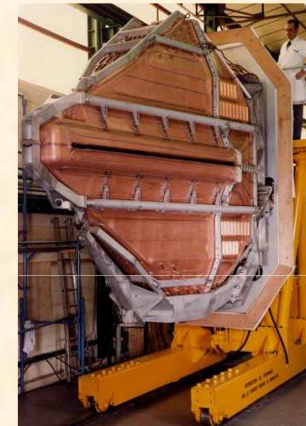
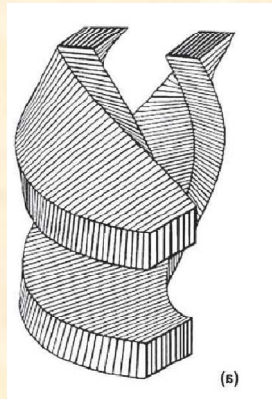


Cyclotrons : specific techniques

- Acceleration and RF cavities

- Injection



- Extraction

(stripping, turn separation, precession...)

Acceleration

- The final energy is independent of the accelerating potential $V = V_0 \cos\varphi$.

If V_0 varies, the **number** of turn varies. ($B\rho_{final} = \langle B \rangle \cdot R_{extraction}$)

- **The energy gain** per turn depends on the peak voltage V_0 , but is constant, if the cyclotron is **isochronous** ($\varphi = \text{const}$):

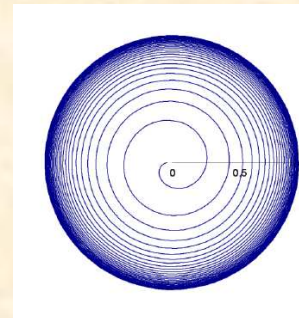
$$\delta E = N_g q V_0 \cos\varphi$$

N_g : number of gaps per turn

- The **radial separation** δr between two turns varies as $1/r$ ($\gamma \sim 1$):

$$\frac{\delta r}{r} = \frac{\delta B\rho}{B\rho} = \frac{\delta p}{p} = \frac{\gamma}{\gamma + 1} \frac{\delta E}{E} \approx \frac{qV_0 \cos\varphi}{2E} \propto \frac{1}{r^2}$$

$$\delta r \propto \frac{1}{r}$$



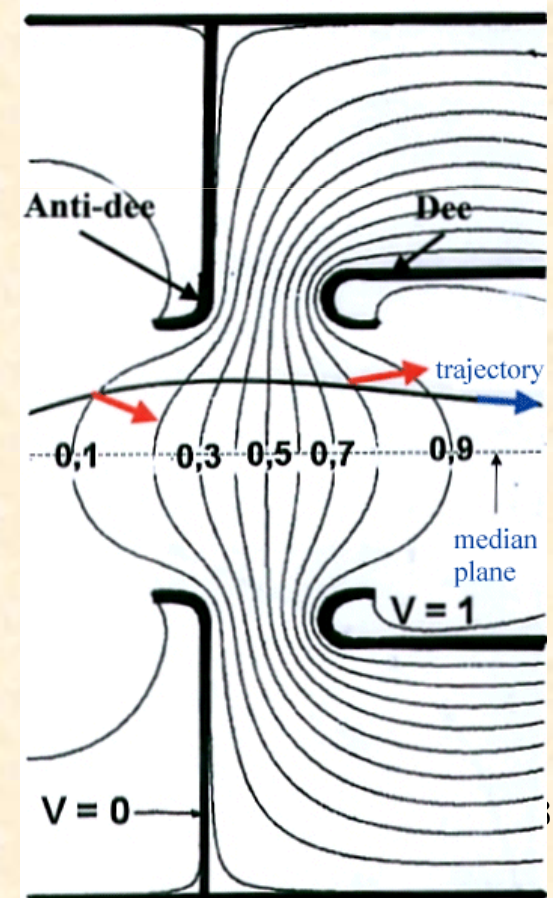
Accelerating gap & Transit Time

The formula $\delta E = QV_0 \cos \varphi$ corresponds to small accelerating gaps
Because of the gap geometry, the efficiency of the acceleration through
the gap (g) is modulated by the **transit time factor τ** :

$$\delta E = QV_0 \tau \cos \varphi$$

$$\tau = \frac{\sin \left\{ \frac{hg}{2r} \right\}}{\frac{hg}{2r}} < 1$$

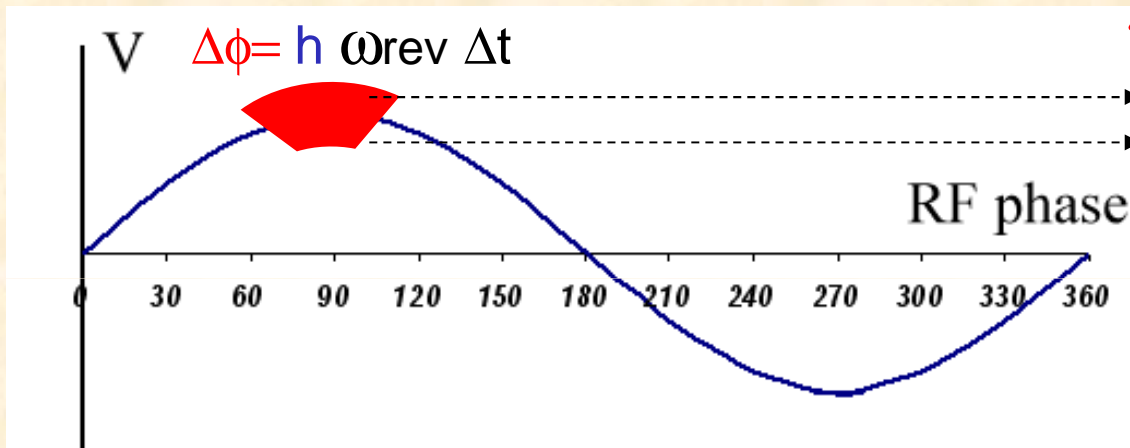
Introduction of pillars into the cavity to reduce the azimuthal field extension



Acceleration & bunch length Δt

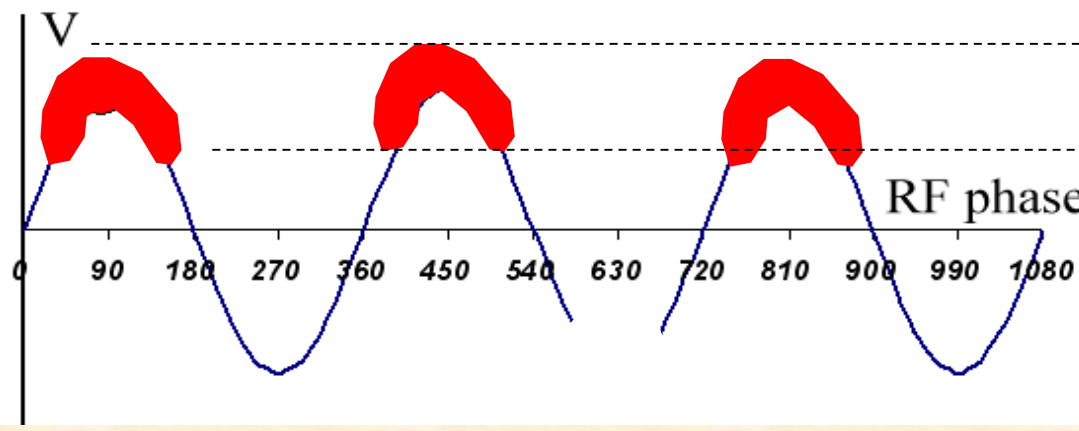
- The bunch length Δt induces energy dispersion

$$\frac{\delta r}{r} = \frac{\delta B \rho}{B \rho} = \frac{\gamma}{\gamma + 1} \frac{\delta E}{E} \approx \frac{1}{2} \frac{\Delta[qV_0 \cos(h \omega_{RF} t)]}{E}$$



harmonics=1

$$\Delta r \sim qV \omega_{rf} \Delta t$$



harmonics > 1

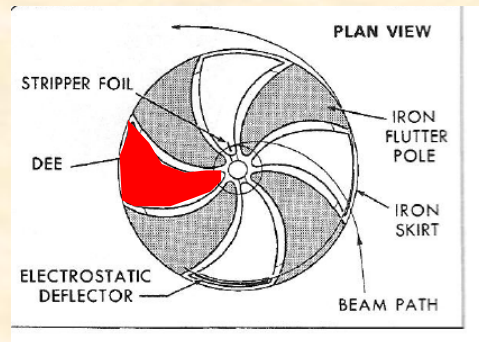
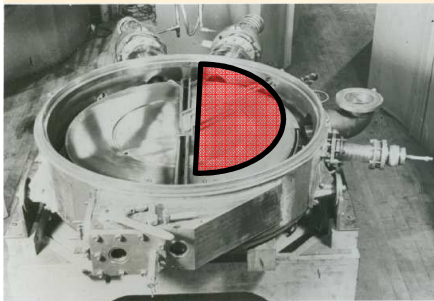
Worst beam quality

$$\Delta r \text{ larger} \sim qV H \omega_{rf} \Delta t$$

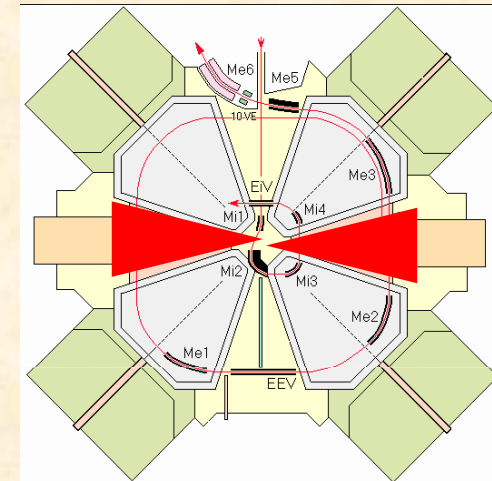
Energy dispersion larger

Acceleration RF Technology

Magnetic structure \Rightarrow RF cavity's shape



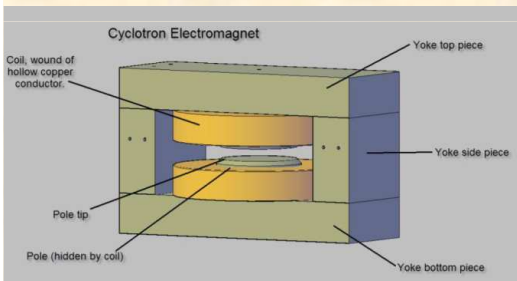
“Curved sector”
For spiral AVF



“Triangle” shape

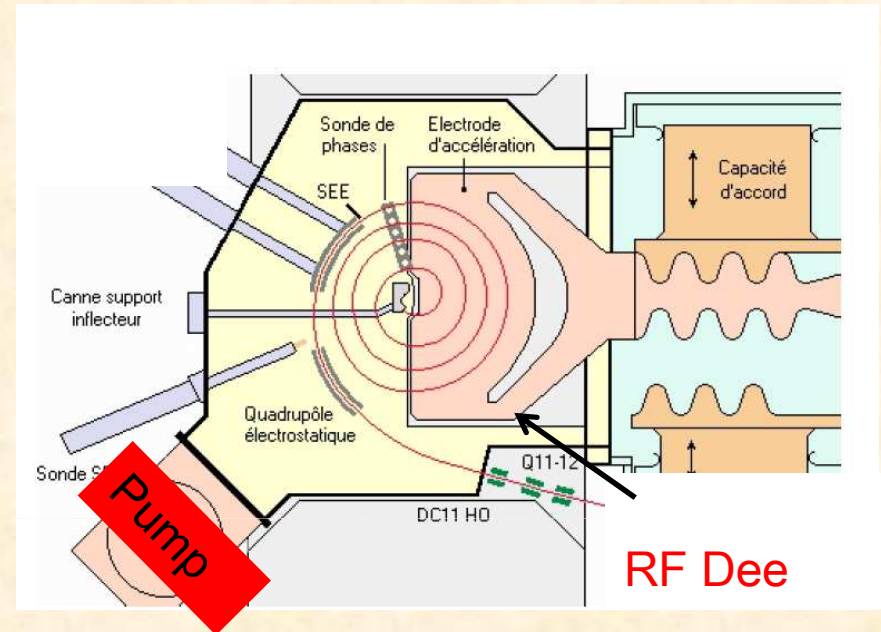
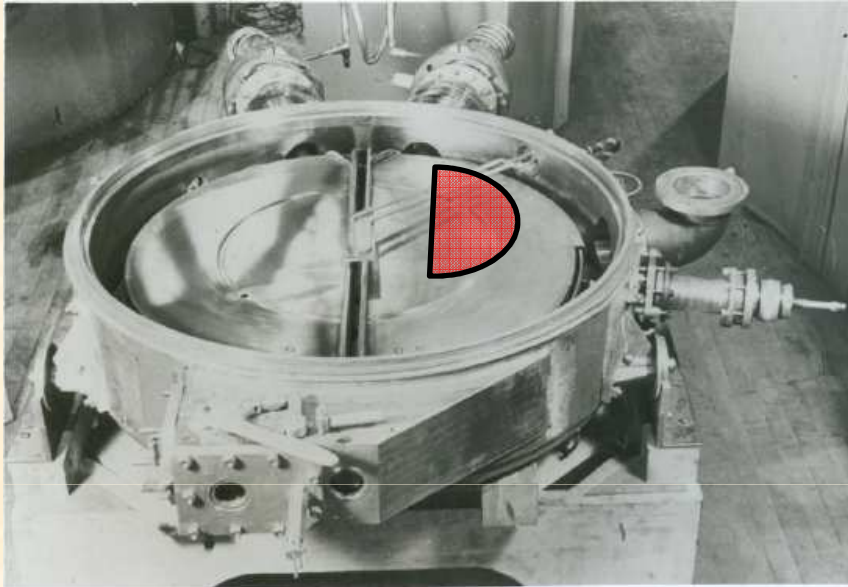
For separated sector cyclo

The classical “D” shape



The choice of the pole shape and the number of sectors have a great impact on the available space for RF systems. Dees, and possibly stems and liners have to fit into the gaps and/or valley sections

RF Cavities : Dees 180°



Dees 180°.

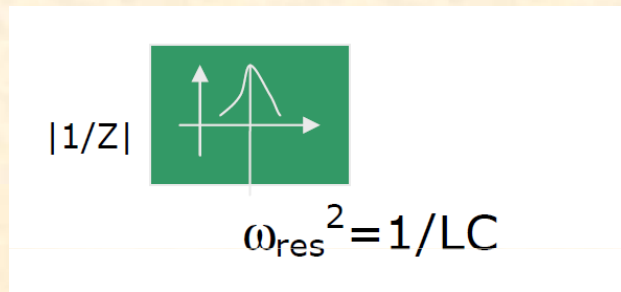
$H=1,3,5$ odd number allowed

$H=2,4$ even number **forbidden**

Dee should change its voltage every half turn of a bunch

RF Cavities : The resonance of the cavity

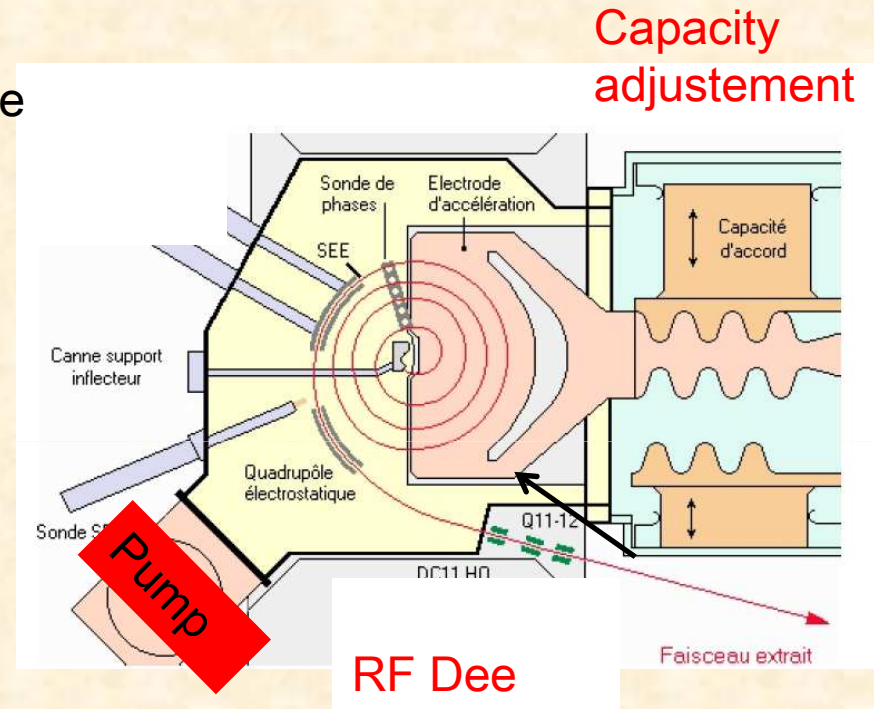
Resonance for a cavity
= minimal impedance (Z) for maximal Voltage



Cyclotron : Variable Energy with
B and Frf variable

$$1/Z = 1/R + j\omega(C - 1/L\omega^2)$$

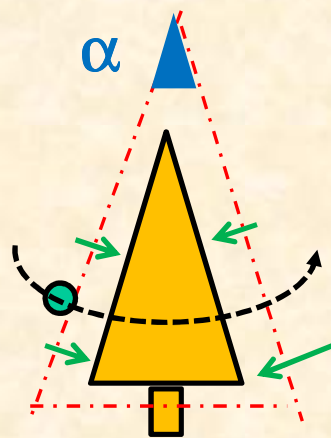
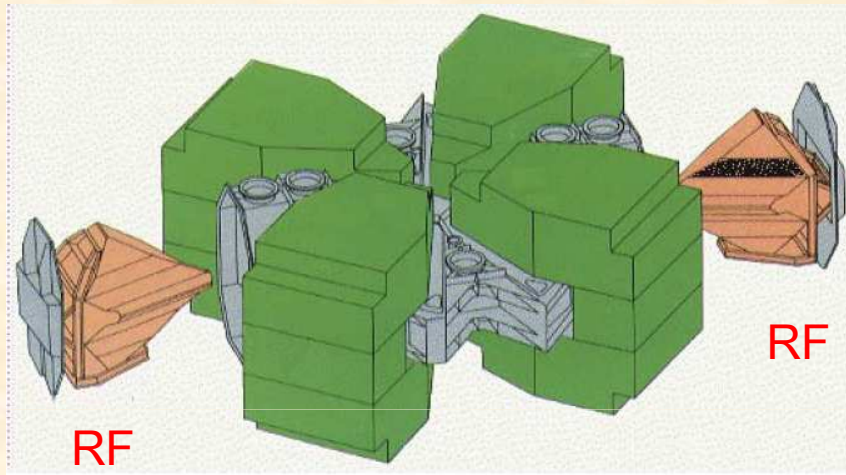
$$\omega_{rev} = \frac{qB}{\gamma m} = H\omega_{RF}$$



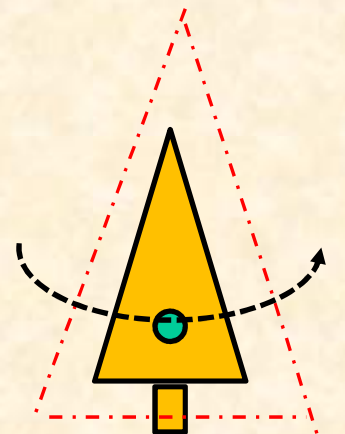
Variation of the Capacity C :
to adjust $\omega_{resonance}$

$$\omega_{resonance} = \omega_{rf} = \omega_{rev} / h$$

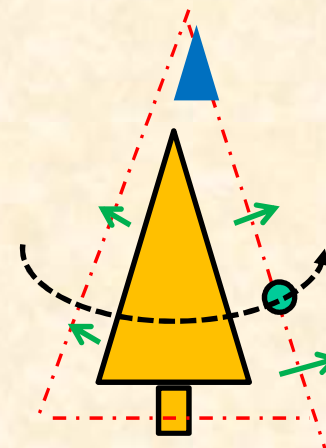
RF Cavities : example 1 for Separated Sectors Cyclo



$$V(t) = V_0 \cos(\phi - h\alpha/2)$$



$$V(t) = V_0 \cos(\phi)$$



$$V(t) = V_0 \cos(\phi + h\alpha/2)$$

Example 1: RF Cavities (not Dees)

Energy gain in 1 gap :

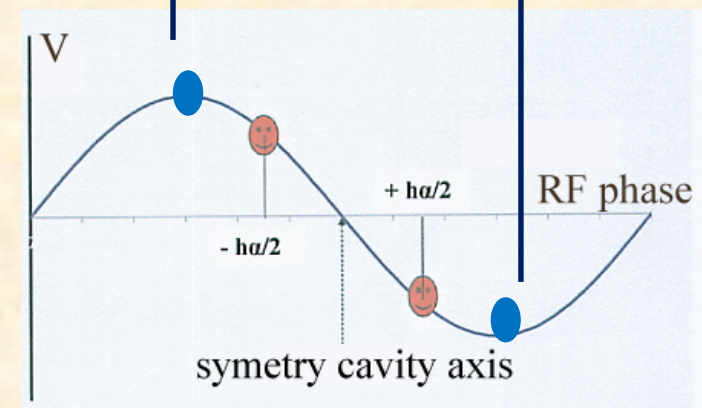
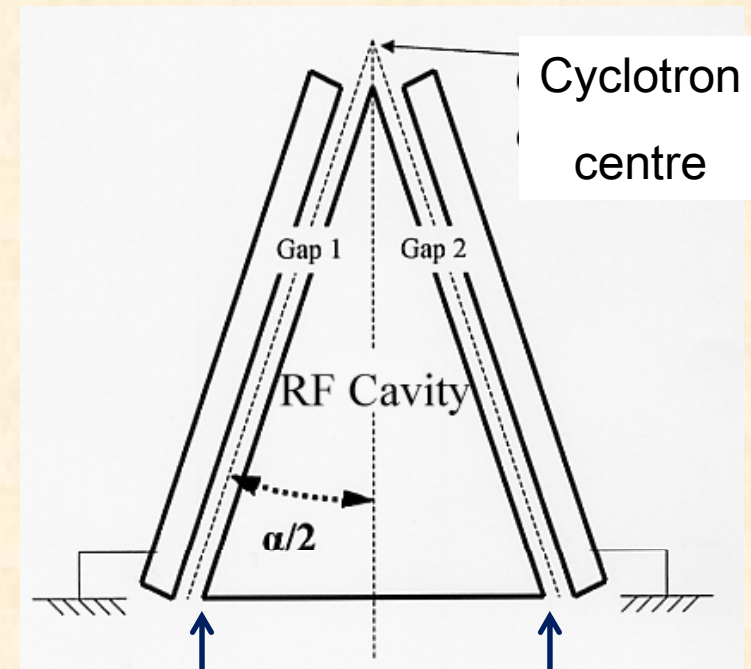
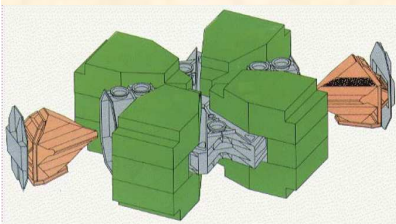
$$\delta E = qV_0 \sin\left(\frac{h\alpha}{2}\right) \cos \varphi$$

- For a maximum energy gain ($\cos\varphi = 1$) the particle passes the symmetry cavity axis
- Energy gain per gap for the various harmonic mode

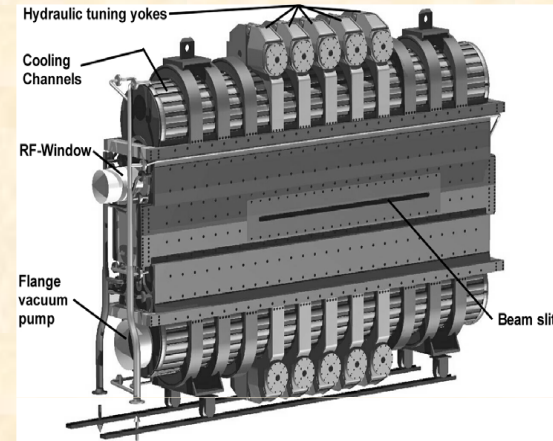
$$\delta E = qV_0 \sin\left(\frac{h\alpha}{2}\right)$$

δE optimum is

for $h.\alpha/2 = 90$ degree



example 2 :separated sector cyclotron:
the PSI ring cyclotron (proton 590 MeV)



$$R_{\text{extraction}} = 4.5 \text{ m}$$

$$K_b = 590 \text{ MeV}$$

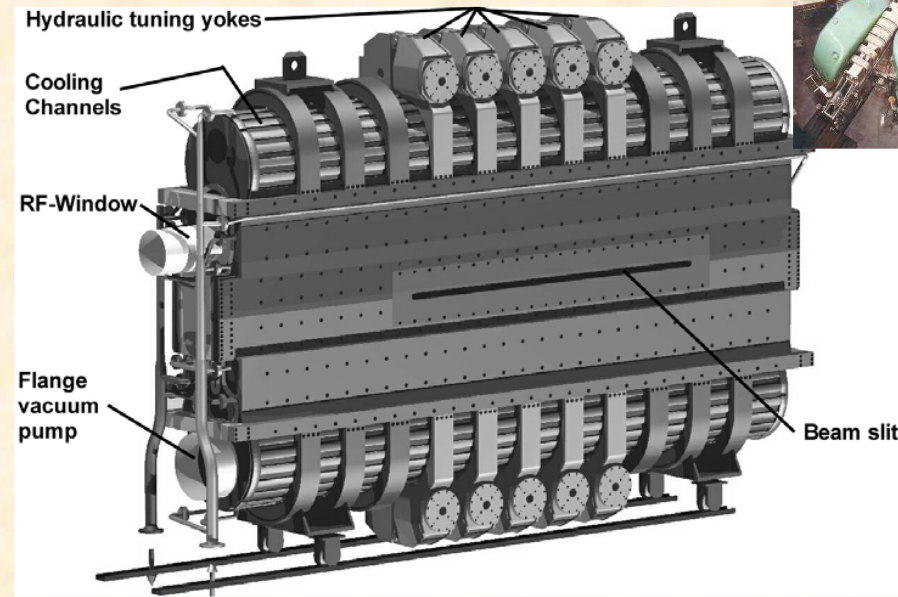
4 RF Cavities

Typical 'Separated Sector Cyclotron' (SSC). the PSI 590 MeV (p) ring cyclotron,
with 8 sector magnets and 4 accelerating cavities

PSI Proton Beam ~1 Mwatt

The Challenge :
Single turn extraction

Turn separation δr large
But Δr small



4 cavities : 50 MHz, CW
Voltage: 0.9-1 MVolt

Harmonics $h=6$

Proton Beam ~1 Mwatt
($I=2$ mA)

if δr ($\sim N_{gap} \cdot V_{rf}$) Large

No beam losses
 $T= 99.99\%$

Beam injection

-THE ION SOURCES (internal and external)

Low energy :

AXIAL INJECTION FOR COMPACT CYCLOTRON

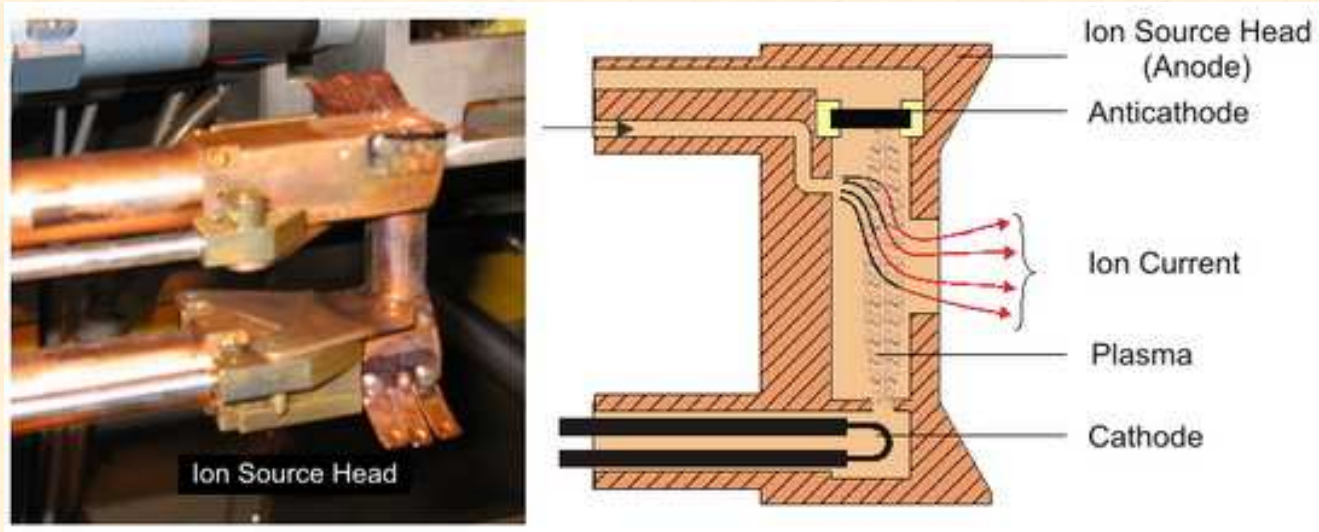
- Infector (spiral, hyperboloid;...)

Higher energy :

RADIAL INJECTION FOR SEPARATED SECTOR
CYCLOTRON

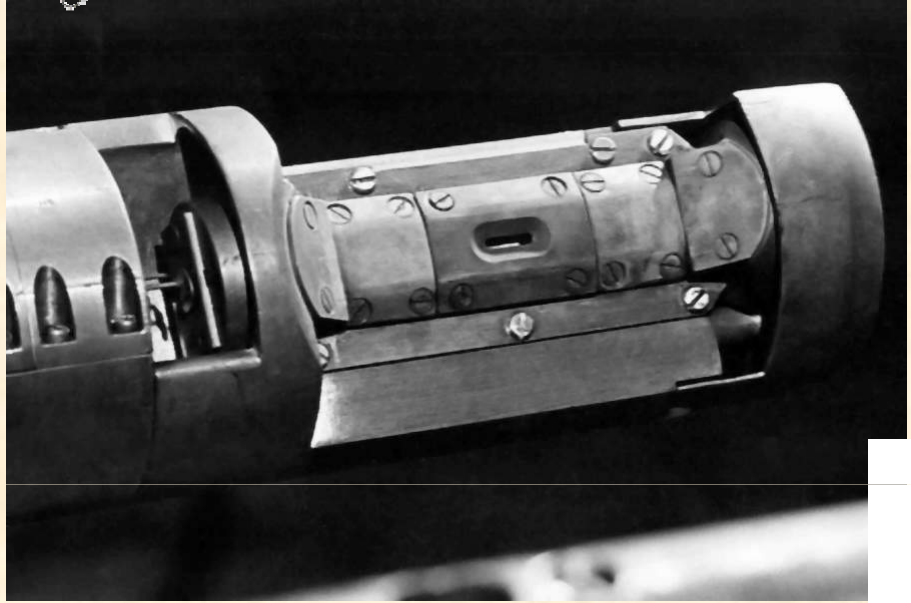
Cold Cathode PIG Ion Source

Penning or Philips Ionization Gauge (*PIG*) ion source



- Electron emission due to electrical potential on the cathodes
- Electron confinement due to the magnetic field along the anode axis
- Electrons produced by thermionic emission and ionic bombardment
 - Start-up: 3 kV to strike an arc
 - At the operating point : 100 V
- Cathodes heated by the plasma (100 V is enough to pull an outer e- off the gas atoms)

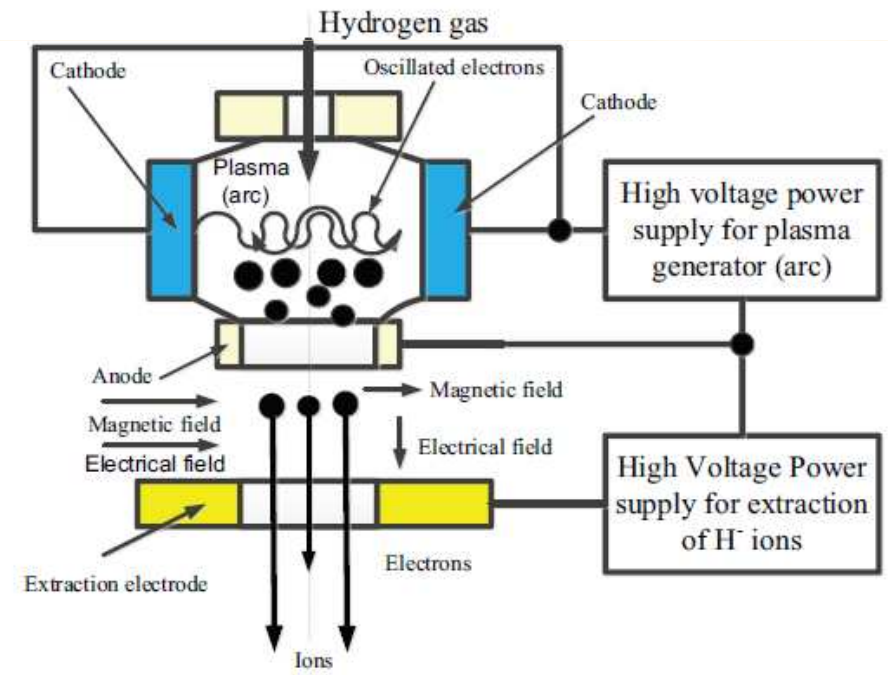
Example of PIG source



Small size

Inserted in the cyclotron gap

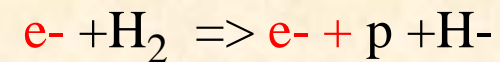
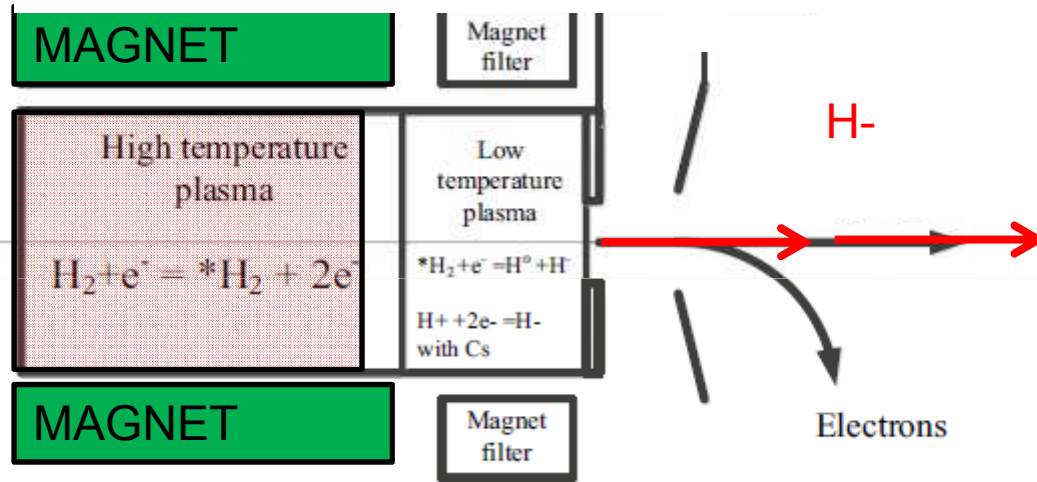
FLNR, PIG test-bed, 1992.
 The head of MC400 cyclotron vertical ion source



Multi-CUSP source

negativ ions : H-//D- with high current

Confinement + filtering + extraction



- Larger Than the PIG source (Magnets)
- Better emittance
- Larger current (Magnet confinement+ Filter)

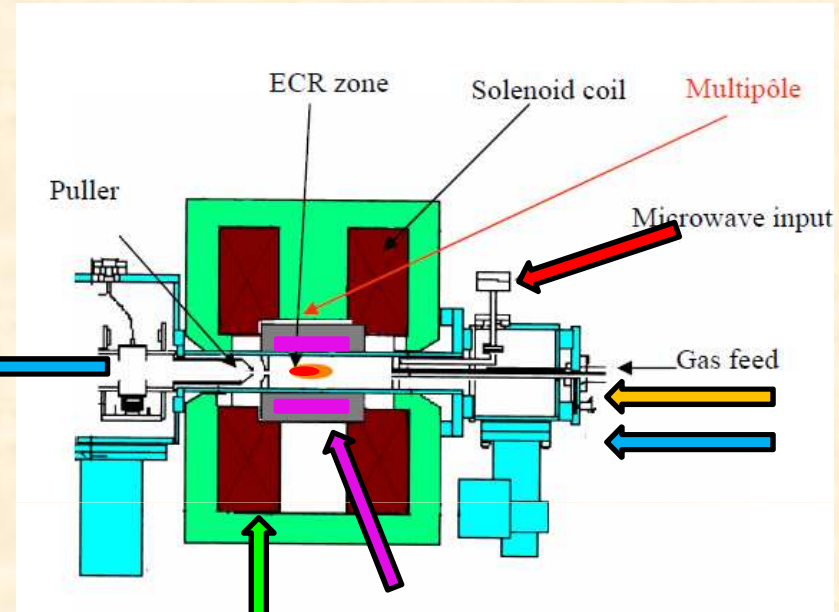
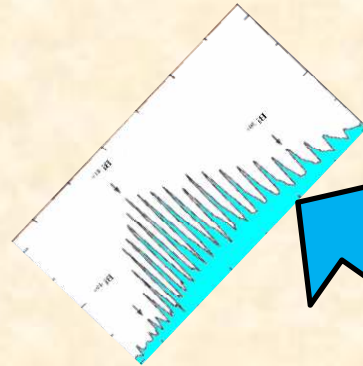
Larger Size \Rightarrow External Source

ECR ion source : positiv Heavy ions



Pantechnic
Nanogan®

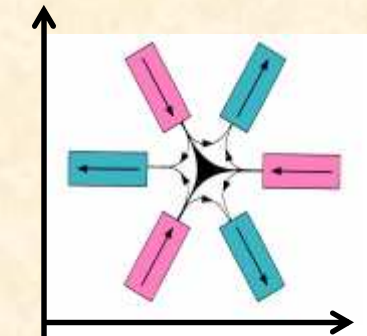
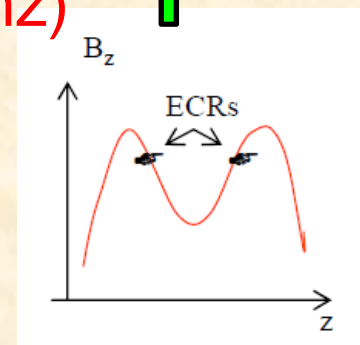
Charge state distribution



Gas (he, O,..) + RF(microWave 10-18Ghz)
Plasma (ions + electrons) :
+ ATOMS

electrons + ions impacts

Ionize any injected heavy atoms
(He, Li,U)



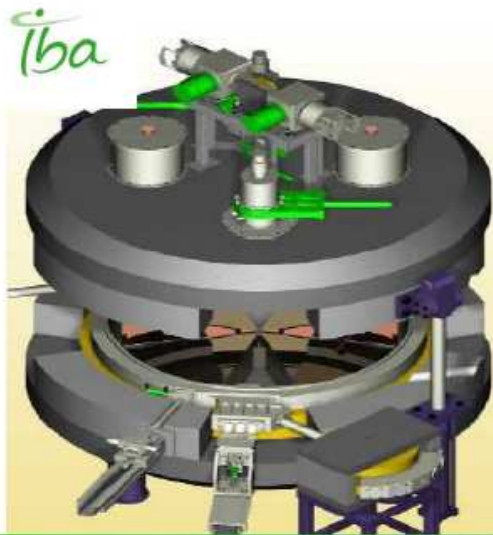
Bz + Br
3D plasma confinement

Exemple ARRONAX (Nantes, Fr)

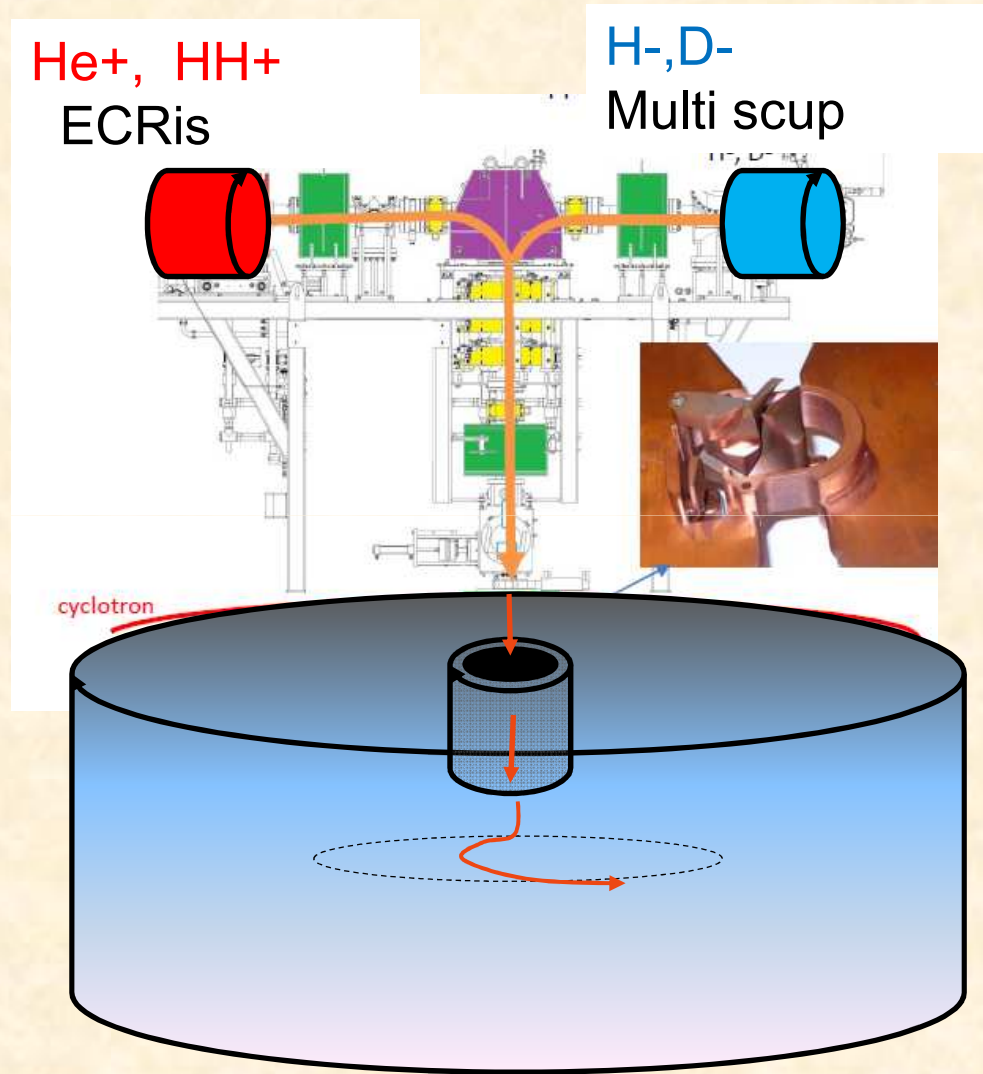
2 external sources in a $K_b=70$ MeV cyclotron

$K_b=70$ MeV

- Radiol isotopes production
- Radio-Biology studies
- Irradiation



Cyclotron ARRONAX



Injection from the Top (AXIAL injection)

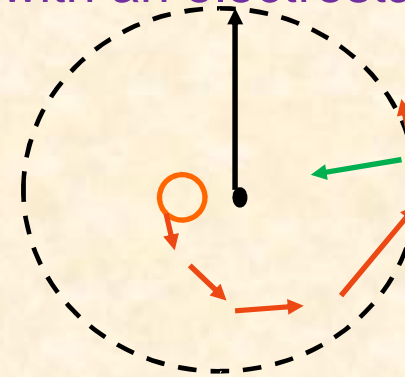
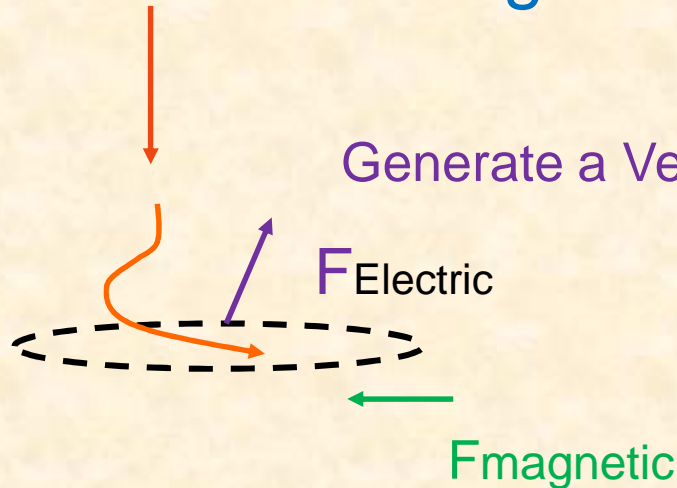
Axial injection with inflector

➤ Goal :

Put **the beam** on the « **good orbit** » at the good phase

with a very compact geometry

Generate a Vertical force with an electrostatic device



Outside cyclotron

axial motion (vertical)

Inside cyclotron (Magnetic force is radial)

radial motion (horizontal)

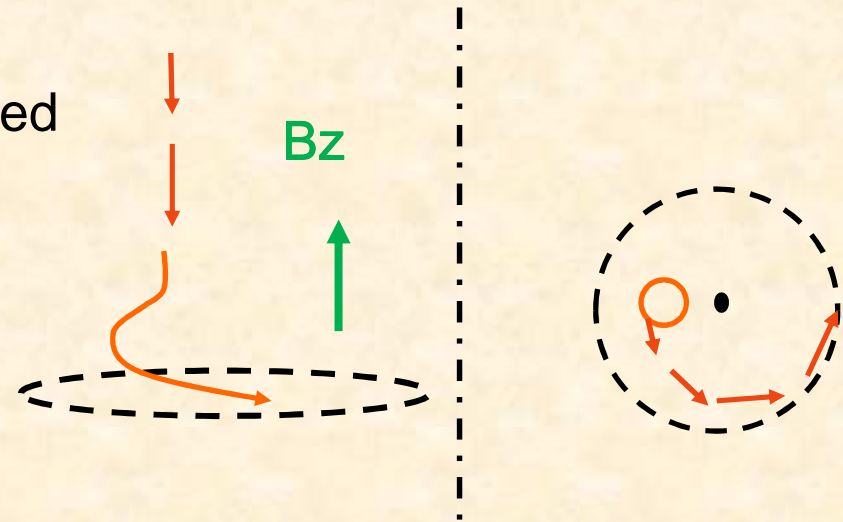
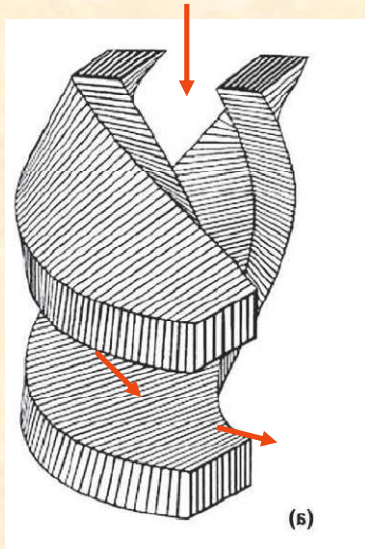
$$R_m = B_p / B_{\text{center}}$$

$$R_E = mV^2/Q / E_{\text{inflector}}$$

Axial injection : Spiral inflector

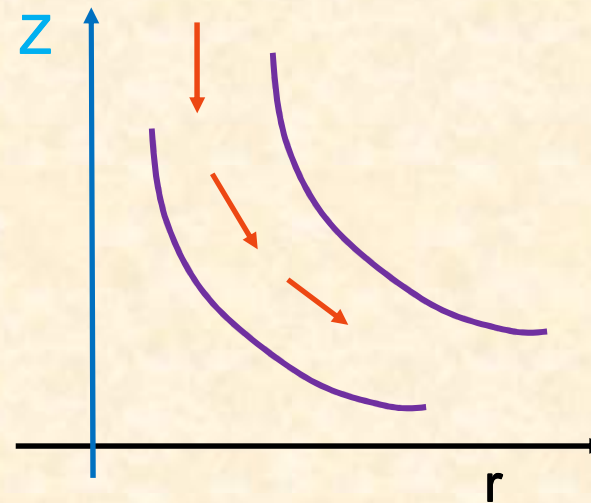
1. Spiral inflector (or helical channel)

principle: 90° electrostatic deflector twisted

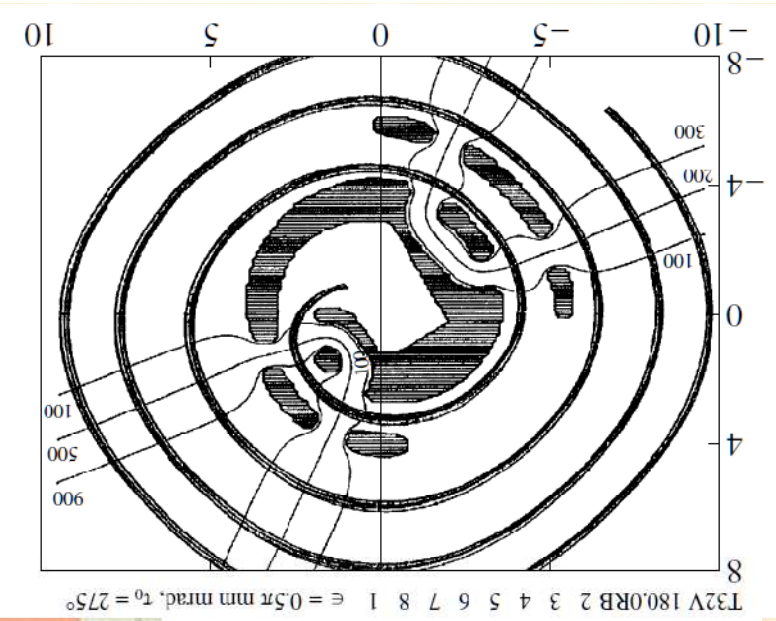
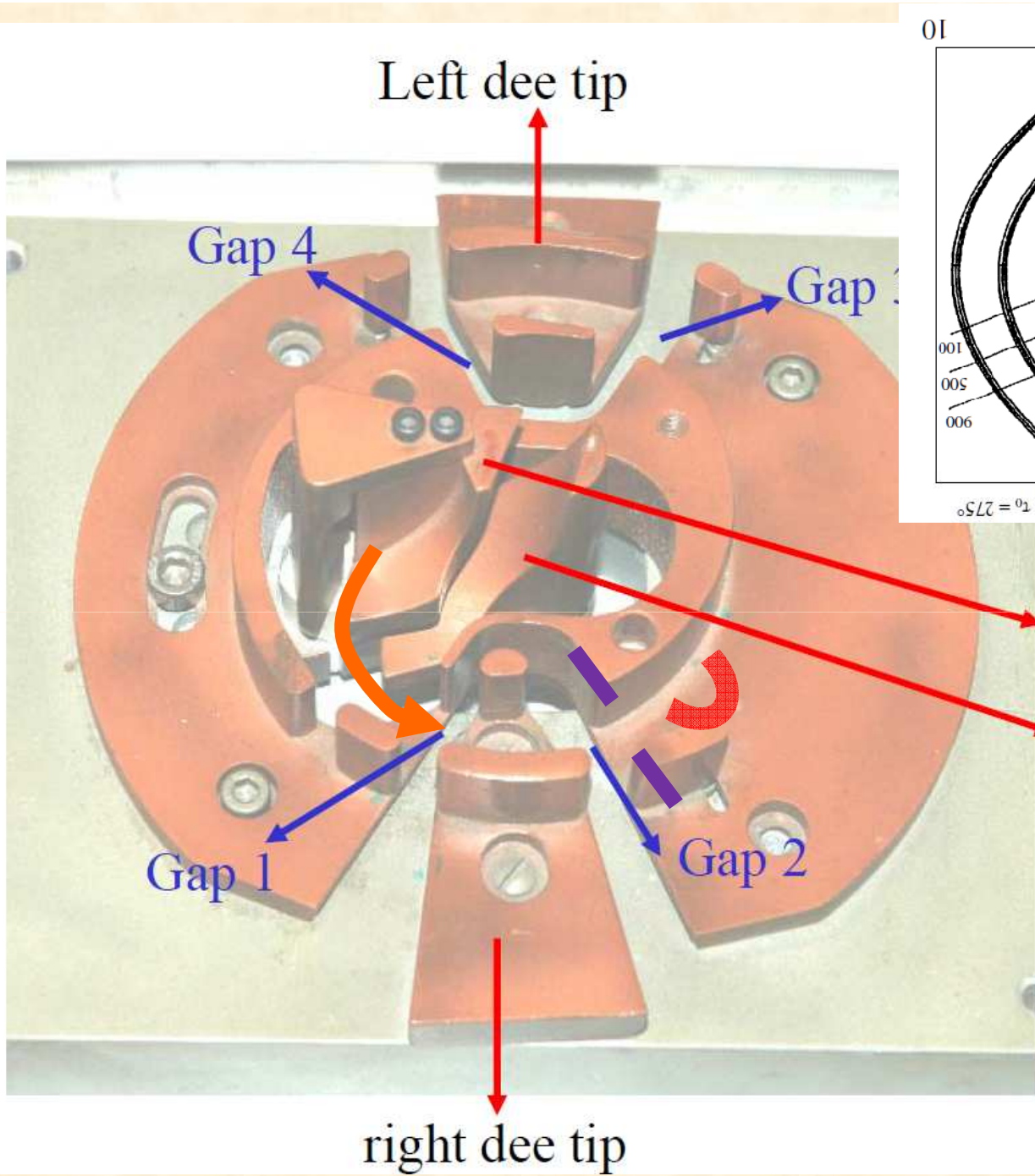


E always perpendicular to v

$B=B_z$ constant (cyclo center)



Complex geometry , very compact

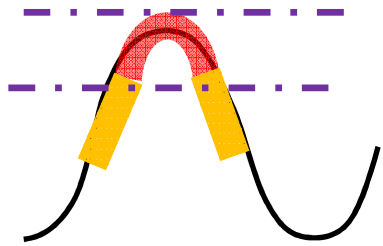


upper electrode

lower electrode

Phase selection

$$\Delta\Phi \sim \Delta E/E \Rightarrow \Delta R$$



Axial injection 1: Spiral inflector

$$m\ddot{x} = qE_x - qv_y B_0,$$

$$m\ddot{y} = qE_y + qv_x B_0,$$

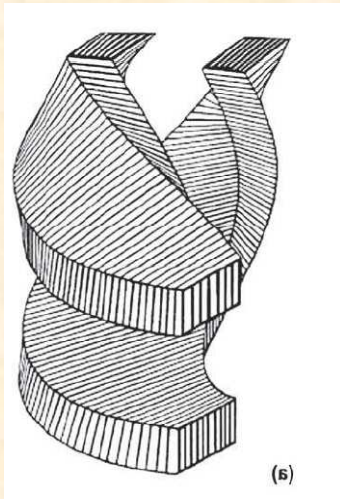
$$m\ddot{z} = qE_z.$$

Trajectory Equations are very funny :

Parametric equation of the trajectory $\theta = [0, \pi/2]$

$$\begin{aligned} x_c &= \lambda(1 - \sin k\theta \sin \theta - \cos k\theta \cos \theta) \\ y_c &= \lambda(\sin k\theta \cos \theta - \cos k\theta \sin \theta) \\ z_c &= A(\sin \theta - 1) \end{aligned} ,$$

$$\begin{aligned} k &= A/R_m + k' \\ \lambda &= A/(k^2 - 1) \end{aligned}$$



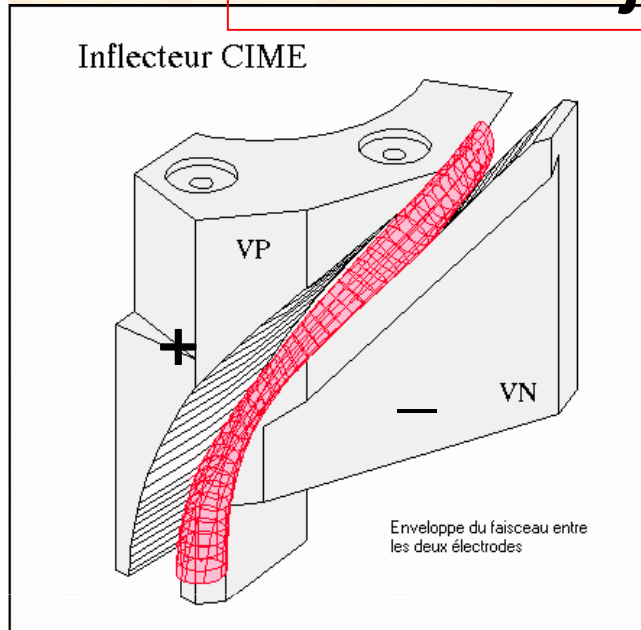
Two parameters : A the inflector Height
k' the tilt

2 forces bend the beam

Electric radius $A = RE = mV^2/Q / E_0$

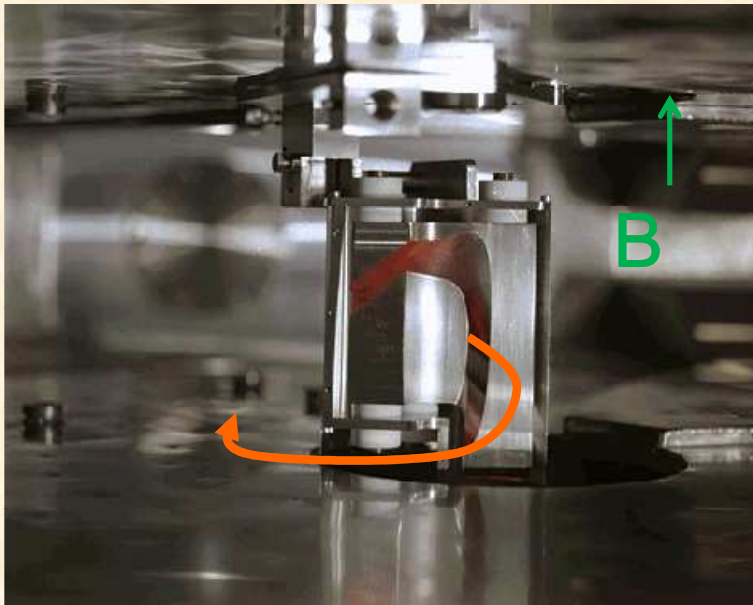
Magnetic radius $R_m = B\rho / B_0$

Axial injection 1: Spiral inflector



- Consists of 2 cylindrical capacitors which have been twisted to take into account the spiraling of the ion trajectory from magnet field.

- $\vec{v}_{beam} \perp \vec{E}$: central trajectory lies on an equipotential surface. Allows lower voltage than with mirrors.



- 2 free parameters (spiral size in z and xy) giving flexibility for central region design
- 100 % transmission

Axial injection 2: hyperboloid inflector

Spiral electrodes are complex :

hyperboloid inflector have simpler electrode

two electrodes equation : $r^2 - 2z^2 = r_1$ $r^2 - 2z^2 = r_2$

$$V = -Kz^2/2 + Kr^2/4 + c$$

Vertical field $Ez = -Kz$

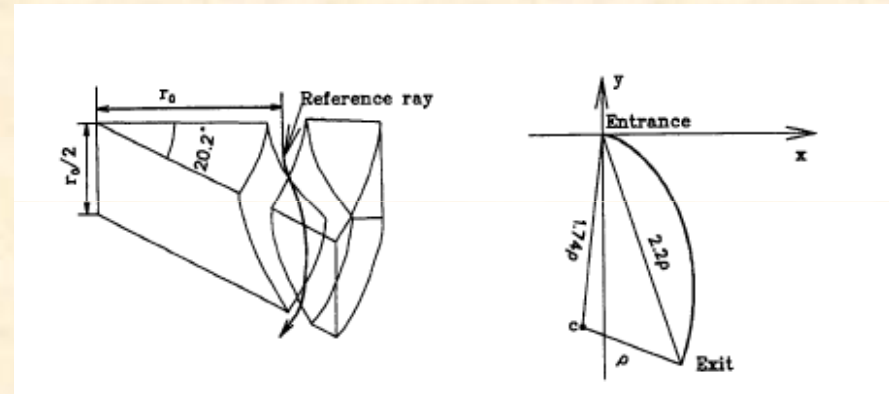
$$\begin{aligned} x &= \frac{r_0}{2} \{-b \cos(akt) + a \cos(bkt)\}, \\ y &= \frac{r_0}{2} \{-b \sin(akt) + a \sin(bkt)\}, \\ z &= \frac{r_0}{2} \sin(kt), \end{aligned}$$

$$k^2 = \frac{qK}{m}$$

$$r_0 = (2\sqrt{6})\rho.$$

$$k^2 = -qv^2/2$$

$$r_0 = 2 \cdot 6^{1/2} Rm$$



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$$\left. \begin{aligned} \varphi_0 &= \frac{1}{2}\pi + \Delta\varphi_0, \\ \Delta\varphi_0 &= k\Delta r_0 \approx \Delta\varphi(1\pi), \\ \Delta r_0 &= -\Delta\varphi(1\pi)/\delta\varphi(1\pi), \end{aligned} \right\} (14)$$

$$\left. \begin{aligned} \zeta_0 &= \Delta r_0, \quad \zeta_0 = f_0/k, \\ \eta_0 &= r_0\delta_0, \quad \eta_0 = r_0\delta_0/k, \quad s_0 = 0, \\ \zeta_0\sqrt{3} - \eta_0\sqrt{2} &= \Delta r_0\sqrt{5}, \\ \zeta_0\sqrt{3} - \eta_0\sqrt{2} &= (f_0/k)\sqrt{5}, \\ -\zeta_0\sqrt{2} - \eta_0\sqrt{3} &= \Delta r_0\sqrt{5}, \\ -\zeta_0\sqrt{2} - \eta_0\sqrt{3} &= (\zeta_0/k)\sqrt{5}, \\ s_0 &= -\frac{1}{2}r_0k\Delta r_0. \end{aligned} \right\} (17a)$$

so

$$\left. \begin{aligned} \dot{r}_0 &= \Delta r_0(1\pi) + \Delta r_0 f_0(1\pi), \\ \dot{z}_0 &= \Delta z_0(1\pi) + \Delta z_0 f_0(1\pi), \\ \dot{r}_0 &= \Delta r_0(1\pi) + \Delta r_0 f_0(1\pi) = \Delta r_0(1\pi), \\ \dot{z}_0 &= \Delta z_0(1\pi) + \Delta z_0 f_0(1\pi) = \Delta z_0(1\pi), \end{aligned} \right\} (15)$$

By transforming eq. (13) we get

$$\xi = [\zeta_0(1 - \frac{1}{2}\sin^2\varphi) + \zeta_0\sin\varphi\cos\varphi - (\frac{1}{2}\delta_0)\eta_0\sin^2\varphi] \cdot (1 + \frac{1}{2}\sin^2\varphi)^{-1},$$

$$\eta = -(\sqrt{6}\delta_0(1 + \frac{1}{2}\sin^2\varphi)^{-1}\sin^2\varphi + (\frac{1}{2}\sqrt{6}\delta_0(1 + \frac{1}{2}\sin^2\varphi)^{-1}\sin^2\varphi\cos\varphi + \eta_0(1 + \frac{1}{2}\sin^2\varphi)\cos\varphi + \eta_0(1 + \frac{1}{2}\sin^2\varphi)^{-1}\sin\varphi),$$

and eqs. (13) yield the transfer matrix

$$\begin{pmatrix} \dot{z}_0 \\ z_0 \\ \dot{r}_0 \\ r_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2}k\sqrt{6} & 0 \\ \frac{1}{2}\sqrt{6} & 0 & 0 & -k^{-1} \\ 0 & 0 & k & 0 \end{pmatrix} \begin{pmatrix} \dot{r}_0 \\ r_0 \\ \dot{z}_0 \\ z_0 \end{pmatrix} \quad (16)$$

$$s = 2\zeta_0\sin\varphi\cos\varphi + \zeta_0^2\sin^2\varphi + (\frac{1}{2}\sqrt{6})\eta_0\sin^2\varphi. \quad (18)$$

Simpler geometry than spiral inflector

But No free parameter (Rinjection=Rm fix all parameters)

Radial injection

Radial Injection for pre-accelerated beam :

- Compact inflector not possible (axial inj. not possible) :

- Higher rigidity (electrostatic field have “low efficiency”)

need space to bend the beam with large magnet !!

1. Injection into separated sector cyclotron (most common)

- More room for injection pieces and excellent transmission

2. Specific examples (not described here)

- Injection with Charge exchange (internal stripper foil)

in a compact superconducting cyclotron NSCL

- 3

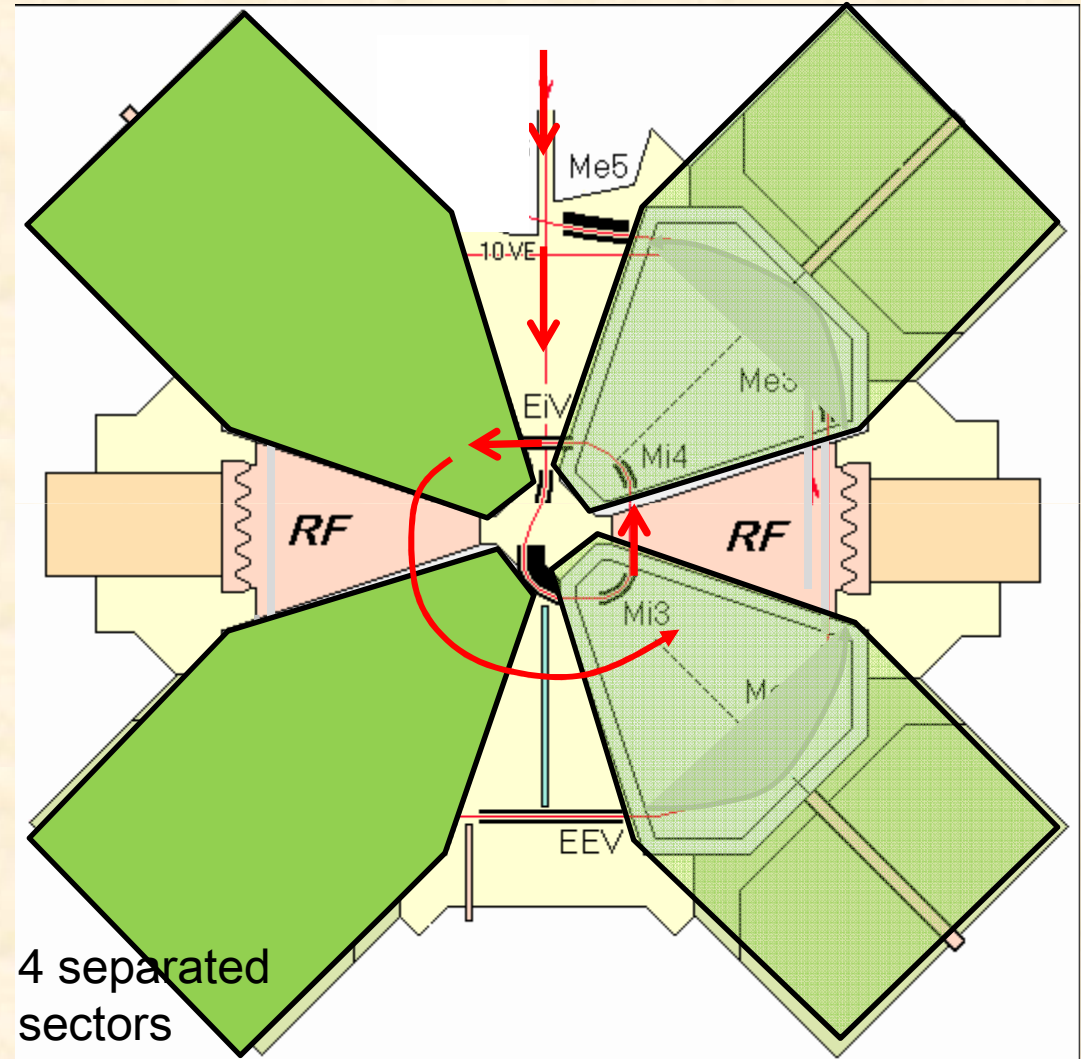
Example: Radial injection in ring cyclotron

- More room to insert bending elements.

Beam injected between sector magnets

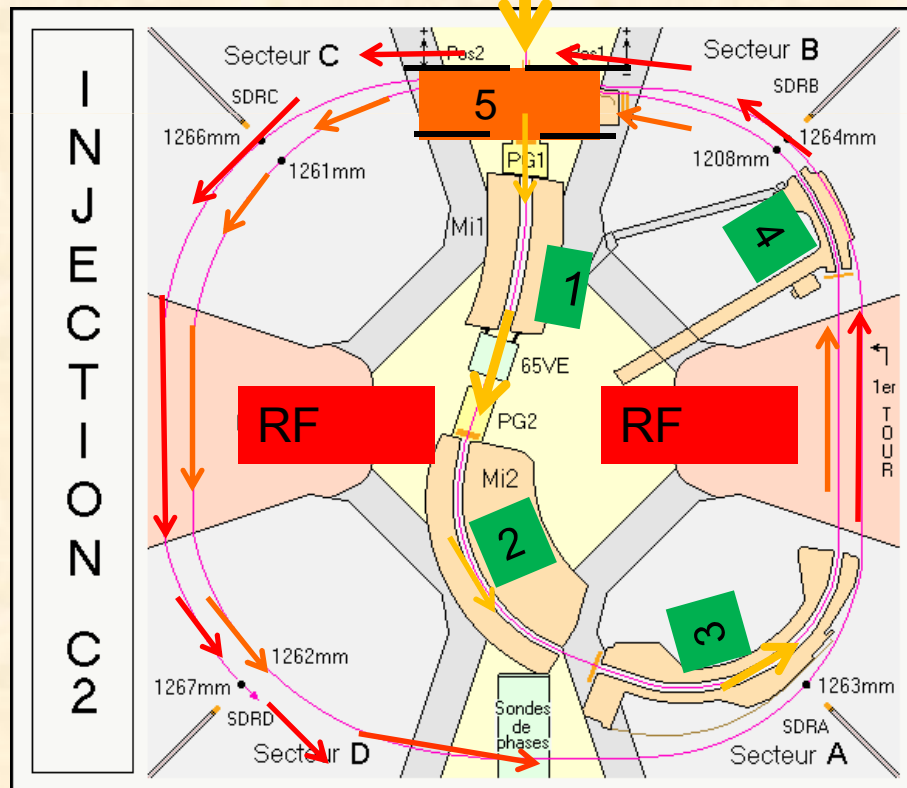
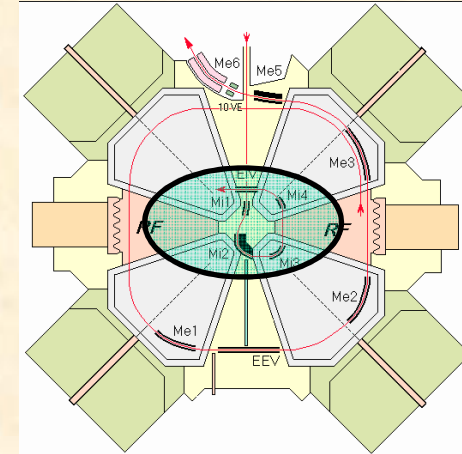
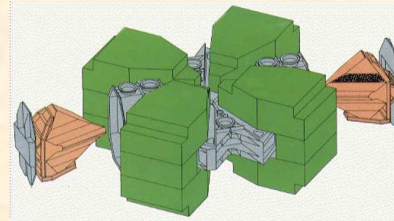
- The beam coming from the pre-injector enters the SSC horizontally.

- It is guided by 4 magnetic dipoles to the “good trajectory”, then an electrostatic inflector deflect the beam behind the dipole yokes.



Example: Radial injection

Beam from coming
an other accelerator



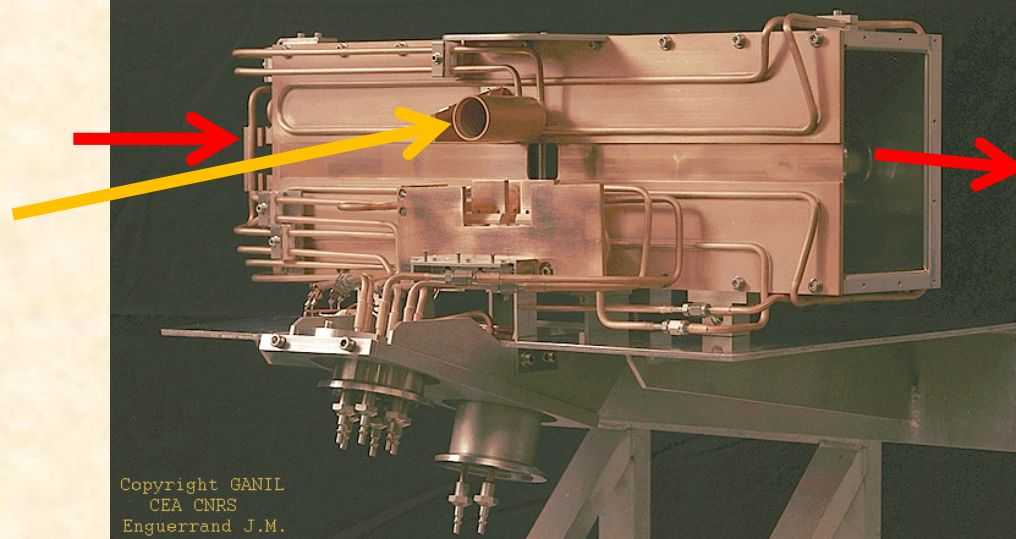
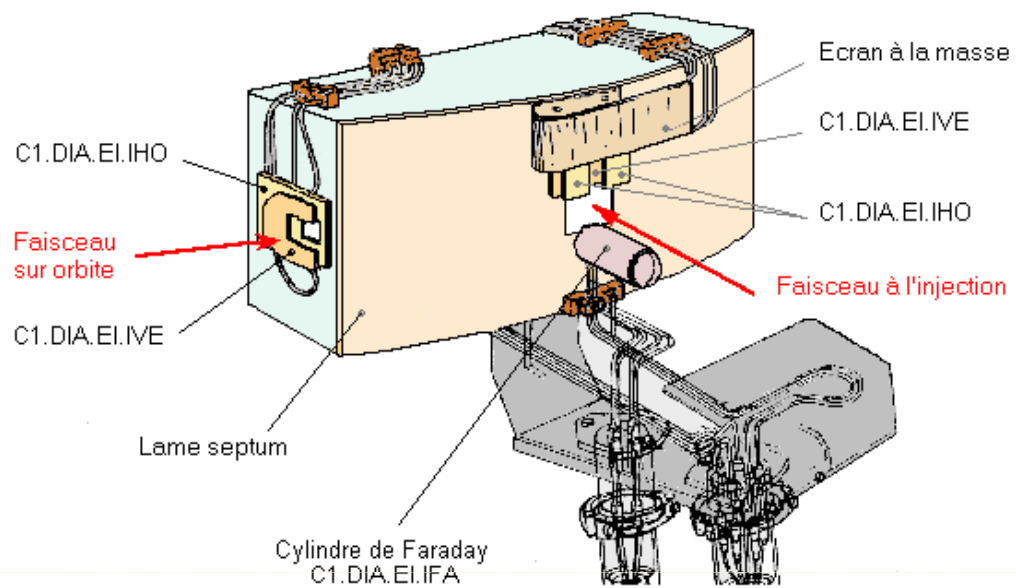
1-4 : magnetic dipoles

5 : electrostatic inflector

- Very carefull centering
- RF Phase ajustement with bunches



INFLECTEUR DE CSS1



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Extraction

1. Extraction by stripping negative ion

simpler and low cost , but restricted to H isotopes

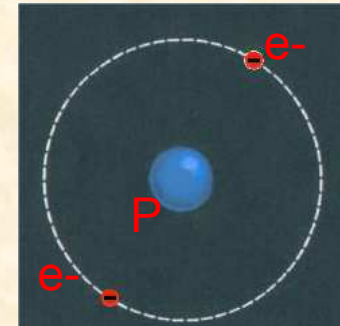
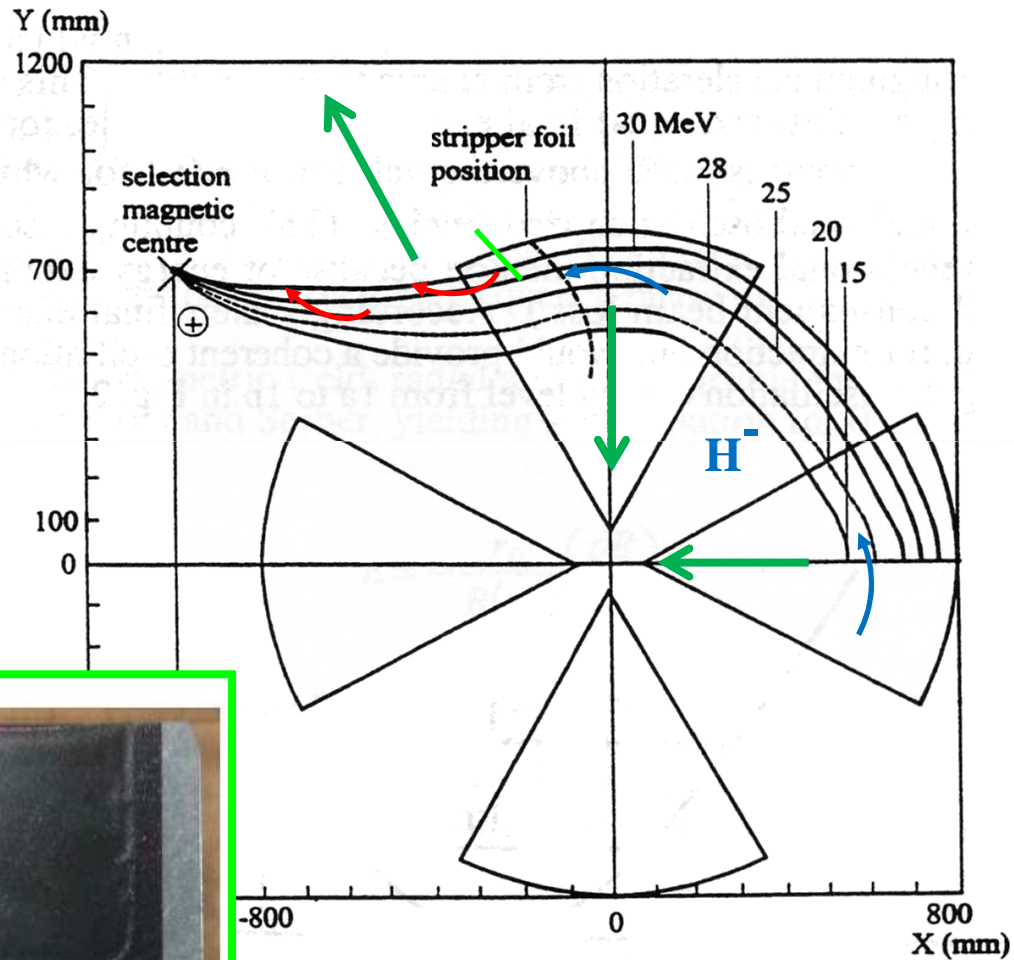
100% efficiency

2. Extraction using the radial separation

between turn n^{th} & $n^{\text{th}}+1$

Extraction by stripping negativ ions

easy and efficient with H^-



The magnetic force is inverted

$$F_r \sim -v.Bz \Rightarrow +v.Bz$$



Carbon foil Stripper

Extraction orbits in the IBA Cyclone 30

H⁻ & D⁻ commercial cyclotrons with two extracted beam

Low cost extraction beam line(s) :

less complex than **electrostatic deflectors**

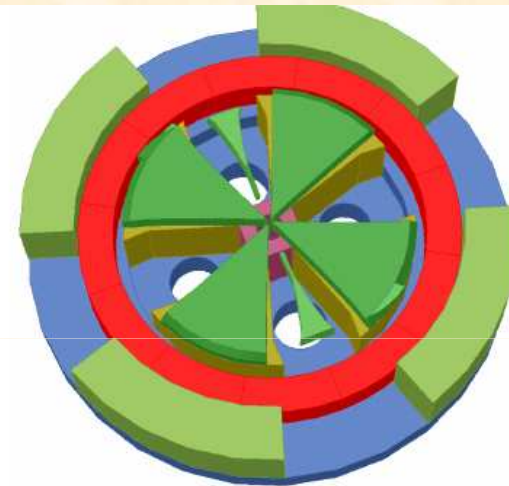
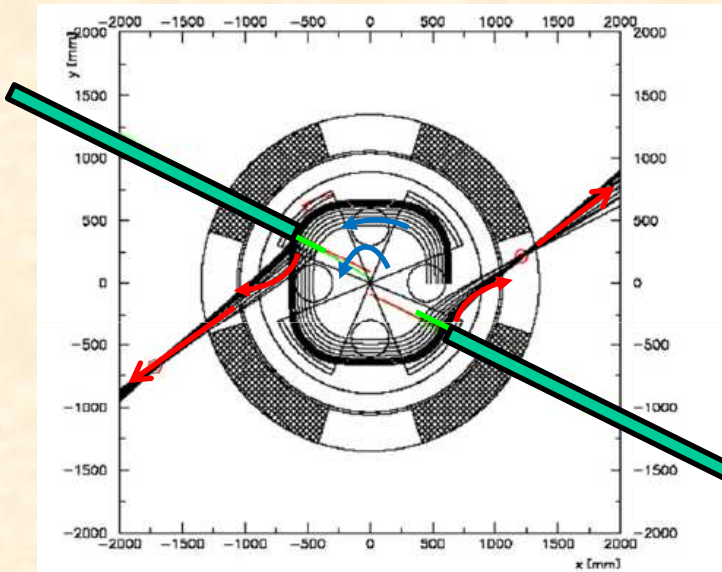
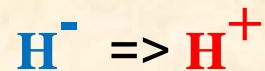


Figure 8: Final Cyclone® 30XP model.

H⁻ production or D⁻ production with an internal source (PIG)

2 strippers at extraction radius :



good beam quality, easy maintenance

Extraction by turn separation

1. Extraction by acceleration (and fringe field)

The orbit radial δr separation between 2 turns is :

$$\delta r = r \times \frac{\delta E}{E} \times \frac{\gamma}{\gamma + 1} \times \frac{1}{v_r^2}$$

- δE : Energy gain per turn as high as possible (RF)
- v_r : Accelerate the beam to fringing field (Bz decrease, $n > 0$, $v_r \searrow$)

Demonstration :

$$\frac{\delta B}{B} = -n \frac{\delta r}{r}$$

$$\begin{aligned} \frac{\delta r}{r} &= \frac{\delta B \rho}{B \rho} = \frac{\delta \langle B \rangle R}{\langle B \rangle R} = \frac{\delta R}{R} \Big|_{acc} - \frac{\delta B}{B} \\ &= \frac{\delta P_{acc}}{P} + n \frac{\delta r}{r} = \frac{\delta P_{acc}}{P} \frac{1}{(1-n)} \approx \frac{\delta P}{P} \frac{1}{v_r^2} \approx \frac{1}{2} \frac{\delta E_{acc}}{E} \frac{1}{v_r^2} \end{aligned}$$

$$v_r = \sqrt{1-n}$$

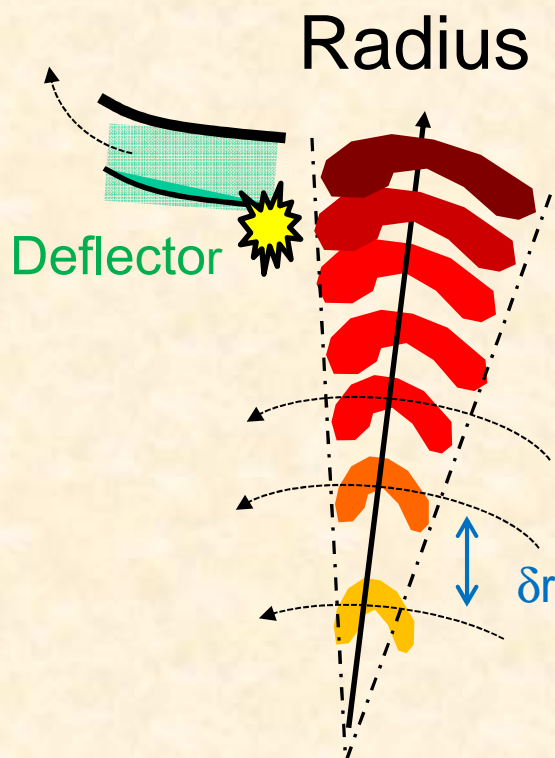
Extraction using turn separation δr

if $\Delta r < \delta r$ Each turn separated

Extraction with an electrostatic deflector

100% efficiency

SINGLE TURN EXTRACTION



$\Delta r > \delta r$ (size > turn separation)

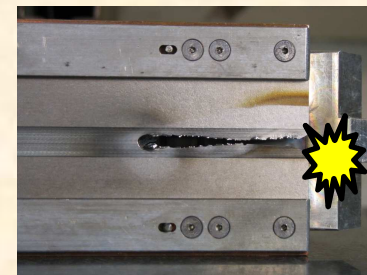
last turns are not separated

Beam losses in the extraction channel

Multi TURN EXTRACTION

Deflector sparking or damaged

$$\delta r \propto \frac{1}{\text{Radius}}$$



Extraction : 3 mechanisms possible

Goal : High extraction efficiency with well separated orbit

$$\Delta r = \text{Acceleration} + \text{Precession} + \text{increase oscillation by a field bump (resonance extraction)}$$

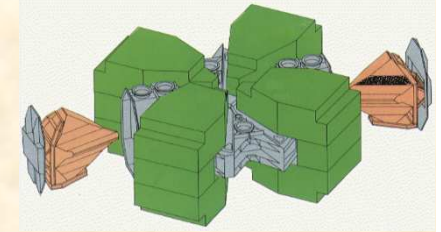
1. Extraction by acceleration (and fringe field + deflector)
 - Energy gain per turn as high as possible...
2. Precession extraction : radial oscillations help to separate orbits

$$r(N) = r_0(N) + x_0 \sin(v_r \cdot \omega_0 t)$$

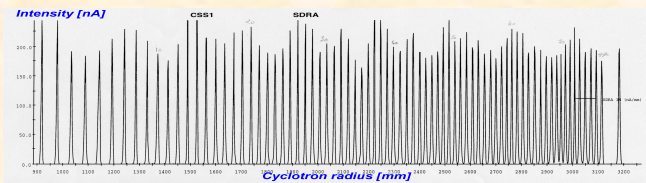
3. Resonant extraction : increase the precession by a field bump

If turn separation not enough then magnetic perturbations are used. Particles are forced to oscillate around their equilibrium orbit with a magnetic bump

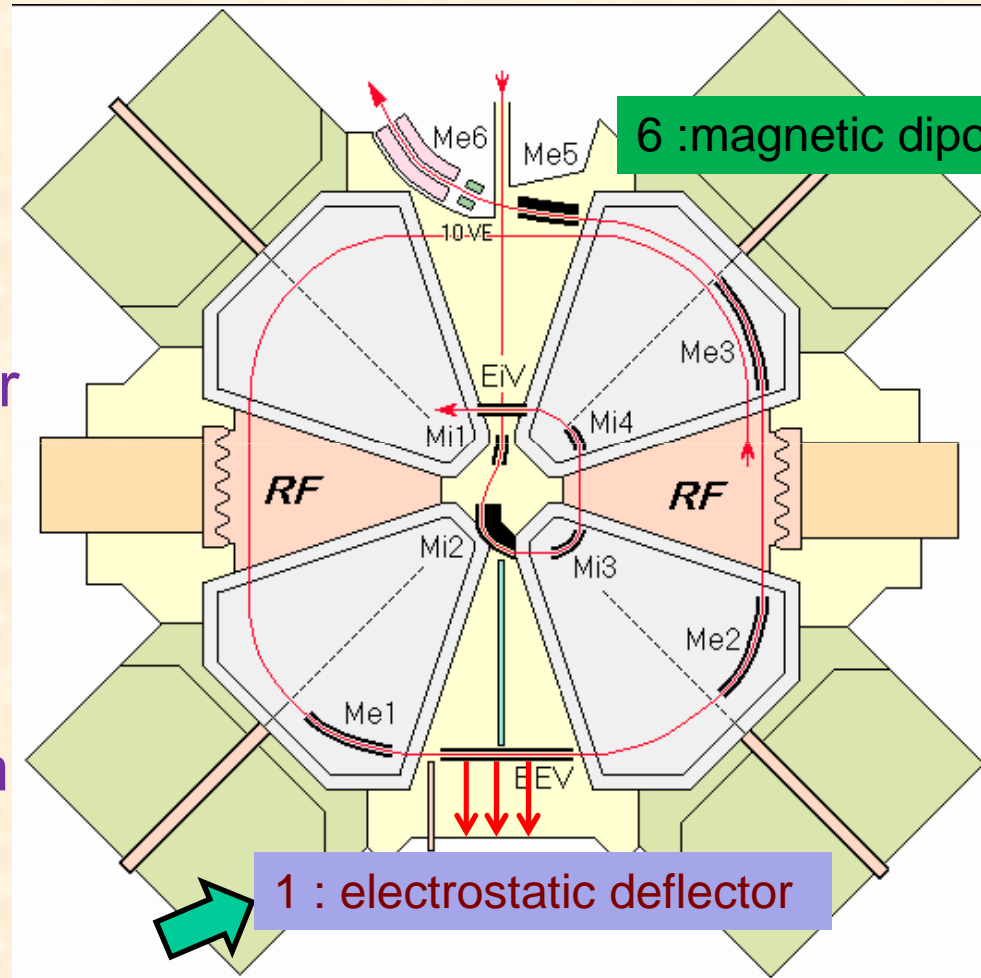
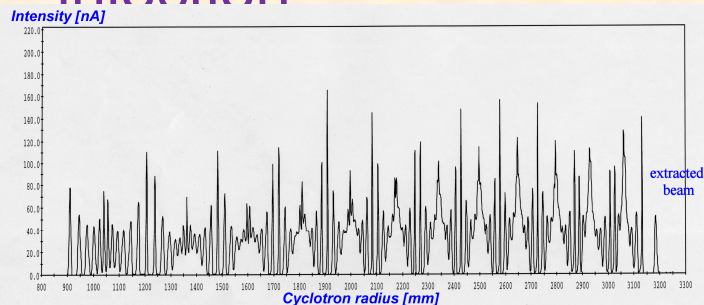
Example: Ejection SSC



- Acceleration (2 RF cavities)
- + Radial kick on the extracted turn With an electrostatic deflector

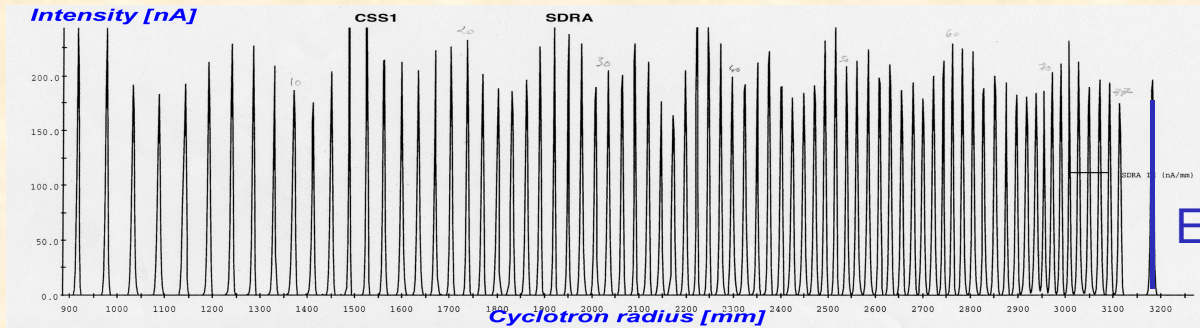


- + Precession : excited from injection



Extraction with precession

Well centered beam orbits **Separated Sector Cyclo N°1 GANIL**



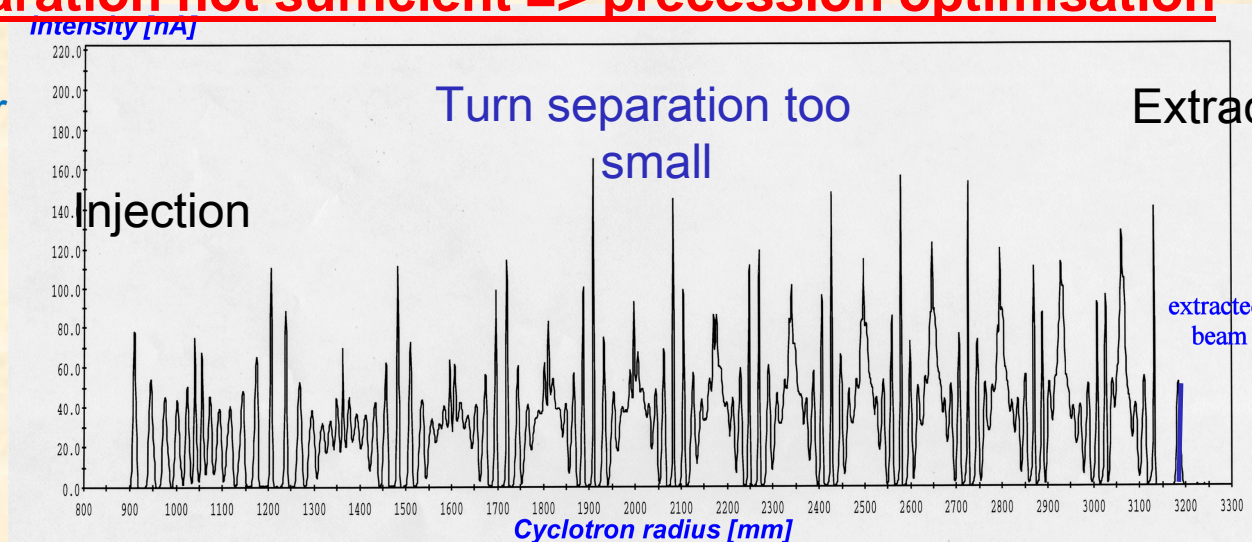
Turn separation sufficient

$$\Delta r < \delta r$$

Precession for optimized extraction **Separated Sector Cyclo N°2 GANIL**

Turn separation not sufficient => precession optimisation

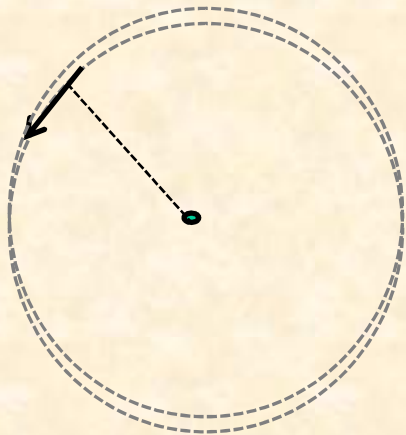
$$\Delta r > \delta r$$



transmission 95%

Resonant extraction : with $Q_x = \nu_r \sim 1$

Step 1 : circular motion
+ small oscillations

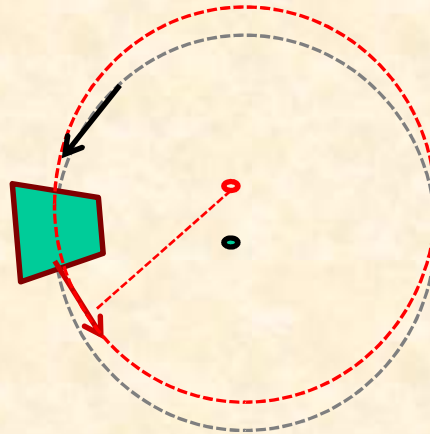


$$\ddot{x} + \nu_r^2 \omega_0^2 \cdot x = 0$$

$\nu_r \sim 1$: 1 oscillation per turn

Step 2 : A magnetic bump
shift of the orbit center

larger deviation

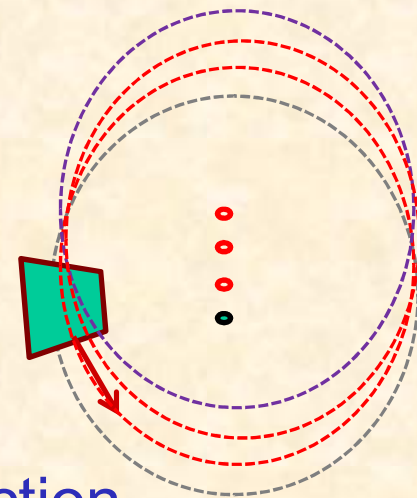


Step 3 : Several turns
produce

Large amplitude oscillation

Larger & Larger & Larger

Large δr = easy extraction



$$\delta r \approx \left[\frac{1}{2} \frac{\delta E_{RF}}{E} \right] + \Delta x_0 \sin(\nu_r \cdot \omega t)$$

Resonant extraction shown by equations

Radial Equation without Perturbation

$$\ddot{x} + \nu_r^2 \omega_0^2 \cdot x = 0$$

Equation with Perturbation $\delta B_z \sim b_M(r) \cos(M\theta)$

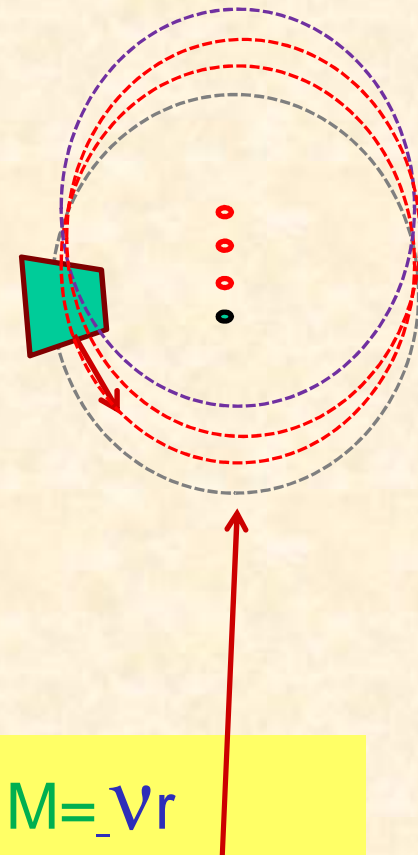
$$\ddot{x} + [\nu_r \omega_0]^2 x = \omega_0^2 \frac{r}{B} \frac{db_M}{dr} \cos(M \omega_0 t)$$

Driven oscillator excited at the « frequency » M

if the excitation is at the resonance frequency $M = \nu_r$
you get Large amplitude oscillations

One field Bump correspond to harmonic $M=1$

$$\sim b_1 \cdot \exp(-\theta^2)$$



Cyclotrons

- End Chapter 2