Beam dynamics for cyclotrons

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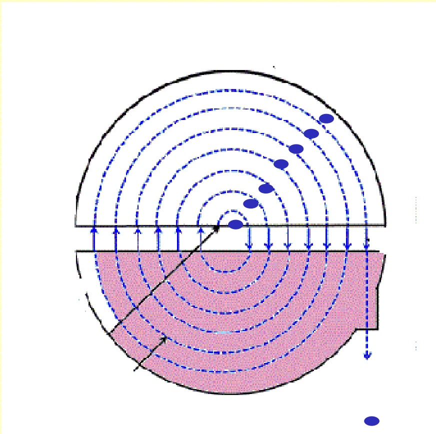
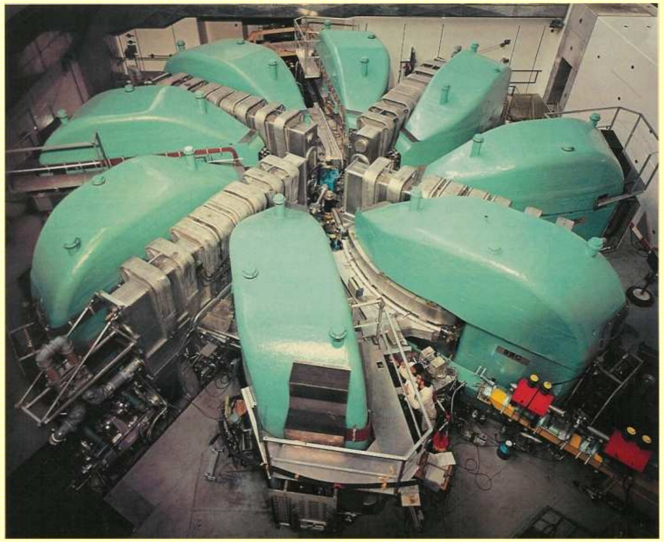




compact cyclotrons

Separated sectors (ring cyclotrons)

Synchrocyclotrons

Fixed energy

Variable energy

Superconducting

Normal conducting

OUTLINE

Chapter 1 : theory 1

- Principle
- Basic equation
- Longitudinal dynamics
- Transverse dynamics

Chapter 2 :specific problems

•Longitudinal dynamics

- Acceleration
- Injection
- Extraction

Chapter 3 :

- Design strategy
- Tracking
- Simulations

Chapter 4 :

-Theory vs reality (tunes,isochronism,...)

Exemples

-Medical cyclotron

-Reseach facility

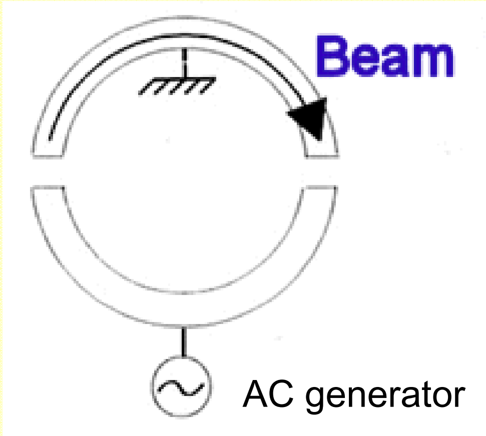
CYCLOTRON HISTORY

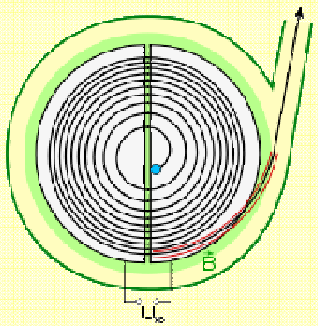
The Inventor, E. Lawrence, get the Nobel in Physics (1939) (first nuclear reactions without alpha source)

- brilliant idea (E. Lawrence, Berkeley, 1929) : RF accelerating field is field is technically complex and expensive.

So Let 's use only 1 RF cavity, but many times

A device is put into a magnetic field, curving the ion trajectories and only one electrode is used several times.





What is a CYCLOTRON ?

- RF accelerator for the ions :

  from proton A=1 to Uranium A=238

- Energy range for proton 1MeV -1GeV (γ close to 1!!!)
- Circular machine : CW (and Weak focusing)
- Size Radius=30cm to R=6m
- RF Frequency : 10 MHz -50 MHz

APPLICATIONS : Nuclear physics

(from fundamental to applied research)

: Medical application

Radio Isotopes production (for PET scan,....)

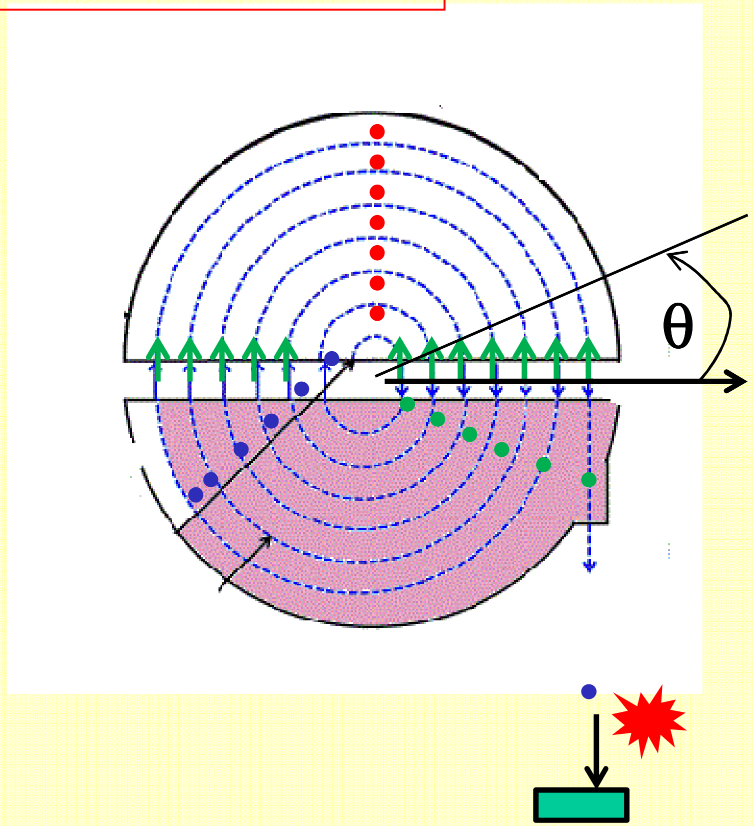
Cancer treatment

Quality : Compact and Cost effective

Usefull words for the cyclotrons

$$B\rho = \frac{P}{q} = \frac{\gamma\, m.v}{q}$$

$$E_K = (\gamma - 1).mc^2$$



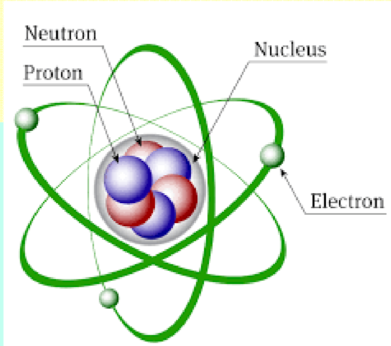
Cyclotron vocable

r Radial = horizontal

z Axial = vertical

θ « Azimuth » = cylindrical angle

MeV/A= kinetic energy unit in MeV per nucleon



Ions : $^{A}_{Z}X^{Q}$

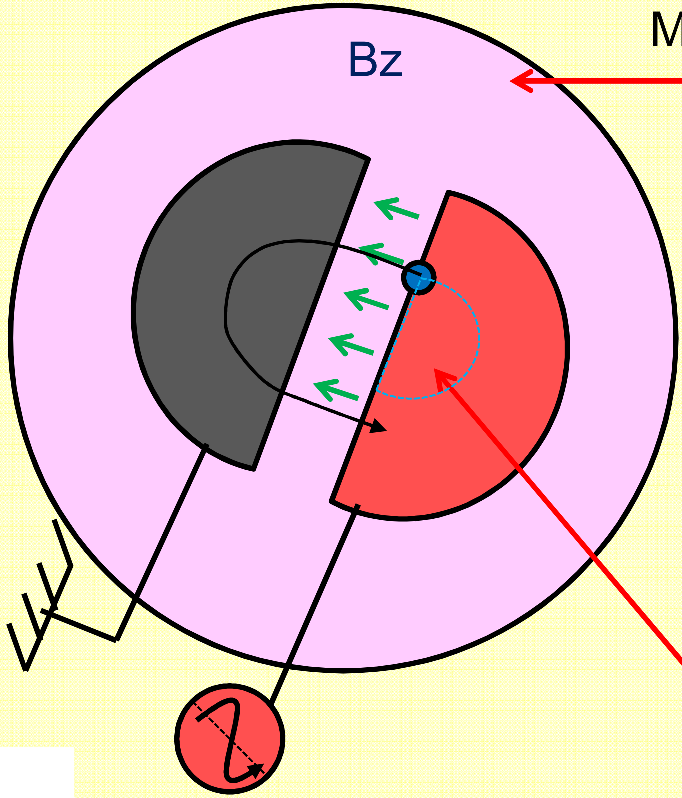
A : nucleons number

Z: protons number

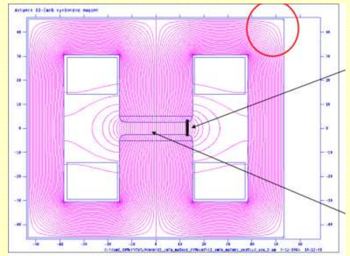
Q : charge state : 0+,1+,2+,…..

Principle :the hardware

$$\vec{E} = [\,V_1(t) - V_0\,]/d$$
$$\sim \cos(\omega t)$$



Magnet



H-type
electro-magnet

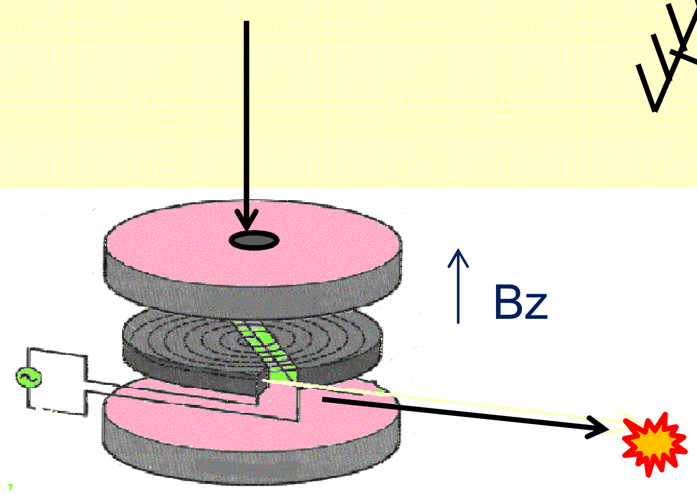
Accelerating Dee's

2 Copper boxes
≠ potential

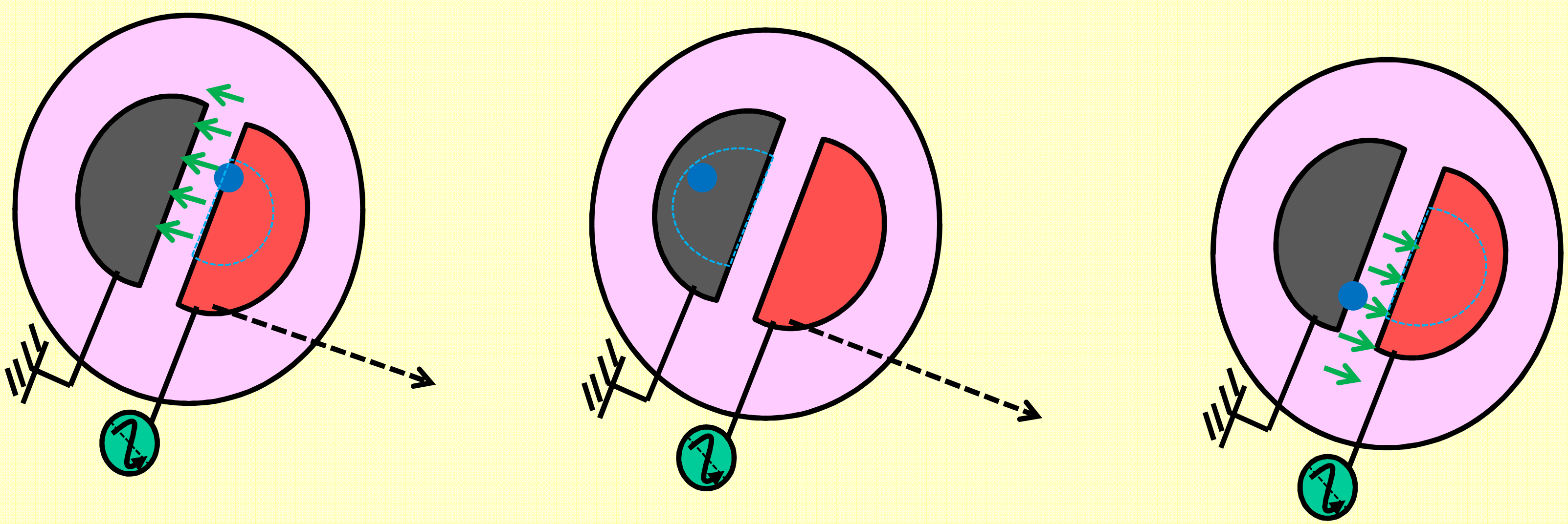
-half is $V1(t) \sim \cos(\omega t)$

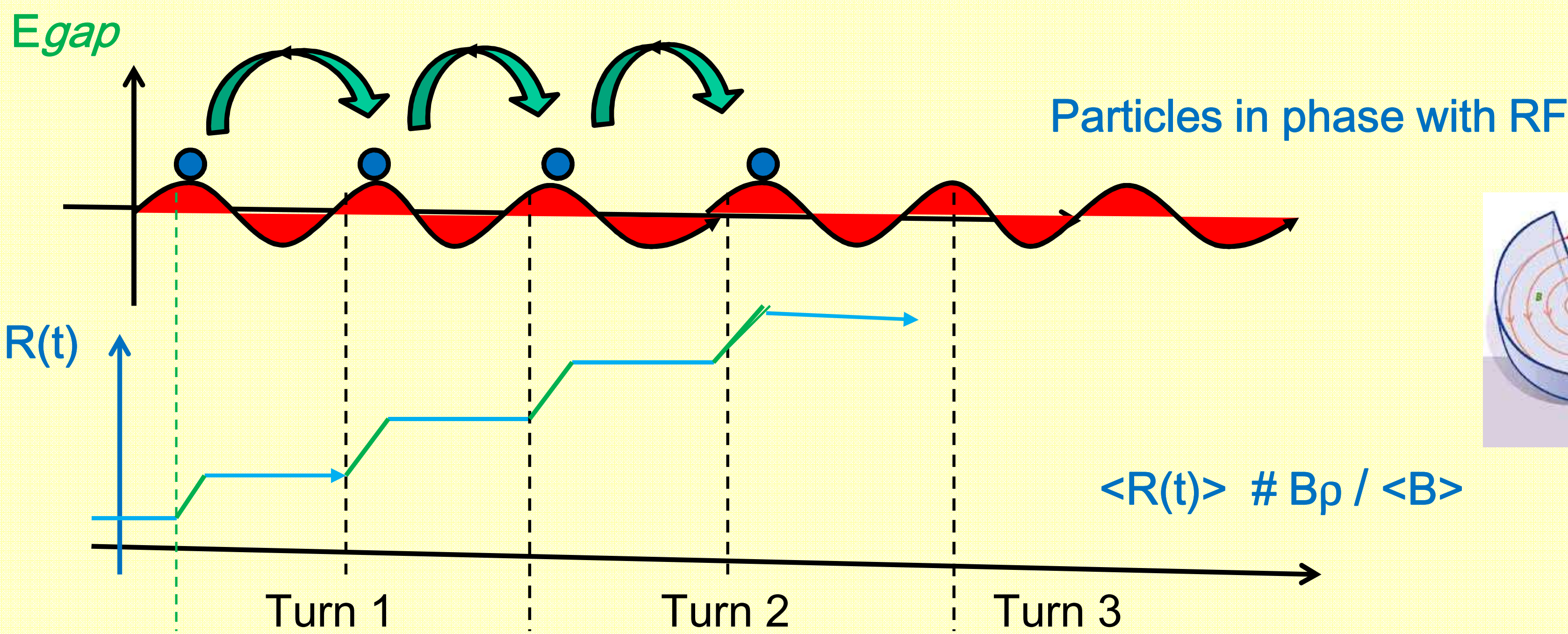
- half is
at the ground potential

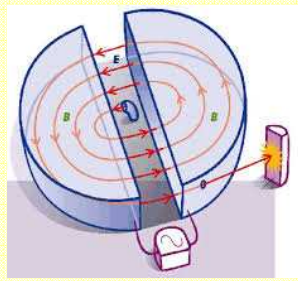
AC generator



Principle B: the trajectories







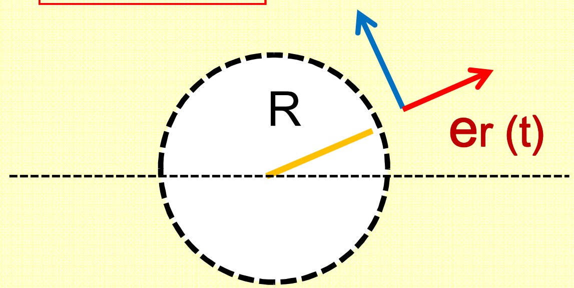
Trajectory in uniform B field

$$\frac{d(\gamma\, m\vec{v})}{dt} = \vec{F}$$

Let's consider an ion with a charge ***q*** and a mass ***m*** circulating at a speed $\boldsymbol{v_\theta}$ in a uniform induction field ***B****.=(0,0,Bz)*

The motion equation can be derived from the **Newton's law** and the **Lorentz force *F*** ***in a cylindrical coordinate system*** (er,eθ,ez):

$$\boxed{\vec{F} \perp \vec{v}}$$



$$\boxed{\frac{d(\vec{v})}{dt} = \vec{a} = \frac{d^2(R.\vec{er})}{dt^2} = \left[\left\|\vec{v}\right\|^2 / R\right].\vec{e_r}}$$

$$\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{v} \times \vec{B}) = -q v_\theta B_z \vec{e_r}$$

$$\boxed{\gamma \frac{m v^2}{R} = -q v_\theta B_z \qquad v_\theta = R\dot{\theta}}$$

$$\boxed{R = \frac{\gamma\, m v}{q B_z}}$$

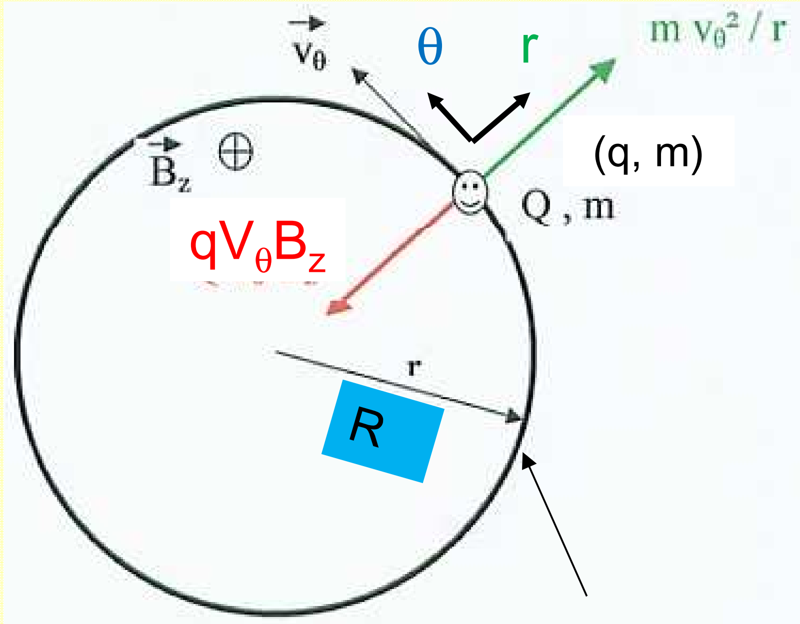
Trajectory in uniform B field

$$R = \frac{\gamma\, mv}{qB_z}$$

$$Frevolution = \frac{\mathrm{v}}{2\pi\, R} = \frac{1}{2\pi}\frac{qB}{\gamma\, m}$$

$$\omega_{rev} = \dot{\theta} = \frac{d\theta}{dt} = \frac{\mathrm{v}_\theta}{R} = \frac{qB}{\gamma\, m}$$

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$



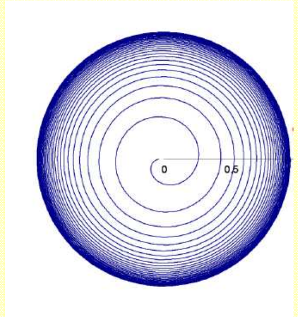
Centrifugal force = Magnetic force

$$\gamma \frac{m\mathrm{v}_\theta^2}{R} = -q\mathrm{v}_\theta B_z$$

Let’s accelerate ions, in a constant vertical field B*z*

The Radius evolves with P/q :

$$R(t) = \frac{P(t)}{qB_z} = \frac{\gamma\, mv}{qB_z}$$



For *non relativistic* ions (low energy) ⇨ $\gamma \sim 1$

In this domain, if B*z* =const ⇨ ω=const

$$\boxed{\omega_{rev} = \frac{qB_z}{\gamma\, m} \approx \quad const}$$

same ΔT for each Turn

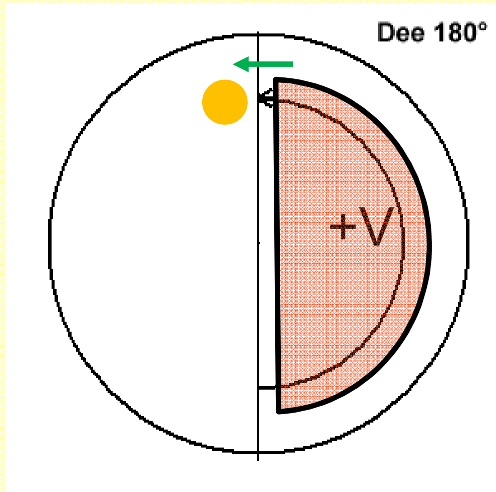
So it is easy to synchronize a RF cavity

having a “D” shape, with accelerated ions

$$\boxed{V = V_0 \cos(\omega_{RF}\, t)}$$

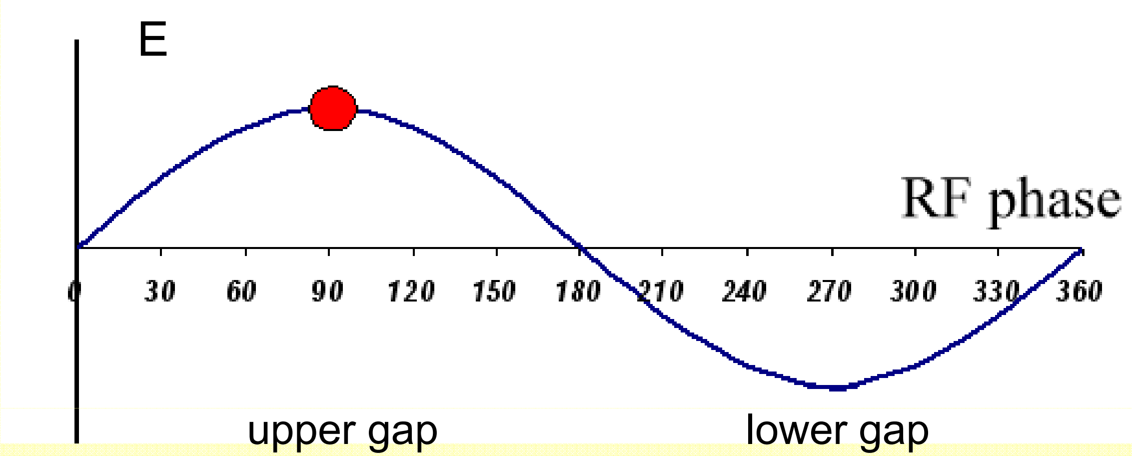
$$\omega_{RF} = \mathbf{h} \;\; \omega_{rev}$$

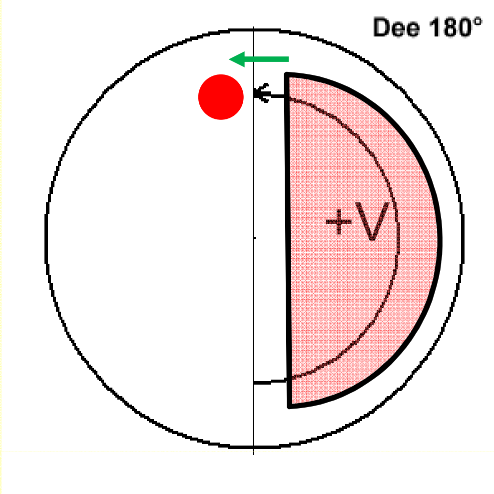
h = 1, 2, 3, … called the RF harmonic number

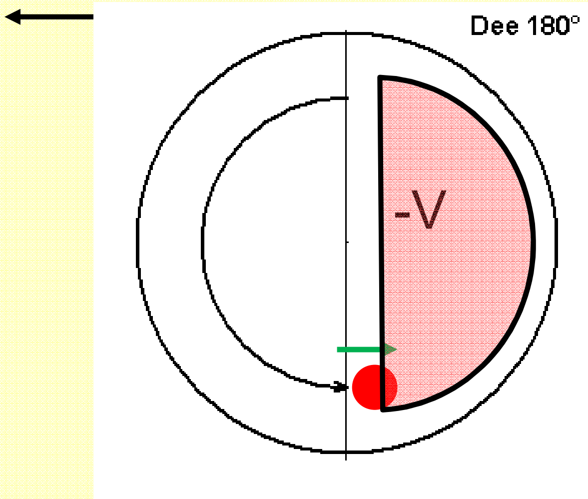


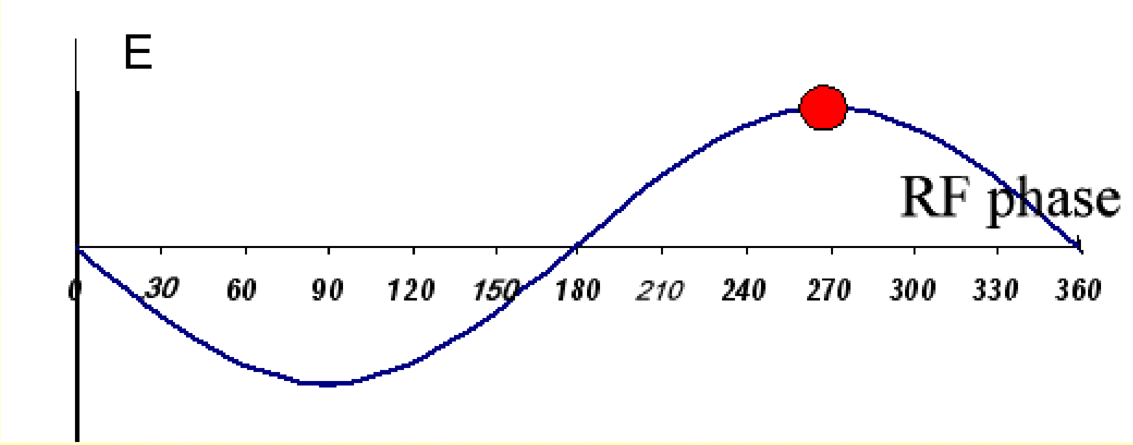
Harmonic number h=FRF/Frev

h = 1 *1 bunch by turn* $\omega_{rf} = \mathbf{h} \ \omega_{rev}$





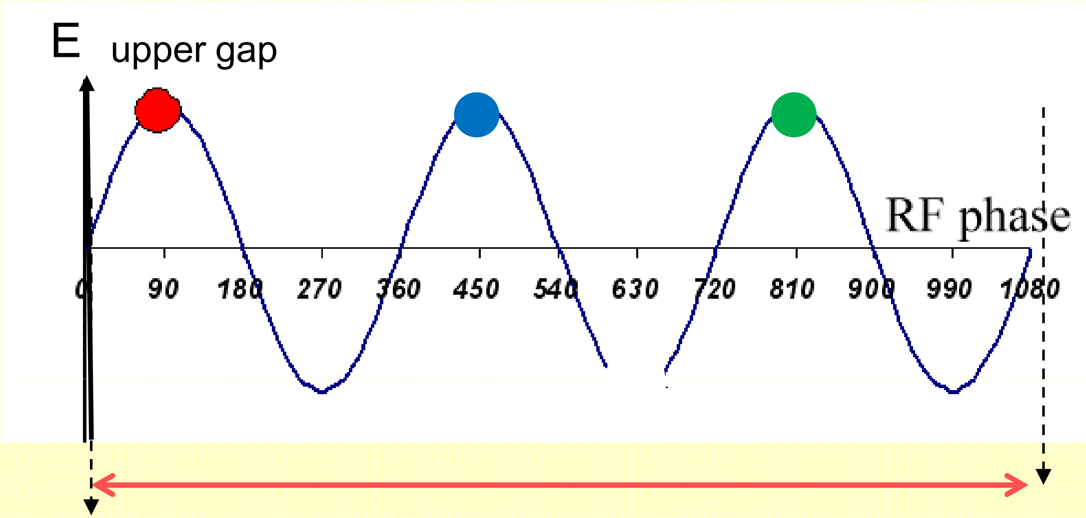




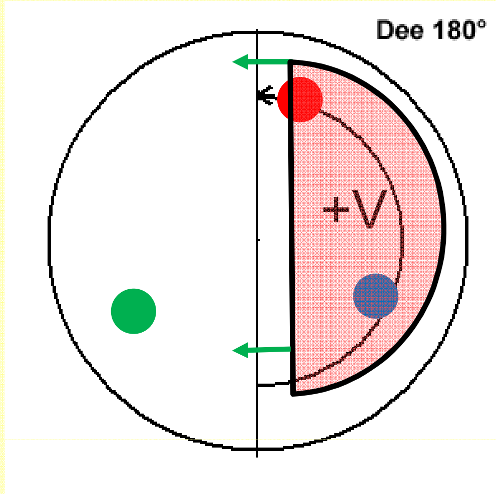
Harmonic number $h = F_{RF} / F_{rev}$

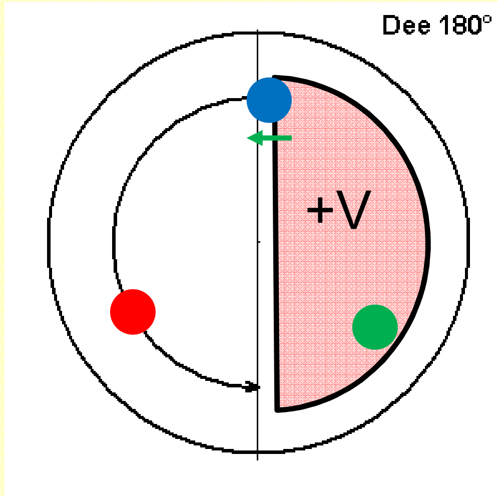
h = 3

3 bunch by turn $\omega_{rf} = \mathbf{h} \ \omega_{rev}$



1 turn RF= 3 x 360°





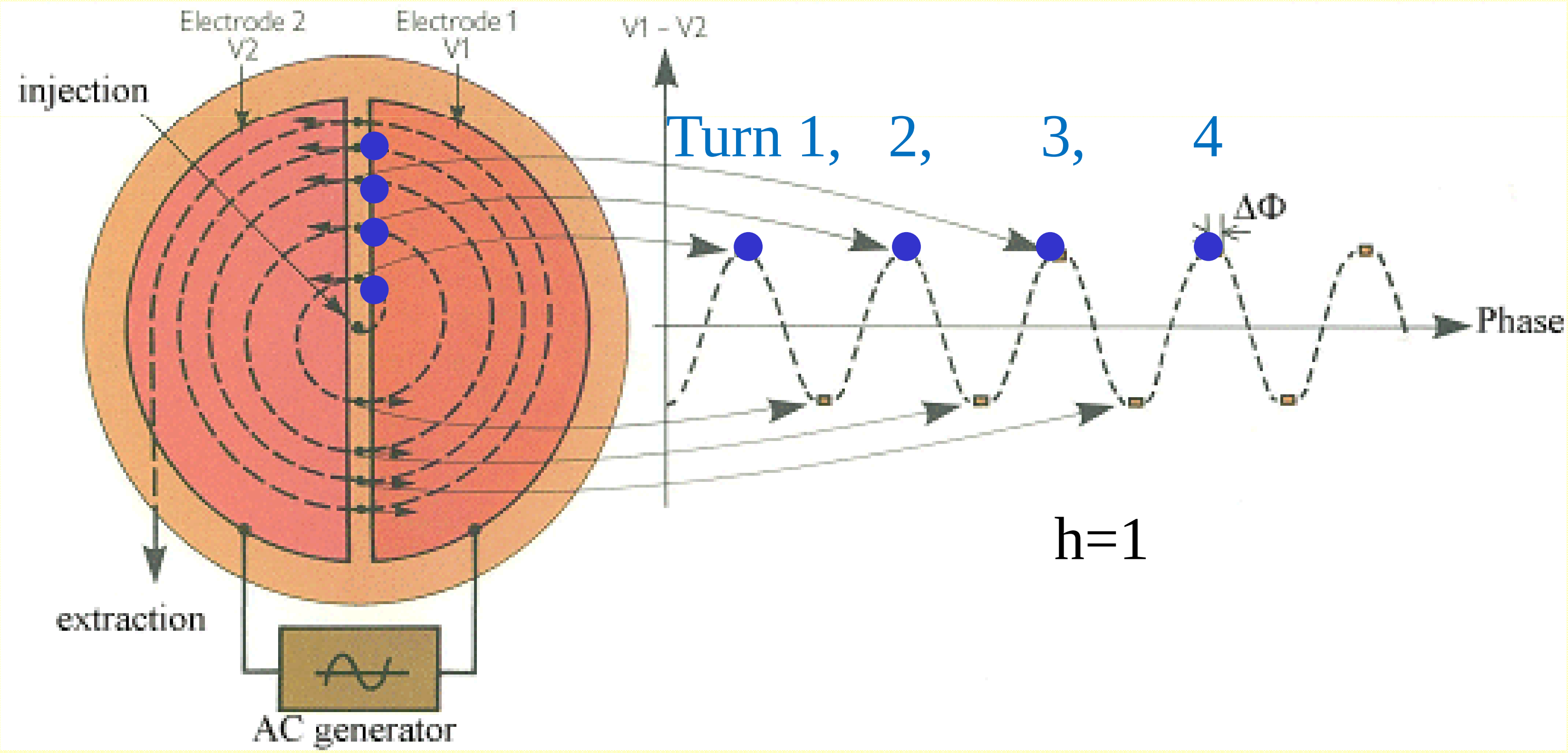
Over 360° the 3 beams are separated by

360°/3 = 120° (beam phase)

Isochronism condition: The particle takes the same amount of time to travel one turn : (constant revolution frequency ω_{rev} =const)

and with $\omega_{rf} = h\ \omega_{rev}$, the particle is synchronous with the RF wave.

In other words, the particle arrives always at the same RF phase in the middle of the accelerating gap.

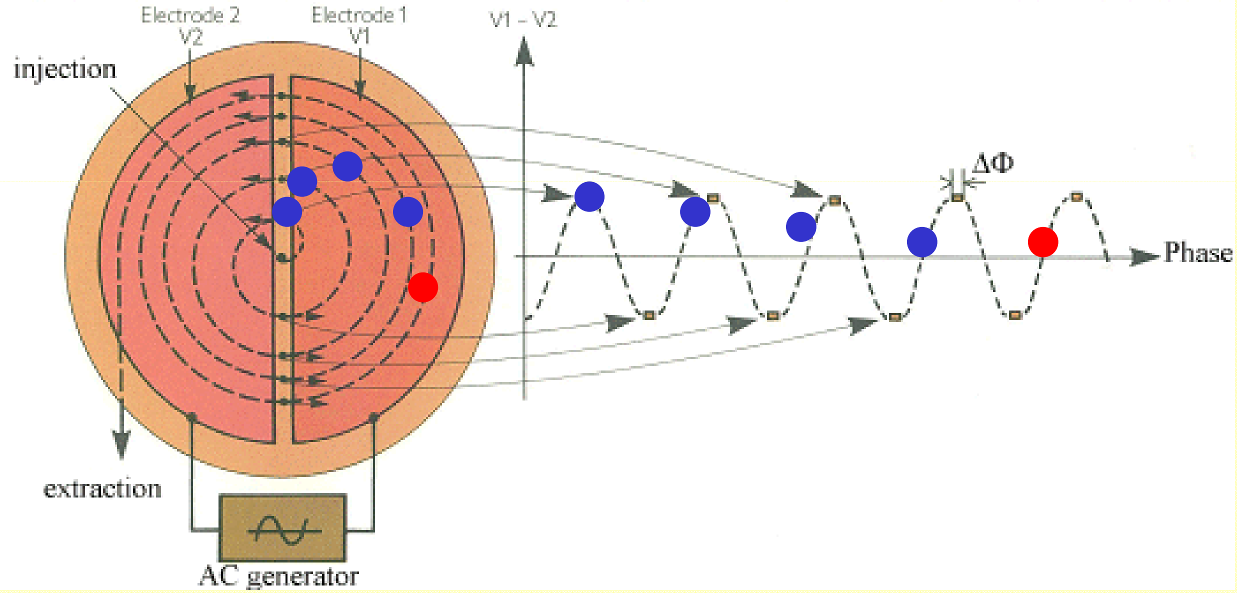


Longitudinals with relativistic particles

With Bz = constant , relativistic γ increases AND ωrev decreases

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$

Isochronism condition not fullfilled



$$\omega_{rev} = \frac{q.B_z(r)}{\gamma(r)\, m}$$

Isochronism condition fullfilled

Bz(R)/γ(R) =CONSTANT

Dynamics in cyclotron

summary

$Qe_0 \hat{V} \cos\phi . N_{gap}$ ***Energy gain per turn***

$\phi_0 \approx 0°$ ***Central RF phase ,***
Ion bunches are centered at 0°

$\omega_{RF} = h\omega_{rev} = const$

RF synchronism = Isochronism
(h - harmonic number)

$$\boxed{\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const}$$

$R = R(t) = R(N°turn\quad)$ ***Orbit evolving***

$B\rho(t) = {}^{P}\!/_{q} \Rightarrow \langle B \rangle = B\rho / R$ ***Average Magnetic field***

Transverse dynamics in the cyclotrons

Isochronism condition
(longitudinal)

Bz =Bz(R)

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

We will show that that isochronism have a bad consequence on vertical oscillations

Vertical oscillations

$$\mathbf{z}(t) = z_0 \cos(\nu_z \omega_0 t)$$

Transverse dynamics with Bz(r)

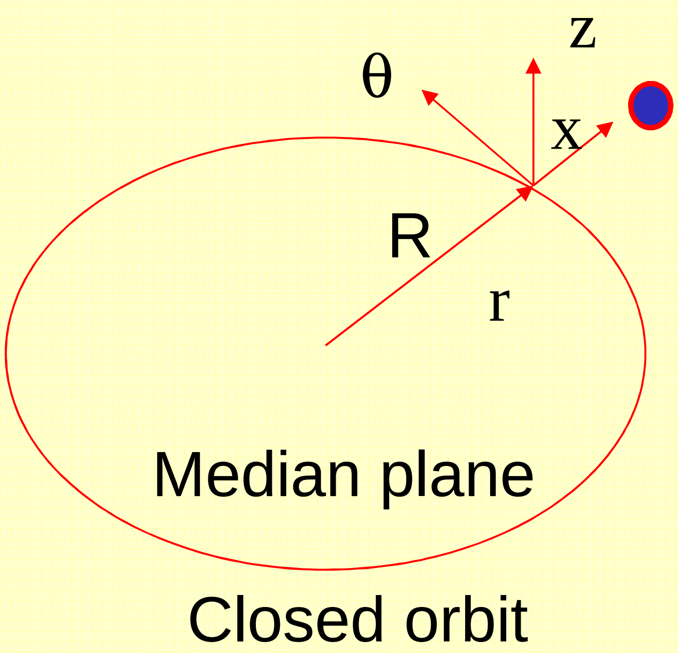
Steenbeck 1935, Kerst and Serber 1941

Horizontal stability : cylindrical coordinates (**er**, **eθ**, **ez**)

and

define **x** a small orbit deviation with Bz=Bz(r) (not constant)

$$\vec{r} = [R+x].\overrightarrow{er} = R\,(1+\frac{x}{R}).\overrightarrow{er}$$



$$x << R$$

$$(\textit{Paraxial or Gauss conditions})$$

$$\boxed{\frac{d(\overrightarrow{\mathrm{v}})}{dt} = \frac{d^2(\mathrm{R}.\overrightarrow{er})}{dt^2} = \left[\mathrm{v}^2 / R\right].\overrightarrow{e_r}}$$

Radial dynamics with Bz(r) *(No RF)*

- Taylor expansion of the field B_z around the median plane:
- $$B_z = B_{0z} + \frac{\partial B_z}{\partial x} x + \ldots = B_{0z}(1 - n\frac{x}{R})$$

n Definition

$$\boxed{with \quad n = -\frac{R}{B_{0z}}\frac{\partial B_z}{\partial x} \quad the \quad field \quad index}$$

•How evolve an ion in this non uniform Bz : *r(t)= R+x(t)*

$$m\gamma\frac{d^2\vec{r}}{dt^2} = -q\,\mathrm{v}_\theta B_z$$

$$m\gamma\frac{d^2(r\cdot\overrightarrow{er})}{dt^2} = m\gamma(\ddot{x} + \frac{\mathrm{v}_\theta^2}{r}) = m\gamma\ddot{x} + m\gamma\frac{\mathrm{v}_\theta^2}{R}(1 - \frac{x}{R})$$

$$\frac{1}{r} = \frac{1}{R\ (1 + \frac{x}{R})} = \frac{1}{R}(1 - \frac{x}{R})$$

$$m\gamma\left(\ddot{x} + \frac{\mathrm{v}_\theta^2}{R}(1 - \frac{x}{R})\right) = q\,v_\theta B_{0z}(1 - n\frac{x}{R})$$

$$m\gamma\left(\ddot{x}+\frac{v_\theta^2}{R}\left(1-\frac{x}{R}\right)\right)=q\,v_\theta B_{0z}\left(1-n\frac{x}{R}\right)$$

After simplification :

and $\omega_{rev} = \frac{v_\theta}{R} = \frac{qB_{0z}}{m} = \omega_0$

$$\ddot{x}+\omega_0^2\cdot(1-n)x=0 \Rightarrow$$

$$\ddot{x}+\omega_r^{\,2}x=0 \qquad \omega_r^{\,2}=\frac{v_\theta^2}{R^2}(1-n)$$

Harmonic oscillator with the frequency $\boxed{\omega_r=\sqrt{1-n}\;\omega_0}$

Horizontal stability condition (ω real) : **n < 1**

n <1 : Bz could decrease//or increase with the radius R

Horizontal stability is generally easy to obtain

Horizontal stability condition (ω real) :

n <1 : Bz could decrease//or increase with the radius R

n < 0 : isochronism Bz **should increase**

Harmonic oscillator with the frequency

$$\ddot{x} + \omega_r^2 x = 0 \qquad \boxed{\omega_r = \sqrt{1-n}\ \omega_0 = \nu_r . \omega_0}$$

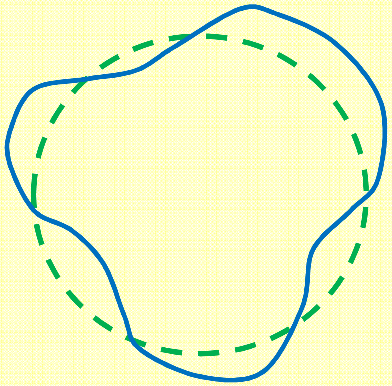
Horizontal stability + isochronism

n <1 + n <0

$$\omega_r^2 > 0$$

IF n <0

$$r(t) = R_0(t) + x_0 \cos(\nu_r \omega_0 t)$$



Vertical dynamics with B (r)

Vertical motion in the non uniform Bz(r)

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

$$m\gamma\frac{d^2z}{dt^2} = F_z = q(\mathrm{v}\times B)_z = -q(\dot{r}\,\cancel{B_\theta} - r\,\dot{\theta}\,B_r)$$

Because $\nabla\times\mathrm{B}=0$ $\quad \dfrac{\partial B_r}{\partial z} - \dfrac{\partial B_z}{\partial x} = 0 \qquad B_r = -n\dfrac{B_{oz}}{R}z$

Motion equation $\boxed{\ddot{z} + \omega_z{}^2 z = 0}$

Harmonic oscillator with the frequency $\boxed{\omega_z = \sqrt{n}.r\,\dot{\theta} = \sqrt{n}\,\omega_0}$

Vertical stability condition : n >0 (ω_z real)

$$\boxed{\omega_z{}^2 = n.\omega_0 > 0}$$

n >0 : Bz could decrease with the radius R

Watch the vertical oscillations !!

Isochronism condition :

n < 0 : Bz(r) increase with r

Vertical stability condition :

n > 0 : Bz(r) should decrease

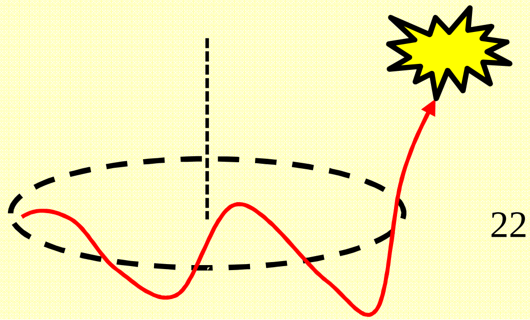
$$\omega_z = \sqrt{n}\ \omega_0$$

Isochronism condition will induce Unstable oscillations

$$z(t) \sim z_0 \exp(-i\,\omega_z t) = z_0 \exp(+|\omega_z t|)$$

Unstable oscillations in Z

= exponential growth =beam losses



Vertical dynamics
let's to refocus in axial plan (z)

Isochronism condition
(longitudinal)

B=Bz(R)

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

Bz should increase with R (n<0)

Unstable Vertical oscillations

Unstable oscillation $z(t) = z_0 \exp(+\sqrt{|n|}\ \omega_0 t\ )$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\,\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

Additive Vertical focusing is needed

Bθ component needed (Fz =-q vr Bθ) : « AVF » Cyclo

Azimuthally Varying Field (AVF)
Vertical weak focusing : $B_z = f(R,\theta)$

Isochronism   n<0   : B_z increase

Vertical stability : $B_z(r)$ Defocus + B_θ Focus
Bz should oscillate with θ to compensate the instability

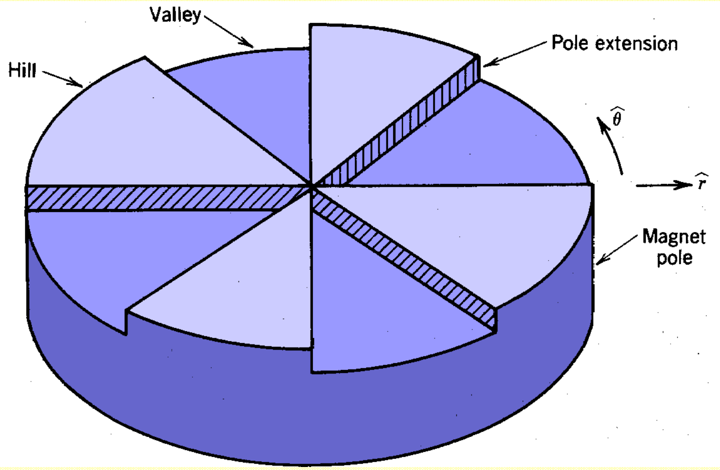
- Vertical force F_z , with radial component **B**r (possible)

•$F_z \sim q\, v_r.\, B_\theta$: Vertical focusing

$$\frac{\partial B_\theta}{\partial z} - \frac{\partial B_z}{\partial \theta} = 0$$

$B_z = f(R,\theta)$

$B_\theta = g(R,\theta)$



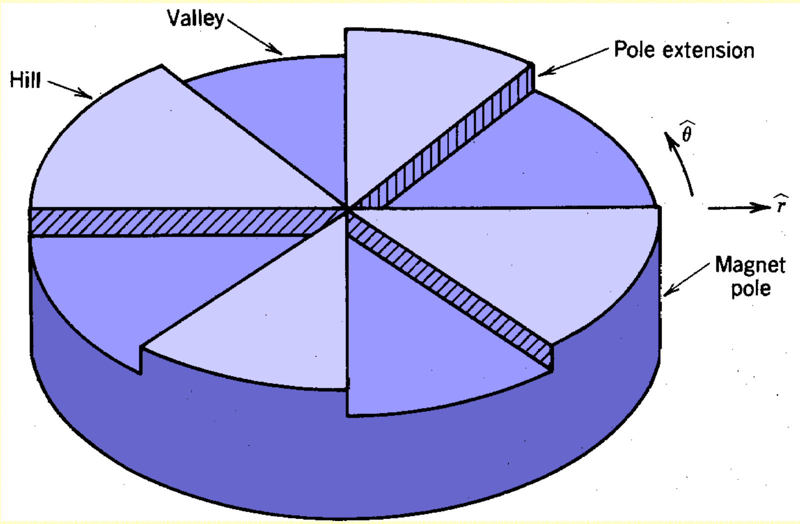
Like edge focusing in dipole magnet :
Bz variation can produce vertical forces

Azimuthally Varying Field : AVF Cyclo

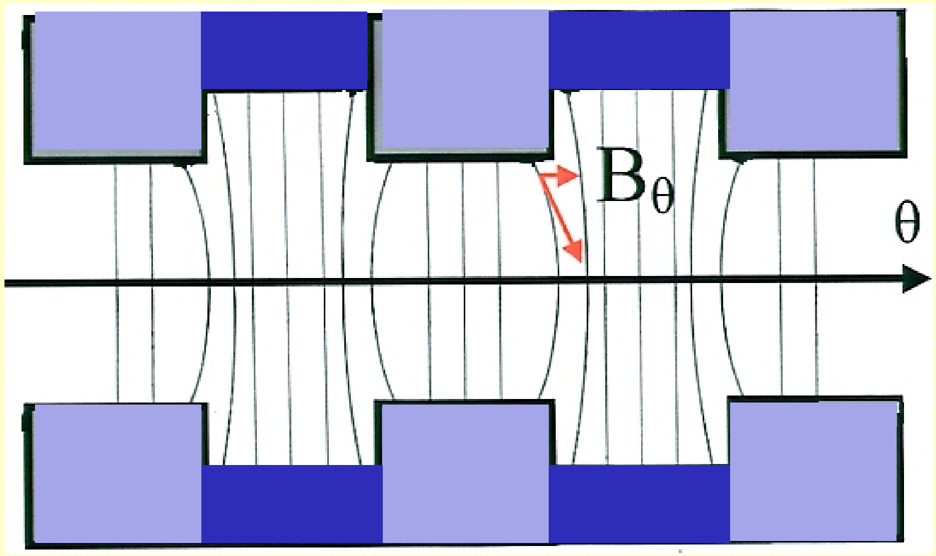
Azimuthally varying Field (AVF)

B_θ created by:

- Succession of high field and low field regions
- B_θ appears around the median plane
  - valley : large gap, weak field
  - Hill : small gap, strong field



N=4 sectors



Hill valley Hill valley

Azimuthally varying Field (AVF) cyclo

V_r created by :

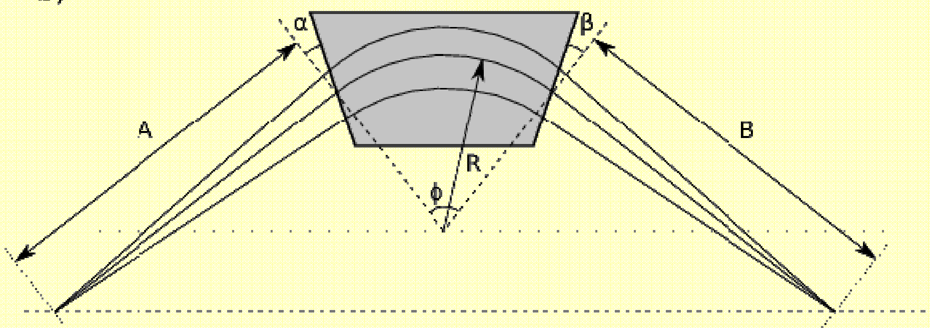
- Valley: weak field, large trajectory curvature
- **Hill : strong field**, small trajectory curvature

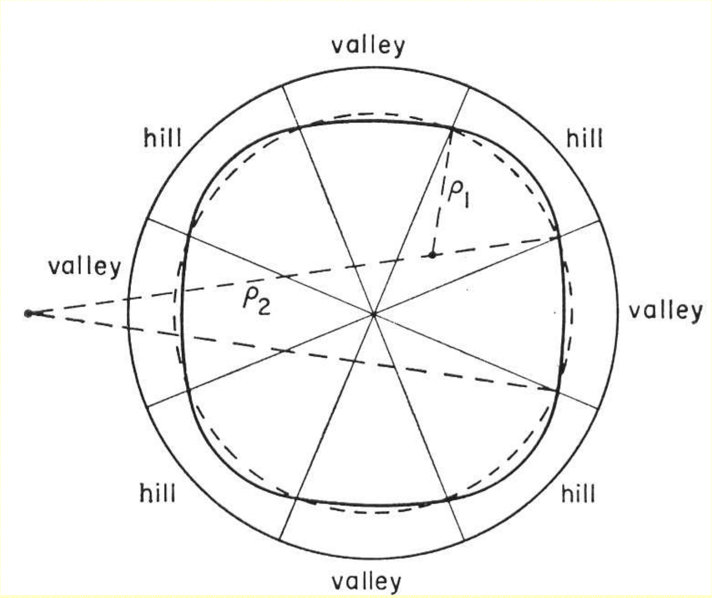
→ Trajectory is not a circle

- Orbit not perpendicular to hill-valley edge

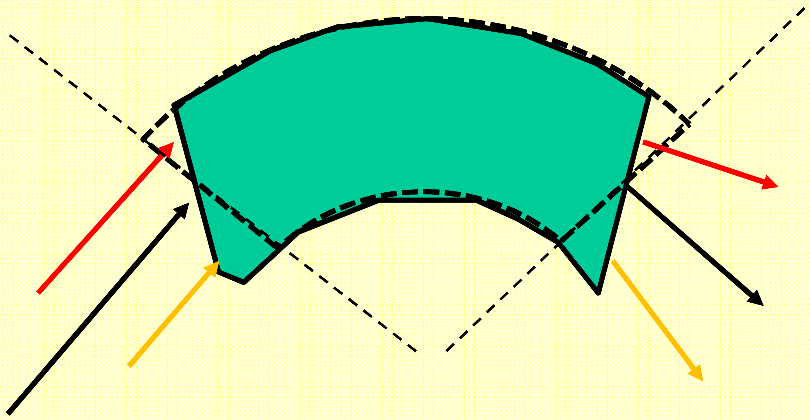
→ Vertical focusing $F_z \propto v_r \,.\, B_\theta$

Like edge focusing in Dipole Magnet





Edge focusing in dipole magnet recap

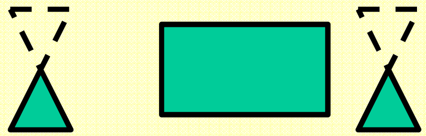


Non perpendicular edge in dipole magnet can provide

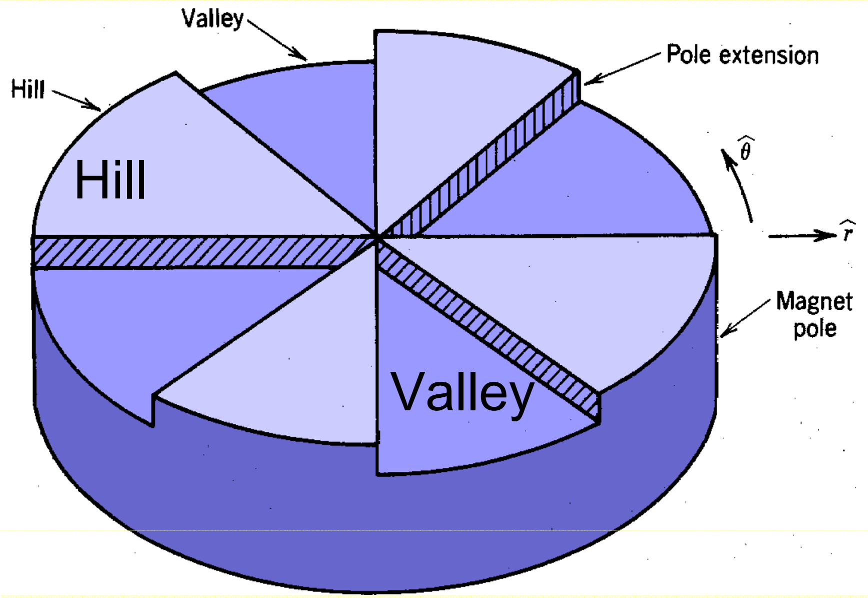
1) additive focusing in vertical + 2) defocusing in horizontal plane

The optical Transfer Matrix is

$$M_{dipole} = M_{edge1} \cdot M_{body} \cdot M_{edge2}$$



Vertical focusing with sectors



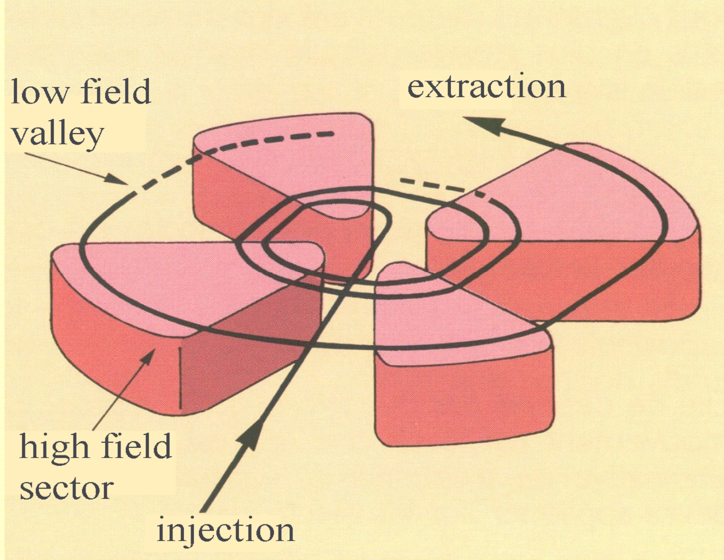
Compact cyclo : pole oscillation in θ

Bz= <B0> [1+ f. cos (N θ)]

f <1 f= 0.5. (Bhill- Bvalley)/<B0>

FLUTTER function (definition)

$$\boxed{F_l = \frac{\left(B_{hill} - B_{valley}\right)^2}{8\left\langle B\right\rangle^2}}$$



Separated sector cyclotron

Separated magnet
generate field oscillation in θ

Bz= <B0> [1+ cos (N θ)]

Separated sector cyclotronq

The FLUTTER is larger

Larger vertical focusing

Separated sectors(ring) cyclotron

Focusing condition limit: (n<0)

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l + \ldots > 0$$

Increase the flutter F_l, using separated sectors where $B_{valley} = 0$

$$F_l = \frac{\left(B_{hill} - B_{val}\right)^2}{8 \left\langle B \right\rangle^2}$$

PSI= 590 MeV proton

γ=1.63

Separated sectors cyclotron needed at "High energies" (n=1-γ^2 <<0)

Vertical focusing and isochronism

2 conditions to fulfill

- **Increase the vertical focusing force strength:** $\nu_z^2 = n + \frac{N^2}{N^2-1}F_l + \ldots > 0$
- **Keep the isochronism condition true: n<0**

$$n = -\frac{R}{B_{0z}}\frac{\partial B_z}{\partial R} = 1-\gamma^2 < 0$$

So we should have: $\frac{N^2}{N^2-1}F_l > \gamma^2 - 1$

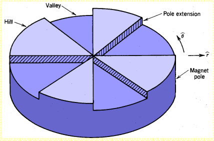
For High Energy cyclotron : 3 solutions for vertical stability

1) Increase $N_{sectors}$ N=3, 4, 6 $\frac{N_{sector}^2}{N_{sector}^2 - 1}F_l \nearrow$

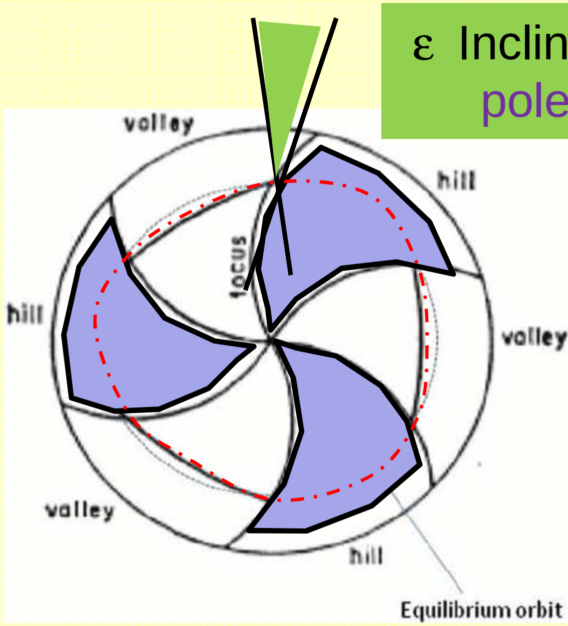
2) Larger Flutter (separated sectors) Fl

3) Other idea ??? Yes (spiralled sectors)

Better vertical focusing : Spiralled sectors



AVF with straight sectors



ε Inclination angle
pole edge AND Trajectory

AVF cyclo with spiralled sectors

Larger vertical focusing

Bz= <B0>(1+ f(r).g (r, θ))

Spiral eq. r = A.(θ+ 2π j /2N)

tan ε =2r/A

Additive vertical focusing : + FLUTTER .(1+ 2 $\tan^2 \varepsilon$)

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \, (1 + 2 \tan^2 \varepsilon)$$

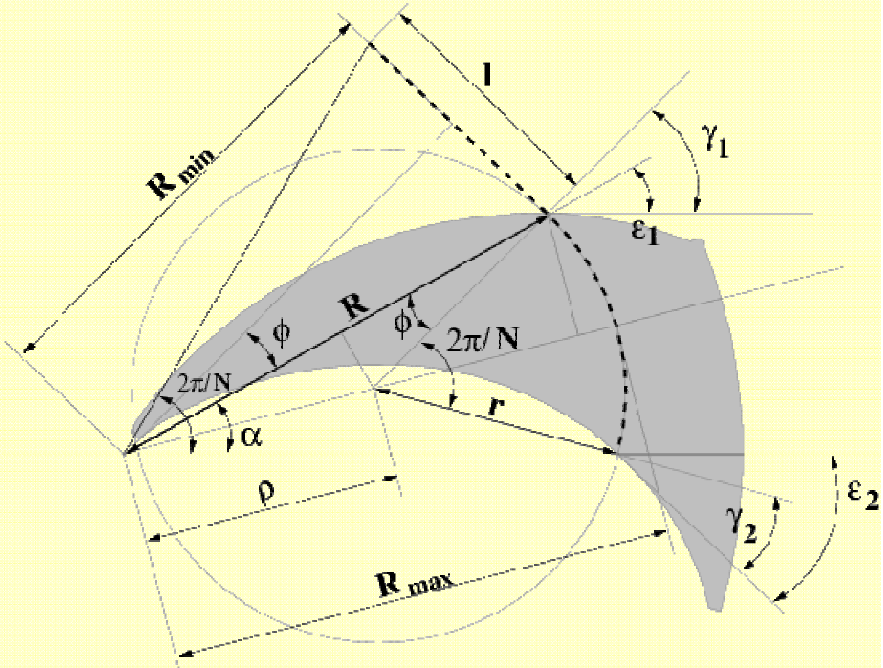
Spiralled sectors

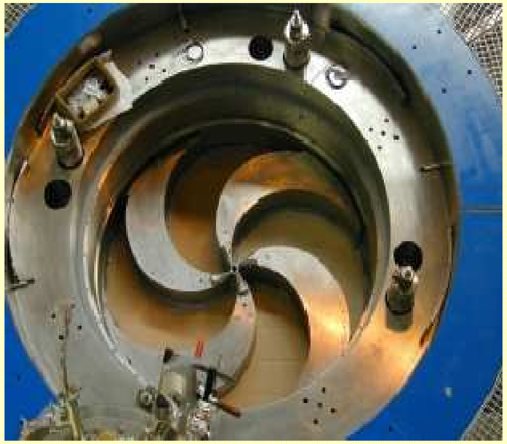
By tilting the edges (ε angle) :

- The valley-hill transition became more focusing
- The hill-valley transition became less focusing

But by the strong focusing principle (larger betatron amplitude in focusing, smaller in defocusing), the net effect is focusing (cf F+D quadrupole).

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \left(1 + 2\tan^2 \varepsilon\right)$$





Beam dynamics in the ISOCHRONOUS cyclotrons

B=Constant ≠Isochronism condition

A STRONG LIMITIATION in energy γ=1

to get the ions synchrone With RF

$$\boxed{\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}}$$

$B_z = B_0 . g(R)$ B_z increase with R (field index n < 0)

Unstable Vertical oscillations

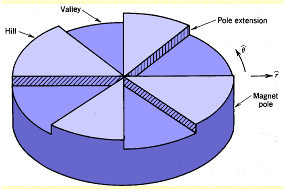
strong limitation in transmission

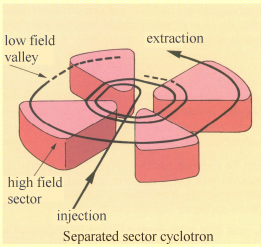
Additive Vertical focusing is needed : N sectors (Hills//valleys)

separated Sectors

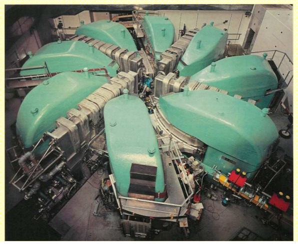
spiralled sector

separated spiralled sectors





Separated sector cyclotron



$B_z = B_0 . g(R,\theta)$

One Other possibilities
SYNCHRO CYCLOTRON (NOT ISOCHRONOUS)

Acceleration condition with Bz decreasing (n>0)

ω_{rev}=not constant

Not isochronous !!

But no vertical instabilities!!

Revolution frequency evolves $F_{rev}(t) = F_{rev}(\text{Radius})$

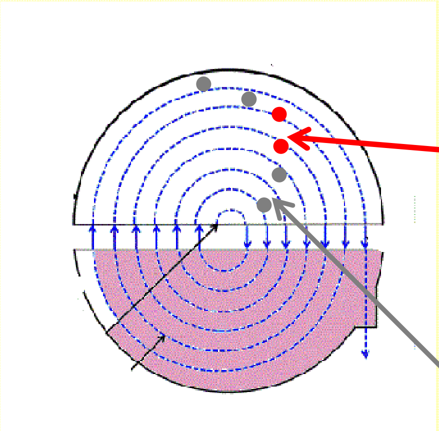
beam has to be synchrone With RF :

$$\boxed{\omega_{rev}(R)/h = \omega_{RF}(R)}$$

Revolution frequency is evolving $F_{RF}(R)$

Pulsed Machine $\omega_{RF}(t)$: SYNCHRO CYCLOTRON

Synchro-cyclotrons

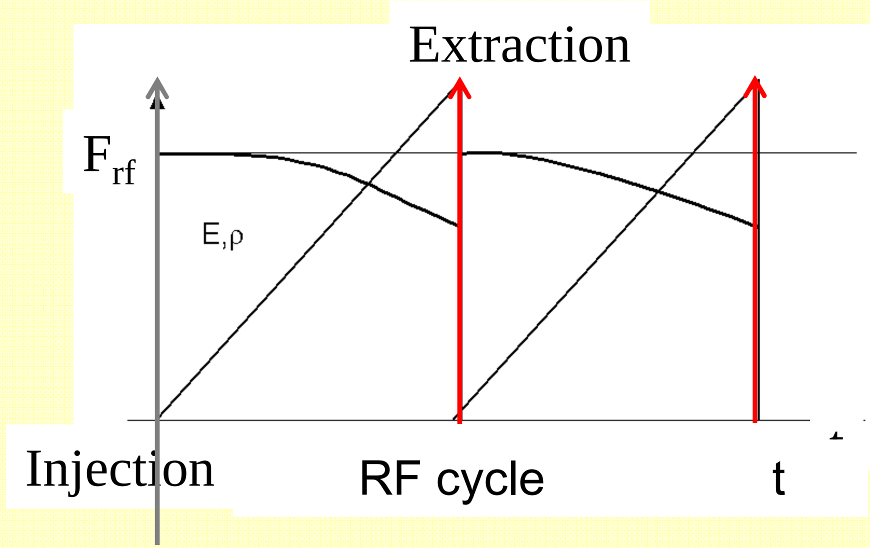


Pulsed beam

Accelerated bunches

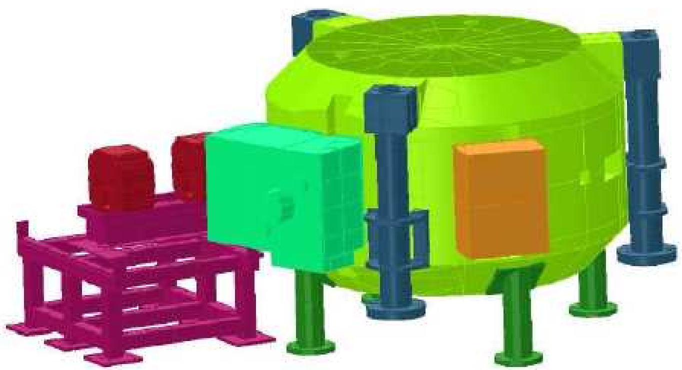
$F_{RF} = h\ F_{rev}$

Not accelerated



Less intensity (pulsed) avalaible (not cw)

Exemple : medical aplication
Superconducting synchrocyclo.



ProteusOne® (IBA) : 250 MeV proton

Bz = 5.7 - 5.0 Tesla (very compact)

Rextraction =0.6 m / harmonics=1

F_{RF}= 93 MHz -63 MHz (Rextraction)

Beam pulse : Every 1 ms

CYCLOTRONS
The Family

$$\omega_{rev} = \frac{qB_z}{\gamma . m}$$

1

Conventional cyclotrons

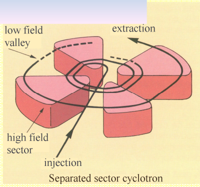
Bz = uniform
F*rev* = evolve with γ !!!
F*RF* = constant

NOT USED anymore
Limited in energy :EK<1MeV

2: isochronous

Compact cyclotrons
1 magnet
with modulation

Separated sectors



Bz = NOT uniform =f(R)
F*revolution* = Constant
F*RF* = constant

Vertical focusing with
Bz = f(R,θ)

Isochronous

$$\omega_{rev} / h = \omega_{RF}$$

3 : non isochronous

Synchrocyclotrons

F*rev* = NOT Constant
F*RF* = NOT Constant = beam pulsed

Not Isochronous

Less intensity (pulsed) ≠ cw

$$\omega_{rev}(R) / h = \omega_{RF}(t)$$

End Chapter 1