Lecture 2: Magnetization, cables and ac losses

Magnetization

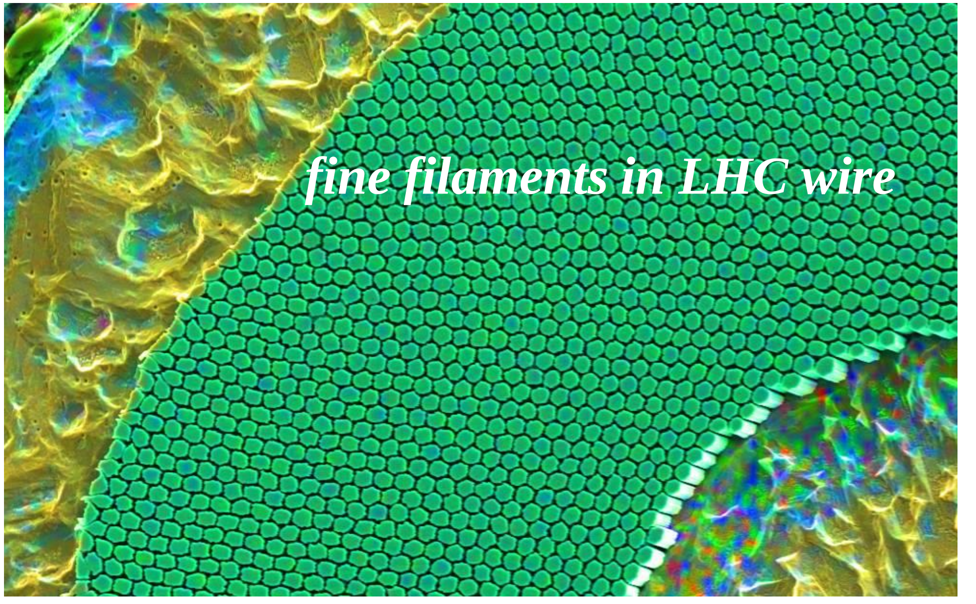
- superconductors in changing fields
- magnetization of wires & filaments
- coupling between filaments
- flux jumping

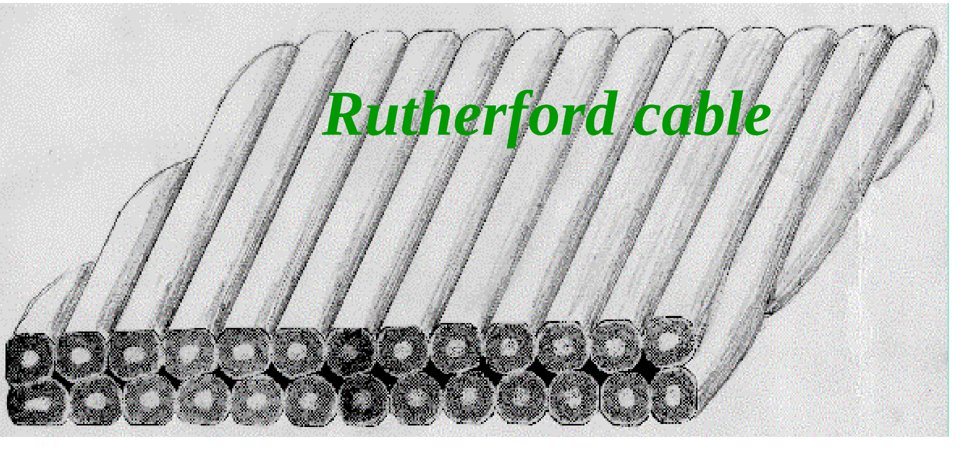
Cables

- why cables?
- coupling in cables
- effect on field error in magnets

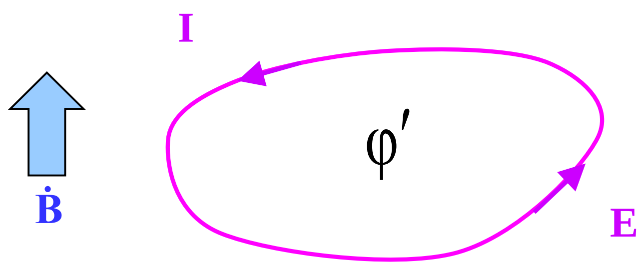
AC losses

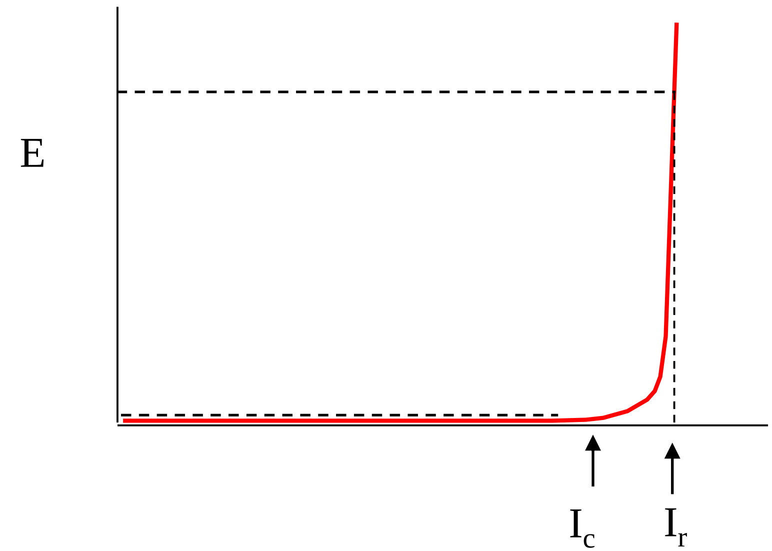
- general expression
- losses within filaments
- losses from coupling





Superconductors in changing magnetic fields





Faraday's Law of EM Induction

$$\oint E dl = \int B' dA = \phi'$$

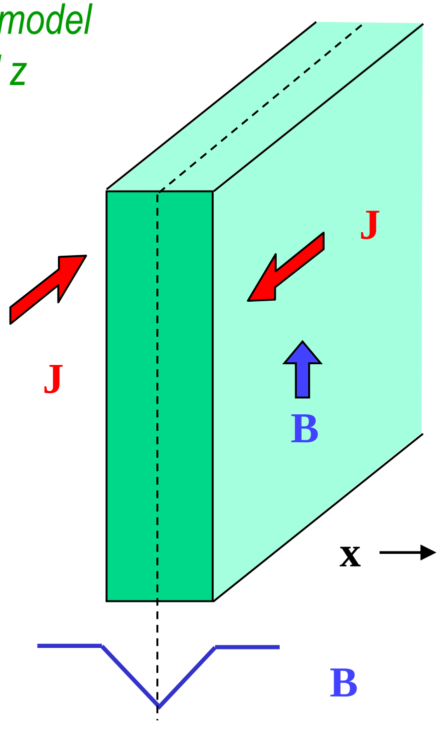
- changing field
  - → changing flux linked by loop
  - → electric field E in superconductor
  - → current I_r flows around the loop
- change stops
  - → electric field goes to zero
  - → superconductor current falls back to I_c (not zero)
  - → current circulates for ever ***persistent current***

changing magnetic fields on superconductors
→ electric field
→ resistance
→ power dissipation

Persistent screening currents

- ***screening currents*** are in addition to the ***transport current***, which comes from the power supply
- like eddy currents but, because no resistance, they don't decay

simplified slab model infinite in y and z



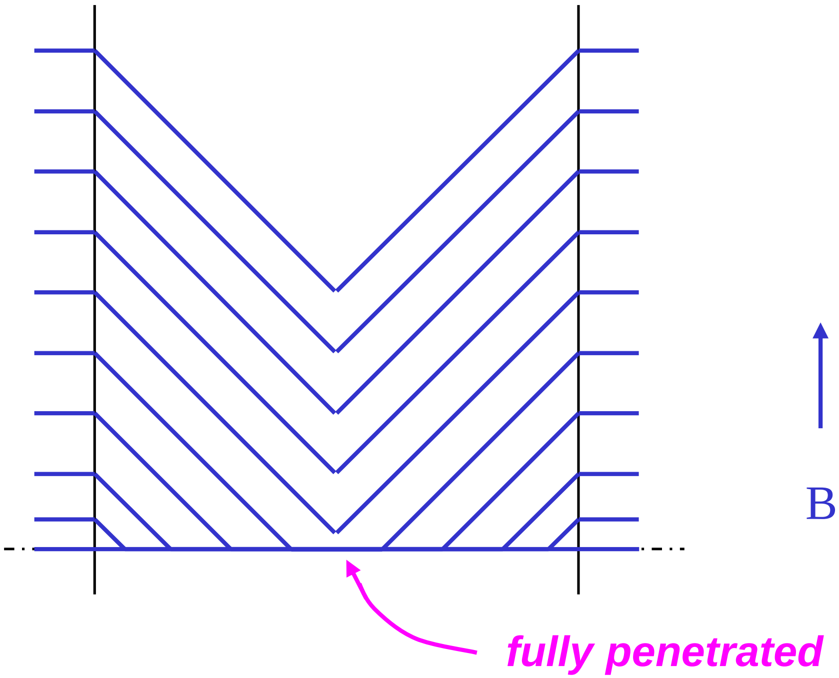
- ***dB/dt*** induces an electric field ***E*** which drives the screening current up to current density J_r
- ***dB/dt*** stops and current falls back to J_c
- so in the steady state we have persistent $J = +J_c$ or $J = -J_c$ or $J = 0$ nothing else
- known as the ***critical state model*** or ***Bean London model***
- in the 1 dim infinite slab geometry, Maxwell's equation says

$$\frac{\partial B_y}{\partial x} = -\mu_o J_z = \mu_o J_c$$

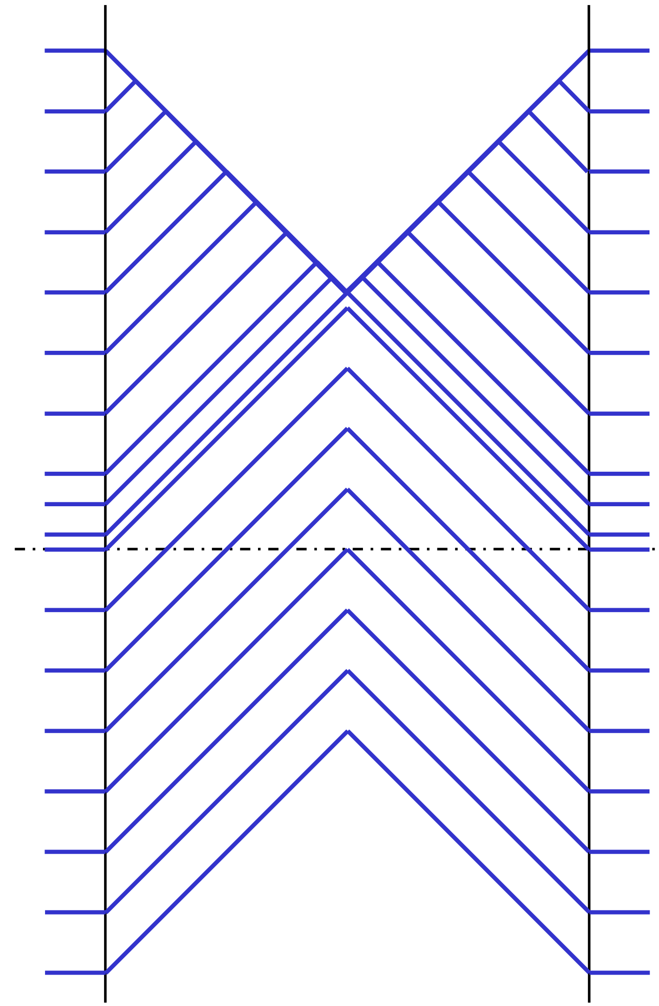
- so a uniform J_c means a constant field gradient inside the superconductor

The flux penetration process

plot field profile across the slab



field increasing from zero



field decreasing through zero

Bean London critical state model

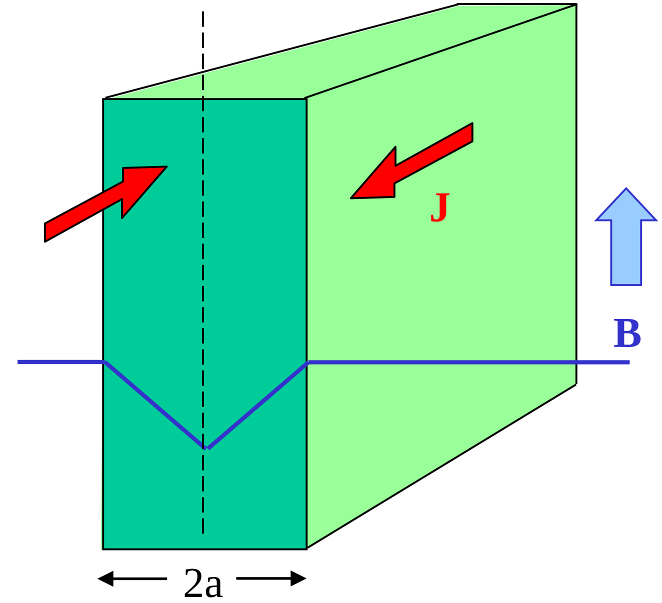
- **current density everywhere is $\pm J_c$ or zero**
- **change comes in from the outer surface**

Magnetization

- screening current are a problem because they
  - are irreversible ⇒ losses in changing field
  - affect the field shape
  - can go unstable (flux jump)

- when viewed from outside the sample, the persistent currents produce a magnetic moment.

- by analogy with a magnetic material can define a magnetization = magnetic moment per unit volume



$$M = \sum_V \frac{I.A}{V}$$

NB units of H

for a fully penetrated slab

(symmetry about centre line)

$$M_s = \frac{1}{a}\int_0^a J_c.x.dx = \frac{J_c.a}{2}$$

to reduce *M* need small 'a'

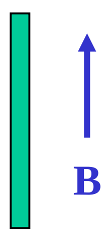
thin layers

fine filaments

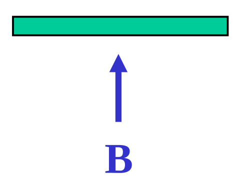
parallel to field

- slab approximation

good for a single tape parallel to field



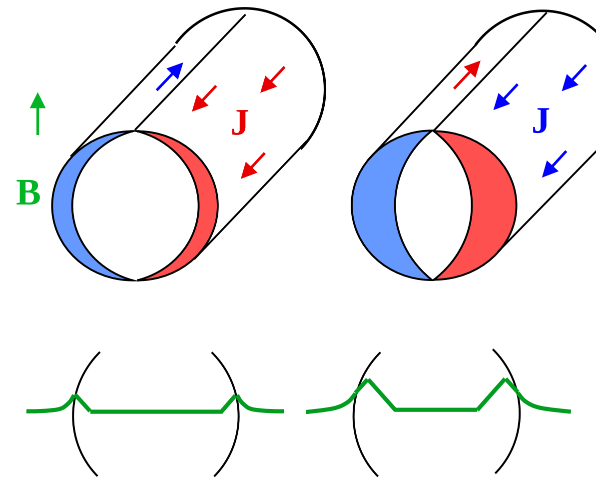
not good for single tape perpendicular to field

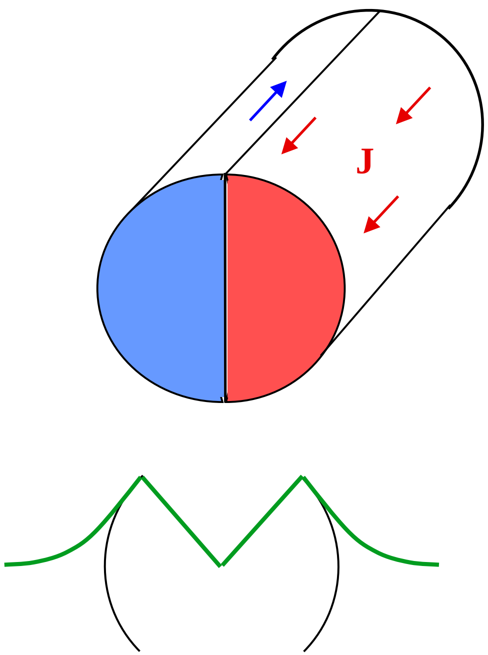


but OK for stack of tapes perpendicular to field

Magnetization of cylindrical filaments

when field has not fully penetrated the inner current boundary is roughly elliptical





when fully penetrated, the calculation is similar to slab but more complicated - magnetization is

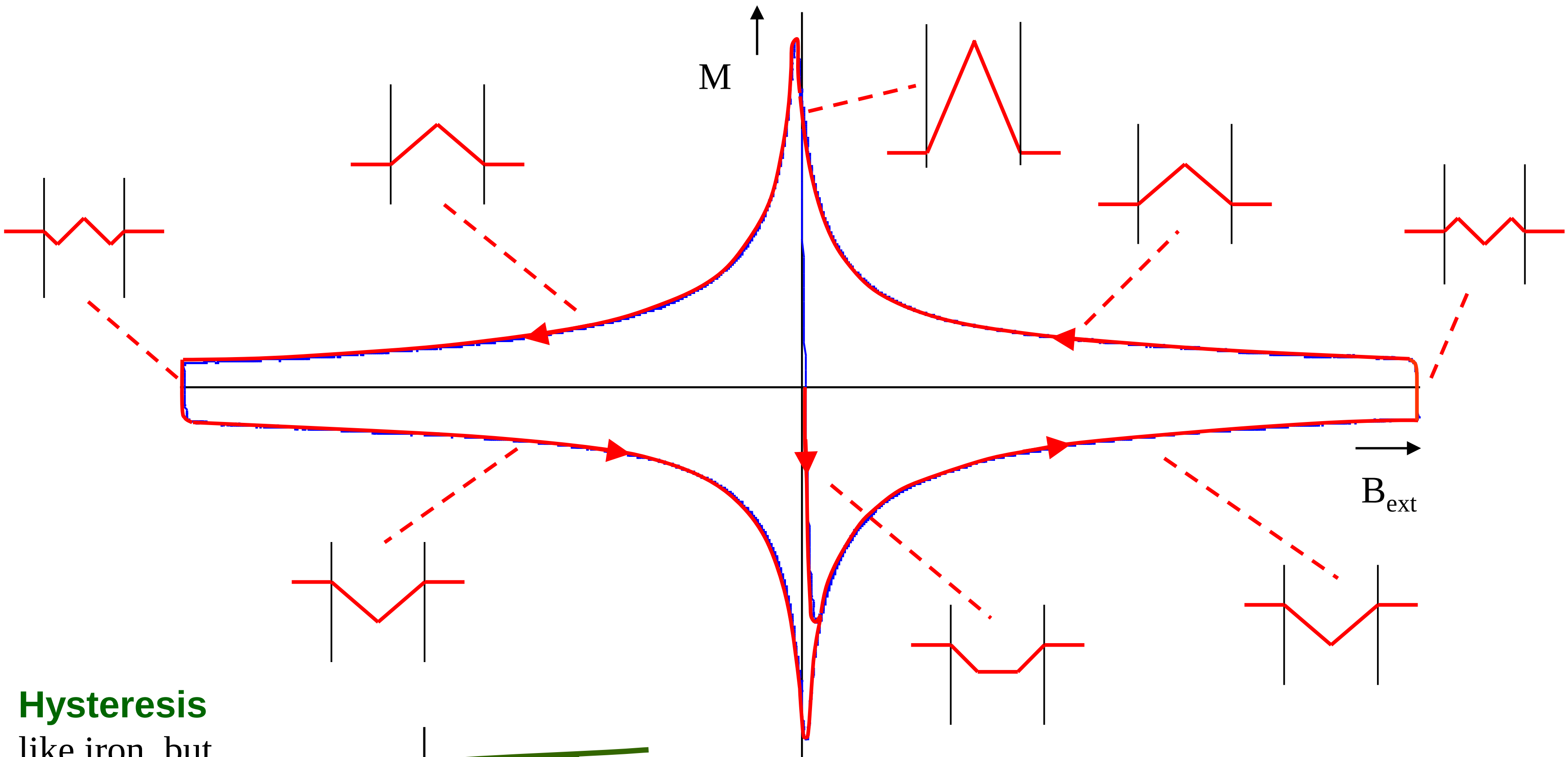
$$M_s = \frac{4}{3\pi} J_c a = \frac{2}{3\pi} J_c d_f$$

where a, d_f = filament radius, diameter

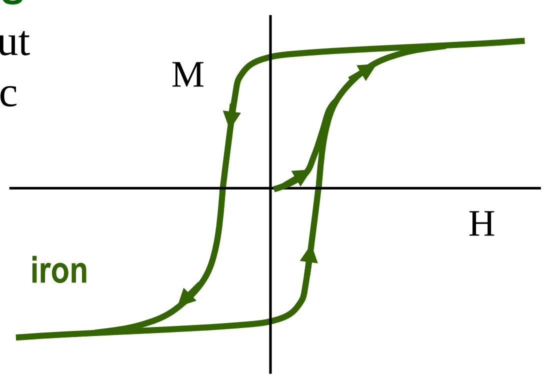
Note: M is defined per unit volume of filament

to reduce M need small d_f - fine filaments

Magnetization of NbTi

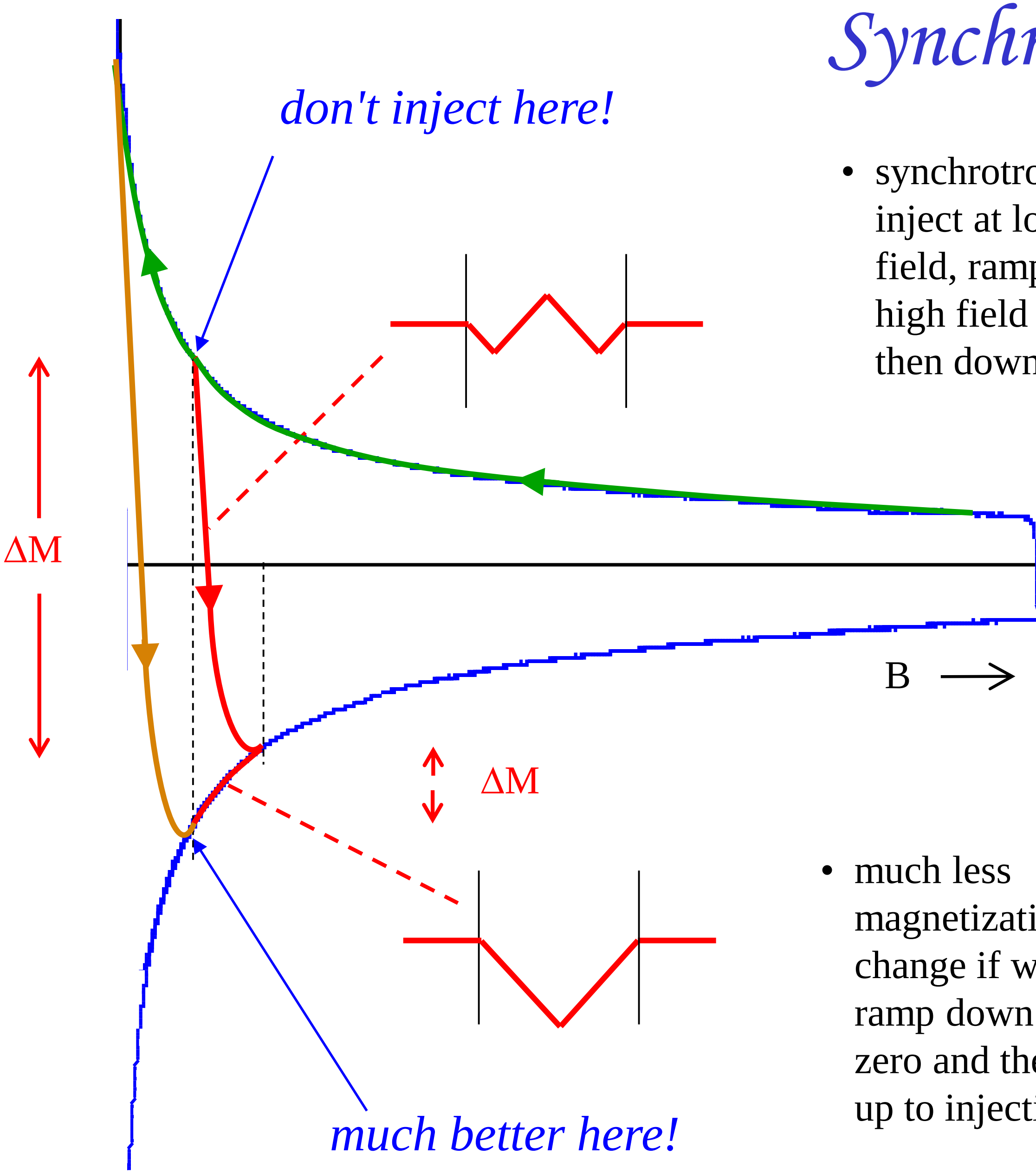


Hysteresis
like iron, but diamagnetic

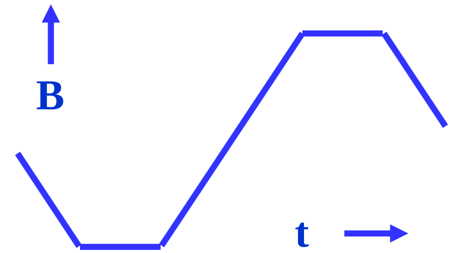


Magnetization is important because it produces field errors and ac losses

Synchrotron injection

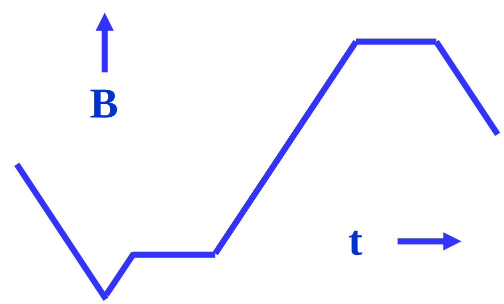


- synchrotrons inject at low field, ramp to high field and then down again



- note how quickly the magnetization changes when we start the ramp up

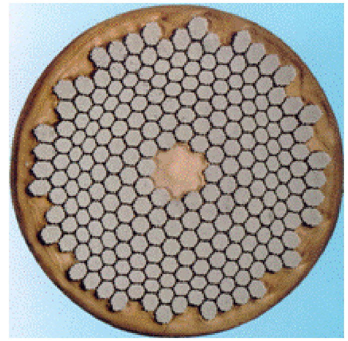
- much less magnetization change if we ramp down to zero and then up to injection

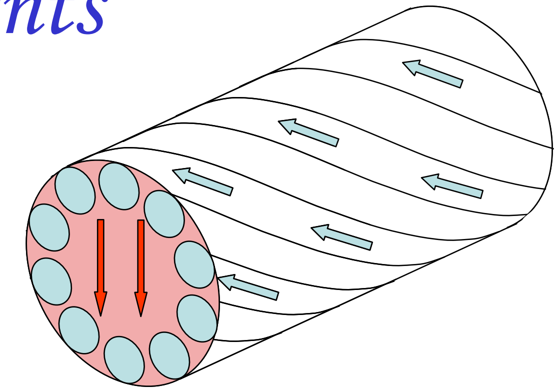


Coupling between filaments

recap $M_s = \frac{2}{3\pi} J_c \, d_f$

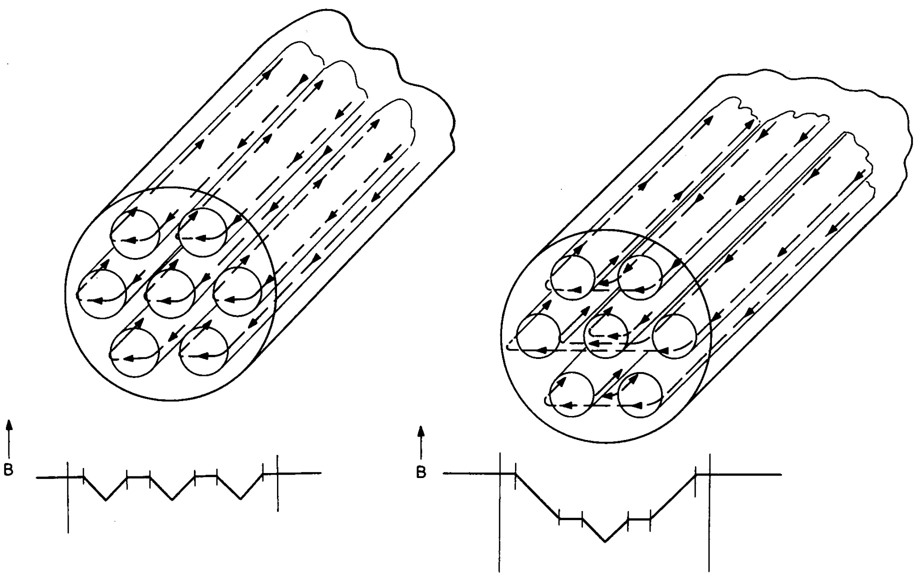
- reduce M by making fine filaments
- for ease of handling, filaments are embedded in a copper matrix





- coupling currents flow along the filaments and across the matrix
- reduce them by twisting the wire
- they behave like eddy currents and produce an additional magnetization

$$M_e = \frac{dB}{dt} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$



- but in changing fields, the filaments are magnetically coupled
- screening currents go up the left filaments and return down the right

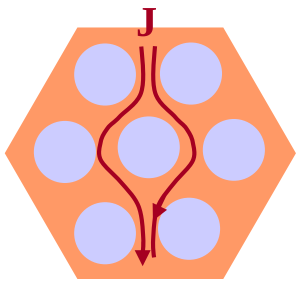
$$M_e = \frac{2}{\mu_o} \frac{dB}{dt} \tau \quad \text{where} \quad \tau = \frac{\mu_o}{2\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

per unit volume of **wire**

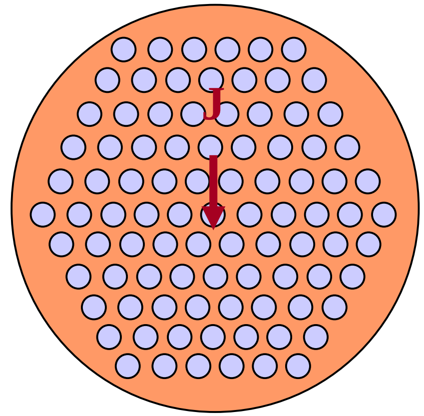
ρ_t = resistivity across matrix, p_w = wire twist pitch

Transverse resistivity across the matrix

Poor contact to filaments

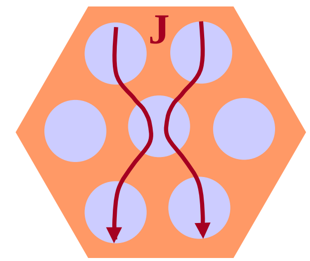


$$\rho_t = \rho_{Cu} \frac{1+\lambda_{sw}}{1-\lambda_{sw}}$$



where λ_{sw} is the fraction of superconductor in the wire cross section (after J Carr)

Good contact to filaments

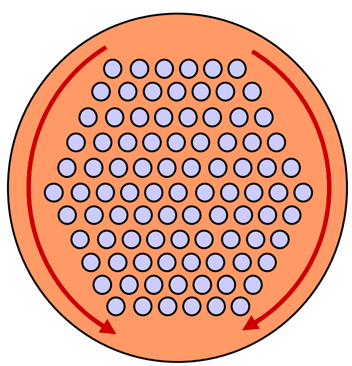


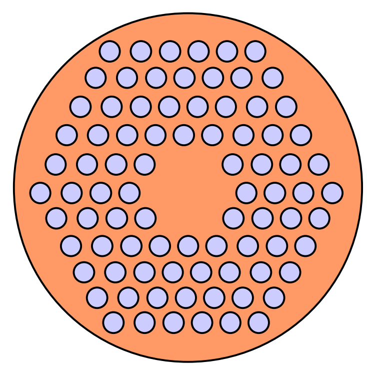
$$\rho_t = \rho_{Cu} \frac{1-\lambda_{sw}}{1+\lambda_{sw}}$$

Some complications

Thick copper jacket

include the copper jacket as a resistance in parallel

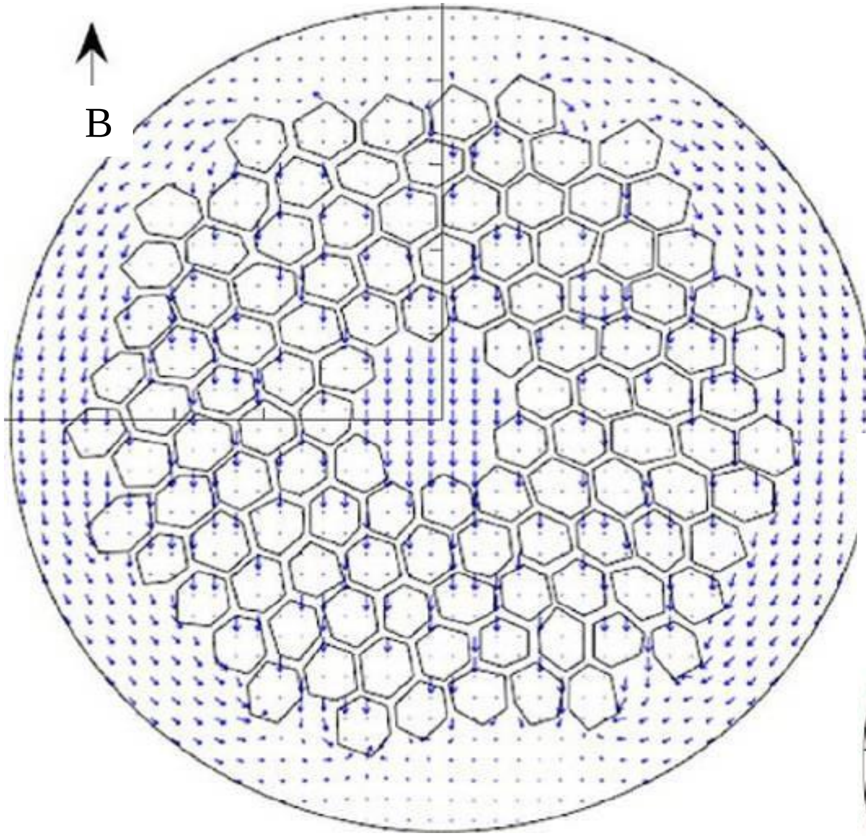




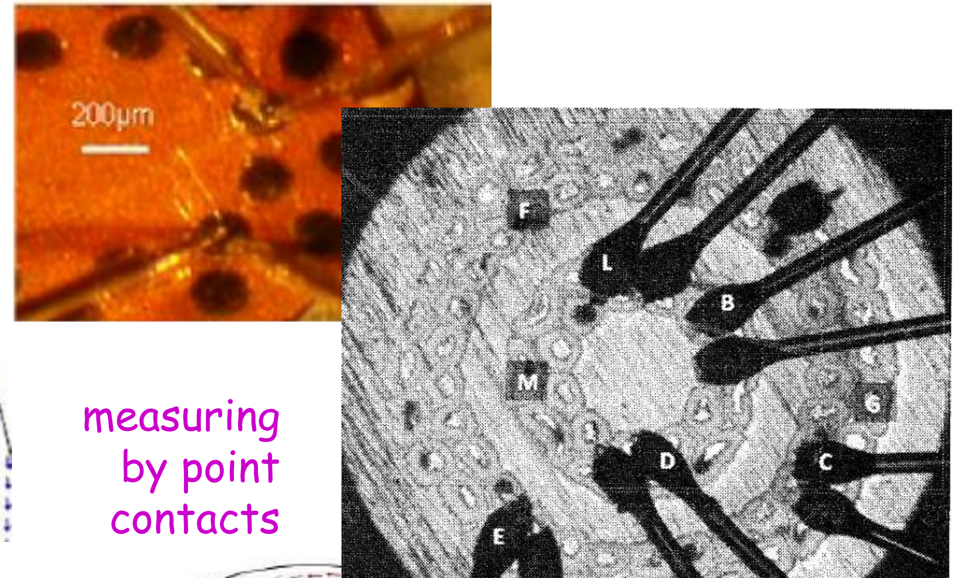
Copper core

resistance in series for part of current path

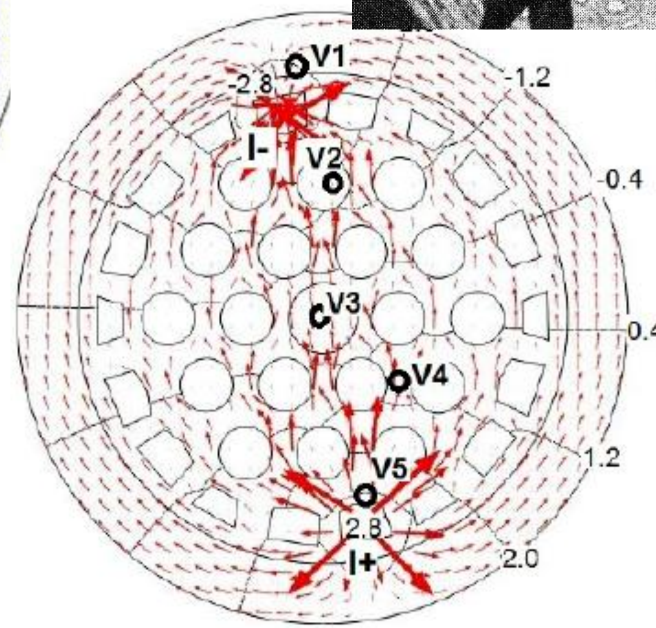
Computing & measuring current flow across matrix



calculated using the COMSOL code
by P.Fabbricatore et al JAP, 106, 083905 (2009)



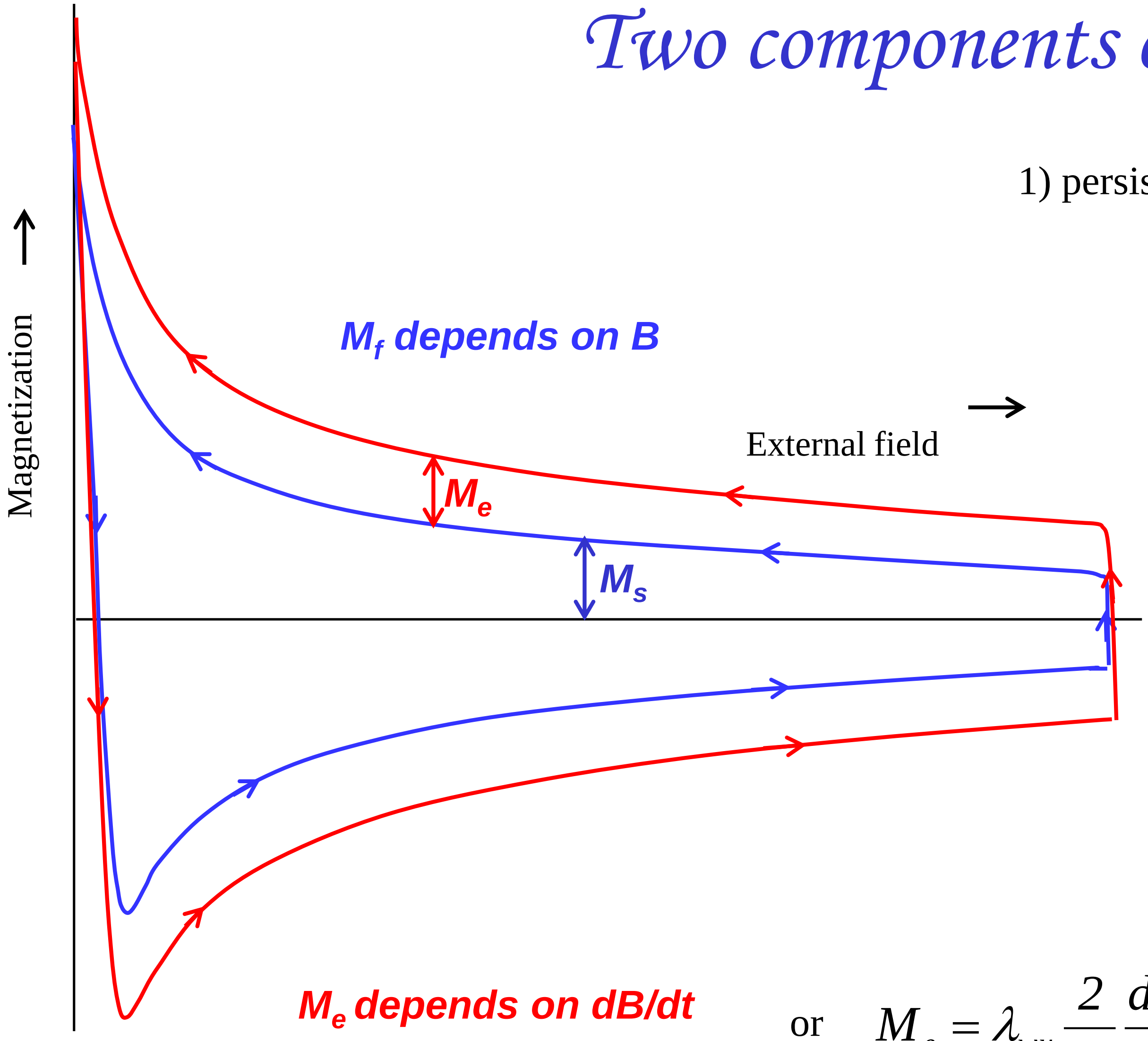
measuring
by point
contacts



simulation
by COMSOL

C Zhou et al IEEE
Trans App Sup **23**
3 p 6000204

Two components of magnetization



1) persistent current within the filaments

$$M_s = \lambda_{su} \frac{2}{3\pi} J_c(B)\, d_f$$

where λ_{su} = fraction of superconductor in the unit cell

2) eddy current coupling between the filaments

$$M_e = \lambda_{wu} \frac{dB}{dt} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

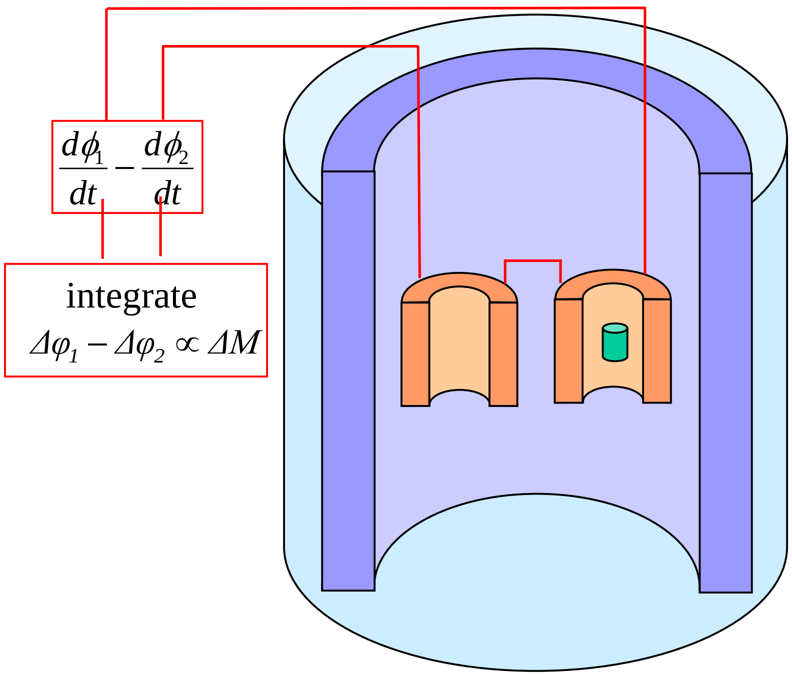
or $$M_e = \lambda_{wu} \frac{2}{\mu_o} \frac{dB}{dt} \tau \quad \text{where} \quad \tau = \frac{\mu_o}{2\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

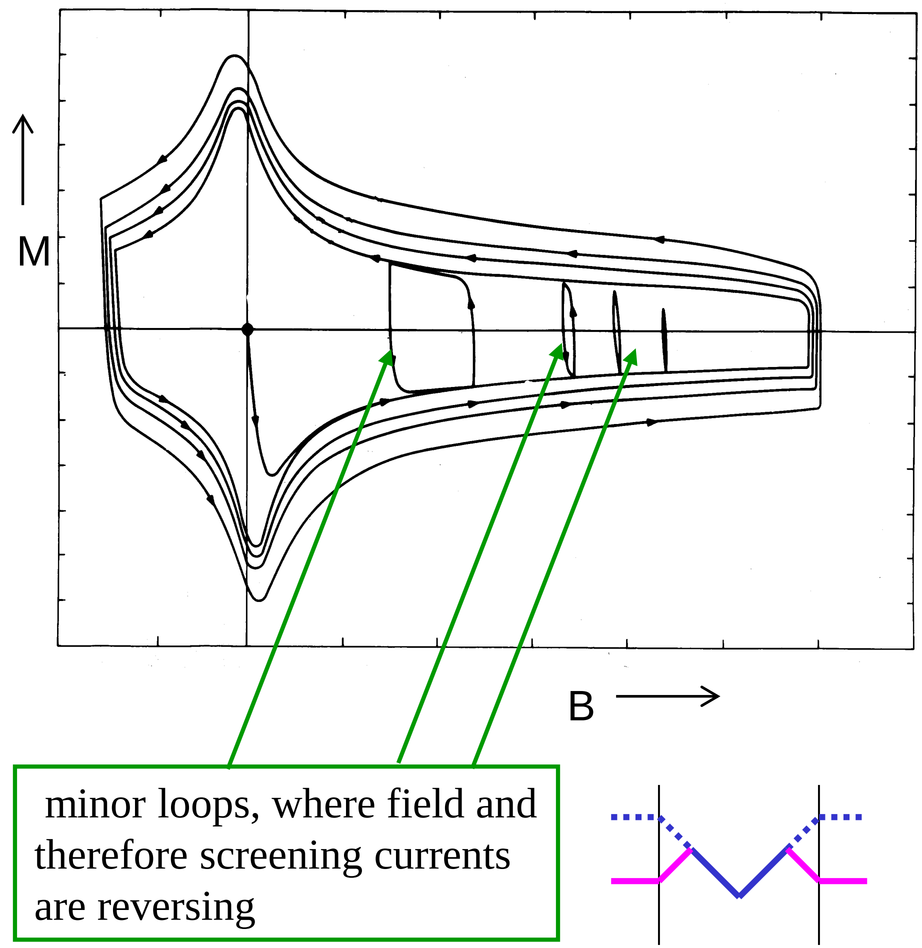
Magnetization is averaged over the ***unit cell***

where λ_{wu} = fraction of wire in the section

Measurement of magnetization

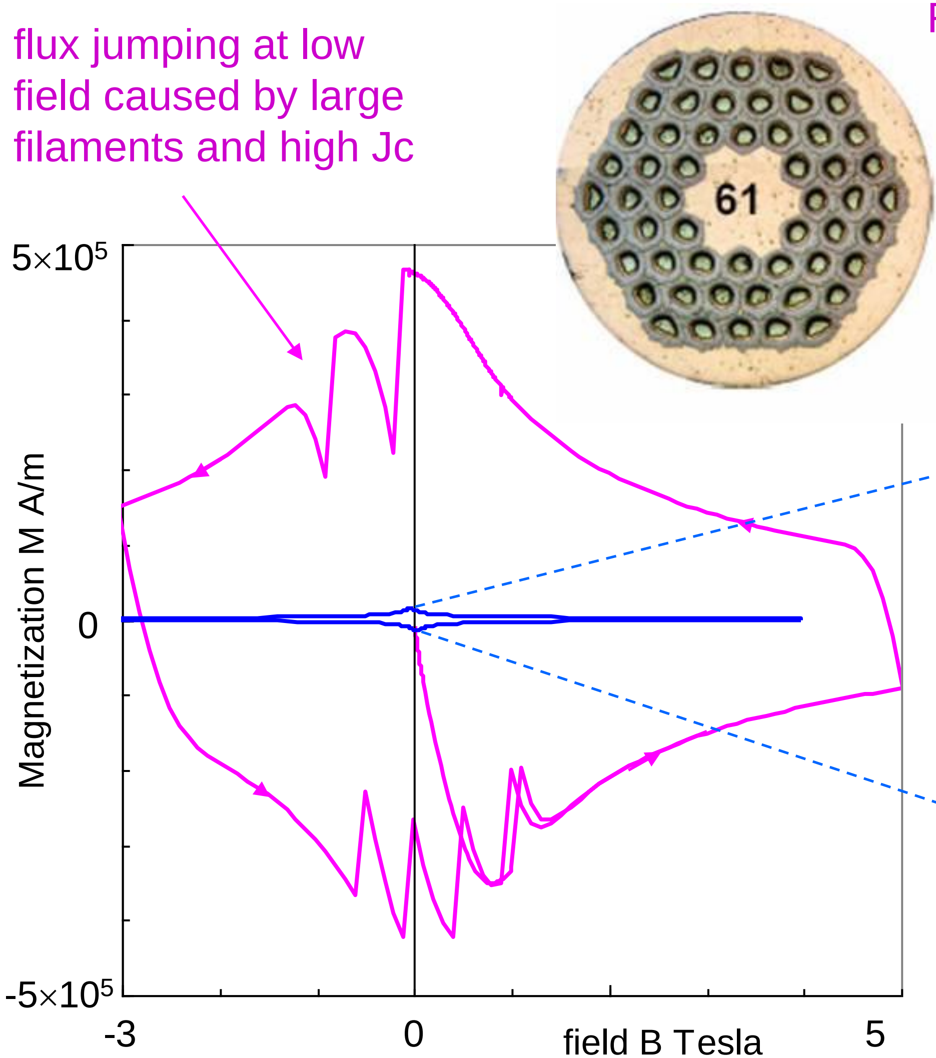
- in field, superconductor behaves like a magnetic material.
- plot the magnetization curve using a magnetometer.
- shows hysteresis - like iron but magnetization is both diamagnetic and paramagnetic.

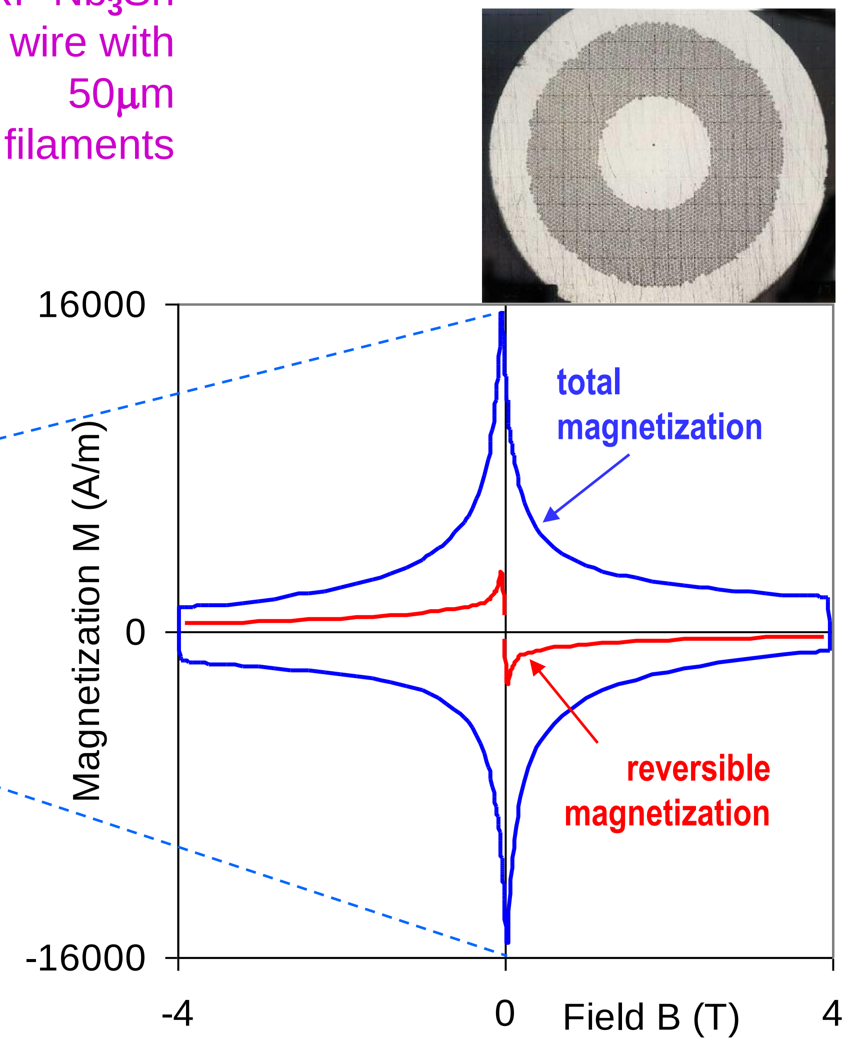




Two balanced search coils connected in series opposition, are placed within the bore of a superconducting solenoid. With a superconducting sample in one coil, the integrator measures ΔM when the solenoid field is swept up and down

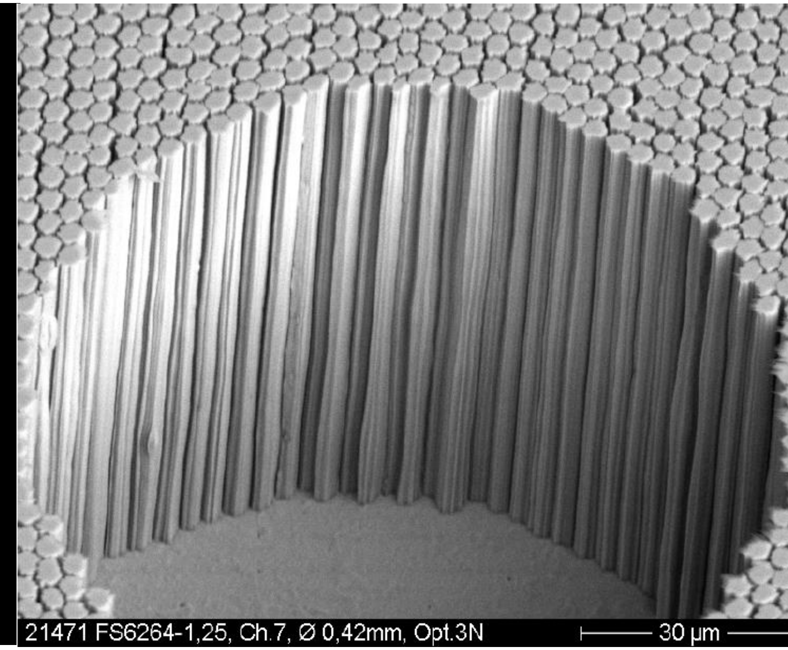
Magnetization measurements

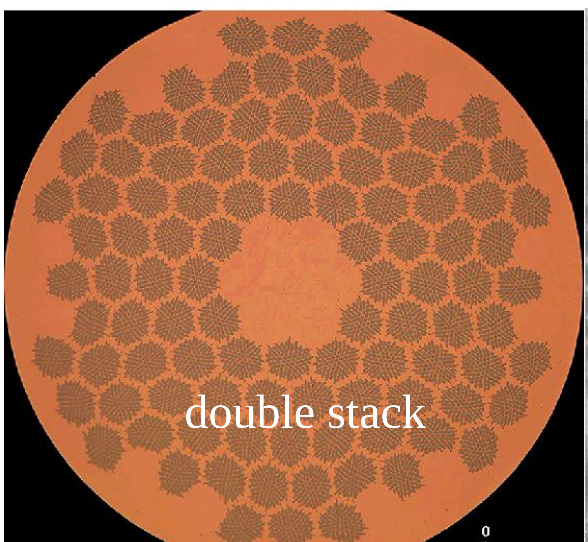


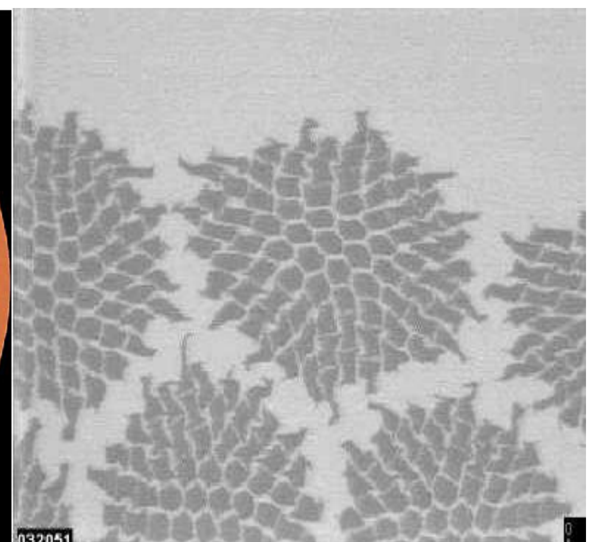


Fine filaments for low magnetization

Accelerator magnets need the finest filaments
- to minimize field errors and ac losses



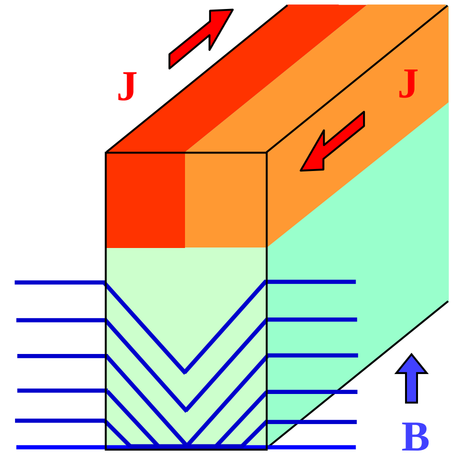




- typical diameters ~ 5 - 10µm.
- smaller diameters ⇨ lower magnetization, but at the cost of lower Jc and more difficult production.

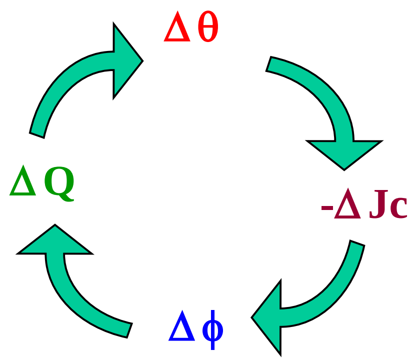
Flux Jumping

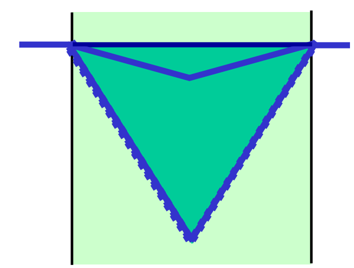
a magnetic thermal feedback instability



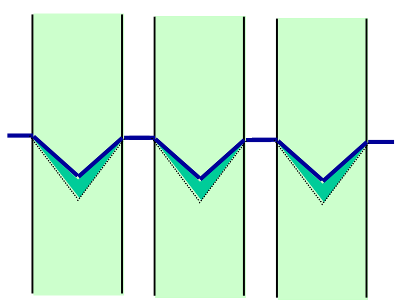
- **screening currents**

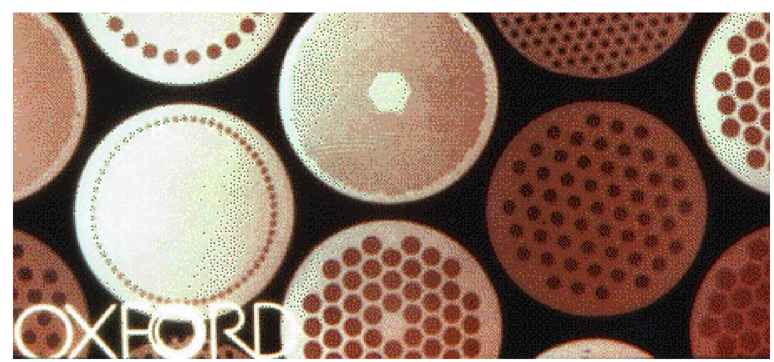
- **temperature rise**
- **reduced critical current density**
- **flux motion**
- **energy dissipation**
- **temperature rise**





- cure flux jumping by weakening a link in the feedback loop
- fine filaments reduce $\Delta\phi$ for a given $-\Delta Jc$
- for NbTi the stable diameter is ~ 50μm





Flux jumping: the numbers

stable against flux jumping when filament diameter d

$$d \le \frac{2}{J_c}\left\{\frac{3\gamma C(\theta_c - \theta_o)}{\mu_o}\right\}^{\frac{1}{2}}$$

NbTi at 4.2K and 1T

J_c *critical current density* = $7.5\times 10^{9}\ Am^{-2}$

γC *specific heat/volume* = $3400\ J.m^{-3}K^{-1}$

θ_c *critical temperature* = 9.0K

⇨ filament diameter d < **52**μm

- least stable at low B because of high J_c
- and at low θ because of high J_c and low C

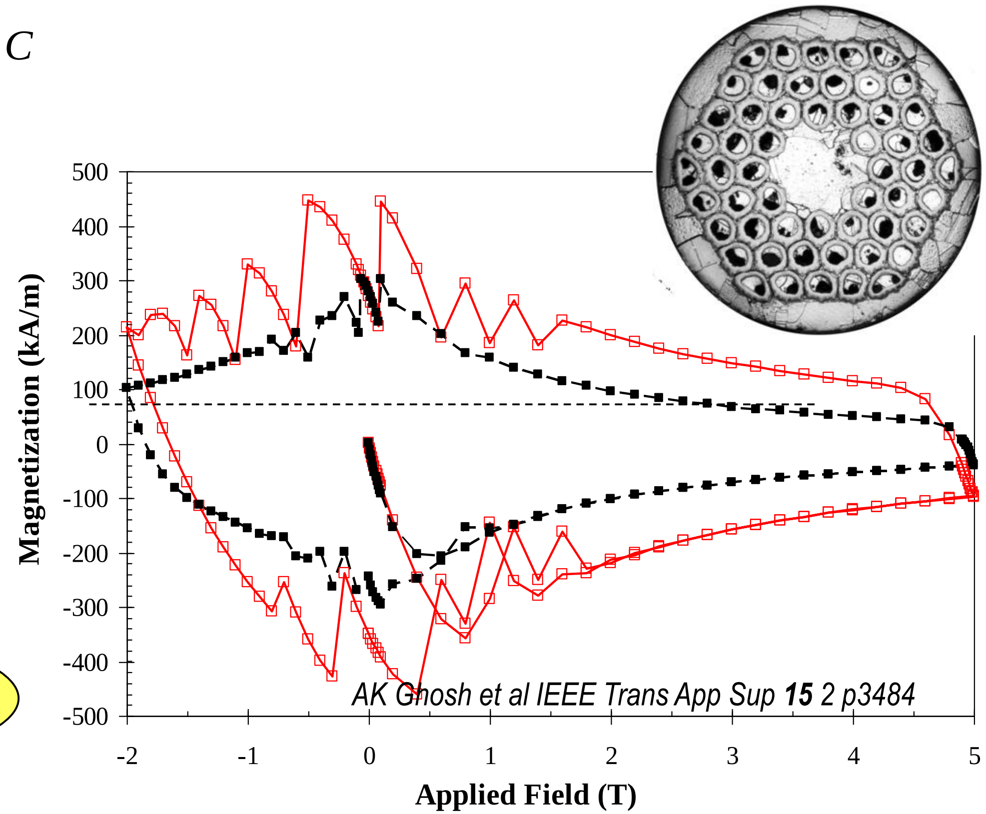
Nb_3Sn at 4.2K and 2T

J_c *critical current density* = $1.7\times 10^{10}\ Am^{-2}$

γC *specific heat/volume* = $1600\ J.m^{-3}K^{-1}$

θ_c *critical temperature* = 15.7K

⇨ filament diameter d < **25**μm



Flux jumping is a solved problem✓

Cables - why do we need them?

- for accurate tracking we connect synchrotron magnets in series
- for rise time *t* and operating current *I* , *c*harging voltage is

$$V = \frac{L\,I}{t} = \frac{2\,E}{I\,t}$$

- for a given field and volume, stored energy of a magnet is fixed, regardless of current or inductance

$$E = \frac{1}{2} L I^2 = \frac{B^2}{2\mu_o} vol$$

RHIC E = 40kJ/m, t = 75s, 30 strand cable

cable I = **5kA**, charge voltage per km = **213V**

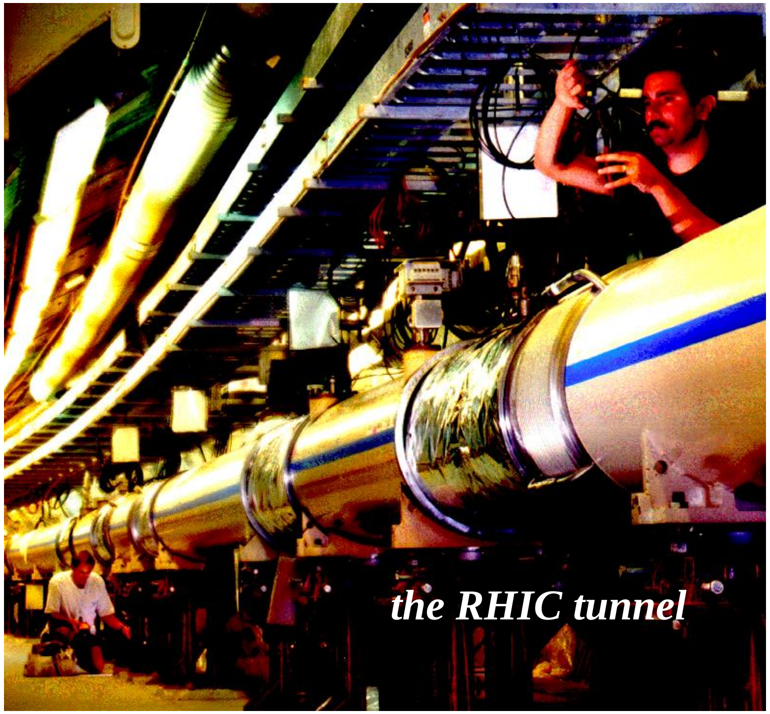
wire I = **167A**, charge voltage per km = **6400V**

FAIR at GSI E = 74kJ/m, t = 4s, 30 strand cable

cable I = **6.8kA**, charge voltage per km = **5.4kV**

wire I = **227A**, charge voltage per km = **163kV**

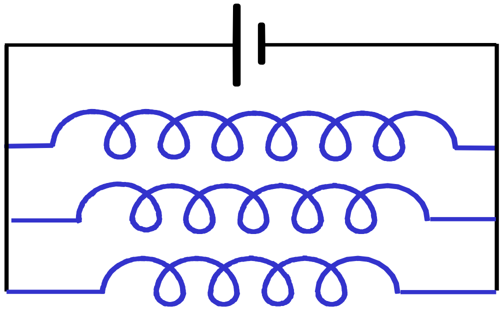
- so we need high currents!

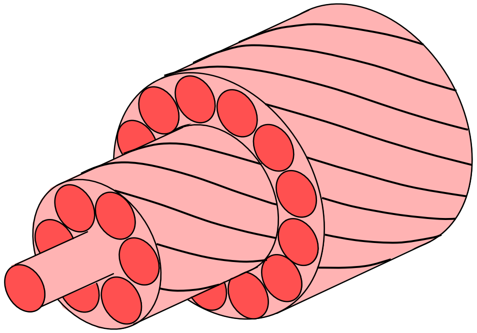


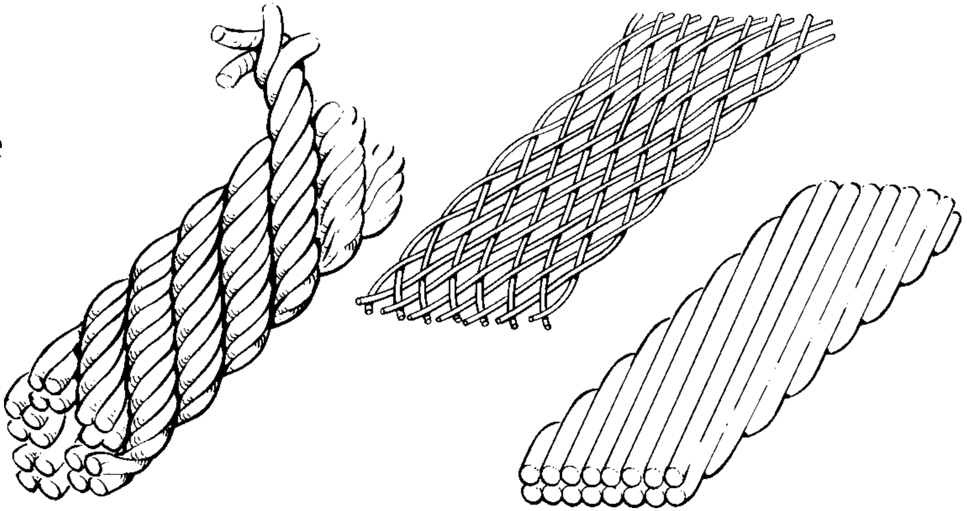
- a single 5μm filament of NbTi in 6T carries 50mA
- a composite wire of fine filaments typically has 5,000 to 10,000 filaments, so it carries 250A to 500A
- for 5 to 10kA, we need 20 to 40 wires in parallel - a cable

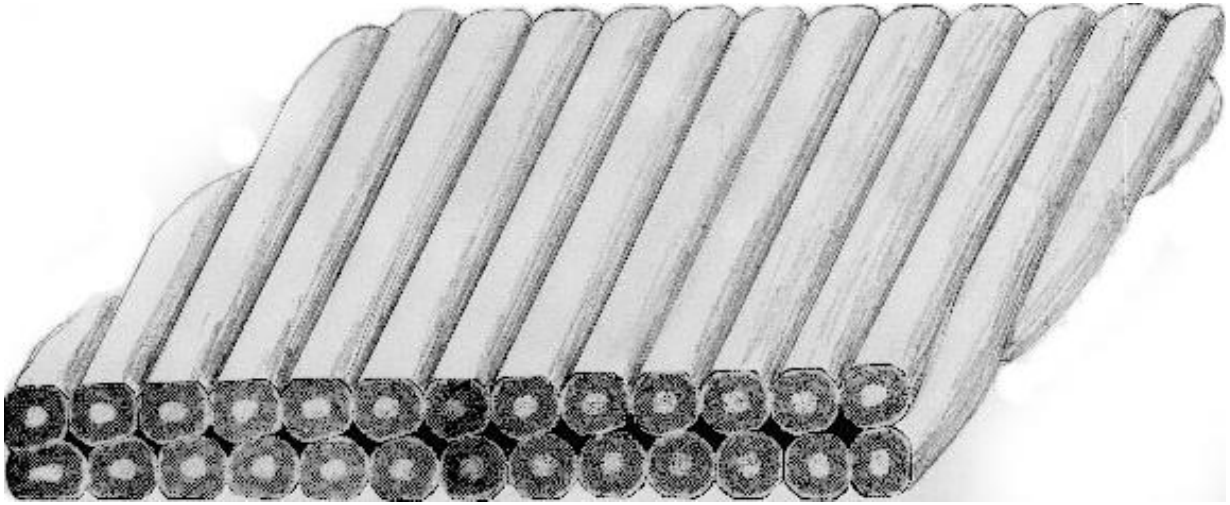
Cable transposition

- many wires in parallel - want them all to carry same current zero resistance - so current divides according to inductance
- in a simple twisted cable, wires in the centre have a higher self inductance than those at the outside
- current fed in from the power supply therefore takes the low inductance path and stays on the outside
- outer wires reach J_c while inner are still empty
- so the wires must be fully **transposed**, ie every wire must change places with every other wire along the length
  inner wires ⇒ outside   outer wire ⇒ inside
- three types of fully transposed cable have been tried in accelerators
  - rope
  - braid
  - Rutherford

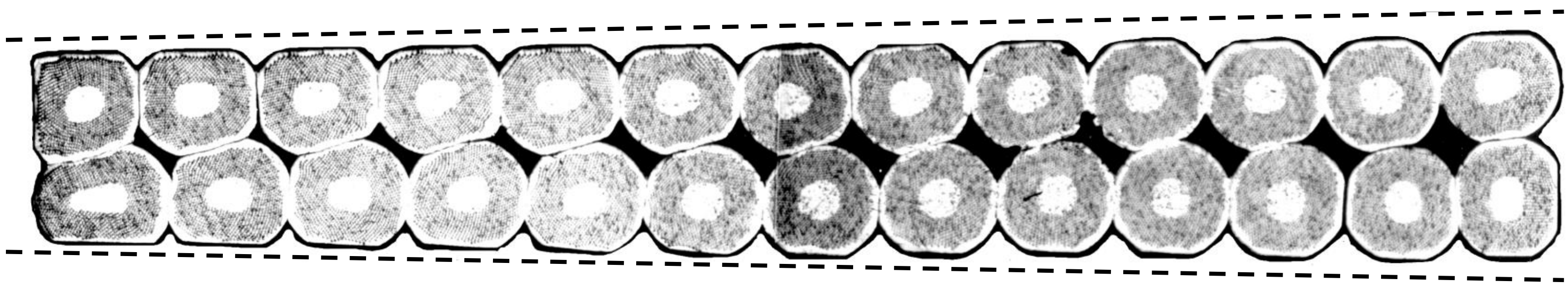






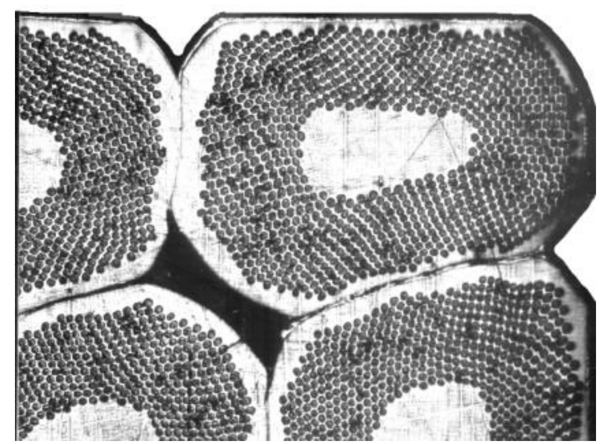


Rutherford cable

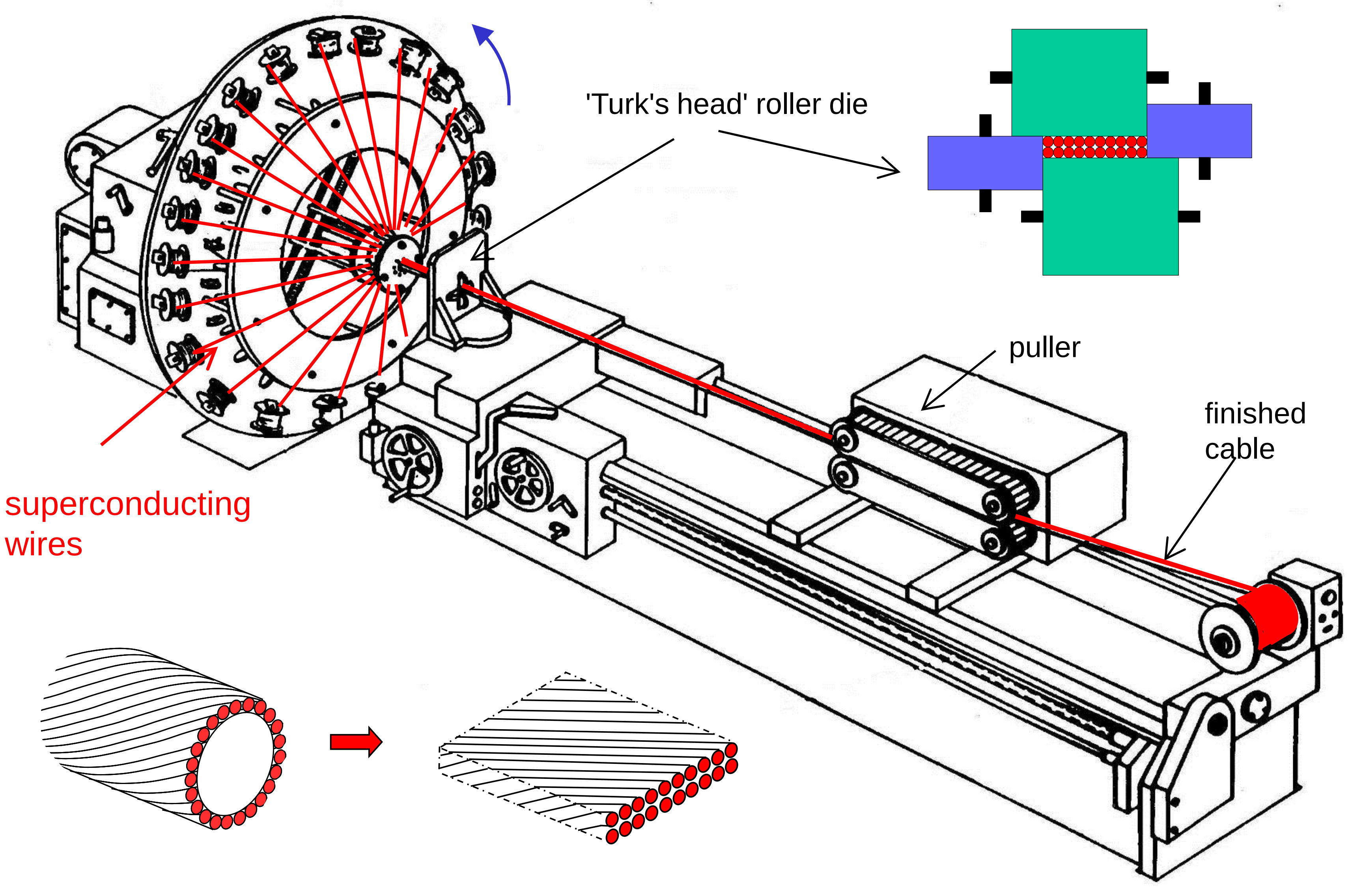


- Rutherford cable succeeded where others failed because it could be compacted to a high density (88 - 94%) without damaging the wires, and rolled to a good dimensional accuracy (~ 10μm).

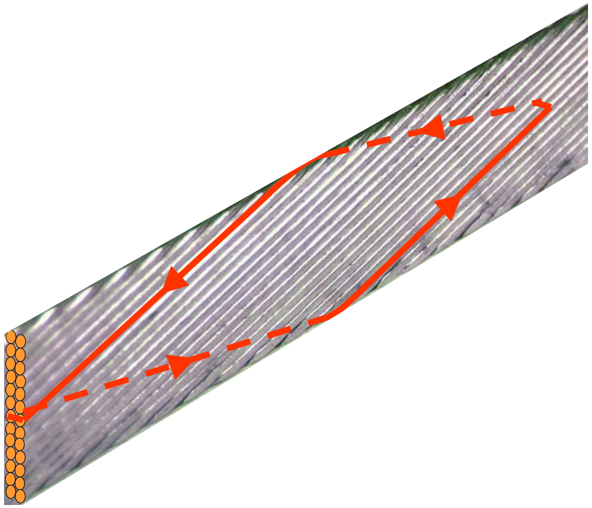
- Note the 'keystone angle', which enables the cables to be stacked closely round a circular aperture



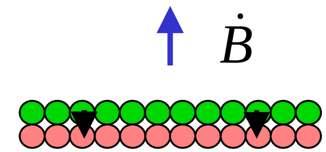
Manufacture of Rutherford cable

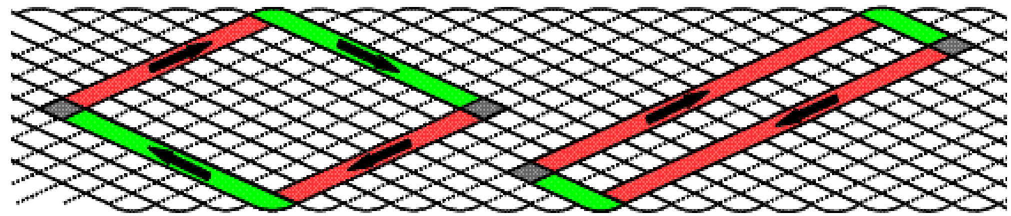


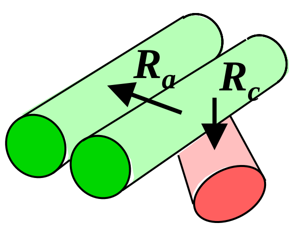
Coupling in Rutherford cables



- Field transverse
  coupling via crossover resistance R_c

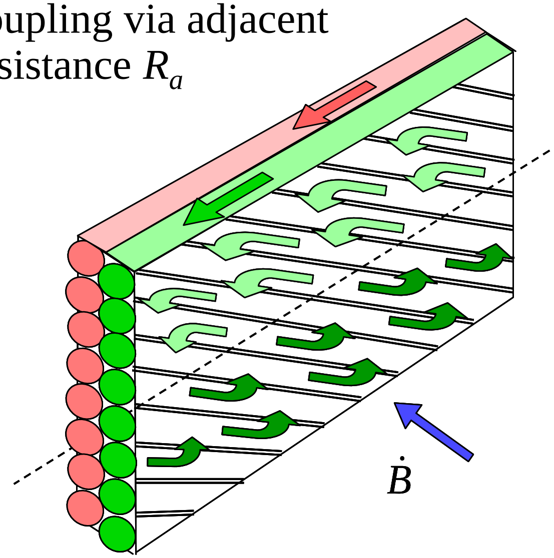




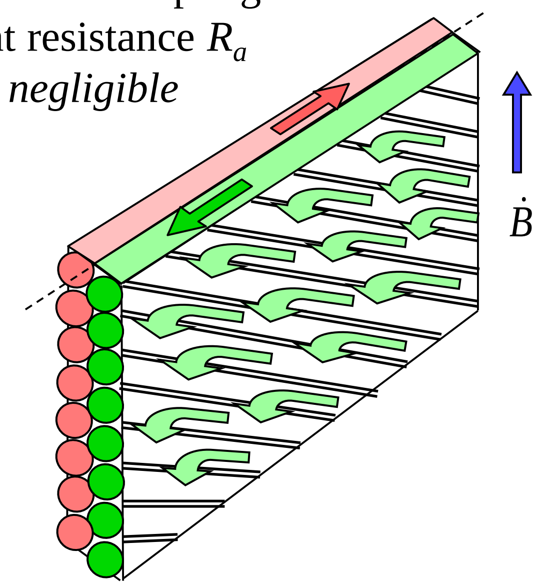


crossover resistance *Rc*
adjacent resistance *Ra*

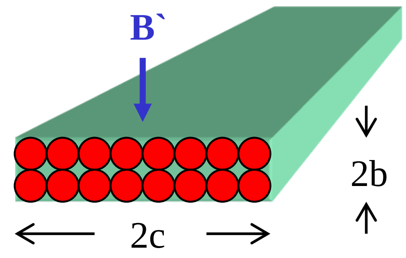
- Field transverse
  coupling via adjacent resistance R_a



- Field parallel coupling via adjacent resistance R_a
  usually negligible



Magnetization from coupling in cables



- Field transverse coupling via crossover resistance R_c

$$M_{tc} = \frac{1}{120}\frac{B_t'}{R_c}\frac{c}{b}\,p\,N(N-1) = \frac{1}{60}\frac{B_t'}{\rho_c}\,p^2\,\frac{c^2}{b^2}$$

where M = magnetization per unit volume of cable, p= twist pitch, N = number of strands
R_c R_a = resistance per crossover ρ_c ρ_a = effective resistivity between wire centres

- Field transverse coupling via adjacent resistance R_a

  where θ = slope angle of wires $Cos\theta$ ~ 1

$$M_{ta} = \frac{1}{6}\frac{B_t'}{R_a}\,p\,\frac{c}{b} = \frac{1}{48}\frac{B_t'}{\rho_a}\frac{p^2}{Cos^2\theta}$$

- Field parallel coupling via adjacent resistance R_a

$$M_{pa} = \frac{1}{8}\frac{B_p'}{R_a}\,p\,\frac{b}{c} = \frac{1}{64}\frac{B_p'}{\rho_a}\frac{p^2}{cos^2\theta}\frac{b^2}{c^2}$$

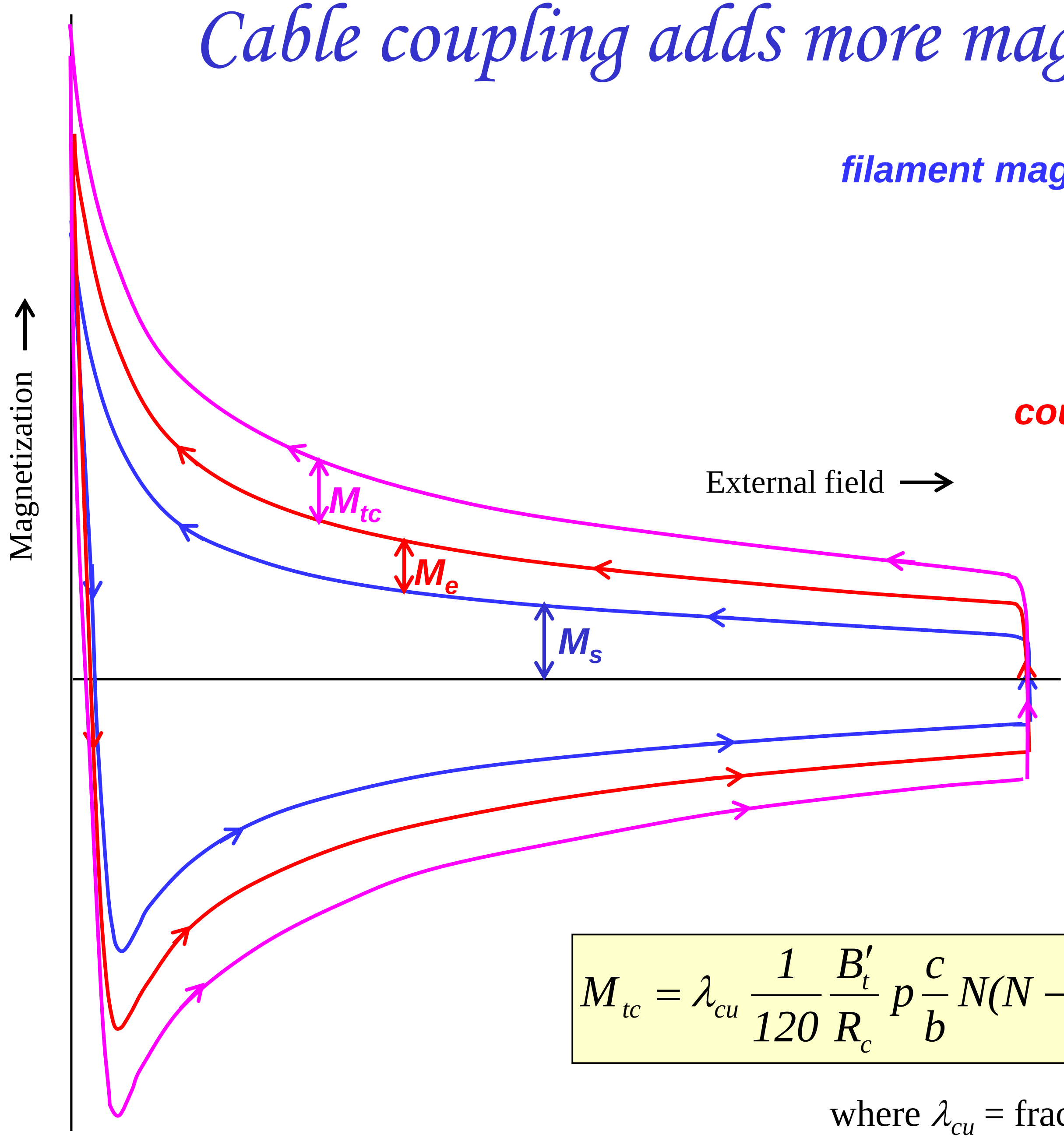
(usually negligible)

- Field transverse ratio crossover/adjacent

$$\frac{M_{tc}}{M_{ta}} = \frac{R_a}{R_c}\frac{N(N-1)}{20} \approx 45\frac{R_a}{R_c}$$

So without increasing loss too much can make $\boldsymbol{R_a}$ 50 times less than $\boldsymbol{R_c}$ - anisotropy

Cable coupling adds more magnetization



filament magnetization M_f depends on B

$$M_s = \lambda_{su} \frac{2}{3\pi} J_c(B) d_f$$

coupling between filaments M_e depends on dB/dt

$$M_e = \lambda_{wu} \frac{dB}{dt} \frac{1}{\rho_t} \left[\frac{p_w}{2\pi} \right]^2$$

coupling between wires in cable depends on dB/dt

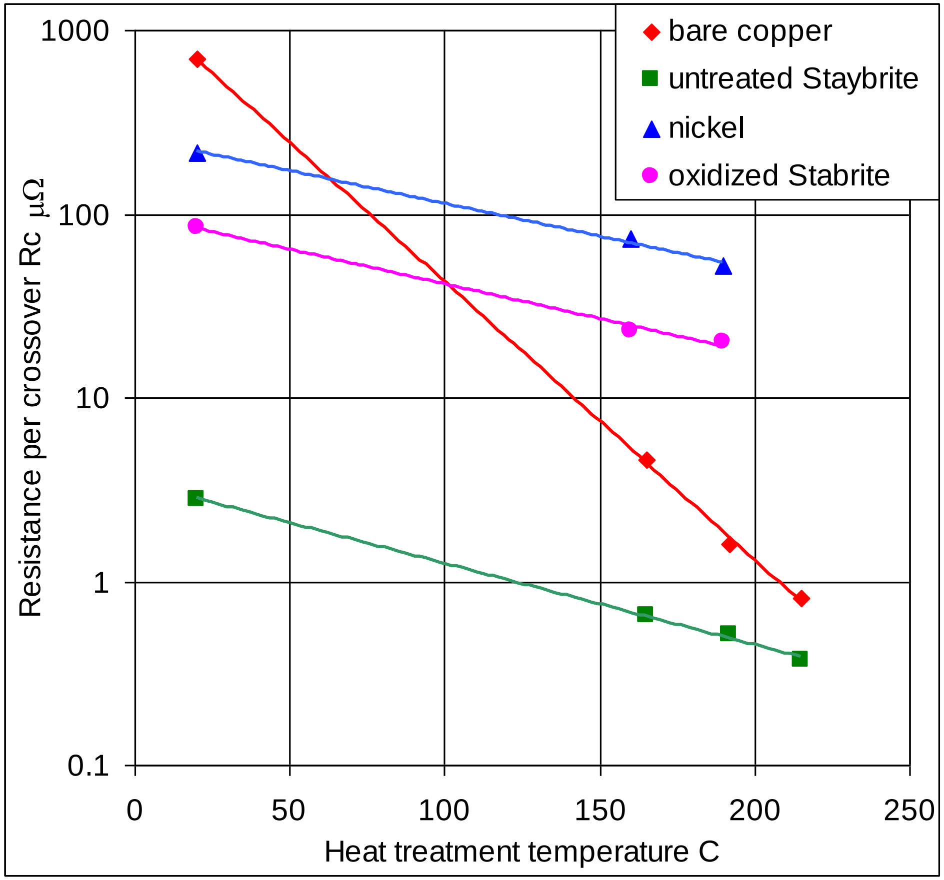
$$M_{tc} = \lambda_{cu} \frac{1}{120} \frac{B_t'}{R_c} p \frac{c}{b} N(N-1)$$

$$M_{ta} = \lambda_{cu} \frac{1}{6} \frac{B_t'}{R_a} p \frac{c}{b}$$

where λ_{cu} = fraction of cable in the section

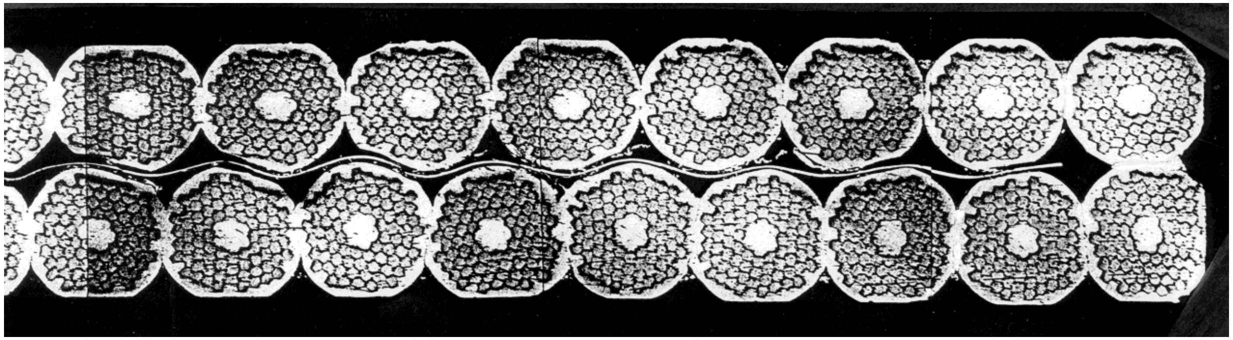
Controlling R_a and R_c

- can adjust contact resistance by surface coatings on the wires
- contact resistance is sensitive to pressure and heat treatments used in coil manufacture (to cure the adhesive between turns)
- *data from David Richter CERN*



Cored Cables

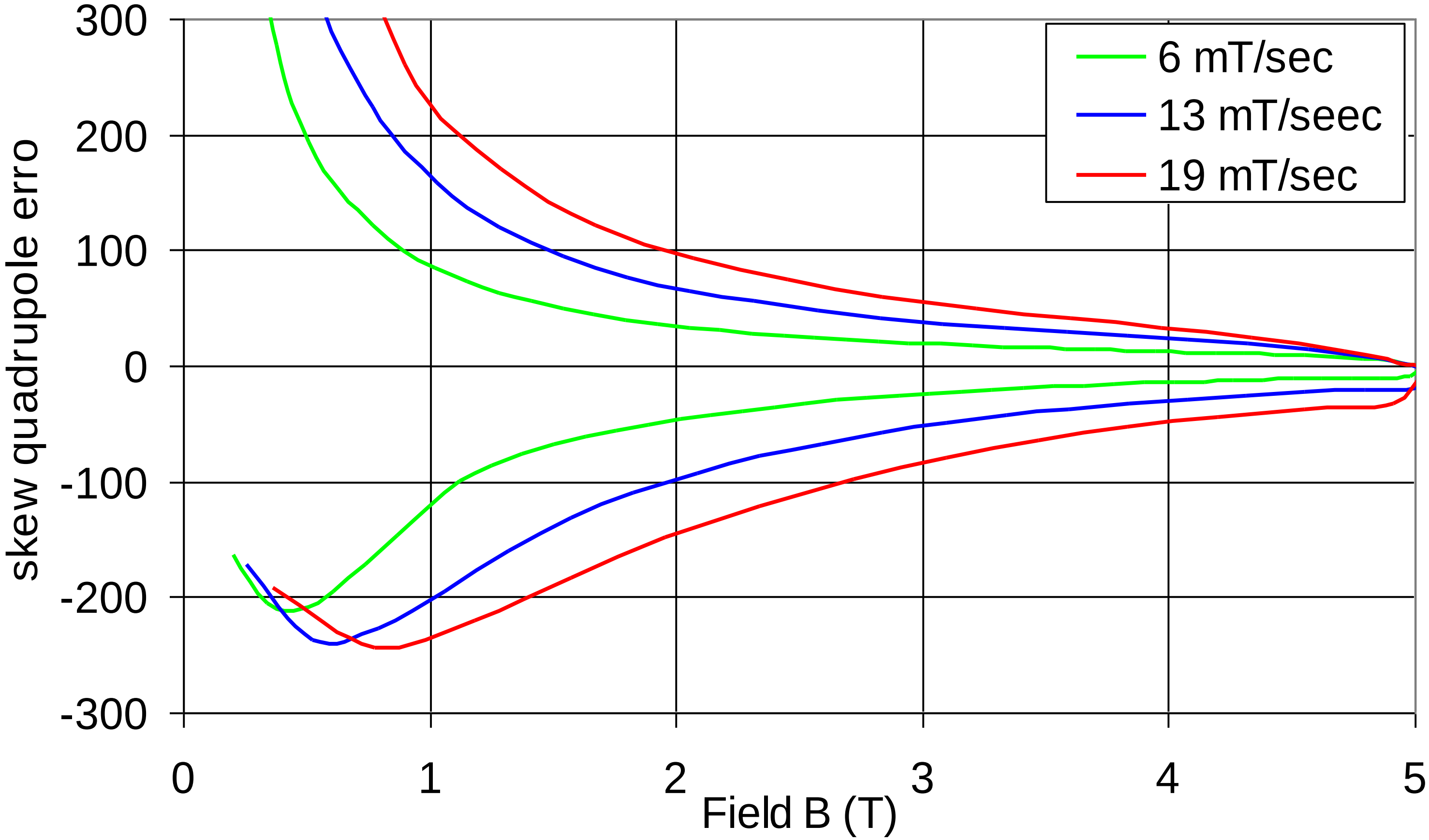
- resistive core foil increases R_c but leaves R_a the same
- reduces magnetization but keeps good current transfer between wires
- not affected by heat treatment



Magnetization and field errors – an extreme case

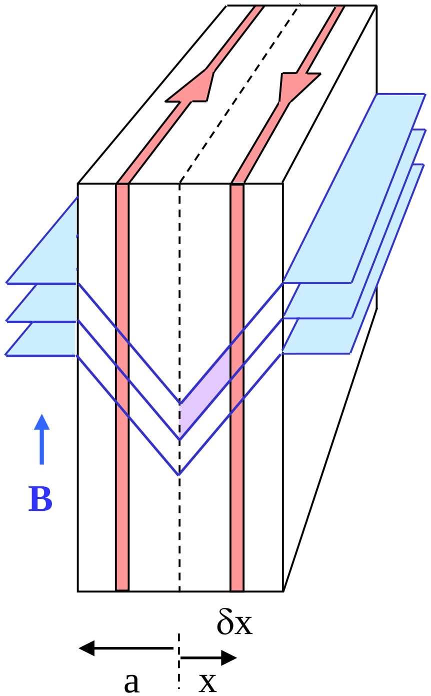
Magnetization is important in accelerators because it produces field error. The effect is worst at injection because

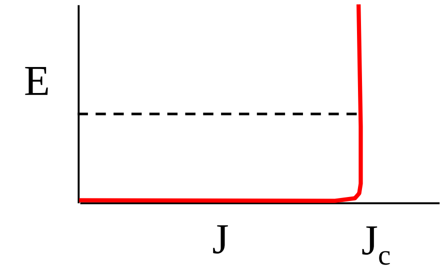
- $\Delta B/B$ is greatest
- magnetization, ie ΔB is greatest at low field



skew quadrupole error in Nb_3Sn dipole which has exceptionally large coupling magnetization (University of Twente)

AC loss power





Faraday's law of induction

$$\oint E dl = \frac{d}{dt}\int_A B dA$$

loss power / unit length in slice of width *dx*

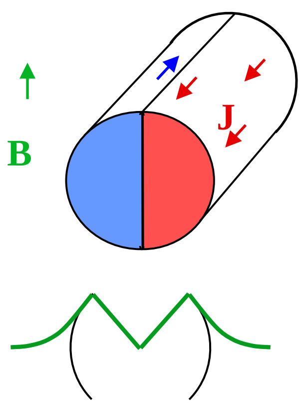
$$p(x) = E J_c \delta x = \frac{d\phi}{dt} J_c \delta x = \frac{dB}{dt} x J_c \delta x$$

total loss in slab per unit volume

$$P = \frac{1}{a}\int_0^a p(x) dx = \frac{1}{a}\frac{dB}{dt} J_c \int_0^a x dx = B' J_c \frac{a}{2} = B' M$$

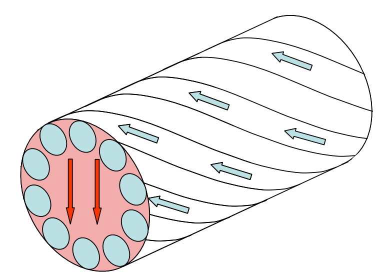
$$P = B' M$$

for round wires (not proved here)



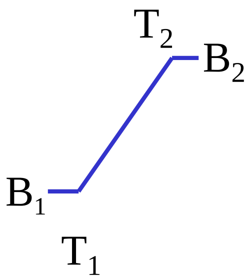
$$P = B' M = \frac{4}{3\pi} B' J_c a = \frac{2}{3\pi} B' J_c d_f$$

also for coupling magnetization



$$P_e = B' M_e = B'^2 \frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2 = \frac{B'^2}{\mu_o} 2\tau$$

Hysteresis loss



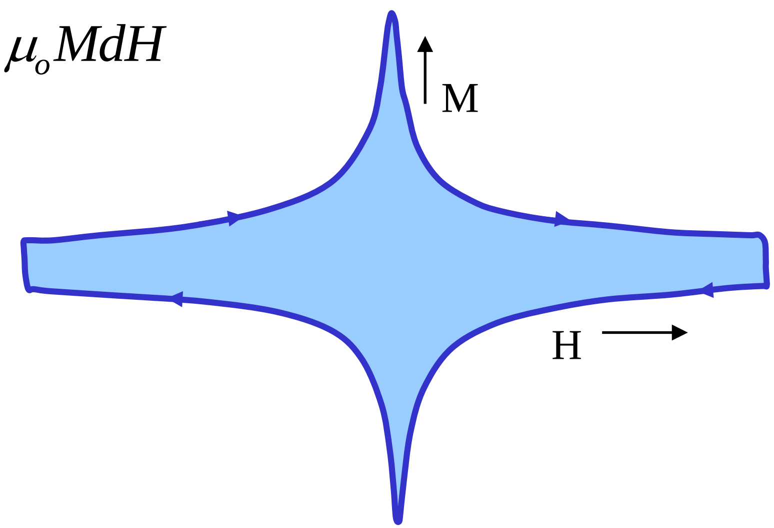
loss over a field ramp $Q = \int_{T_1}^{T_2} M \frac{dB}{dt} dt = \int_{B_1}^{B_2} M dB$

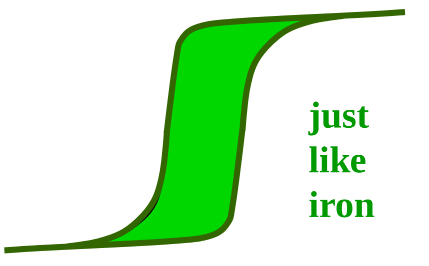
loss per ramp independent of $\dot{B}$

- in general, when the field changes by δB the magnetic field energy changes by $\delta E = H \delta B$ *(see textbooks on electromagnetism)*
- so work done by the field on the material $W = \int \mu_o H dM$
- around a ***closed loop***, this integral must be the energy dissipated in the material

$$Q = \int \mu_o H dM = \int \mu_o M dH$$

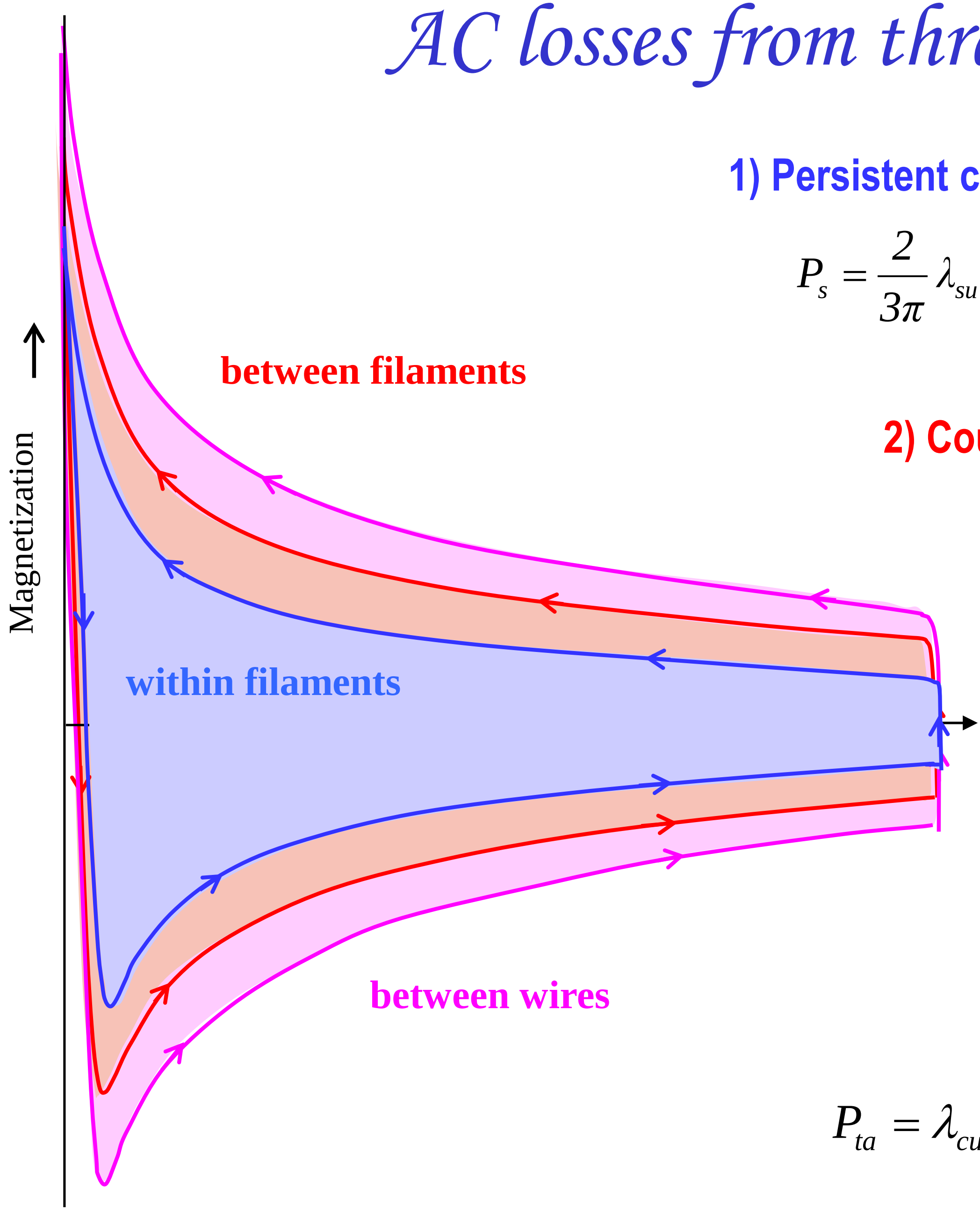
hysteresis loss per cycle (not per ramp)





just like iron

AC losses from three sources



1) Persistent currents within filaments

$$P_s = \frac{2}{3\pi}\lambda_{su} B' \; J_c(B) d_f \qquad Q_s = \frac{2}{3\pi}\lambda_{su} d_f \int J_c(B) dB$$

2) Coupling between filaments within the wire

$$P_e = \lambda_{wu} B'^2 \frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

$$Q_e = \lambda_{wu} B' \Delta B \frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

3) Coupling between wires in the cable

$$P_{tc} = \lambda_{cu} \frac{1}{120} \frac{B_t'^2}{R_c} \frac{c}{b} p_c N(N-1)$$

$$P_{ta} = \lambda_{cu} \frac{1}{6} \frac{B_t'^2}{R_a} p_c \frac{c}{b} \qquad P_{pa} = \lambda_{cu} \frac{1}{8} \frac{B_p'^2}{R_a} p_c \frac{b}{c}$$

Summary of losses - per unit volume of winding

1) Persistent currents in filaments

power W.m^{-3}

$$P_s = \lambda_{su} M_f B' = \lambda_{su} \frac{2}{3\pi} J_c(B)\, d_f B'$$

where λ_{su} , λ_{wu} , λ_{cu} = fractions of superconductor, wire and cable in the winding cross section

2) Coupling currents between filaments in the wire

power W.m^{-3}

$$P_e = \lambda_{wu} M_e B' = \lambda_{wu} \frac{B'^2}{\rho_t} \left(\frac{p}{2\pi} \right)^2$$

3) Coupling currents between wires in the cable

transverse field crossover resistance power W.m^{-3}

$$P_{tc} = \lambda_{cu} \frac{1}{120} \frac{B_t'^2}{R_c} p \frac{c}{b} N(N-1)$$

transverse field adjacent resistance power W.m^{-3}

$$P_{ta} = \lambda_{cu} \frac{1}{6} \frac{B_t'^2}{R_a} p \frac{c}{b}$$

parallel field adjacent resistance power W.m^{-3}

don't forget the filling factors

Concluding remarks

- changing magnetic fields drive superconductor into resistive state ⇒ losses – leave persistent currents
- screening currents produce magnetization (magnetic moment per unit volume)
  ⇒ lots of problems - field errors and ac losses
- in a synchrotron, the field errors from magnetization are worst at injection
- we reduce magnetization by making fine filaments - for practical use embed them in a matrix
- in changing fields, filaments are coupled through the matrix ⇒ increased magnetization
  - reduce it by twisting and by increasing the transverse resistivity of the matrix
- flux jumping is an electromagnetic/thermal instability that afflicts all high field superconductors
  - solved problem – fine filaments
- accelerator magnets must run at high current because they are all connected in series
  - combine wires in a cable, it must be fully transposed to ensure equal currents in each wire
- wires in cable must have some resistive contact to allow current sharing
  - in changing fields the wires are coupled via the contact resistance
    - different coupling when the field is parallel and perpendicular to face of cable
      - coupling produces more magnetization ⇒ more field errors
- irreversible magnetization ⇒ ac losses in changing fields
  - coupling between filaments in the wire adds to the loss
    - coupling between wire in the cable adds more

never forget that magnetization and ac loss are defined per unit volume - ***filling factors***