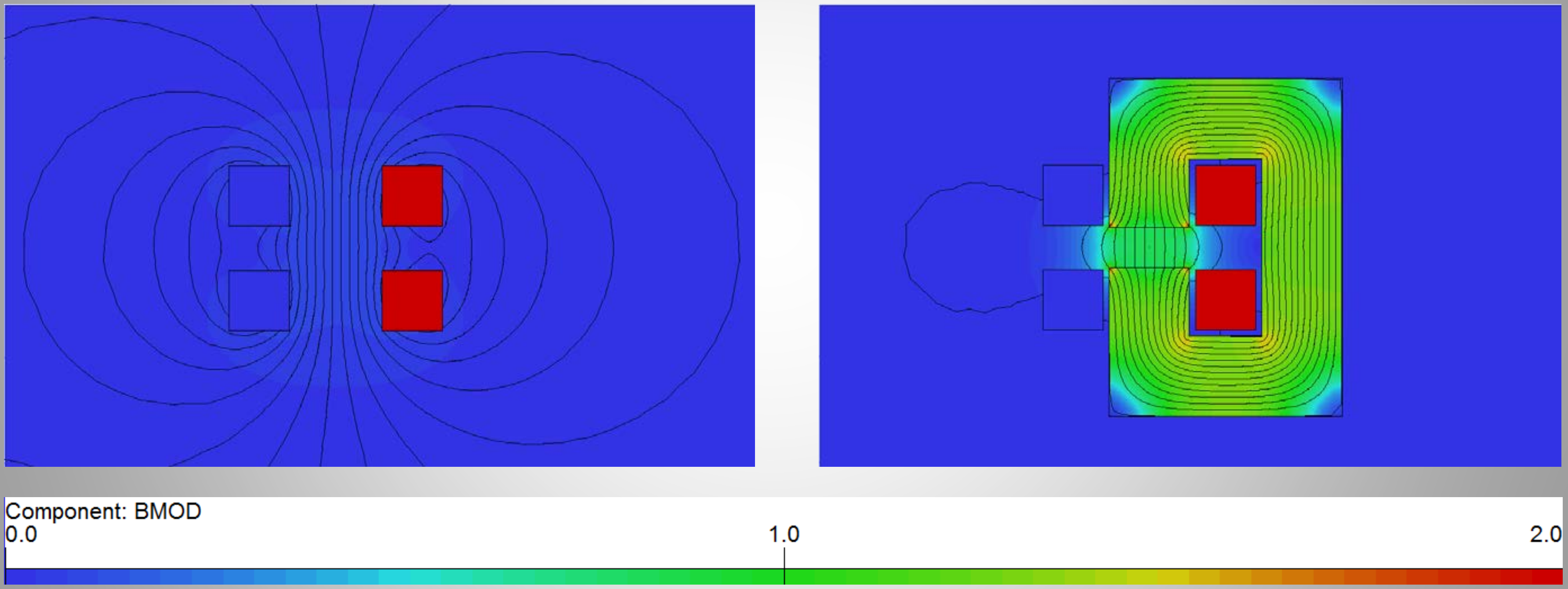
Coils hold the electrical current
Iron holds the magnetic flux

→ *"iron-dominated magnet"*

Magnetic circuit

$I = 32$ kA
$B_{centre} = 0.09$ T

$I = 32$ kA
$B_{centre} = 0.80$ T



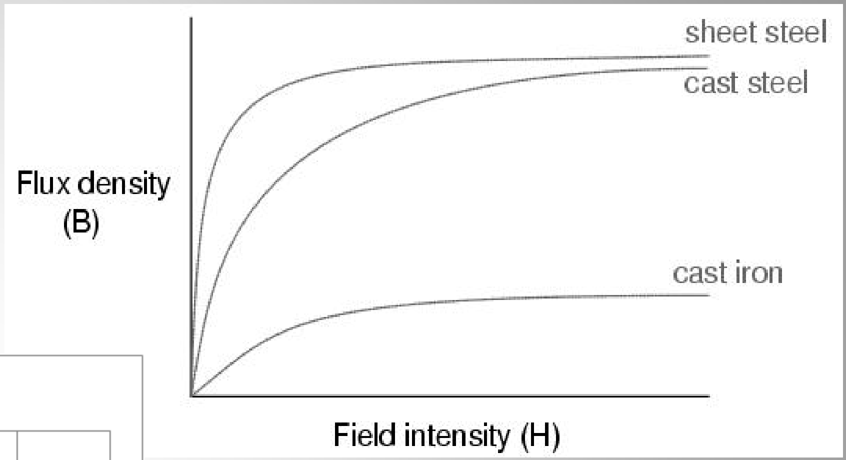
The presence of a magnetic circuit can increase the flux density in the magnet aperture by factors

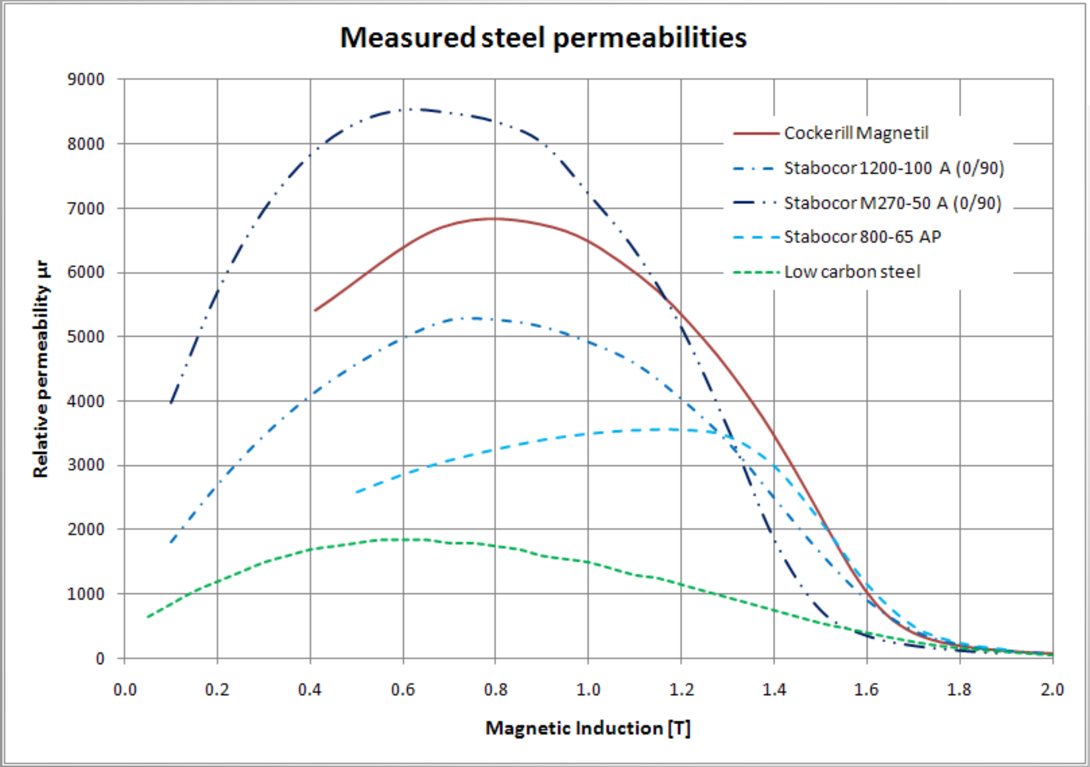
Note: the asymmetric field distribution is an artifact from the FE-mesh

Permeability

$$\vec{B} = \mu \vec{H} \qquad \mu = \mu_0 \mu_r$$

Permeability: correlation between field strength H and flux density B





Ferro-magnetic materials: high permeability (μ_r >>1), but not constant

Excitation current in a dipole

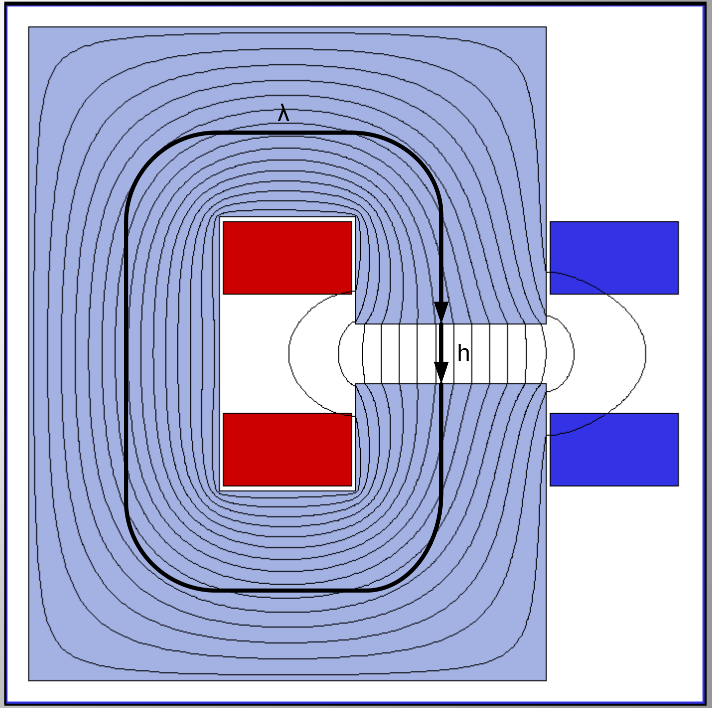
Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$

leads to $$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

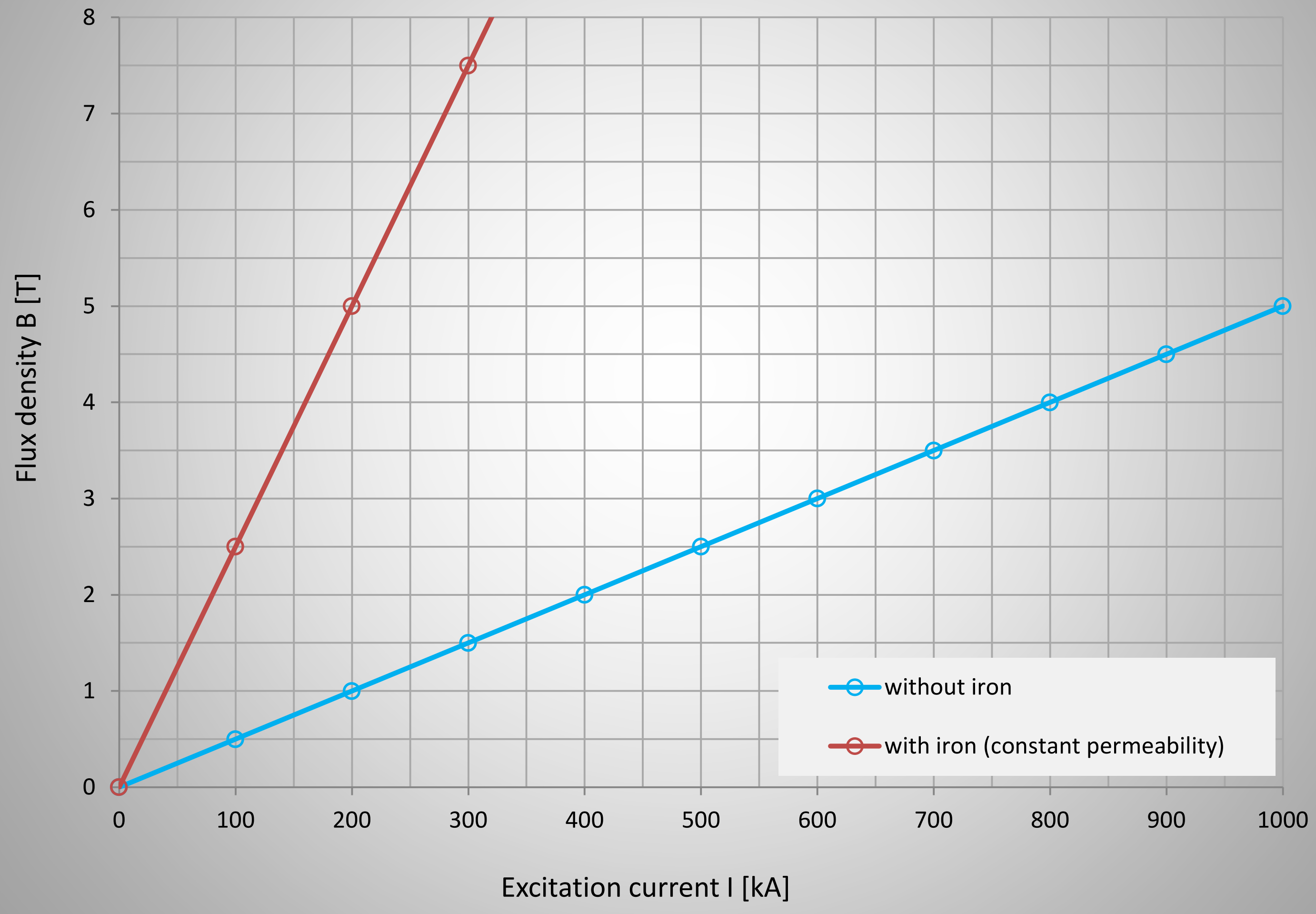
assuming, that B is constant along the path.

If the iron is not saturated: $$\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$$

then: $$NI_{(per\ pole)} \approx \frac{Bh}{2\mu_0}$$



Transfer function



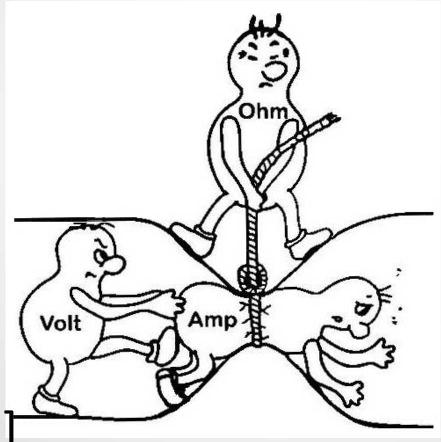
Reluctance and saturation

Similar to electrical circuits, one can define the ‘resistance’ of a magnetic circuit, called ‘reluctance’:

Ohm’s law:

$$R_E = \frac{U}{I} = \frac{l_E}{A_E \sigma}$$

- Voltage drop U [V]
- Resistance R_E [Ω]
- Current I [A]
- El. conductivity σ [S/m]
- Conductor length l_E [m]
- Conductor cross section A_E [m^2]



Hopkinson’s law:

$$R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$$

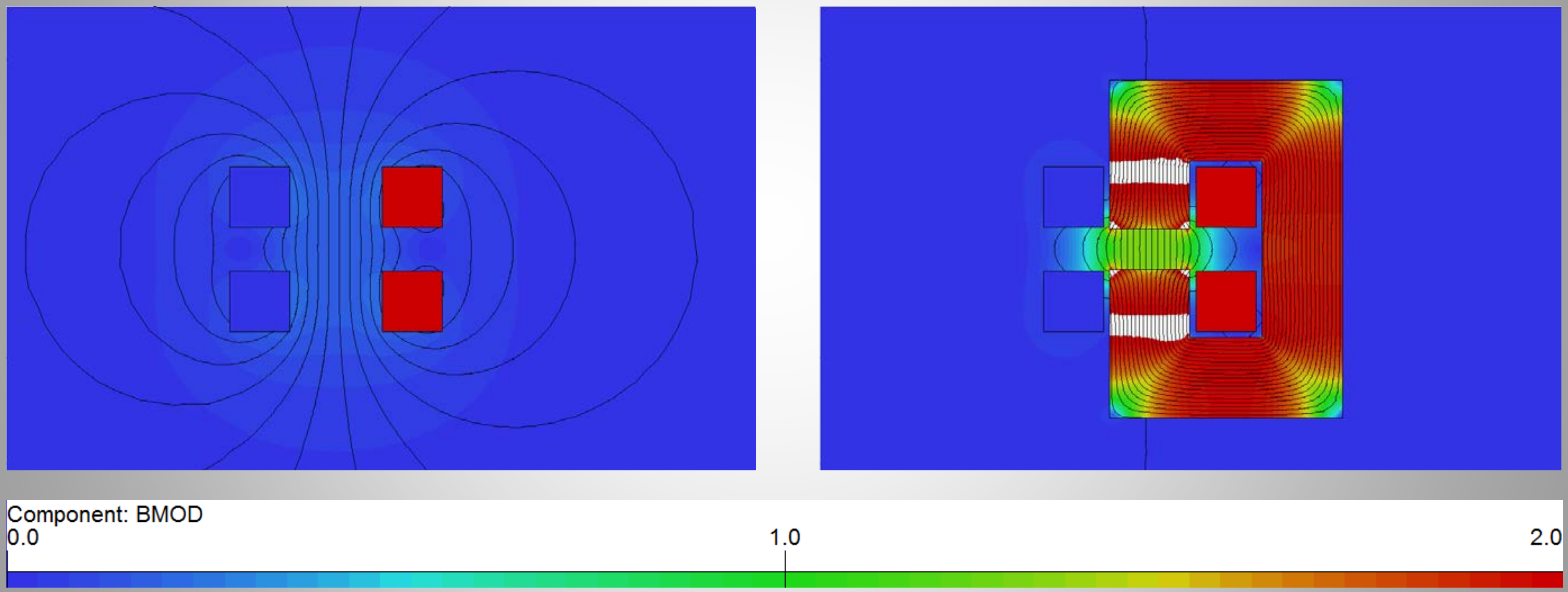
- Magneto-motive force NI [A]
- Reluctance R_M [A/Vs]
- Magnetic flux Φ [Wb]
- Permeability μ [Vs/Am]
- Flux path length in iron l_M [m]
- Iron cross section A_M [m^2] (perpendicular to flux)

…but: μ_{iron} is in general <u>not</u> constant!

Reluctance and saturation

$I = 32$ kA, $B_{centre} = 0.09$ T → $I = 64$ kA, $B_{centre} = 0.18$ T

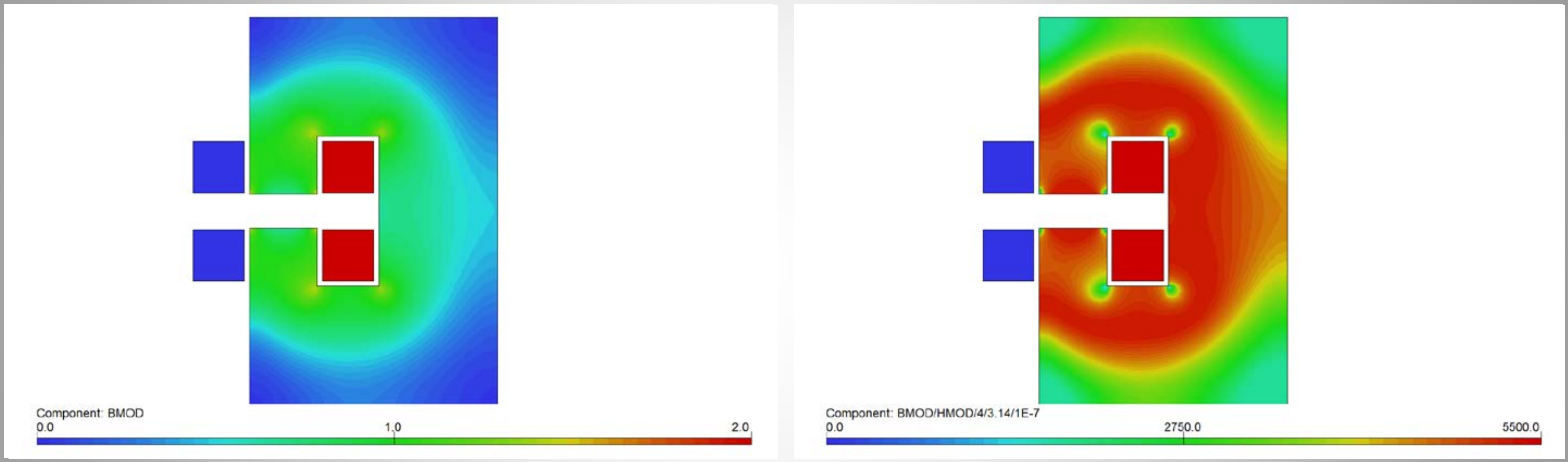
$I = 32$ kA, $B_{centre} = 0.80$ T → $I = 64$ kA, $B_{centre} = 1.30$ T



Increase of B above 1.5 T in iron requires non-proportional increase of H

Iron saturation (small μ_{iron}) leads to inefficiencies

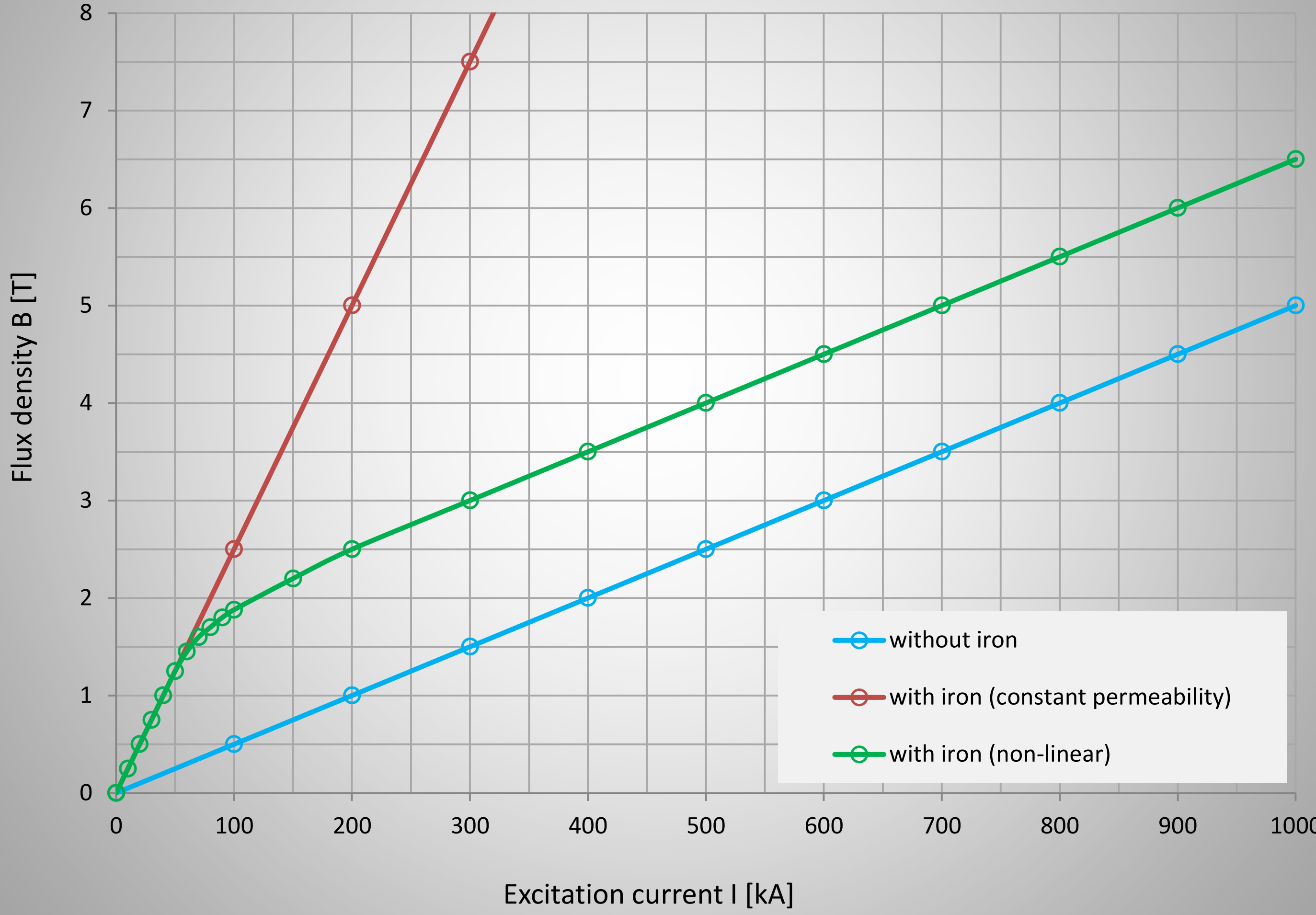
Reluctance and saturation



Keep yoke reluctance small by providing sufficient iron cross-section!

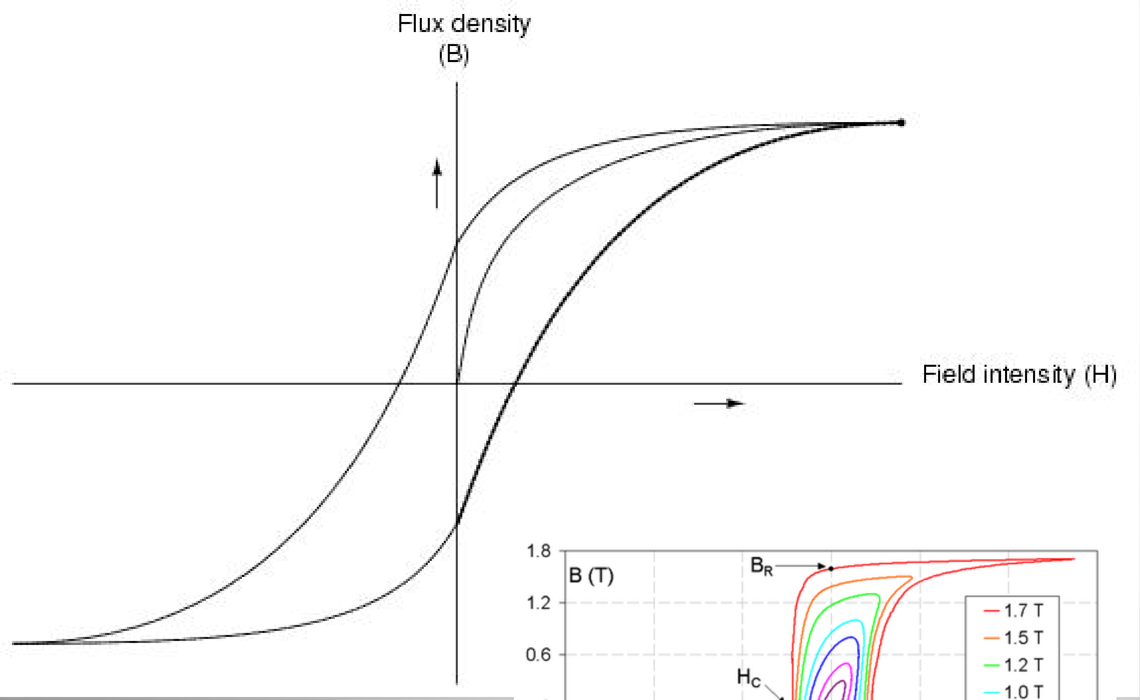
Reluctance and saturation

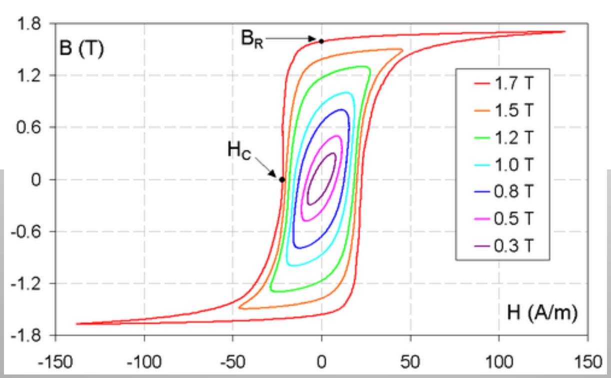
$$\vec{B} = \mu_0 \vec{H} + \vec{J} = \mu_0 \mu_r \vec{H}$$



Steel hysteresis

Flux density $B(H)$ as a function of the field strength is different, when increasing and decreasing excitation



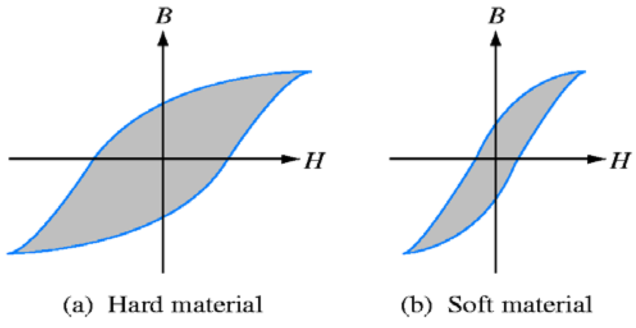


Remanent field (Retentivity):

$H = 0 \rightarrow B = B_r > 0$

Coercivity or coercive force:

$B = 0 \rightarrow H = H_c < 0$



Residual field in a magnet

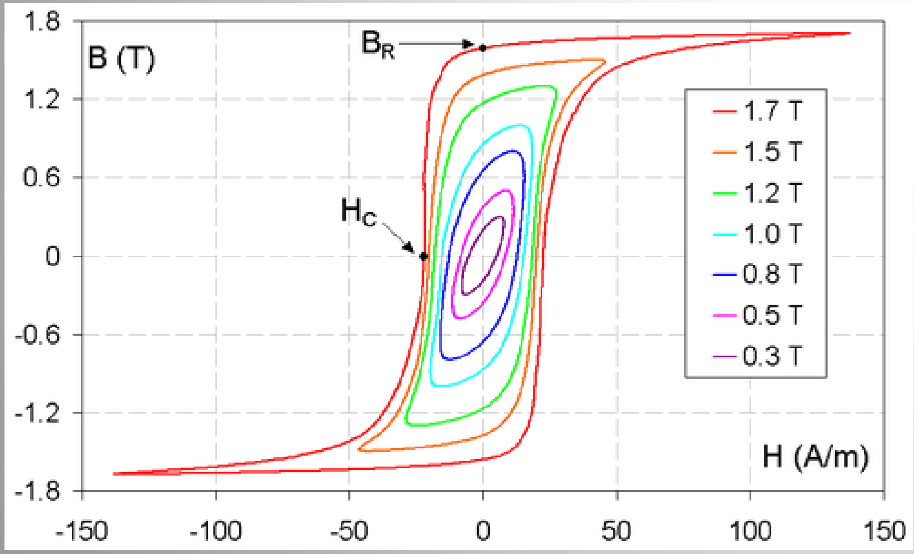
In a continuous ferro-magnetic core (transformer) the residual field is determined by the remanent field B_r

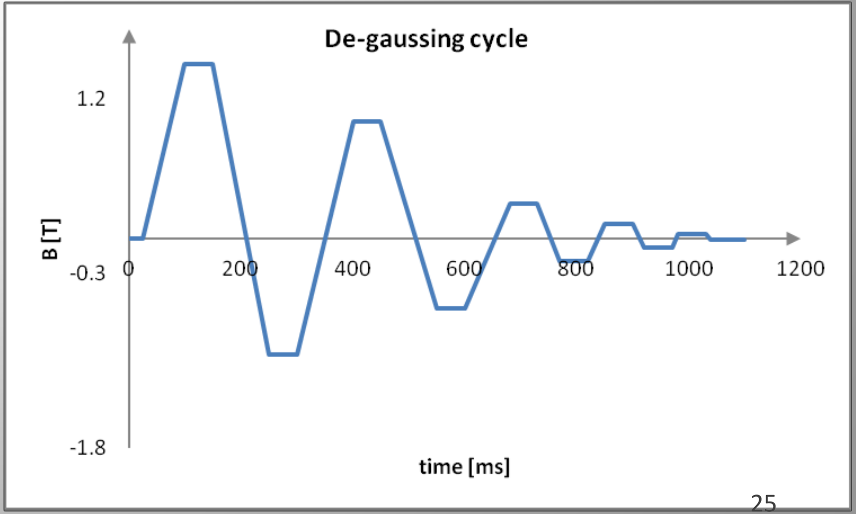
In a magnet core (gap), the residual field is determined by the coercivity H_c

Assuming the coil current $I=0$:

$$\oint \vec{H} \cdot \vec{dl} = \int_{gap} \vec{H}_{gap} \cdot \vec{dl} + \int_{yoke} \vec{H}_c \cdot \vec{dl} = 0$$

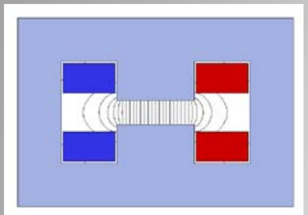
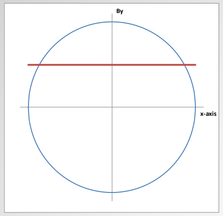
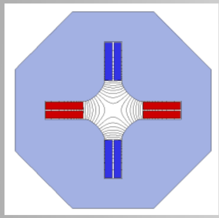
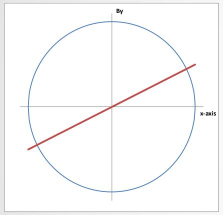
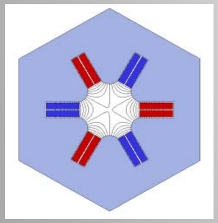
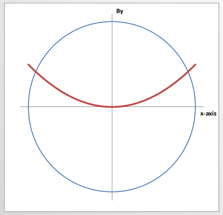
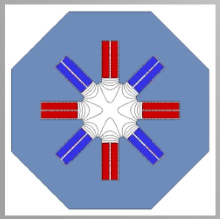
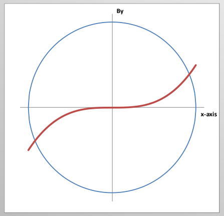
$$B_{residual} = -\mu_0 H_C \frac{l}{g}$$





Demagnetization cycle!

Magnet types

Pole shape	Field distribution	Pole equation	B_x, B_y
		$y = \pm r$	$B_x = 0$ $B_y = B_1(r_0) = \text{const.}$
		$2xy = \pm r^2$	$B_x = \frac{B_2(r_0)}{r_0} y$ $B_y = \frac{B_2(r_0)}{r_0} x$
		$3x^2y - y^3 = \pm r^3$	$B_x = \frac{B_3(r_0)}{r_0^2} xy$ $B_y = \frac{B_3(r_0)}{r_0^2} (x^2 - y^2)$
		$4(x^3y - xy^3) = \pm r^4$	$B_x = \frac{B_4(r_0)}{6r_0^3} (3x^2y - y^3)$ $B_y = \frac{B_4(r_0)}{6r_0^3} (x^3 - 3xy^2)$

Normal-conducting accelerator magnets
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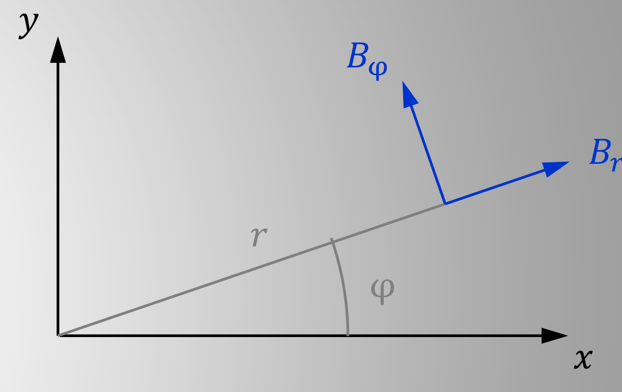
Field description

The 2D vector field of B can be expressed as a series of multipole coefficients $B_n(r_0)$, $A_n(r_0)$ with r_0 being the reference radius:

$$B_r(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} [B_n \sin(n\varphi) + A_n \cos(n\varphi)]$$

$$B_\varphi(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} [B_n \cos(n\varphi) - A_n \sin(n\varphi)]$$



$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{r_0}\right)^{n-1}$$

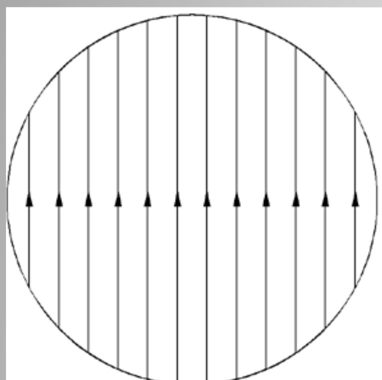
$$z = x + iy = re^{i\varphi}$$

This 2D decomposition holds only in a region of space:

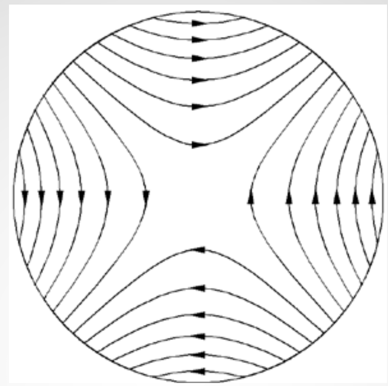
- without currents
- without magnetic materials ($\mu_r = 1$)
- where B_z is constant

Field description

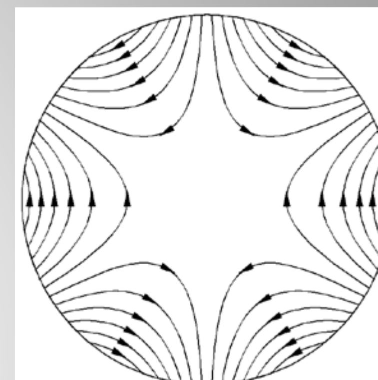
B_1: normal dipole



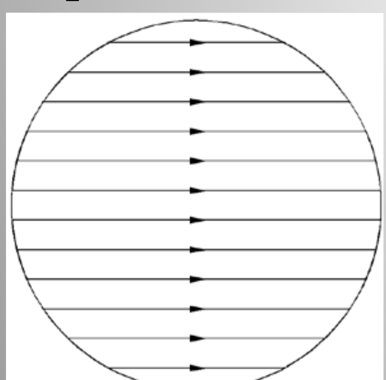
B_2: normal quadrupole



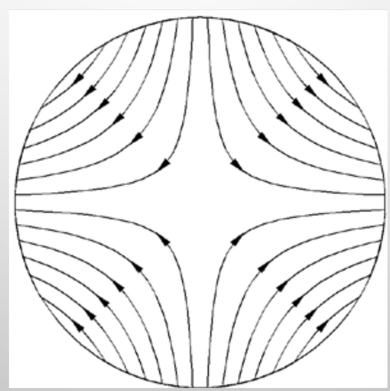
B_3: normal sextupole



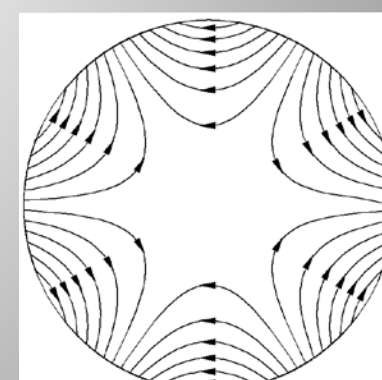
A_1: skew dipole



A_2: skew quadrupole



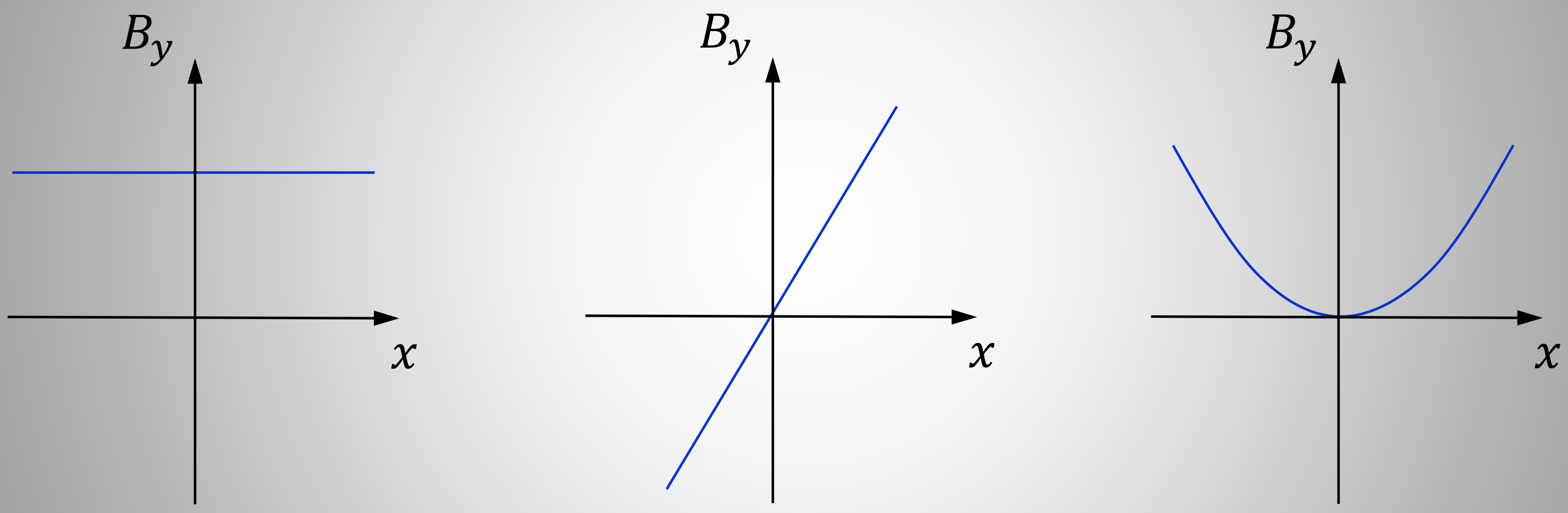
A_3: skew sextupole



Each multipole term has a corresponding magnet type

Field description

Field expansion along x: $B_y(x) = \sum_{n=1}^{\infty} B_n \left(\frac{x}{r_0}\right)^{n-1} = B_1 + B_2\frac{x}{r_0} + B_3\frac{x^2}{r_0^2} + \cdots$



B_1: dipole

B_2: quadrupole

B_3: sextupole

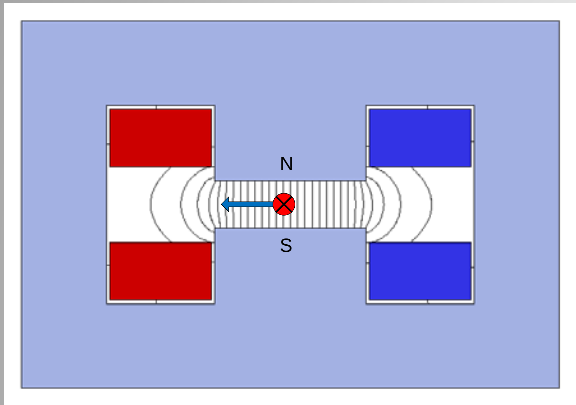
$$G = \frac{B_2}{r_0} = \frac{\partial B_y}{\partial x}$$

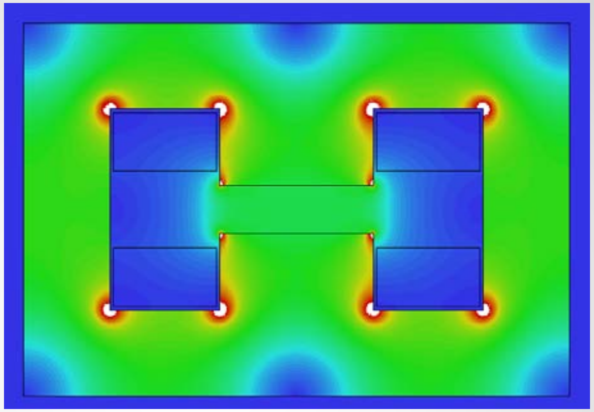
The field profile in the horizontal plane follows a polynomial expansion

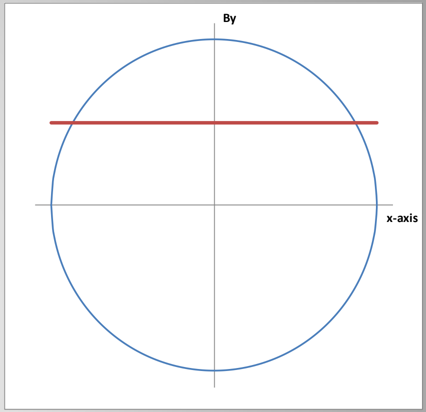
The ideal poles for each magnet type are lines of constant scalar potential

Dipoles

Purpose: bend or steer the particle beam







Equation for normal (non-skew) ideal (infinite) poles:

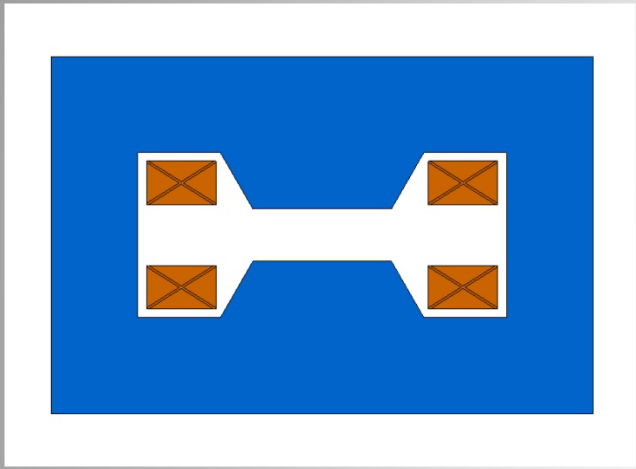
- Polar coordinates: $\rho \sin(\varphi) = \pm h/2$
- Cartesian coordinates: $y = \pm h/2$
- Straight line ($h =$ gap height)

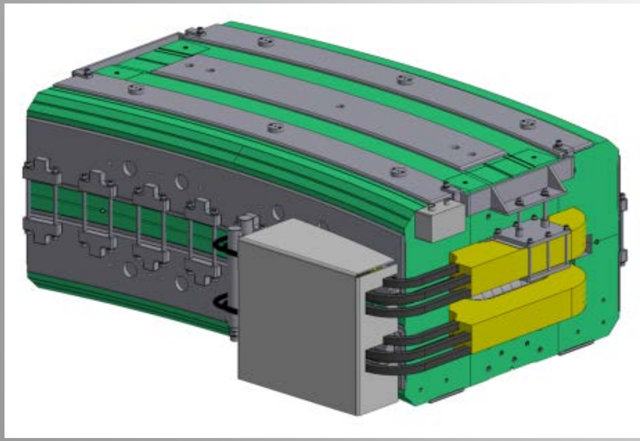
Magnetic flux density: $B_x = 0$; $B_y = B_1(r_0) = \text{const.}$

Applications: synchrotrons, transfer lines, spectrometry, beam scanning

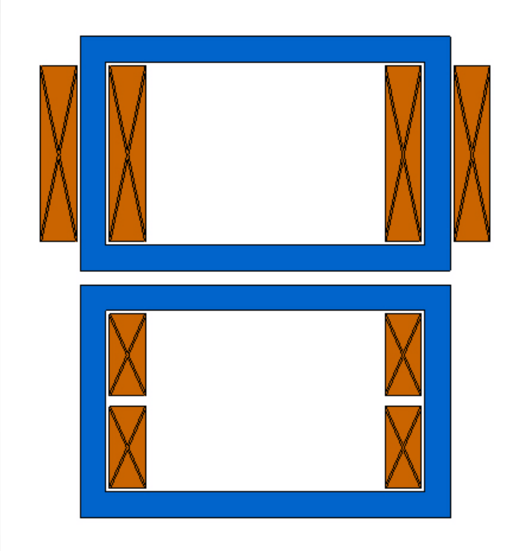
Dipole types

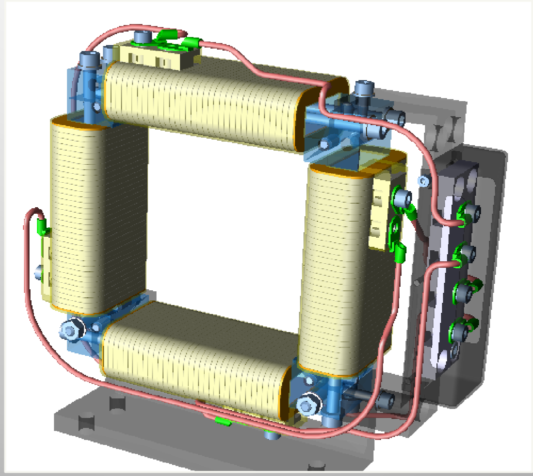
H-Shape



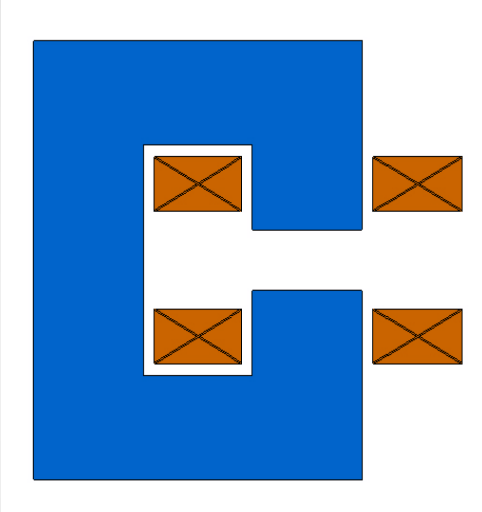


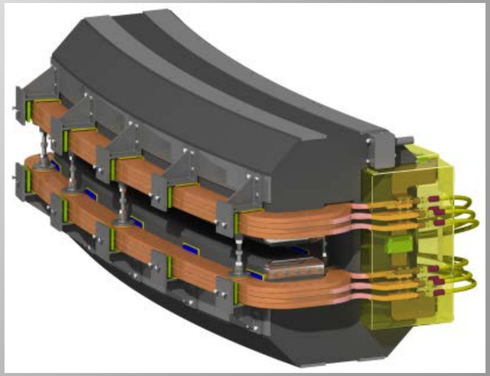
O-Shape





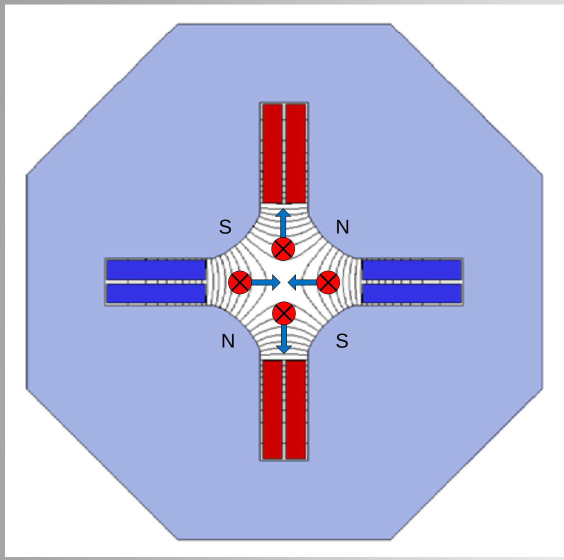
C-Shape

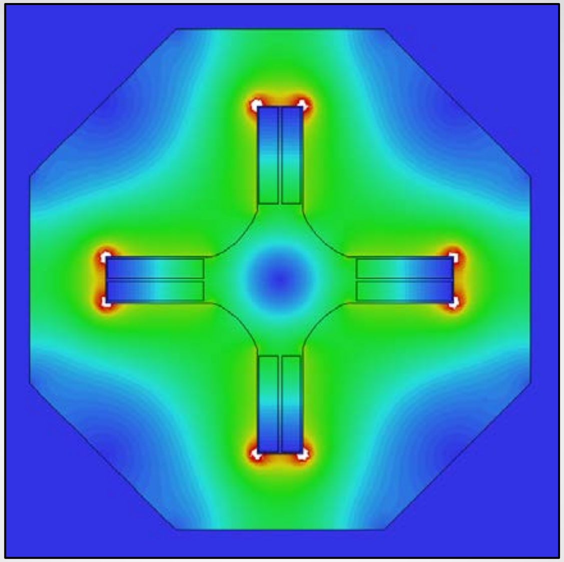


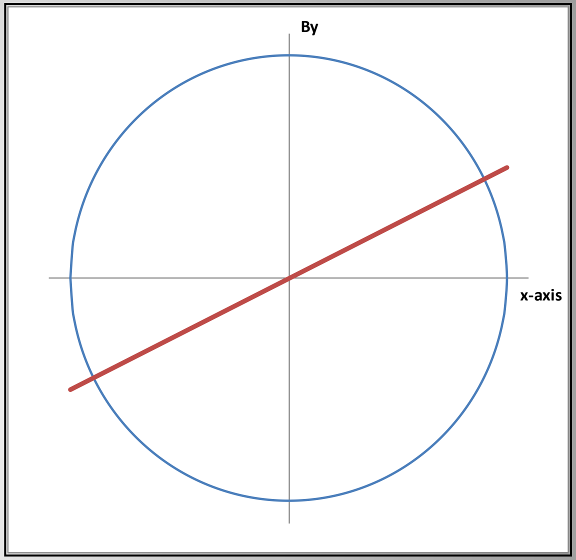


Quadrupoles

Purpose: focusing the beam (horizontally focused beam is vertically defocused)







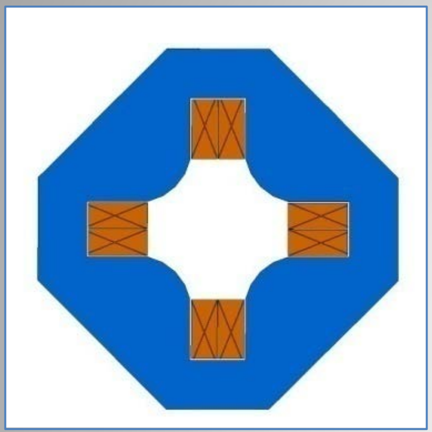
Equation for normal (non-skew) ideal (infinite) poles:

- Polar coordinates: $\rho^2 \sin(2\varphi) = \pm r^2$
- Cartesian coordinates: $2xy = \pm r^2$
- Hyperbola ($r =$ aperture radius)

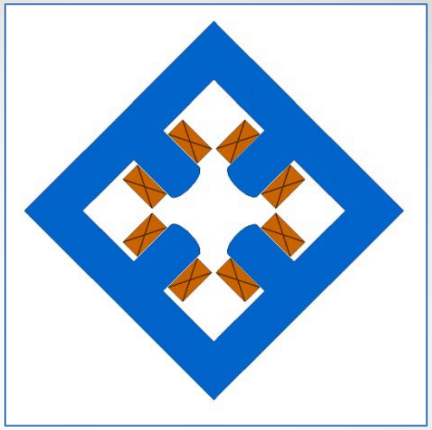
Magnetic flux density: $B_x = \frac{B_2(r_0)}{r_0} y\,; B_y = \frac{B_2(r_0)}{r_0} x$

Quadrupole types

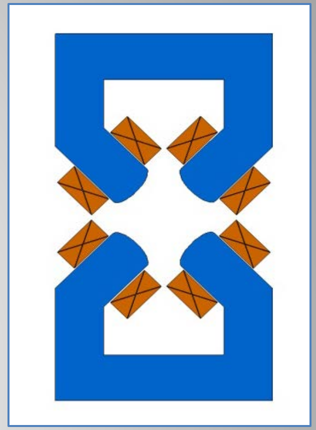
Standard quadrupole I

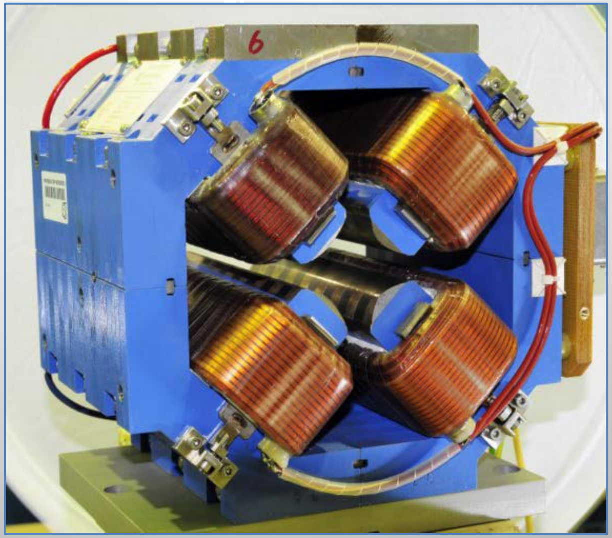


Standard quadrupole II



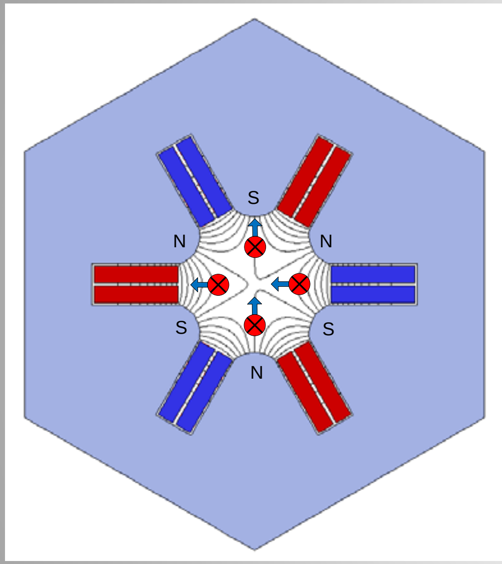
Collins or Figure-of-Eight

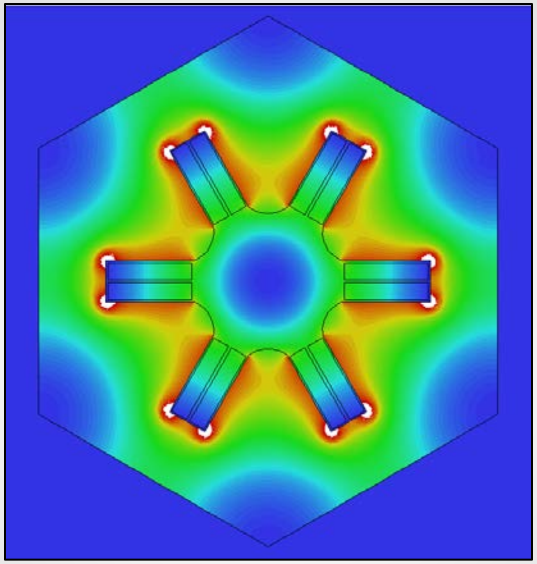


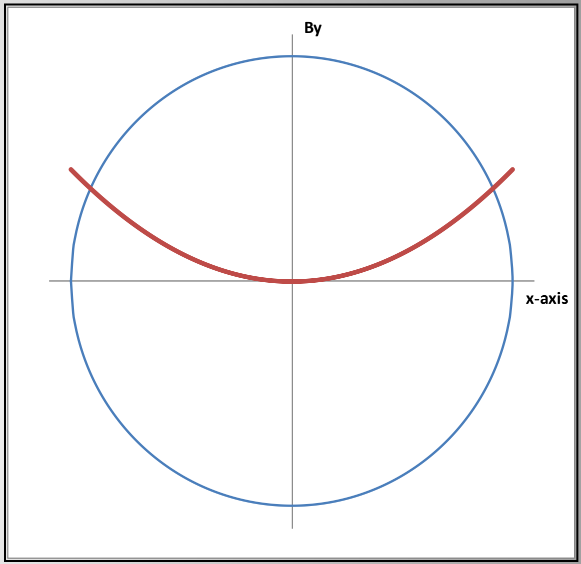


Sextupoles

Purpose: correct chromatic aberrations of 'off-momentum' particles







Equation for normal (non-skew) ideal (infinite) poles:

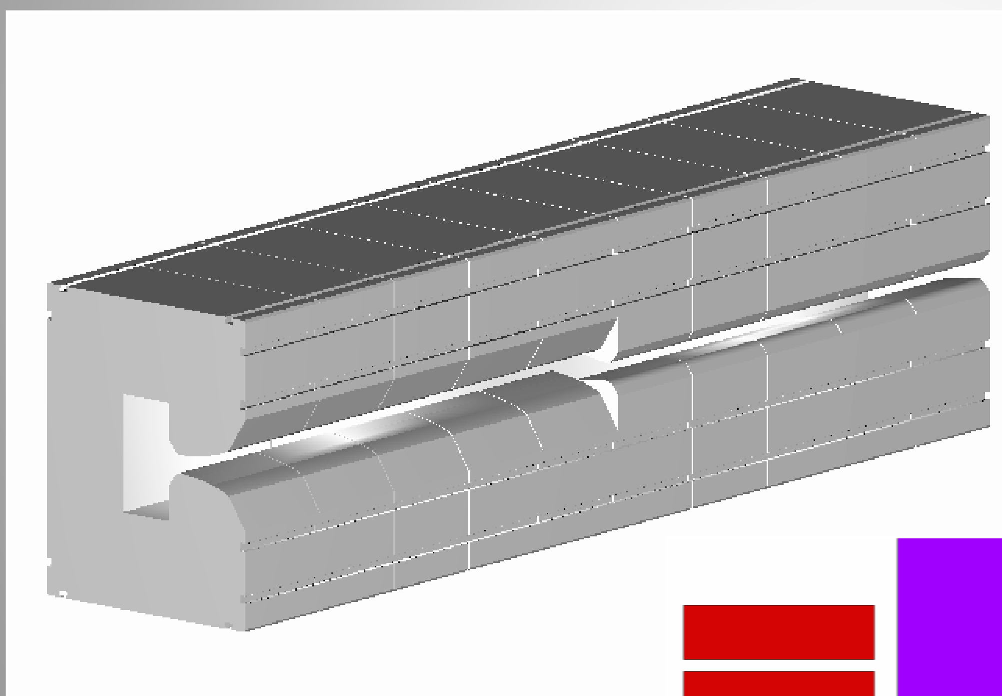
- Polar coordinates: $\rho^3 \sin(3\varphi) = \pm r^3$
- Cartesian coordinates: $3x^2y - y^3 = \pm r^3$

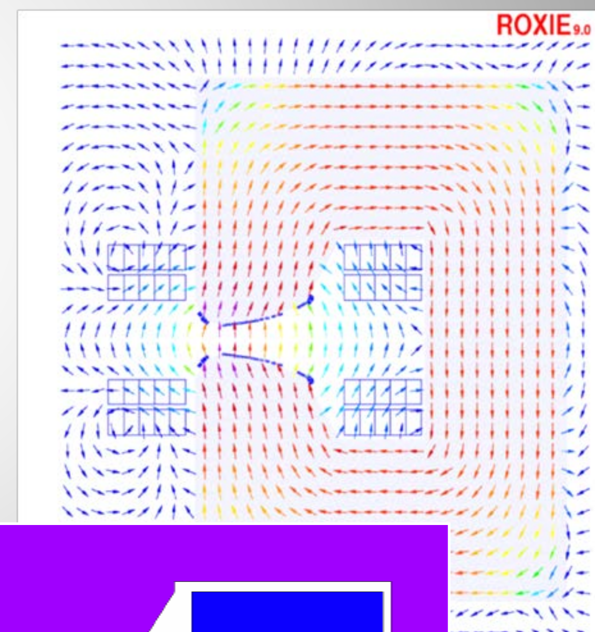
Magnetic flux density: $B_x = \frac{B_3(r_0)}{r_0^2} xy\,; B_y = \frac{B_3(r_0)}{r_0^2}(x^2 - y^2)$

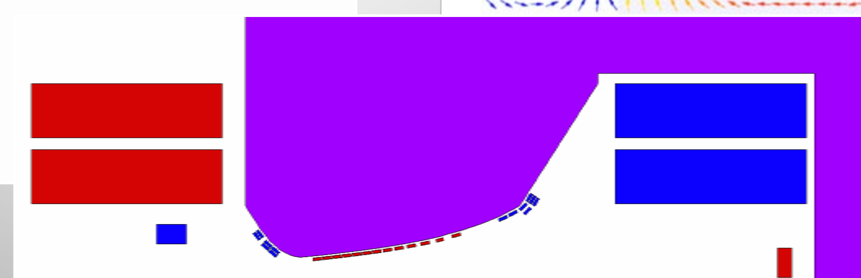
Combined function magnets

Functions generated by pole shape (sum a scalar potentials):

- Amplitudes cannot be varied independently
- Dipole and quadrupole: PS main magnet (PFW, Fo8…)



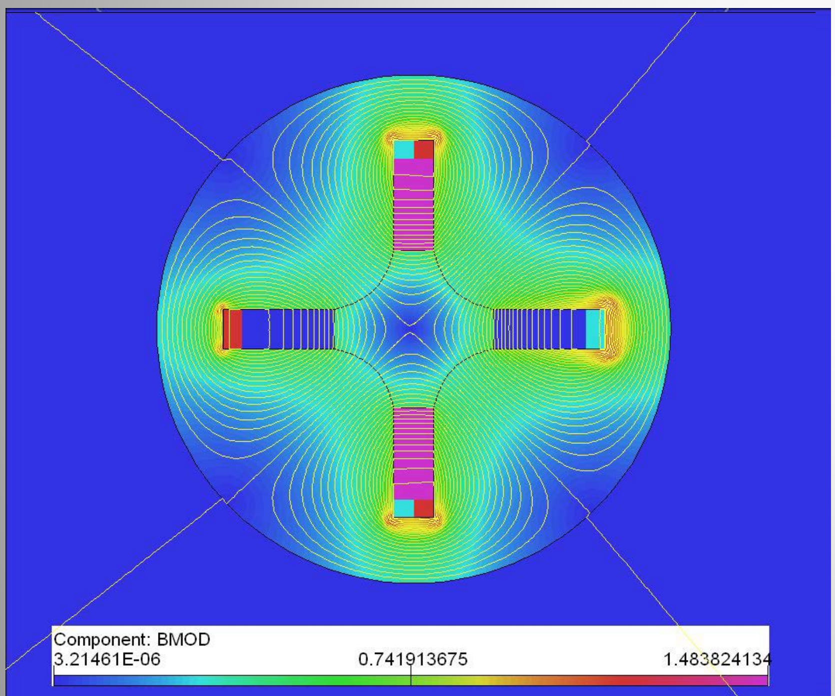


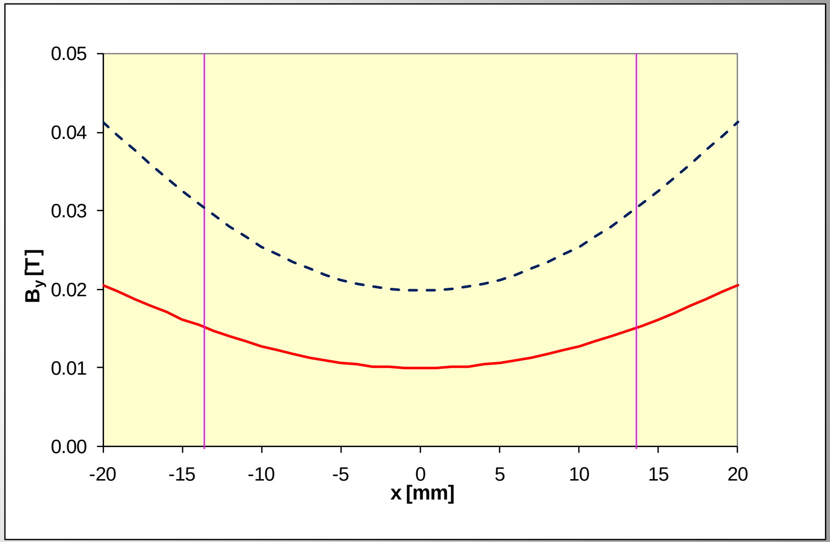


Combined function magnets

Functions generated by individual coils:

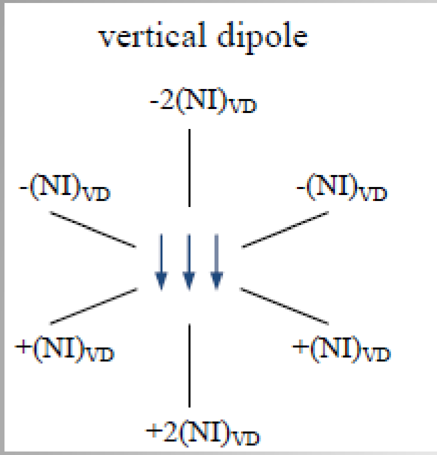
Amplitudes can be varied independently

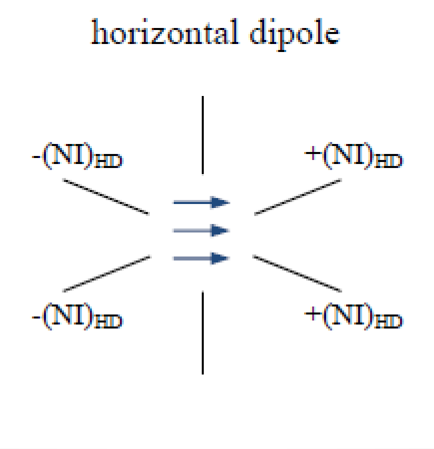


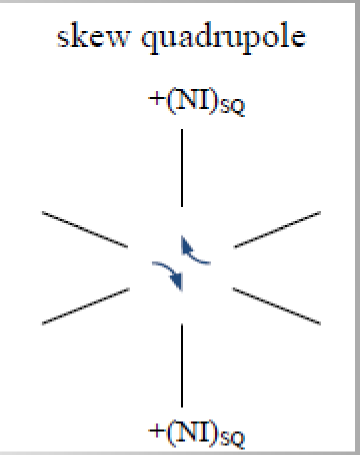


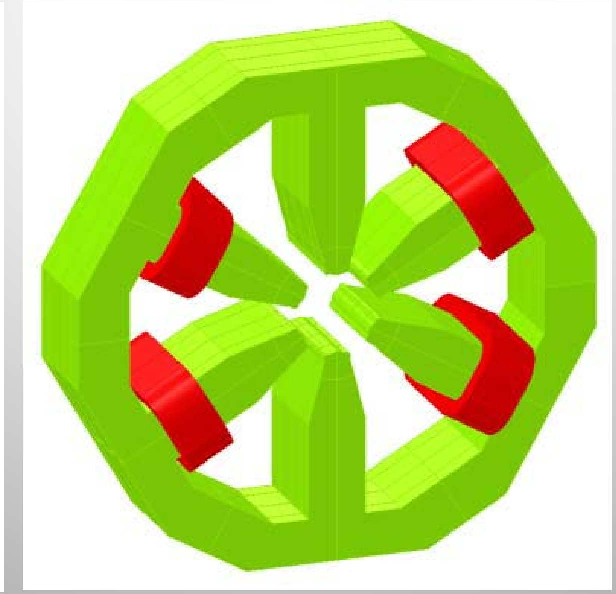
Quadrupole and corrector dipole (strong sextupole component in dipole field)

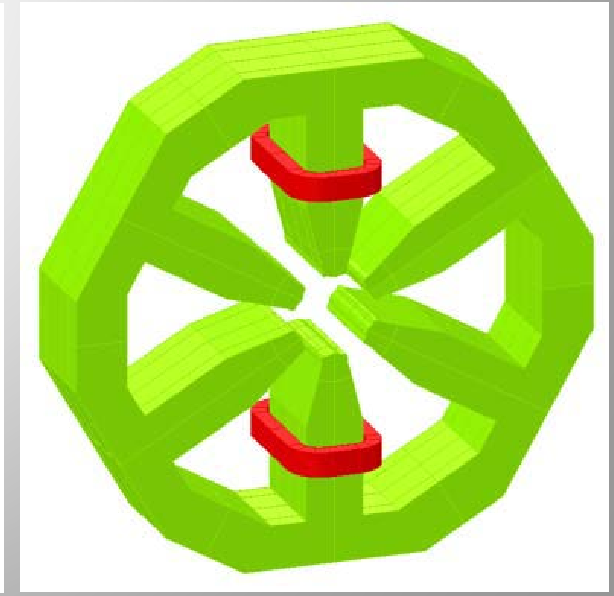
Combined function magnets







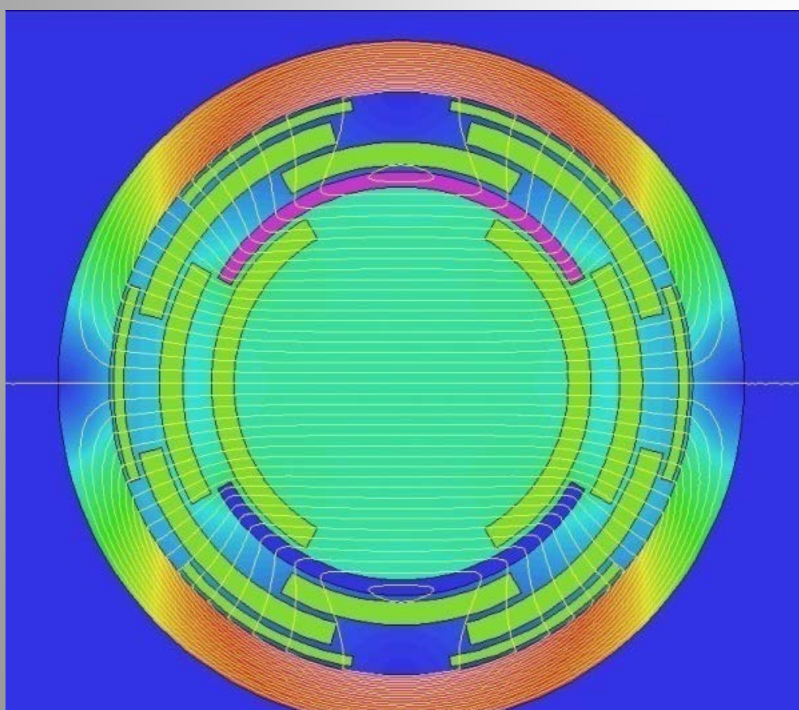




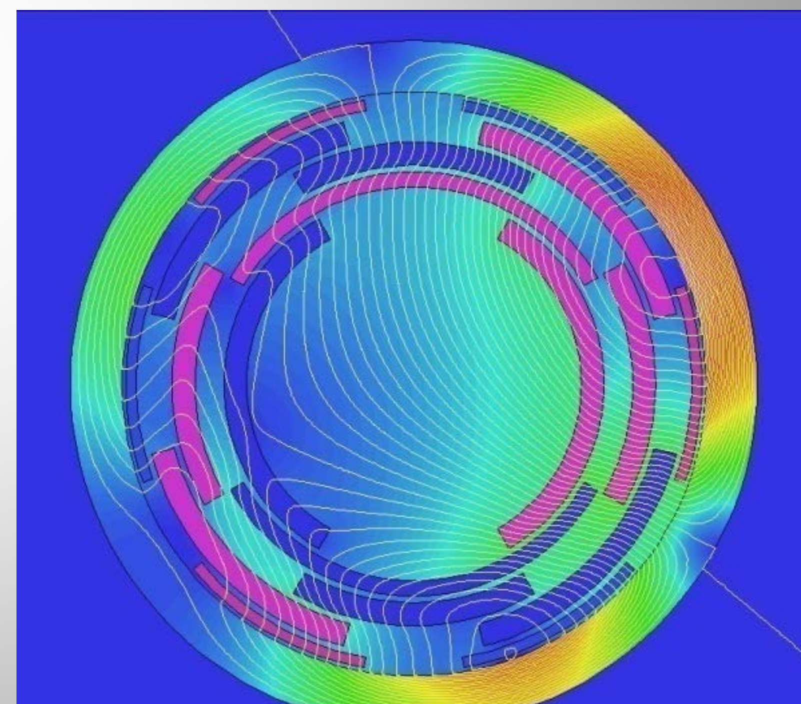
Courtesy of A. Milanese

Coil dominated magnets

- Nested multi-pole corrector (moderate field levels)
- Iron for shielding only
- Field determined by current distribution



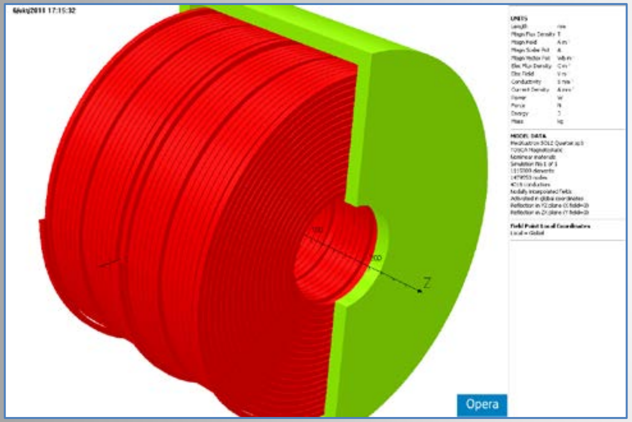
Vertical dipole configuration

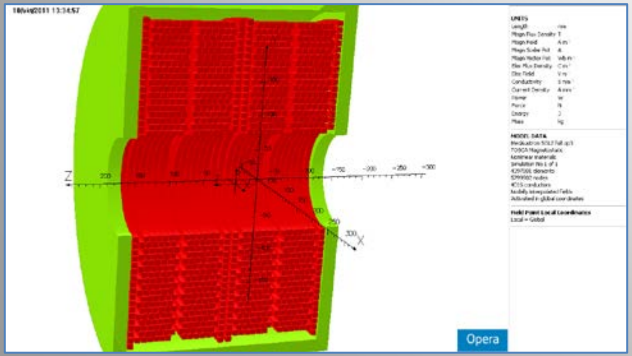


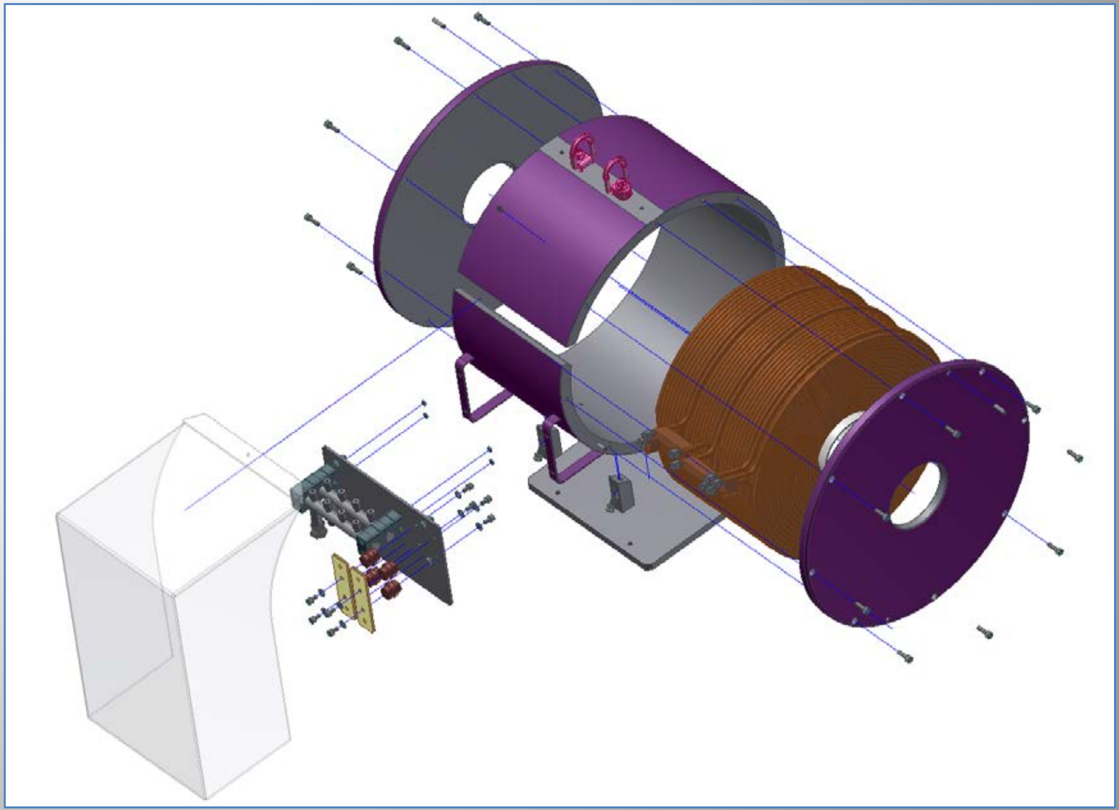
Multi-pole configuration

Solenoids

- Weak focusing, non-linear elements
- Main field component in z-direction, focusing by end fields
- Usually only used in experiments or low-energy beam lines







Summary

- Magnets are needed to guide and shape particle beams
- Coils carry the electrical current, the iron yoke carries the magnetic flux
- Steel properties and yoke geometry have a significant influence on the magnet performance
- Iron saturation infleunce the efficiency of the magnetic circuit and has to be taken into account in the design
- The 2D (magnetic) vector field can be expressed as a series of multipole coefficients
- Different magnet types for different functions