

Joint Universities Accelerator School

JUAS 2017

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Normal-conducting accelerator magnets

Lecture 2: Analytical design

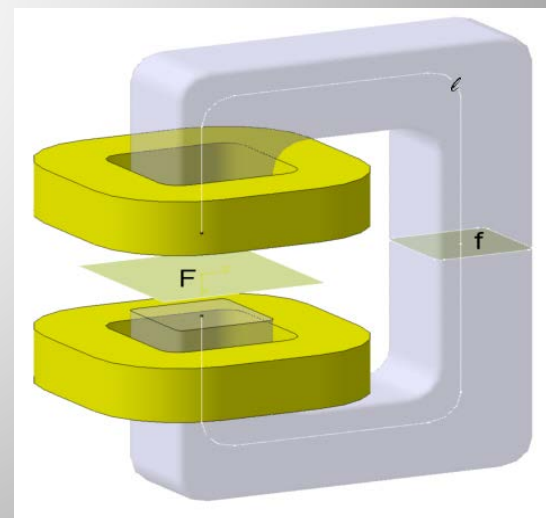
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CERN



Lecture 2: Analytical design

- Goals in magnet design
- What do we need to know before starting?
- Defining the requirements & constraints
- Deriving the magnet main parameters
- Coil design and cooling





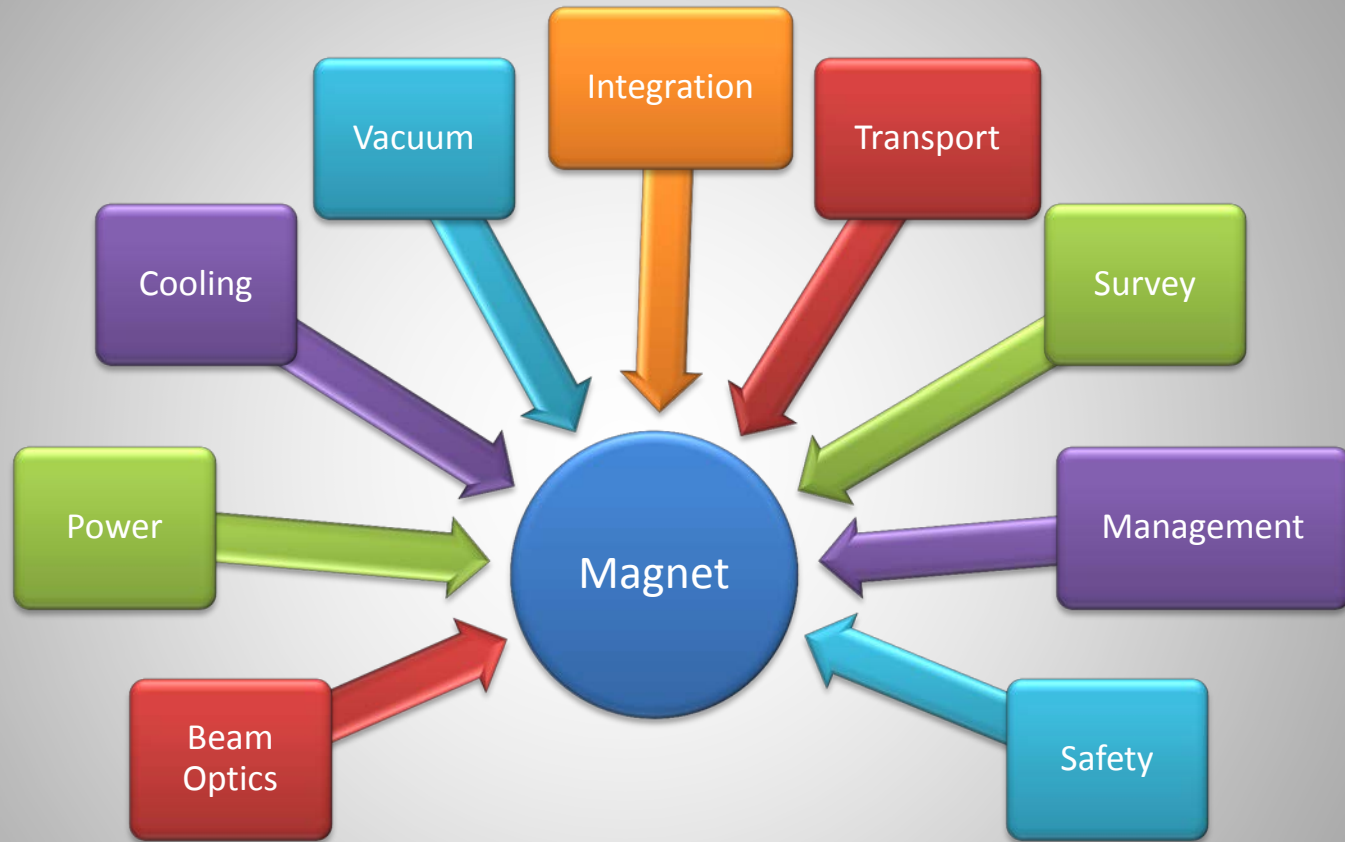
Goals in magnet design

The goal is to produce a product just **good enough** to perform **reliably** with a sufficient **safety factor** at the **lowest cost** and on **time**.

- Good enough:
 - Obvious parameters are clearly specified, but tolerance difficult to define
 - Tight tolerances lead to increased costs
- Reliability:
 - Get MTBF and MTTR reasonably low
 - Reliability is usually unknown for new design
 - Requires experience to search for a compromise between extreme caution and extreme risk (expert review)
- Safety factor:
 - Allows operating a device under more demanding condition as initially foreseen
 - To be negotiated between the project engineer and the management
 - Avoid inserting safety factors a multiple levels (costs!)



Magnet interfaces

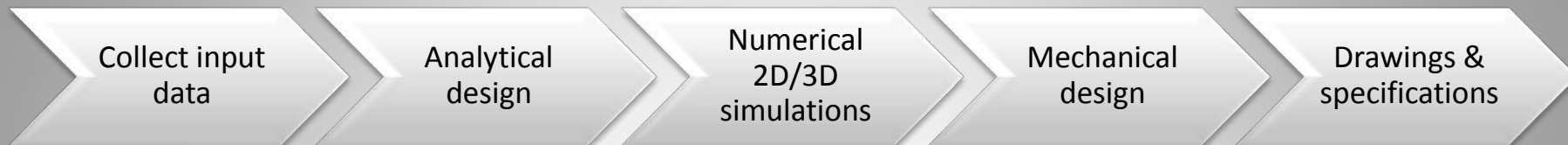


A magnet is not a stand-alone device!

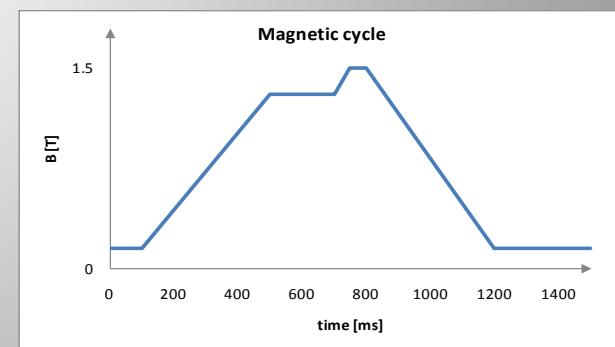
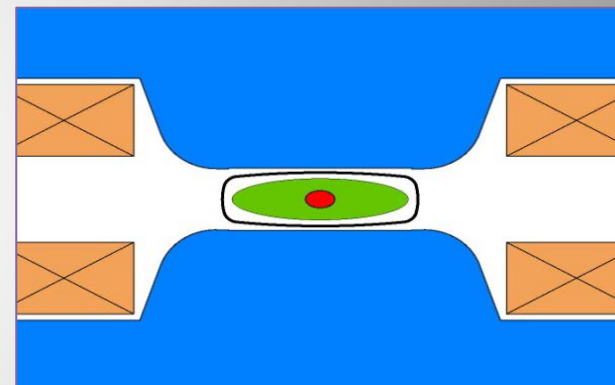


Design process

Electro-magnetic design is an iterative process:



- Field strength (gradient) and magnetic length
- Integrated field strength (gradient)
- Aperture and ,good field region‘
- Field quality:
 - field homogeneity
 - maximum allowed multi-pole errors
 - settling time (time constant)
- Operation mode: continuous, cycled
- Electrical parameters
- Mechanical dimensions
- Cooling
- Environmental aspects





General requirements

Magnet type and purpose

- Dipole: bending, steering, extraction
- Quadrupole, sextupole, octupole
- Combined function, solenoid, special magnet

Installation

- Storage ring, synchrotron light source, collider
- Accelerator
- Beam transport lines

Quantity

- Installed units
- Spare units (~10 %)



Performance requirements

Beam parameters

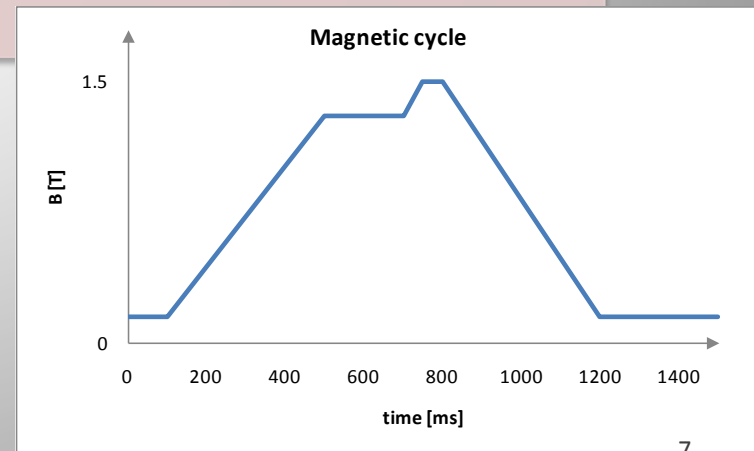
- Type of beam, energy range, deflection angle
- Integrated field (gradient)
- Local field (gradient) and magnetic length

Aperture

- Physical aperture
- 'Good field region'

Operation mode

- Continuous
- Pulse-to-pulse modulation (ppm)
- Ramp rate (T/s)

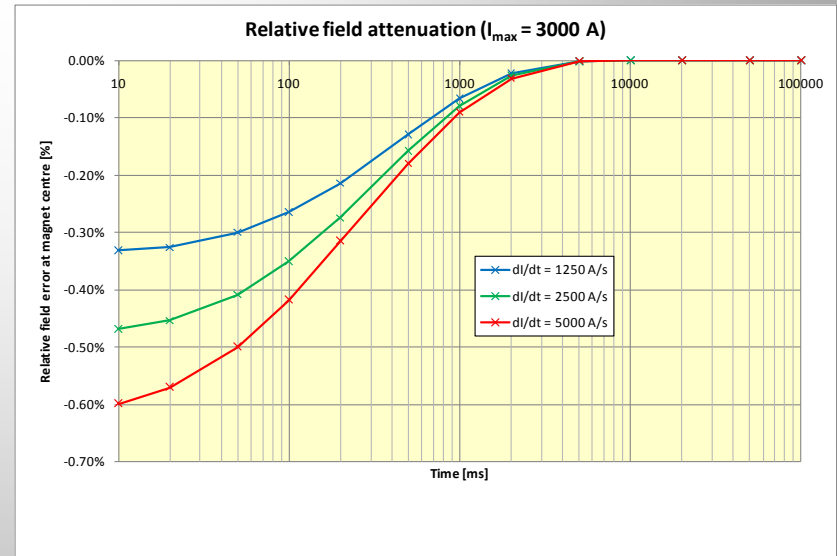
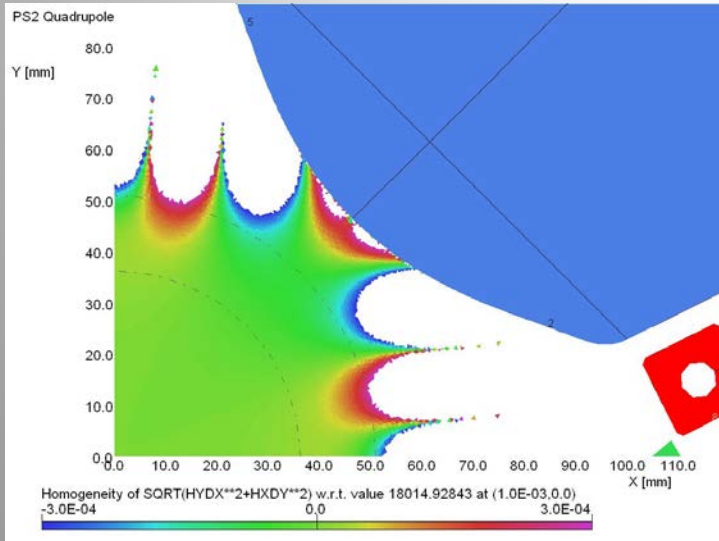




Performance requirements

Field quality

- Homogeneity (uniformity)
- Maximum allowed multipole errors
- Stability & reproducibility
- Settling time (time constant)
- Allowed residual field





Physical requirements

Geometric boundaries

- Available space
- Transport limitations
- Weight limitations

Accessibility

- Crane
- Connections (electrical, hydraulic)
- Alignment targets





Interfaces

Equipment linked to the magnet is defining the boundaries and constraints

Power converter

- Max. current (peak, RMS)
- Max. voltage
- Pulsed/dc

Cooling

- Max. flow rate and pressure drop
- Water quality (aluminium/copper circuit)
- Inlet temperature
- Available cooling power

Vacuum

- Size and material of vacuum chamber
- Space for pumping ports, bake-out
- Captive vacuum chamber



Environmental aspects

Other aspects, which can have an influence on the magnet design

Environment temperature

- Risk of condensation
- Heat dissipation into the tunnel

Ionizing radiation

- High radiation levels require radiation hard materials
- Special devices to allow fast repair/replacement

Electro-magnetic compatibility

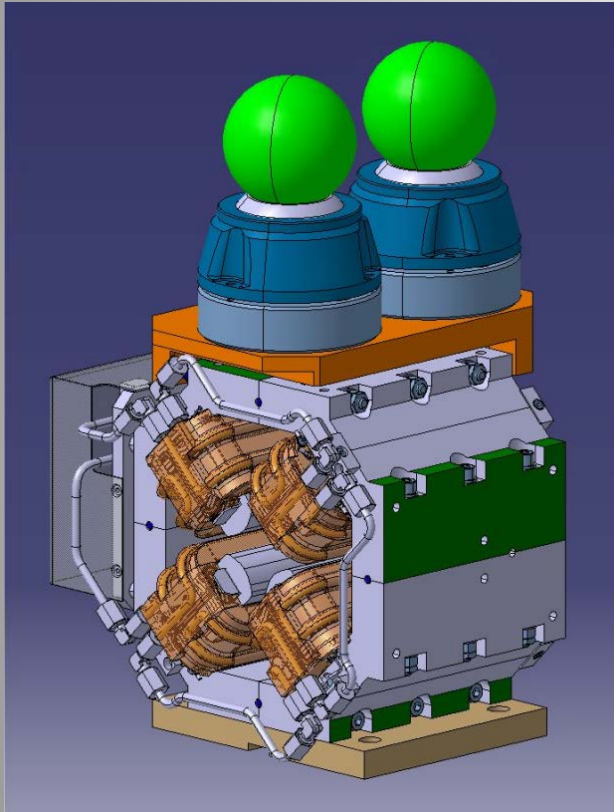
- Magnetic fringe fields disturbing other equipment (beam diagnostics)
- Surrounding equipment perturbing field quality

Safety

- Electrical safety
- Interlocks



Magnet Components



Alignment targets

Yoke

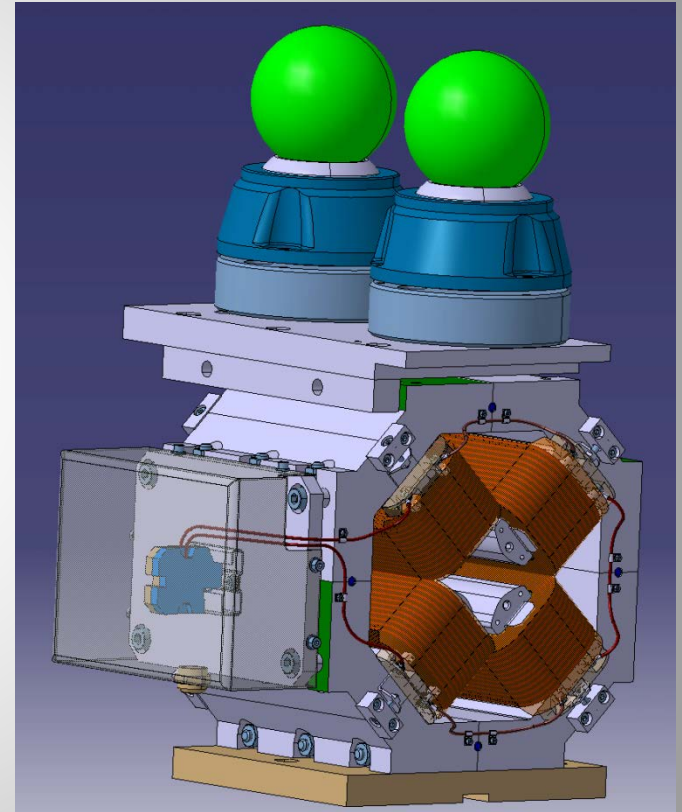
Coils

Sensors

Cooling circuit

Connections

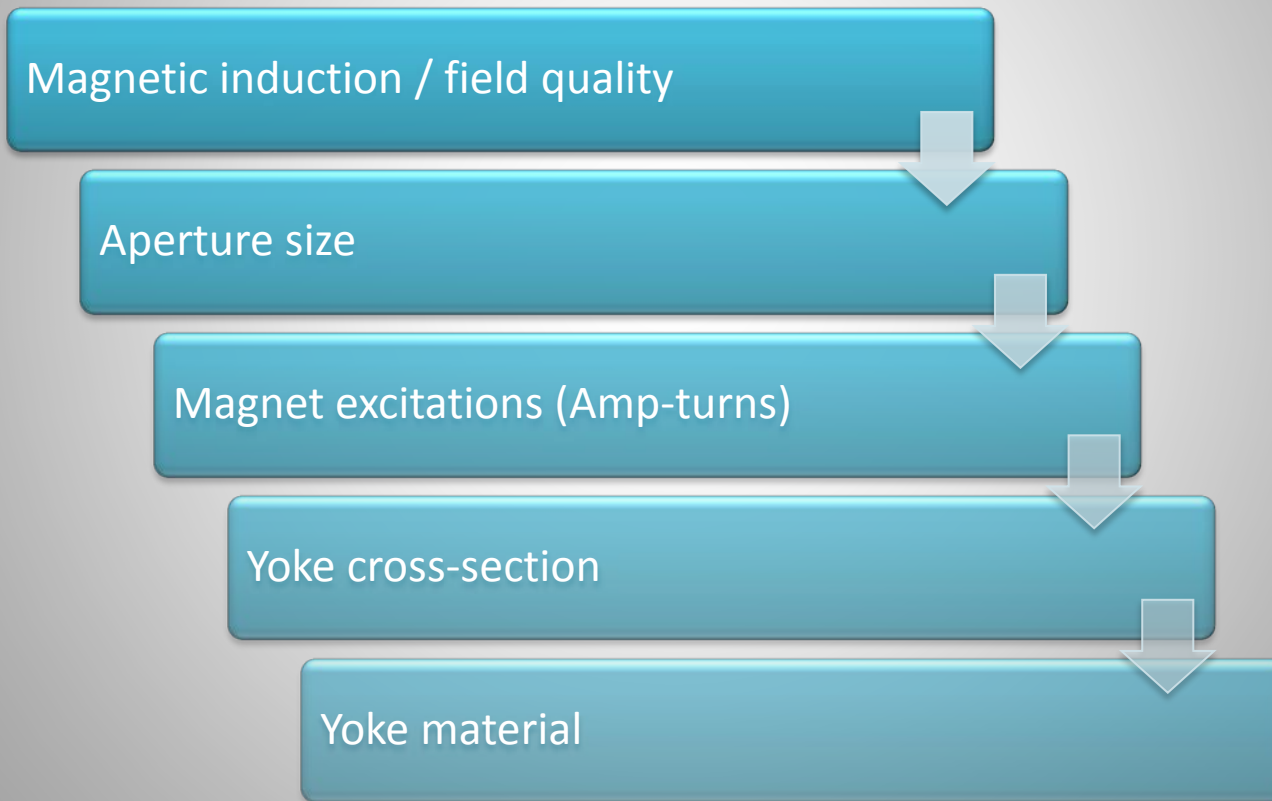
Support





Magnetic design

Translate the beam optic requirements into a magnetic design





Beam rigidity

$$\text{Beam rigidity } (B\rho) \text{ [Tm]: } (B\rho) = \frac{p}{q} = \frac{1}{qc} \sqrt{T^2 + 2T E_0}$$

p : particle momentum [kg m/s]

q : particle charge number [Coulombs]

c : speed of light [m/s]

T : kinetic beam energy [eV]

E_0 : particle rest mass energy [eV]

(0.51 MeV for electrons, 938 MeV for protons)

“ ...resistance of the particle beam against a change of direction when applying a bending force...”



Magnetic induction

Dipole bending field B [T]:

$$B = \frac{(B\rho)}{r_M}$$

B : Flux density or magnetic induction
(vector) [T]

r_M : magnet bending radius [m]

Quadrupole field gradient B' [T/m]:

$$B' = (B\rho)k$$

k : quadrupole strength [m^{-2}]

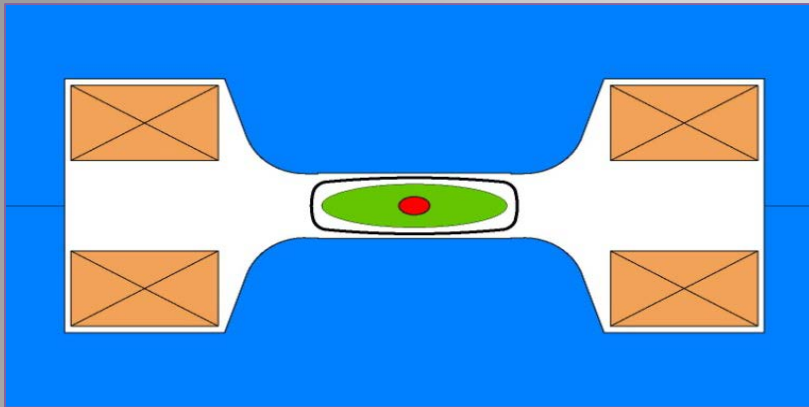
Sextupole differential gradient B'' [T/ m^2):

$$B'' = (B\rho)m$$

m : sextupole strength [m^{-3}]



Aperture size



Max. beam size envelope (typical 3-sigma)

- Lattice functions: beta functions and dispersion
- Geometrical transverse emittances (energy depended)
- Momentum spread

$$\sigma = \sqrt{\varepsilon \beta + \left(D \frac{\Delta p}{p} \right)^2}$$

Closed orbit distortions (few mm)

Good field region

Aperture

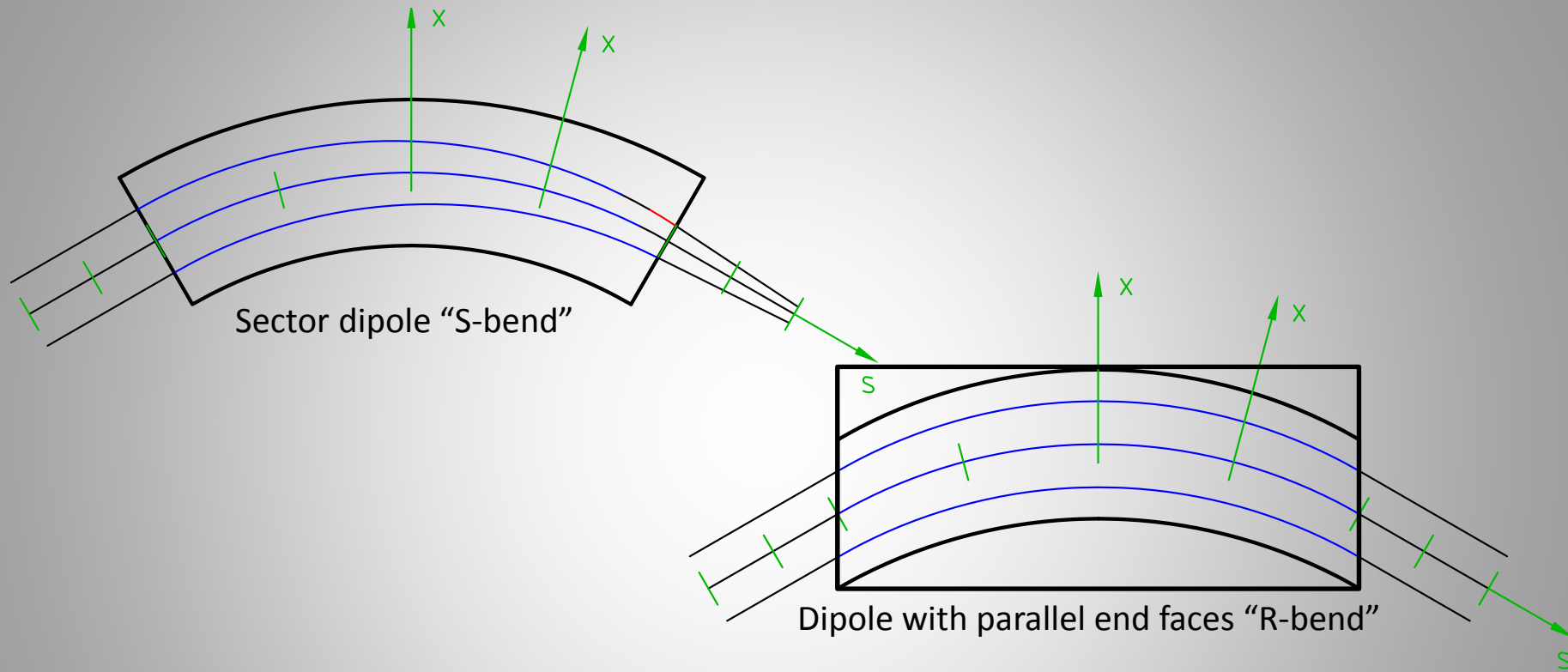
Vacuum chamber thickness (0.5 – 5 mm)

Installation and alignment margin (0 – 10 mm)

“...good field region: region where the field quality has to be within certain tolerances...”



Excursion: S-bend vs. R-bend



The two types are slightly different in terms of focusing:

- S-bend: focuses horizontally
- R-bend: no horizontal focusing, but small vertical defocusing at the edges

Note: the curvature has no effect, it is just for saving material, otherwise the pole would have to be wider ("*sagitta*").



Excitation current in a dipole

Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

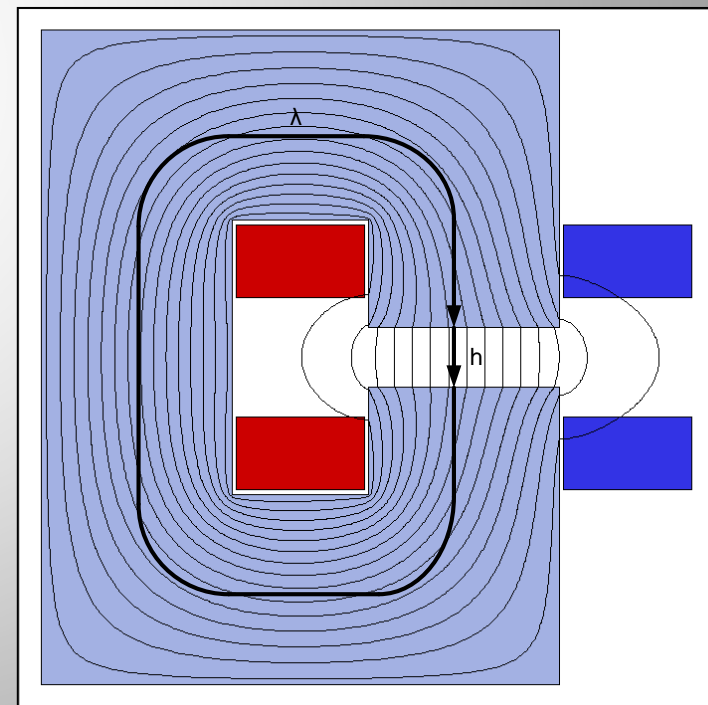
$$\text{leads to } NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{\text{gap}} \frac{\vec{B}}{\mu_{\text{air}}} \cdot d\vec{l} + \int_{\text{yoke}} \frac{\vec{B}}{\mu_{\text{iron}}} \cdot d\vec{l} = \frac{Bh}{\mu_{\text{air}}} + \frac{B\lambda}{\mu_{\text{iron}}}$$

assuming, that B is constant along the path

If the iron is not saturated: $\frac{h}{\mu_{\text{air}}} \gg \frac{\lambda}{\mu_{\text{iron}}}$

$$\text{then: } NI_{(\text{per pole})} \approx \frac{Bh}{2\eta\mu_0}$$

h : gap height [m]
 η : efficiency (typically 95% - 99 %)





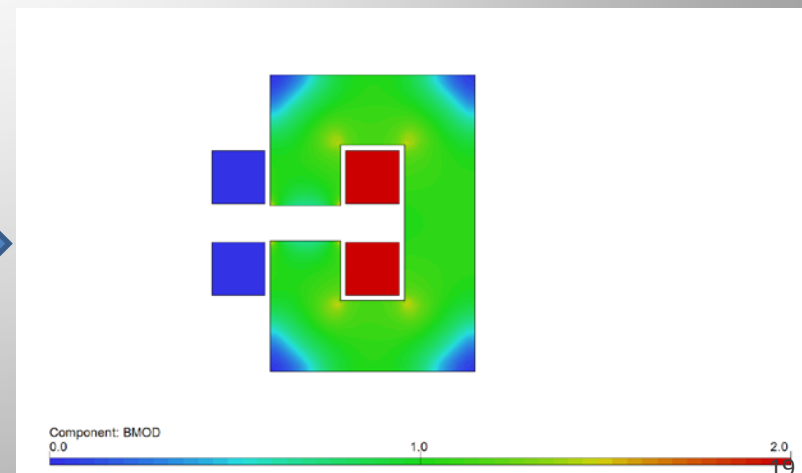
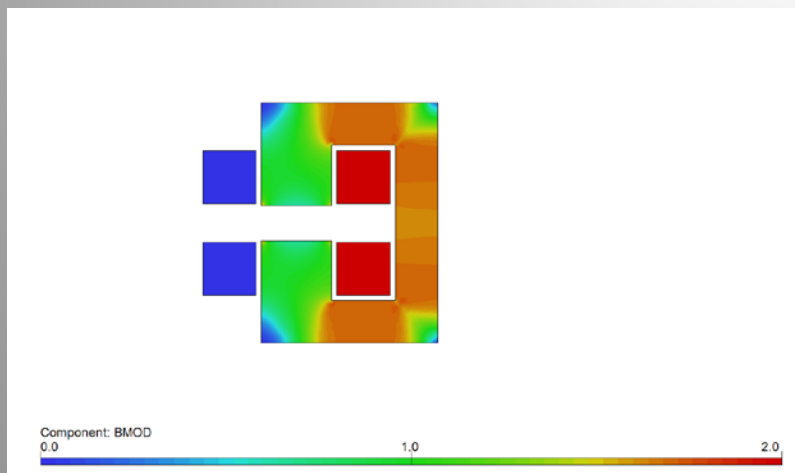
Reluctance and efficiency

Reluctance:
$$R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$$

Term $\left(\frac{\lambda}{\mu_{iron}}\right)$ in previous slide is called 'normalized reluctance' of the yoke

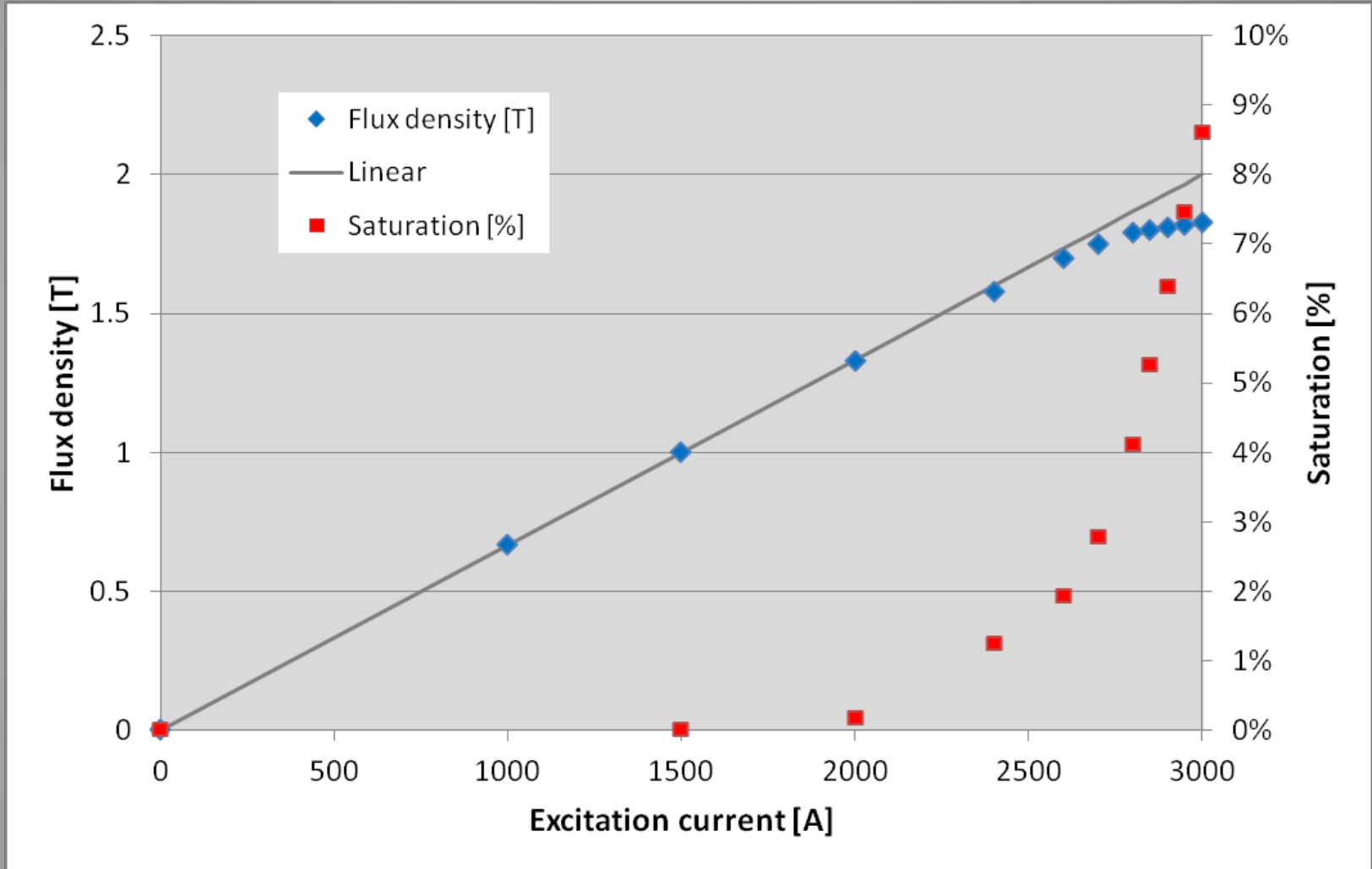
Keep iron yoke reluctance less than a few % of air reluctance $\left(\frac{h}{\mu_0}\right)$ by providing sufficient iron cross section ($B_{iron} < 1.5$ T)

Efficiency:
$$\eta = \frac{R_{M,gap}}{R_{M,gap} + R_{M,yoke}} \approx 99\%$$





Saturation





Pole design

It is easy to derive perfect mathematical pole configurations for a specific field configuration

In practice poles are not ideal: finite width and end effects result in multipole errors disturbing the main field

The uniform field region is limited to a small fraction of the pole width

Estimate the size of the poles and calculate the resulting fields

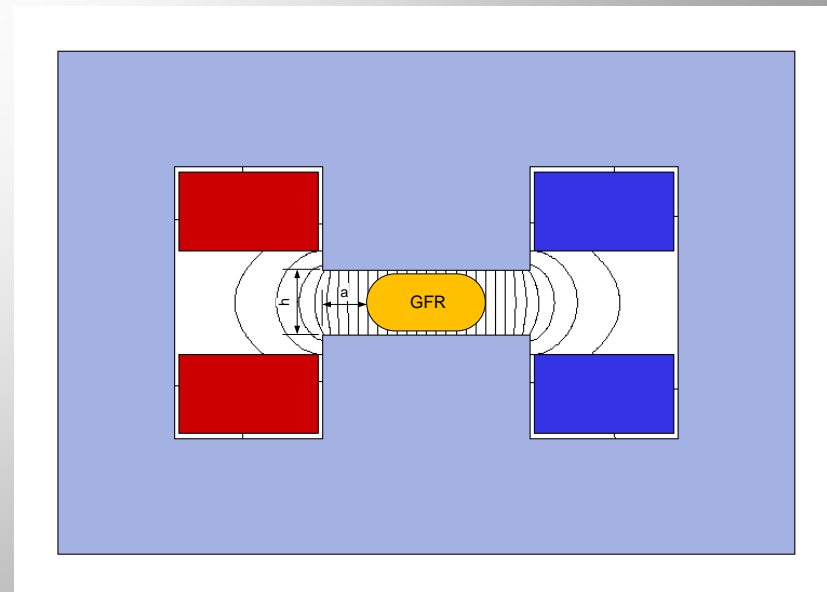
Better approach: calculate the necessary pole overhang for an un-optimized* design

$$x_{unoptimized} = 2 \frac{a}{h} = -0.36 \ln \frac{\Delta B}{B_0} - 0.90$$

x : pole overhang normalized to the gap

a : pole overhang: excess pole beyond the edge of the good field region to reach the required field uniformity

h : magnet gap

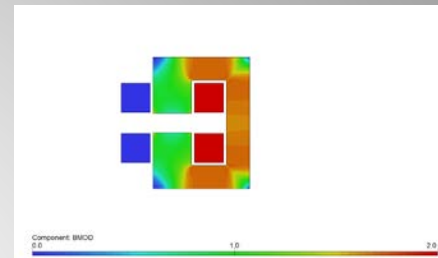


*) see Lecture 4 for corresponding formula using an optimized pole design



Magnetic flux

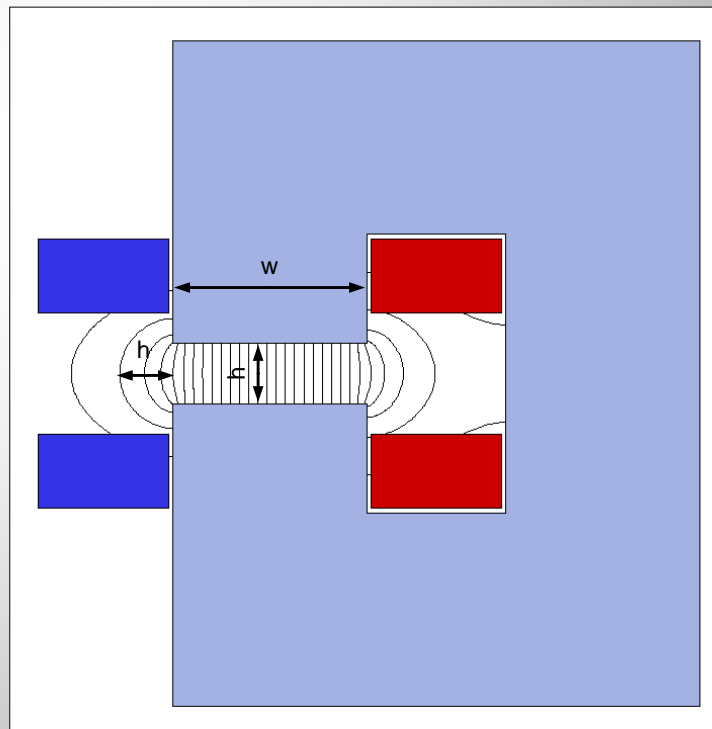
Flux in the yoke includes the gap flux and stray flux



Total flux in the return yoke:

$$\Phi = \int_A B \cdot dA \approx B_{gap} (w + 2h) l_{mag}$$

$$B_{leg} \cong B_{gap} \frac{w + 2h}{w_{leg}}$$





Magnetic length

Coming from ∞ , B increases towards the magnet center (stray flux)

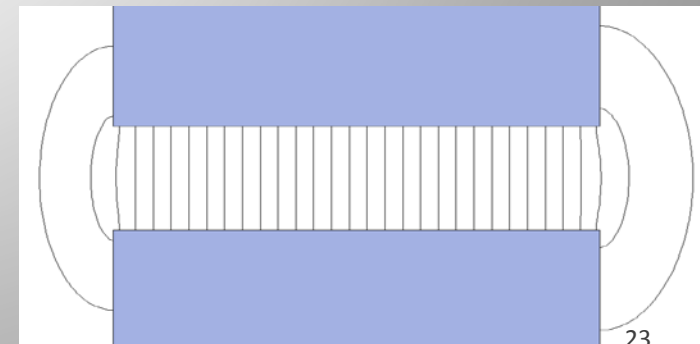
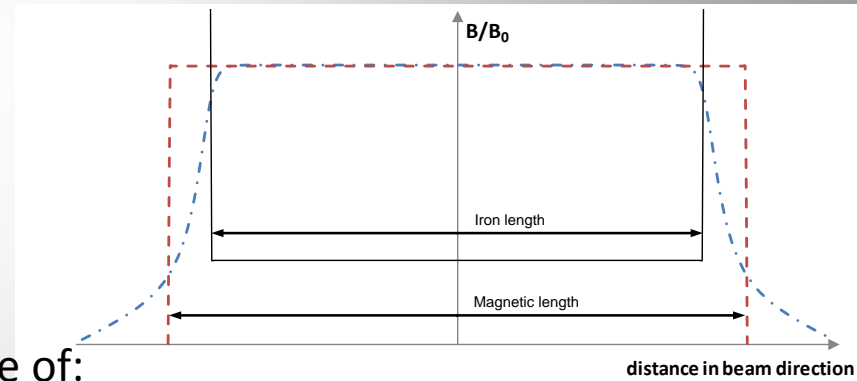
Magnetic length:
$$l_{mag} = \frac{\int_{-\infty}^{\infty} B(z) \cdot dz}{B_0}$$

'Magnetic' length > iron length

Approximation for a dipole:
$$l_{mag} = l_{iron} + 2hk$$

Geometry specific constant k gets smaller in case of:

- pole length < gap height
- saturation
- precise determination only by measurements or numerical calculations





Excitation current in a Quadrupole



Choosing the shown integration path gives:

$$NI = \oint \vec{H} \cdot d\vec{l} = \int_{s1} \vec{H}_1 \cdot d\vec{l} + \int_{s2} \vec{H}_2 \cdot d\vec{l} + \int_{s3} \vec{H}_3 \cdot d\vec{l}$$

For a quadrupole, the gradient $B' = \frac{dB}{dr}$ is constant and $B_y = B'x$ $B_x = B'y$

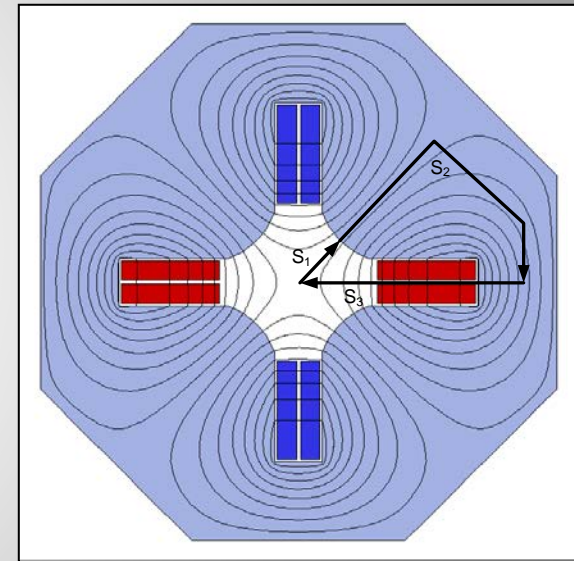
Field modulus along s_1 : $H(r) = \frac{B'}{\mu_0} \sqrt{x^2 + y^2} = \frac{B'}{\mu_0} r$

Neglecting H in s_2 because: $R_{M,s2} = \frac{s_2}{\mu_{iron}} \ll \frac{s_1}{\mu_{air}}$

and along s_3 : $\int_{s3} \vec{H}_3 \cdot d\vec{l} = 0$

Leads to: $NI \approx \int_0^R H(r) dr = \frac{B'}{\mu_0} \int_0^R r dr$

$$NI_{(per\ pole)} = \frac{B' r^2}{2\eta\mu_0}$$





Magnetic length

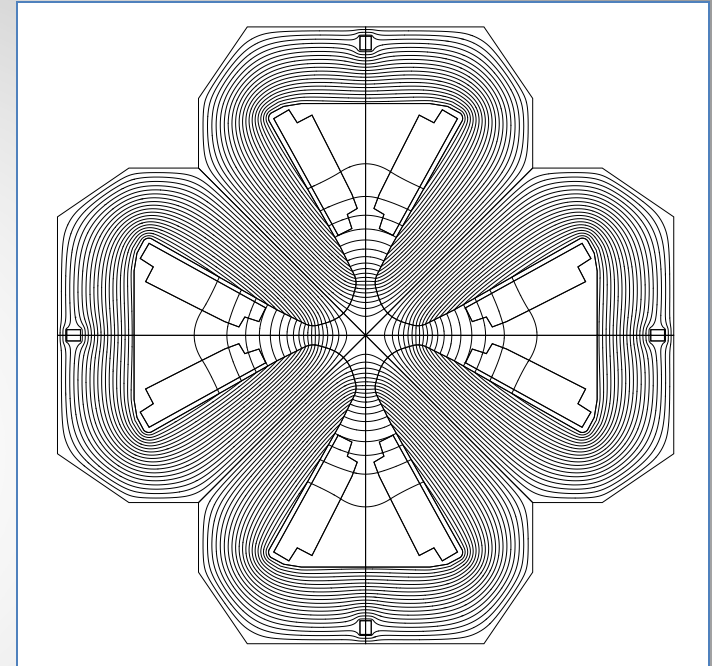
Magnetic length for a quadrupole:

$$l_{mag} = l_{iron} + 2rk$$

NI increases with the square of the quadrupole aperture:

$$NI \propto r^2$$

$$P \propto r^4$$



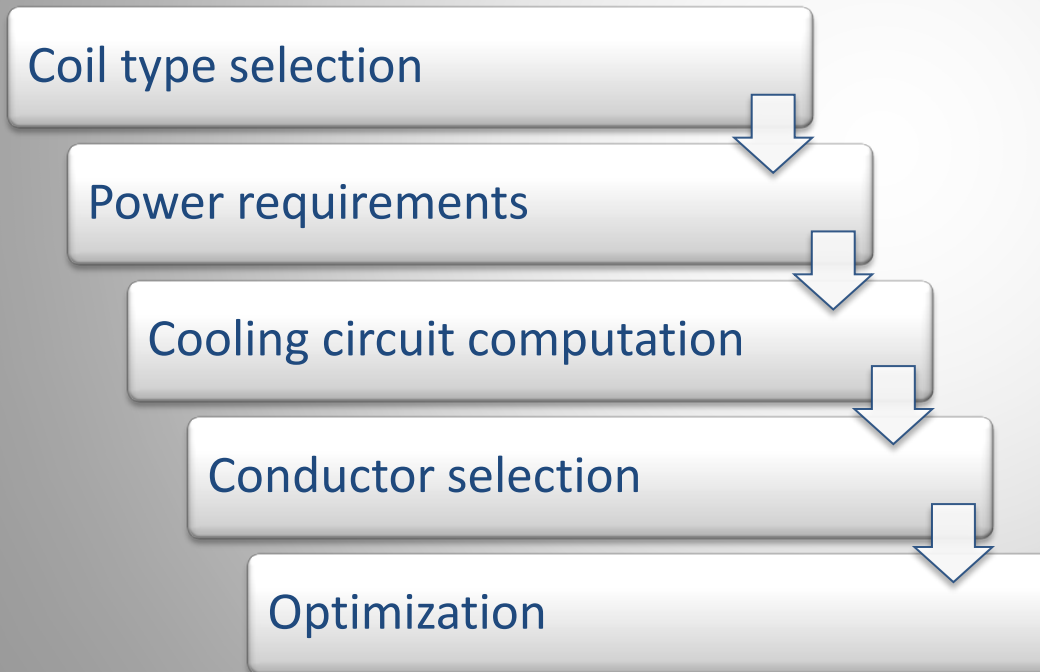
More difficult to accommodate the necessary Ampere-turns (= coil cross section)

→ truncating the hyperbola leads to a decrease in field quality



Coil design

Ampere-turns NI are determined, but the current density j , the number of turns N and the coil cross section need to be defined



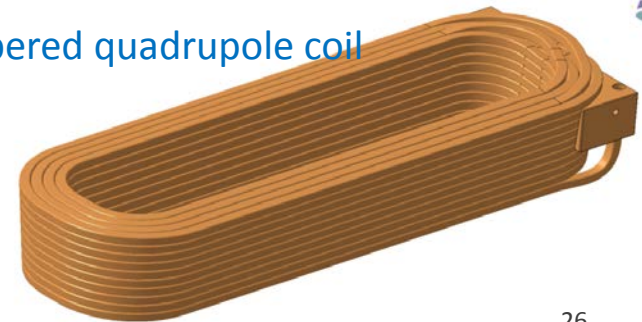
Bedstead or saddle coil



Racetrack coil



Tapered quadrupole coil





Current density

Assuming the magnet cross-section and the yoke length are known, one can estimate the total dissipated power per magnet:

$$P_{dipole} = \rho \frac{Bh}{\eta\mu_0} j l_{avg} 10^6$$

$$P_{quadrupole} = 2\rho \frac{B' r^2}{\eta\mu_0} j l_{avg} 10^6$$

- For a constant geometry, the power loss P is **proportional** to the current density j
- The current density j has a **direct impact** on coil size, coil cooling, power converter choice, operation costs and investment costs

$$j: \quad \text{current density [A/mm}^2\text{]}; \quad j = \frac{NI}{f_c A} = \frac{I}{a_{cond}}$$

ρ : resistivity [Ωm] of coil conductor

l_{avg} : average turn length [m]; approximation: $2.5 l_{iron} < l_{avg} < 3 l_{iron}$ for racetrack coils

a_{cond} : conductor cross section [mm^2]

A : coil cross section [mm^2]

f_c : filling factor = $\frac{\text{net conductor area}}{\text{coil cross section}}$

(includes geometric filling factor, insulation, cooling duct, edge rounding)

Note: If the magnet is not operated in dc, the rms power has to be considered.



Number of turns

The determined ampere-turns NI have to be divided into N and current I

Basic relations: $P_{magnet} \propto j$ $V_{magnet} \propto Nj$ $R_{magnet} \propto N^2 j$

Large N = low current = high voltage

- Small terminals
- Small conductor cross-section
- Thick insulation for coils and cables
- Less good filling factor in the coils
- Large coil volume
- Low power transmission loss

Small N = high current = low voltage

- Large terminals
- Large conductor cross-section
- Thin insulation in coils and cables
- Good filling factor in the coils
- Small coil volume
- High power transmission loss

The number of turns N are chosen to match the impedances of the power converter and connections

Attention when ramping the magnet: $V_{tot} = RI + L \frac{dI}{dt}$





Practical exercise

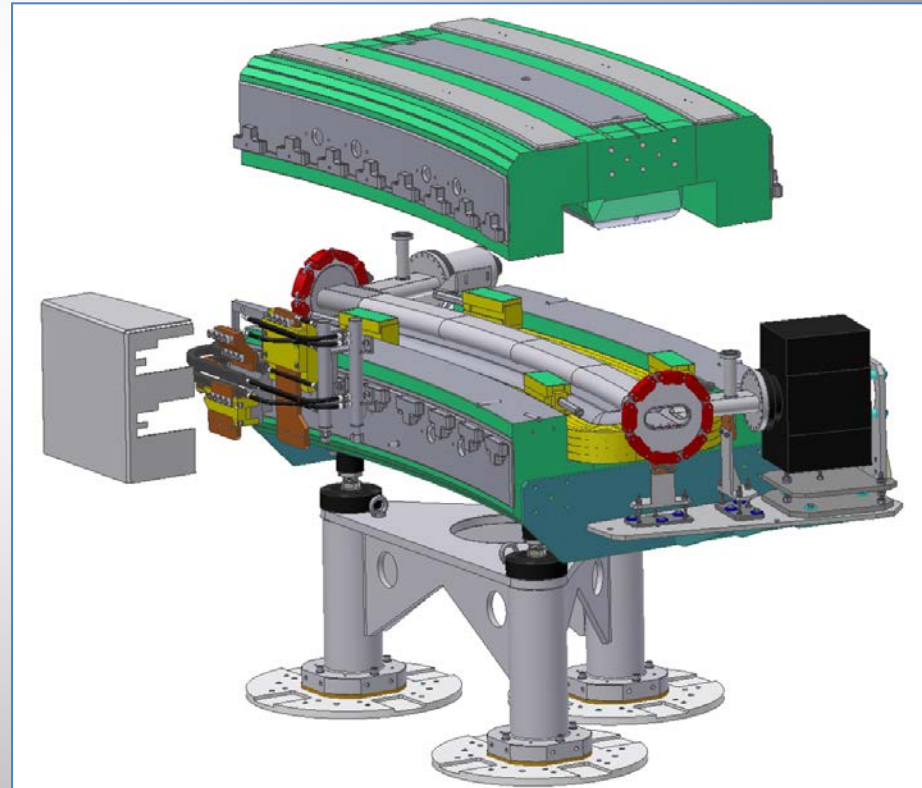
MedAustron: ion therapy facility near Vienna/Austria

Providing beam energies from 120 to 400 MeV/u for carbon ions and from 60 to 220 MeV for protons

16 synchrotron bending magnets:

- Bending angle: 22.5°
- Bending radius: 4.231 m
- Field ramp rate: 3.75 T/s
- Max. current: 3000 A
- Overall length: < 2 m
- Good field region: 120 x 56 mm
- Field quality:

$$\frac{\Delta \int B \cdot dl}{\int B \cdot dl} = 2 \cdot 10^{-4}$$





Practical exercise

Magnet aperture:

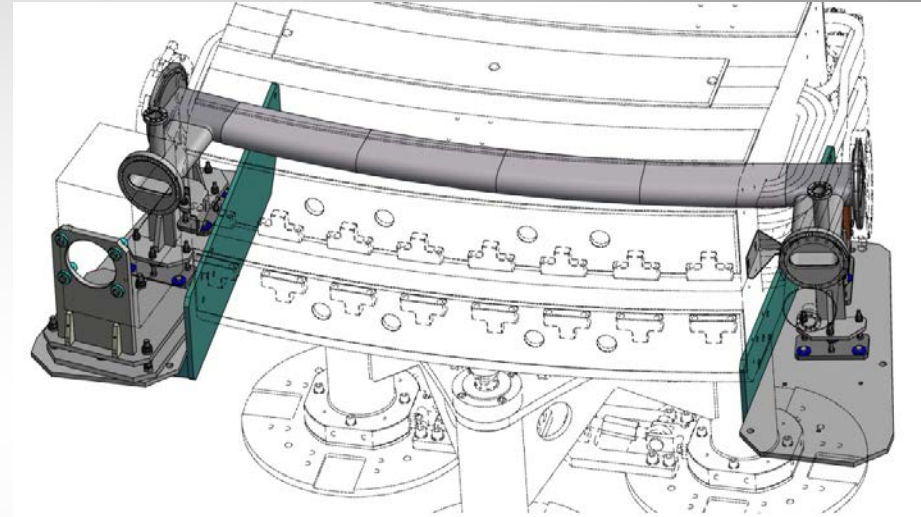
Horizontal GFR: ± 60 mm

Vertical GFR: ± 28 mm

Vacuum chamber thickness: 5 mm

Tolerances for installation: 2.5 mm

Insulation thickness: 0.5 mm



- Excitation current = ?
- Current density j = ?
- Current I = ?
- Number of turns N (per pole) = ?



Coil cooling

Air cooling by natural convection:

- Current density
 - $j \leq 2 \text{ A/mm}^2$ for small, thin coils
- Cooling enhancement:
 - Heat sink with enlarged radiation surface
 - Forced air flow (cooling fan)
- Only for magnets with limited strength (e.g. correctors)

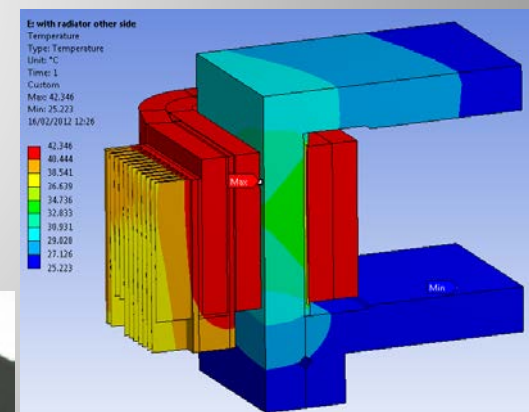


Direct water cooling:

- Typical current density $j \leq 10 \text{ A/mm}^2$
- Requires demineralized water (low conductivity) and hollow conductor profiles

Indirect water cooling:

- Current density $j \leq 3 \text{ A/mm}^2$
- Tap water can be used





Direct water cooling

Practical recommendations and canonical values:

- Water cooling: $2 \text{ A/mm}^2 \leq j \leq 10 \text{ A/mm}^2$
- Pressure drop: $1 \leq \Delta p \leq 10 \text{ bar}$ (possible up to 20 bar)
- Low pressure drop might lead to more complex and expensive coil design
- Flow velocity should be high enough so flow is turbulent
- Flow velocity $u_{avg} \leq 4 \text{ m/s}$ to avoid erosion and vibrations
- Acceptable temperature rise: $\Delta T \leq 30^\circ\text{C}$
- For advanced stability: $\Delta T \leq 15^\circ\text{C}$

Assuming:

- Long, straight and smooth pipes without perturbations
- Turbulent flow = high Reynolds number ($Re > 4000$)
- Good heat transfer from conductor to cooling medium
- Temperature of inner conductor surface equal to coolant temperature
- Isothermal conductor cross section

Note: practical (non-SI) units are used in the following slides for convenience



Direct water cooling

Useful simplified formulas using **water** as cooling fluid:

Water flow Q [litre/min] necessary to remove power P : $Q_{water} = 14.3 \frac{P}{\Delta T} 10^{-3}$

P : dissipated power [W]

ΔT : temperature increase [°C]

Average water velocity u_{avg} [m/s] in a round tube: $u_{avg} = \frac{Q}{A} = 66.67 \frac{Q}{\pi d^2}$

$A = \frac{\pi d^2}{4}$: tube section [mm²]

d : hydraulic diameter [mm]

Pressure drop Δp [bar]: $\Delta p \approx 60 l \frac{Q^{1.75}}{d^{4.75}}$ (from Blasius' law)

l : cooling circuit length [m]

Reynolds number Re []: $Re = d \frac{u_{avg}}{\nu} 10^{-3}$

Re : dimensionless quantity used to help predict similar flow patterns in different fluid flow situations

ν : kinematic viscosity of coolant is temperature depending, for simplification it is assumed to be constant ($6.58 \cdot 10^{-7}$ m²/s @ 40°C for water)



Cooling circuit design

Already determined: current density j , power P , current I , number of turns N

1. Select number of layers m and number of turns per layer n
2. Round up N if necessary to get reasonable (integer) numbers for n and m
3. Define coil height c and coil width b : $A = bc = \frac{NI}{jf_c}$ (Aspect ratio $c : b$ between 1 : 1 and 1 : 2 and $0.6 \leq f_c \leq 0.8$)
4. Calculate average turn length $l_{avg} = \text{pole perimeter} + 4b$
5. The total length of cooling circuit $l = \frac{K_c N l_{avg}}{K_w}$ (start with single cooling circuit per coil)
6. Select ΔT , Δp and calculate cooling hole diameter $d = 0.5 \left(\frac{P}{\Delta T K_w} \right)^{0.368} \left(\frac{l}{\Delta p} \right)^{0.21}$
7. Change Δp or number of cooling circuits, if necessary
8. Determine conductor area $a = \frac{I_{nom}}{j} + \frac{d^2 \pi}{4} + r_{edge} (4 - \pi)$
9. Select conductor dimensions and insulation thickness
10. Verify if resulting coil dimensions, N , R , I , V , ΔT are still compatible with the initial requirements (if not, start new iteration)
11. Compute coolant velocity and coolant flow
12. Verify if Reynolds number is inside turbulent range ($Re > 4000$)

K_c : Number of coils

K_w : Number of cooling circuits per coil



Cooling circuit design

Number of cooling circuits per coil: $\Delta p \propto \frac{1}{K_w^3}$

→ Doubling the number of cooling circuits reduces the pressure drop by a factor of eight for a constant flow

Diameter of cooling channel: $\Delta p \propto \frac{1}{d^5}$

→ Increasing the cooling channel by a small factor can reduce the required pressure drop significantly



Summary

- Before starting the design, all input **parameters**, **requirements**, **constraints** and **interfaces** have to be known and well understood (prepare a checklist or functional specification!)
- **Analytical** design is necessary to derive the main parameters of the future magnet **before** entering into a detailed design using **numerical** methods
- Magnet design is an **iterative process** often requiring a high level of experience and/or educated guessing
- Critically **review** your final design and compare it with the initial requirements