



UNIVERSITÀ DEGLI STUDI DI NAPOLI  
**FEDERICO II**



# **Not All But a Bit of All About Accelerators**

a personal perspective from backstage

**Prof. Vittorio Giorgio VACCARO**

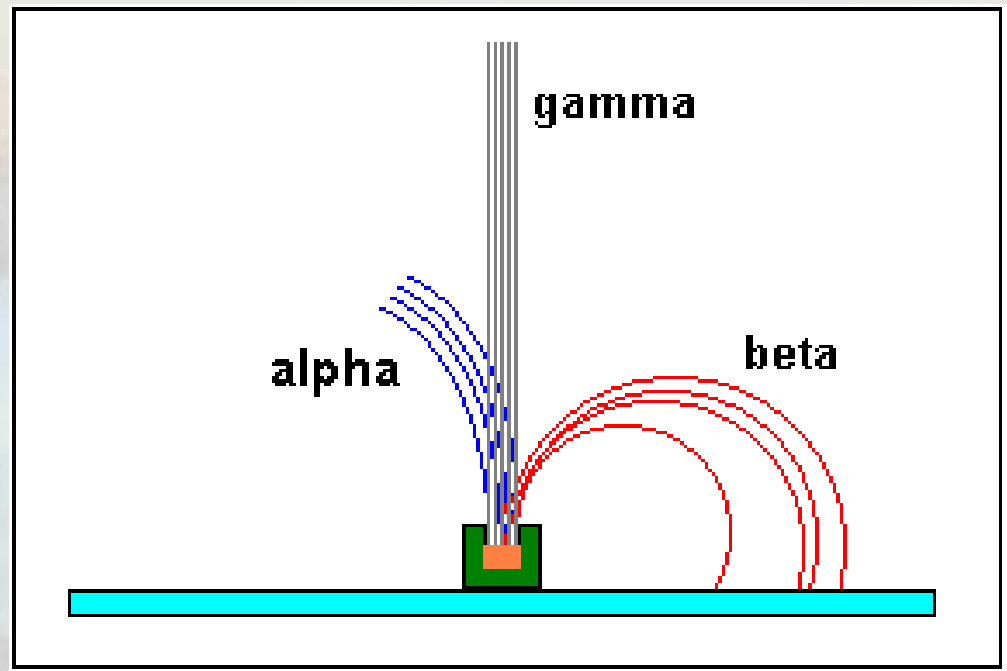
# AT THAT TIME.....(around 1910)

- At that time (about a century ago) radioactivity was most popular among the physics community. From about ten years it was discovered by Becquerel in France. The Curie's, and then the widowed Mrs. Curie-Sklodowska alone, succeeded with a titanic work to isolate various radioactive elements. Within a very short time the majority of physicists and chemists of the world rushed in research on radioactivity.



# AT THAT TIME

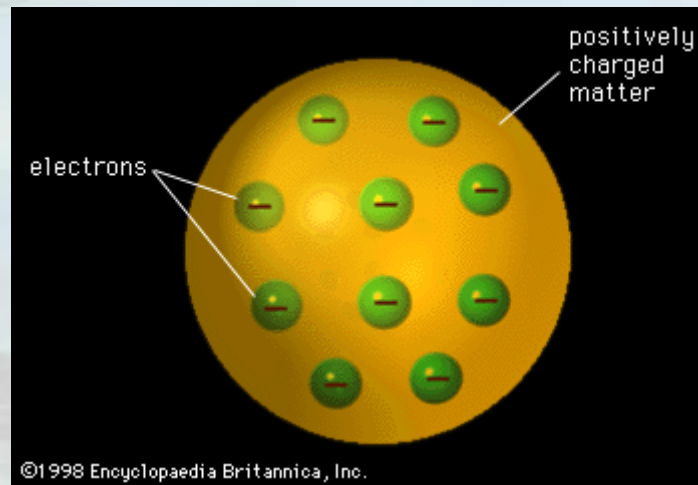
Almost on all High School Physics books, a schematic representation of the experiment which led to the discovery of radioactivity can be found.



The magnetic field is orthogonal to the picture plane

# AT THAT TIME

Among the physicists, it was common belief that matter was made of atoms and that these consisted of positive and negative charges, but nothing was known about the arrangement of positive charges within the atom. (Thomson's atom)



# The Prehistory: Rutherford

In 1908 Ernest Rutherford was awarded of the Nobel Prize for chemistry. The award citation states: "for his investigations into the disintegration of the elements, and the chemistry of radioactive substances." Among others, he had discovered that the radioactive element thorium released a *radioactive* gas. Its *activity* consisted in *radiating* energetic alpha particle (ionized Helium atoms of about 4GeV) as projectiles for investigating on the atom structure:

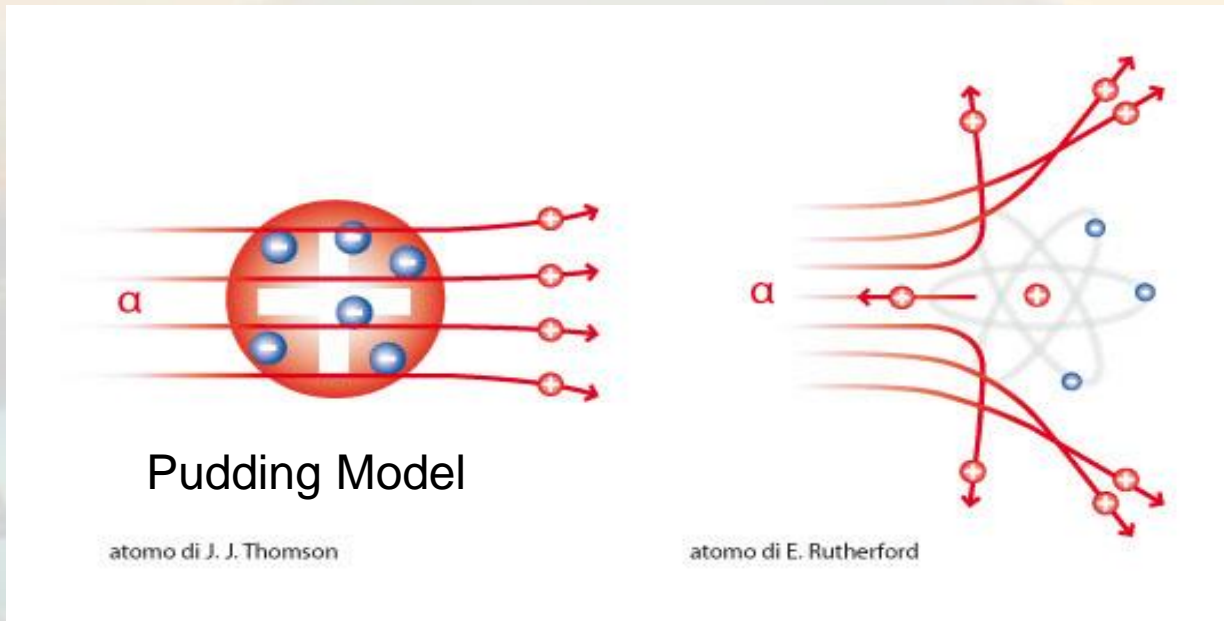
**BRUTE FORCE METHOD**

**THE ACCELERATOR ERA WAS BORN**

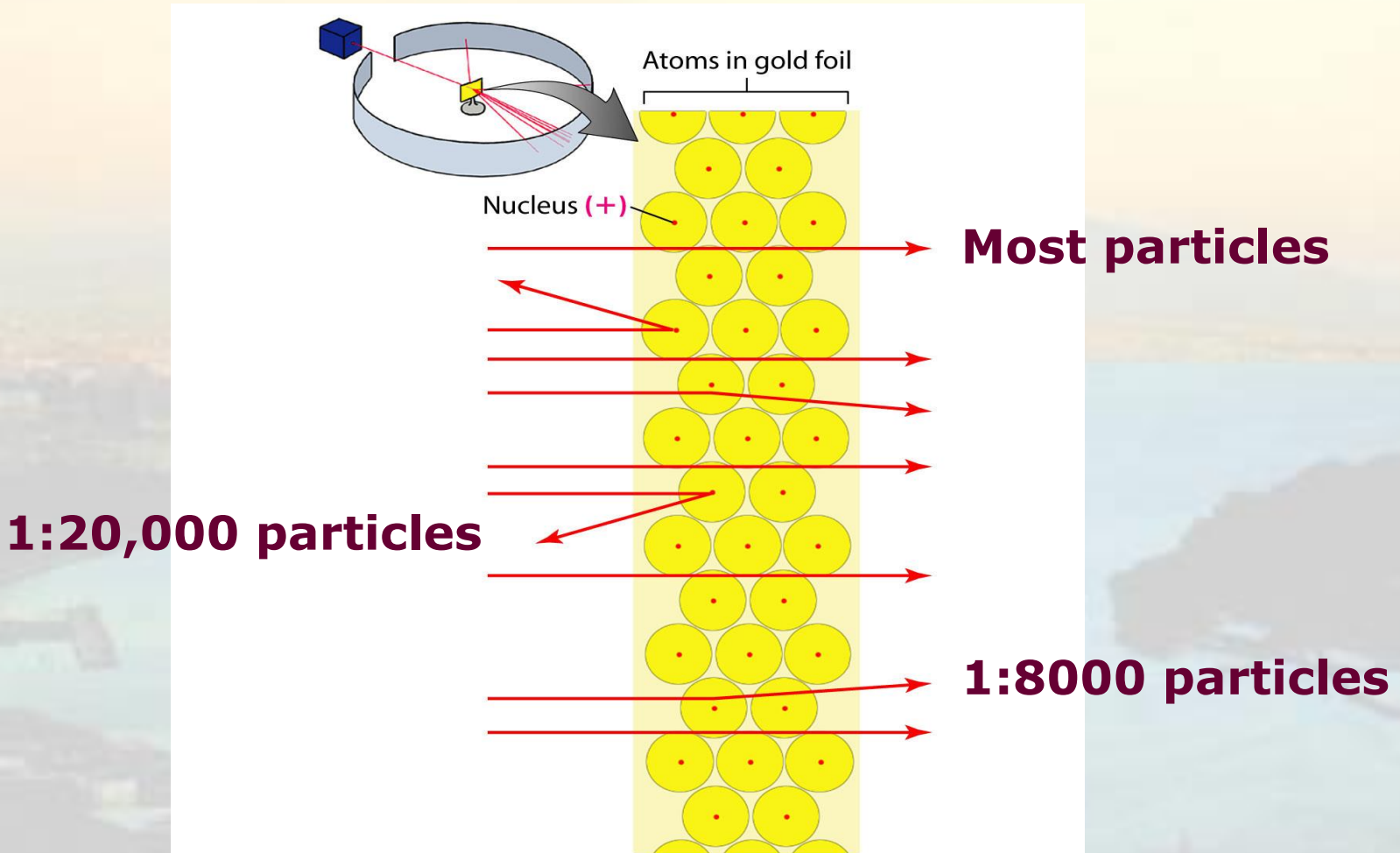
# Two models in comparison

This classic diffraction experiment was conducted in 1911 by Hans Geiger and Ernest Marsden at the suggestion of Ernest Rutherford

*[Rutherford was] a "tribal chief", as a student said.*



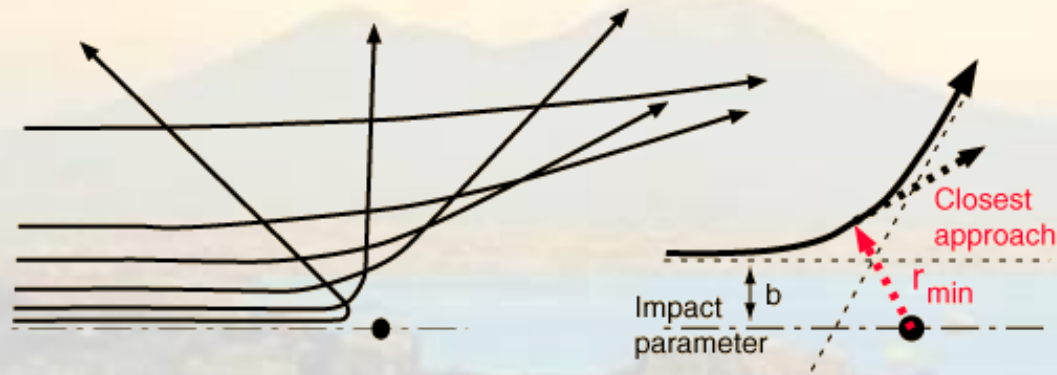
# The crucial experiment



From *Conceptual Chemistry*, Second Edition by John Suchocki. Copyright © 2004 Benjamin Cummings, a division of Pearson Education.

# Impact Parameter

- Besides the masses of the target and projectile, the scattering angle depends upon the force and upon the impact parameter. The impact parameter is the perpendicular distance to the closest approach if the projectile were undeflected.



Rutherford already knew that the trajectory of particle subjected to a repulsive force (proportional to the inverse of the squared distance) was an hyperbola.

**This problem was in the program of studies of the secondary school he attended in New Zeland!**



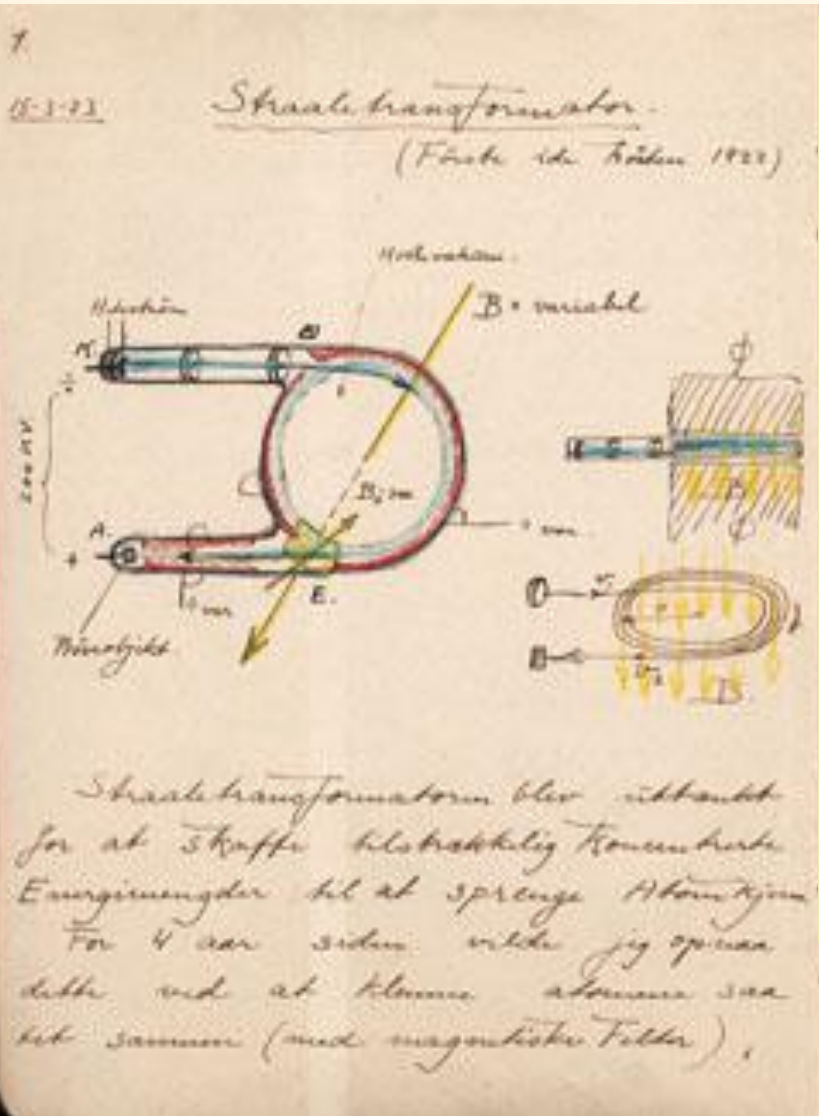
# Rolf Wideroe

- during his studies at the University of Aachen, Rolf Wideroe , barely 20 years old, had already dreamed of making a "radiation transformer ", **later called betatron**, which was very important in the production of X-rays in hospitals.



# Rolf Wideroe

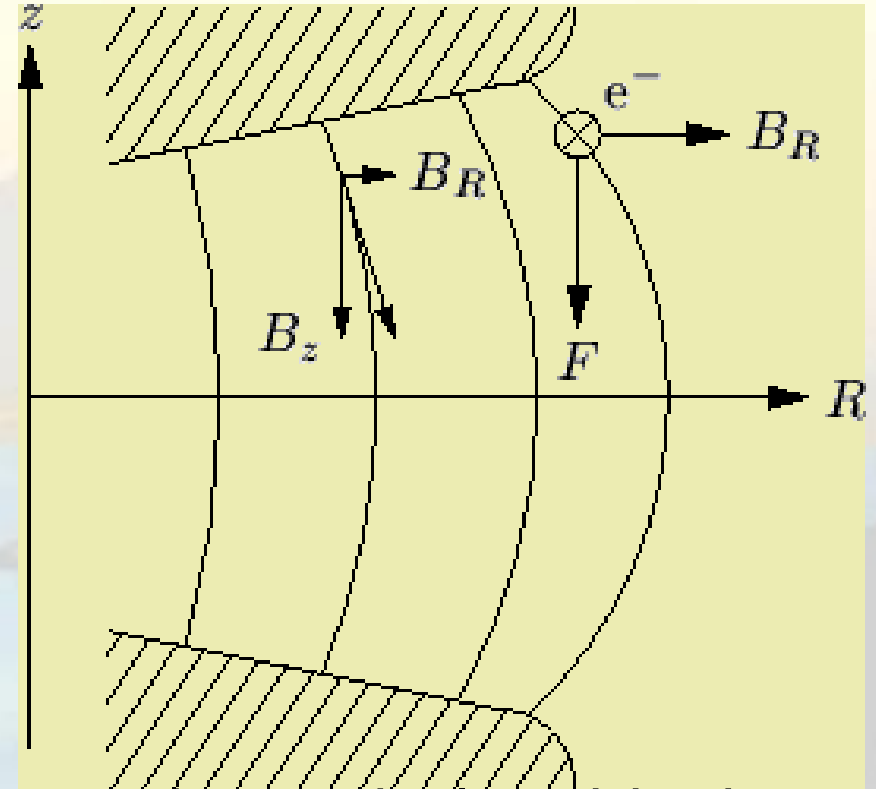
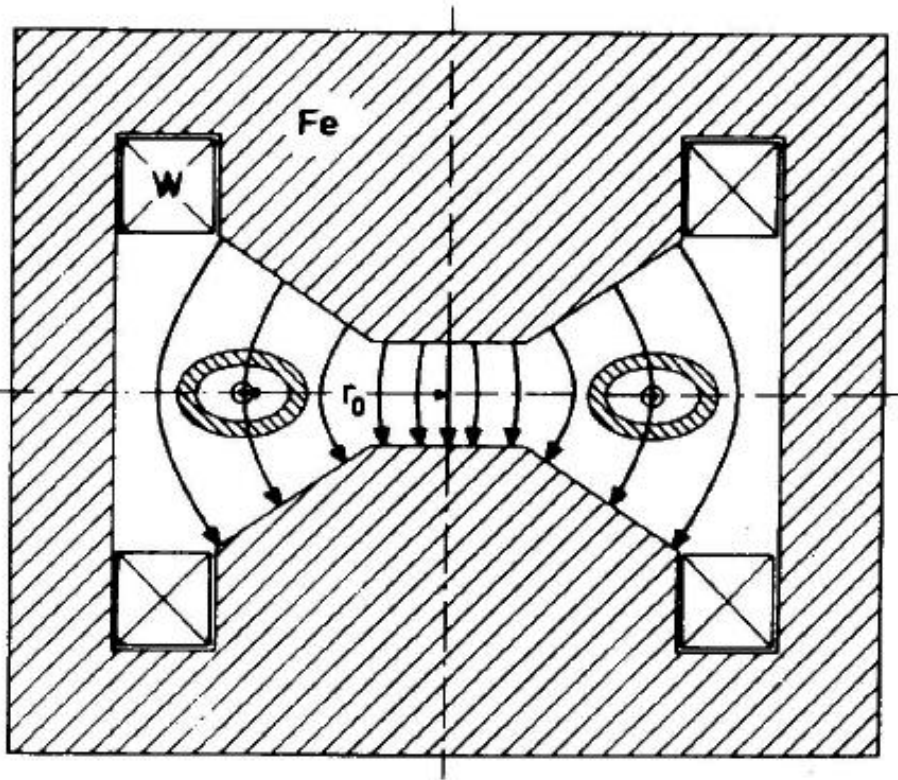
The genius of this idea lies in the fact that the magnetic field bends the charged particles trajectory keeping them on a circular orbit (Lorentz force), and that it may accelerate them, if intensified (Faraday law). He presented this project as a PhD thesis in engineering at the University of Aachen.



Rolf Wideroe  
Formula

$$B_{orbit} = \frac{1}{2} B_{gap}$$

Weak Focusing  $\frac{\partial B_z}{\partial R} = \frac{\partial B_R}{\partial z}$



To find a good compromise between the vertical and horizontal focusing. Particles which are not on the right orbit oscillate around this orbit: **BETATRON OSCILLATION**

# The Betatron

- The transformer was built, but the experiment failed for insufficient vertical focusing

The Betatron, **reinvented by Donald Kerst in 1940**, was historically employed in particle physics experiments to provide high energy electron beams up to about 300 MeV. If the electron beam is directed on a metal plate, the betatron can be used as a source of energetic X-rays or gamma rays; X-rays may be used in industrial and medical applications (mainly in radiation oncology)

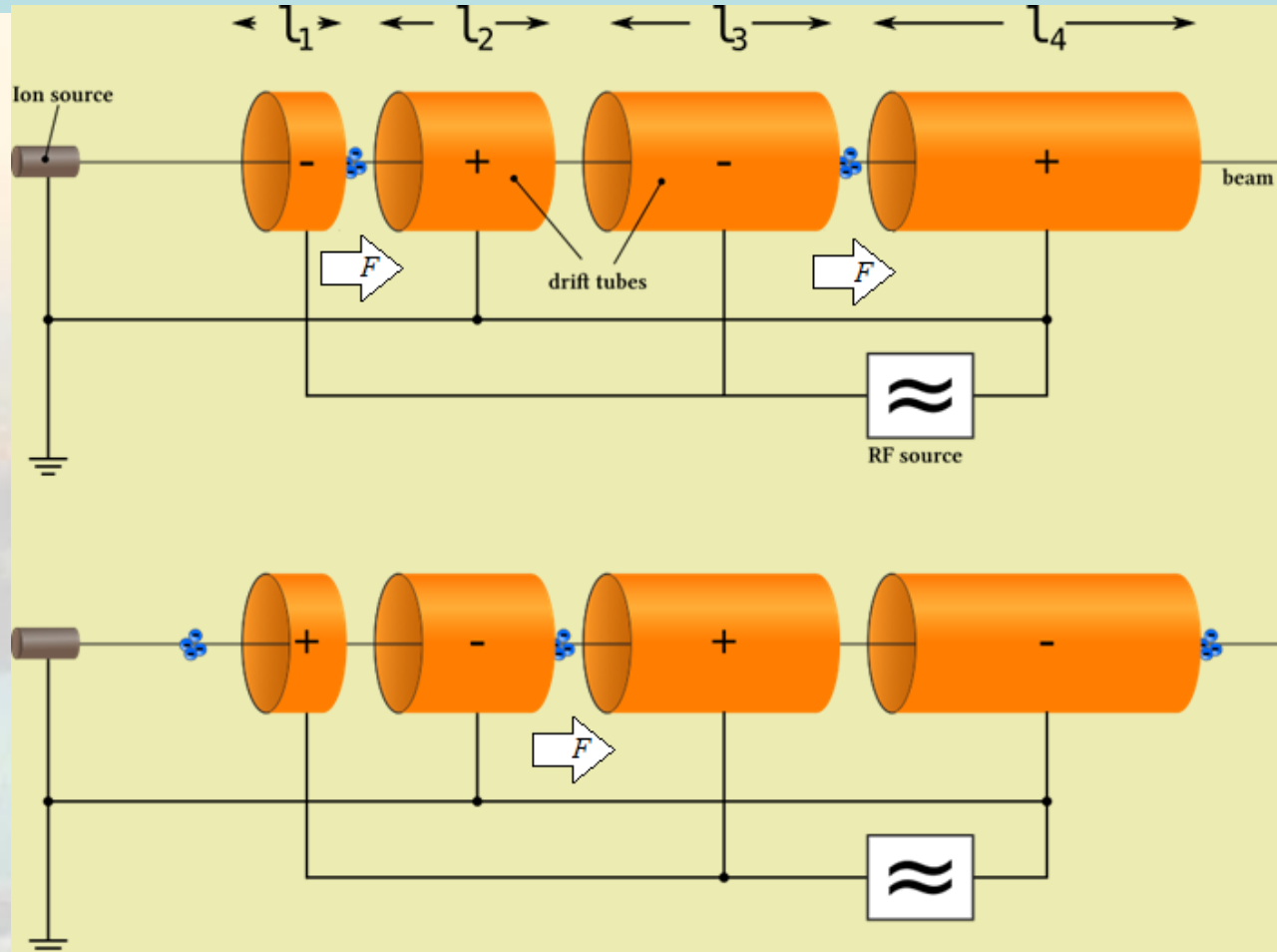
*Ausser-ordentlich-hoch-geschwindigkeit-elektronen-entwickelnden-schwer-arbeits-bei-gollitron,*

# Donald William Kerst



Vintage atmosphere: Kerst is wearing a double-breasted suit.

# Wideroe 2nd attempt



Sketch of the **Ising-Widerøe** linear accelerator concept, employing oscillating fields (1928)

# Rolf Wideroe and Bruno Touschek

- Rolf Wideroe was a volcanic mind: in scientific circles he is credited of inventing the betatron, synchrotron, linear accelerator, the storage rings for **colliding beams**. The latter invention pioneered about 20 years the realization that as we will see later was carried out in Italy at the laboratories of Frascati.
- Bruno Touschek was born and attended school in Vienna. Because of racial reasons, he was not allowed to finish high school. However, he could continue his studies in a precarious way..

# Hamburger Intermezzo

- After *Anschluss*, he moved to Hamburg, where nobody knew of his origins
- There he met Rolf Widerøe with whom he started cooperating in building a betatron and discussing on Widerøe's visionary thoughts. However, Touschek was discovered and arrested by the Gestapo in 1945, Widerøe visited him in prison, bringing cigarettes, food and, during these meetings, and they continued to talk about the betatron.
- Incidentally, in that context Touschek conceived the idea of radiation damping for electrons and of its crucial role.



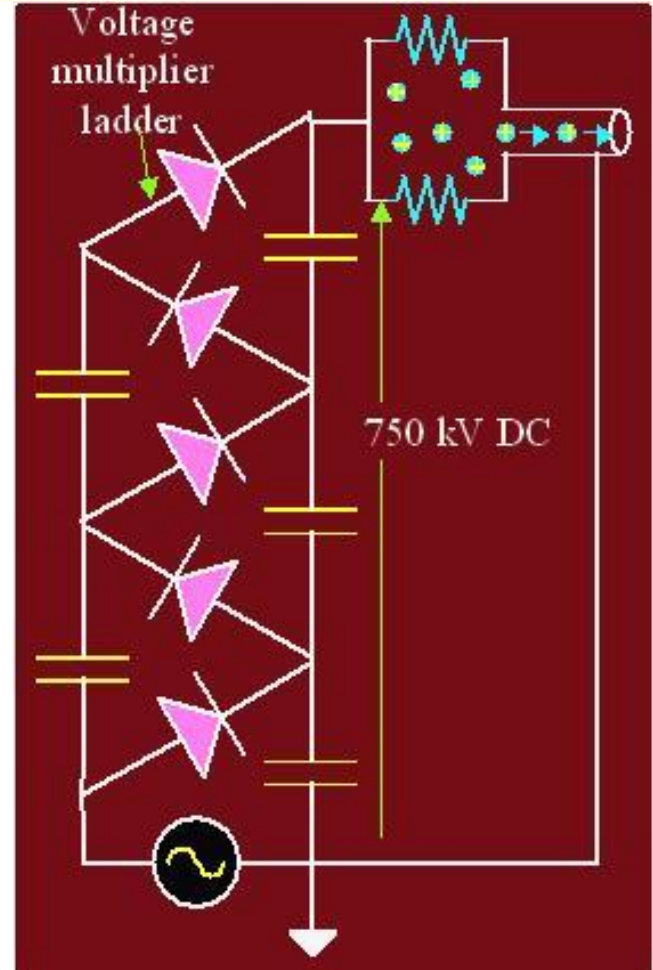
# Hamburger Intermezzo

When the Allied army reached Hamburg Wideroe, suspected of collaboration, was arrested. The "totenstraal" (death ray). Sometime after, he was found not guilty and released. After the war, Touscheck roamed about Europe. Finally, in 1952 he decided to stay in Rome permanently, receiving the position of researcher at the National laboratories of the Istituto Nazionale di Fisica Nucleare in Frascati, near Rome..

# Meanwhile... Cockcroft & Walton

- The thrust impressed by Rutherford bore fruit that matured after the break of the First World War: accelerators based on **potential drop**.
- In the late 20's, Cockcroft & Walton, two researchers belonging to Rutherford's School, made use of a device that technology recently offered them: the diode. They built the apparatus shown in the following page. Despite the complexity, seeming only, the apparatus is simple

# Cockcroft & Walton ..



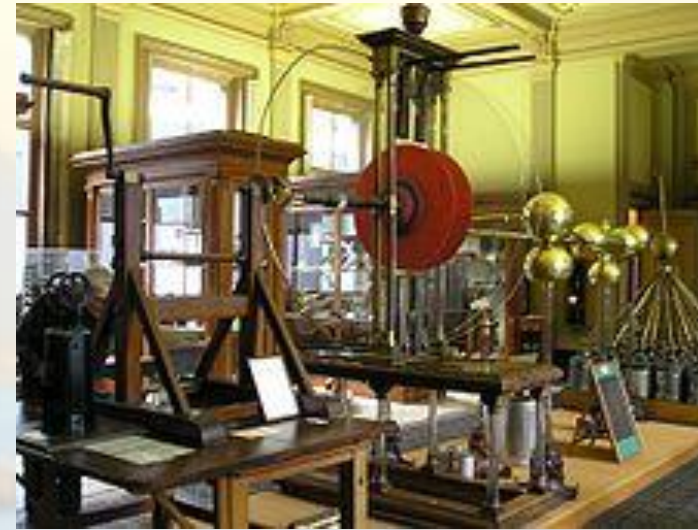
# Cockcroft & Walton

- Cockcroft and Walton in 1932 used this circuit design to power their particle accelerator, performing the first artificial nuclear disintegration in history. On '52 they got the Nobel Prize.
- It is interesting to remark that soon it found a technological use, for instance, for testing devices to be exposed to high electric fields; use that is still practiced.
- During the 30's Cockcroft- Walton accelerator has quickly had its time
- **REMARK:** The circuit was invented much earlier, in 1919, by Heinrich Greinacher, a Swiss physicist.

# Back to Thales: Van Der Graaf Accelerator

Ancient cultures around the Mediterranean knew that certain objects, such as rods of amber, could be rubbed with cat's fur to attract light objects like feathers. Thales of Miletus made a series of observations on static electricity around 600 BC.

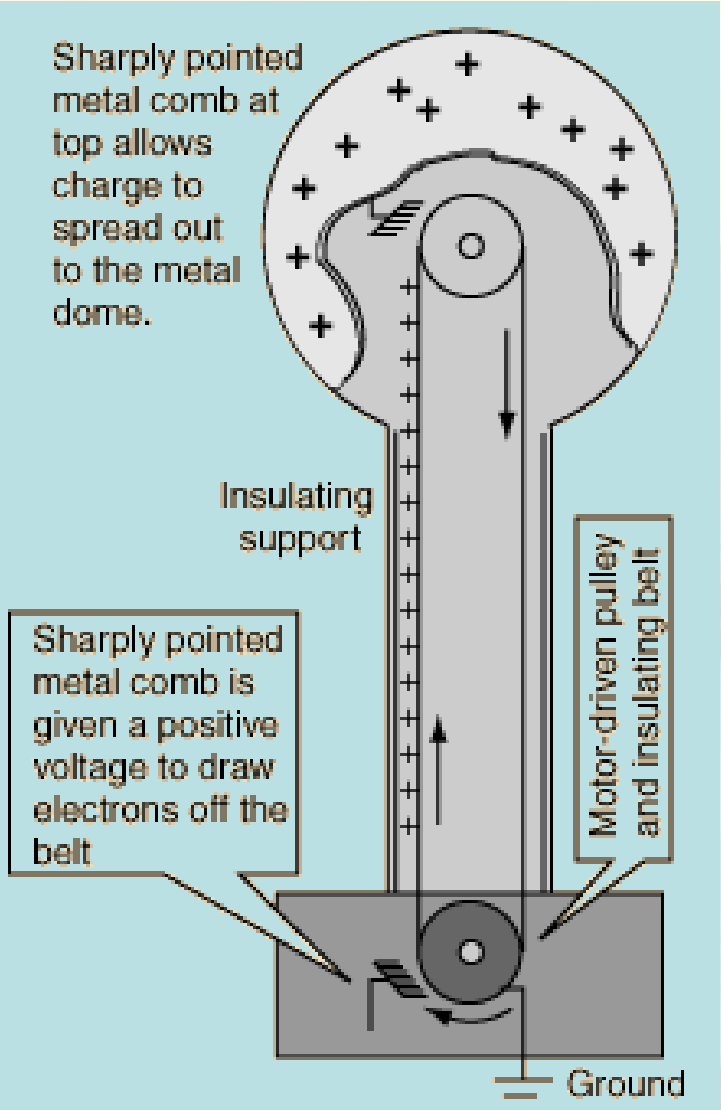
In the eighteen century a number of devices were conceived and carried out which were able to produce very large difference of potential resorting to the same effect described by Thales.



Martinus van Marum's (1784)  
Electrostatic generator at  
Teylers Museum (Nederland)

Last century, at the end of 20's **Van Der Graaf** at Princeton University built an electrostatic generator based on the phenomenon described by Thales, which got its name from the inventor. As an accelerator it was more long-lived than Cockcroft-Walton.

# Van der Graaf Generator



This system is simple and at the same time ingenious: the positive charge is "sprayed" at low potential (10kV) on the belt which drags it inside the sphere; here is neutralized by negative charges "sprayed" **As the belt continues to move, a constant 'charging current' travels via the belt, and the sphere continues to accumulate positive charge until the rate that charge is being lost (through leakage and corona discharges) equals the charging current.** The larger the sphere and the farther it is from ground, the higher will be its peak potential. A student of high school can try to build an accelerator. **The cost was \$90**

**Van de Graaff generator at The  
Magic House, St. Louis  
Children's Museum**



Today it is still used as an accelerator to generate energetic particle and X-ray beams in fields such as nuclear medicine..

# A Van Der Graaf generator for class room demonstrations



09/01/2017

24



# Van Der Graaf Accelerator

- Many microamps of electrons can be accelerated by Van Der Graaf. Currently Van Der Graaf are commercial machines with potential ranging from one million to 25 MV (Mega Volt = million volts). By way of comparison, the short pulses used in research on lightning reach 10 MV and the potential in the clouds just before the lightning discharge is about 200 MV. The Van Der Graaf are often used in the analysis and modification of materials, especially in environmental research
- The need to investigate more and more profound aspects of matter behavior imposed to seek accelerators with increasing current intensity and larger and larger energy.

# Ernest Orlando Lawrence: Mr Cyclotron



In Berkeley **Ernest Orlando Lawrence** had raised the issue of meeting physicist needs of simple and relatively inexpensive research engine. He was a man of harsh frugality. He was used to not hesitate to fire his co-workers, as he did twice with R. Wilson, who later become director of Fermilab. He got the Nobel price for the invention of the cyclotron.

# WIDEROE tells about us E. O. Lawrence

- At this point let us give the floor to Wideroe that, much later, met Lawrence who told him about how he conceived the cyclotron: "Lawrence, once told me that while attending a conference in Berkeley, bored by presentations, went to the library and found my thesis in the journal "Archiv für Elektrotechnik". He watched just the drawings and formulas because he understood little or nothing of German. From the illustrations, he immediately understood the principle of my drift tube. Luckily, his ignorance about German did not let him to understand my misgivings, reported in the paper, about the stability of the orbits in circular accelerators. "

# Lawrence catchall...

- Rolf Wideroe continues: “Back to (Lawrence) Radiation Laboratory, together with his co-worker David Sloan, built his first linear accelerator for mercury ions, which reached the energy of 1,26 MeV. It was an amazing success!” **From then on and until recently, the paternity of linac was attributed by some literature to Lawrence.**
- Lawrence had chosen to play it safe, but meanwhile he was thinking of a structure that would return the beam back on his track in order to save on the number of RF power supplies

# Lawrence catchall...

- Physicists started requiring particles **energy comparable** to those of unstable nuclei used by Rutherford (**4 MeV**), but with **much higher intensity** and heavily collimated beams. Lawrence's goals were very ambitious: to generate intense beams of tens of MeV.
- Resorting to linear accelerators concept would mean to add more drift tubes and then further expensive power supplies.
- Only afterwards, with the development of powerful microwave generators such those used for Second War radars, the idea of linear accelerators for both for electrons to protons was revived and extensively developed. This solution is still very up to date, and has witnessed impressive achievements: the Stanford Linear Accelerator, 3 km long, in 1966 accelerated electrons to 20 GeV and electrons and positrons to 50 GeV in 1989.

# The Cyclotron

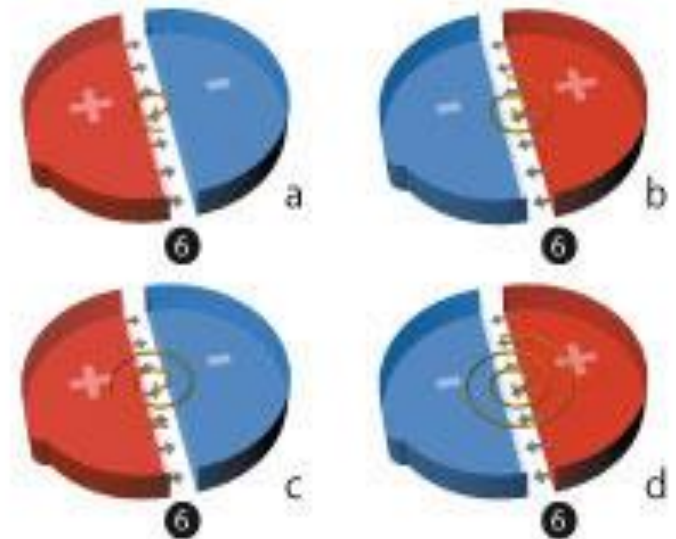
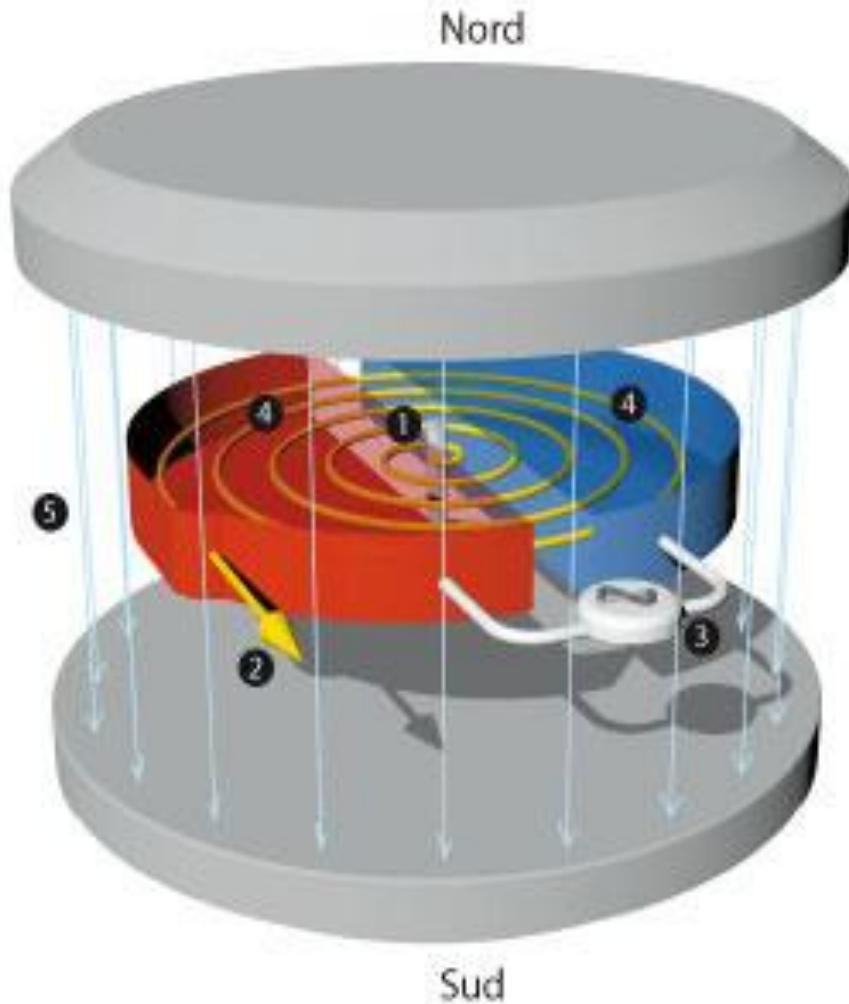
- Therefore, there was need of a magnet which would bend the trajectory of the beam. As accelerating electrodes, he devised a structure formed by the two halves of a very flattened conductive cylinder (pillbox): one half was connected to ground and the other to an RF generator.
- The charged particles, accelerated after each crossing through the gap between the electrodes, travel along the half circumferences of increasing radius. Nevertheless, their angular velocity stay constant.

Lawrence, who had noticed that non-relativistic ions of mass  $m_0$  and charge  $e$  moving in a uniform magnetic field  $B$  circulate at a constant frequency independent of energy:

$$\omega = \frac{eB}{m_0}$$

From then on named cyclotron frequency.

# The Cyclotron



1. Positive ions source
2. Positive ions beam
3. RF power generator
4. D shaped electrodes (Dee's)
5. Magnetic Field
6. Electric Field

# The Cyclotron

- The cyclotron, built by Lawrence in collaboration with his student Milton Stanley Livingston, accelerated **protons** at expected, though modest, energy of 80 keV. It was insisted to apply some changes that, according to his thought, could solve the issue. Lawrence disagreed and did not give his consent. Profiting of Lawrence temporary leave, Livingston applied the changes and they resulted in a successful test.



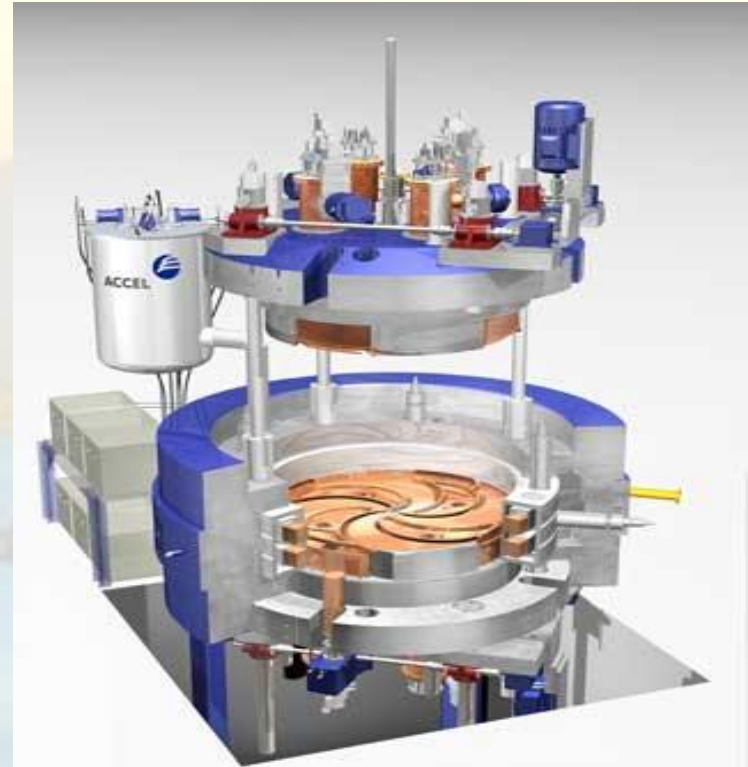
# Cyclotron's limits

$$\omega = \frac{eB}{\gamma m_0}$$

- The researchers began building cyclotrons increasingly large and increasingly higher performance. However, for relativistic energies, some limitations appeared. **The cyclotron frequency decreases with the increment of the relativistic mass; this effect causes the loss of synchronism between the bunch gap crossing and the phase of the RF power:**
- By means of spiral shaping of the Dees one can introduce a compensating correction to the RF phase delay. However, that remedy can work only to some extent (in terms of energy).
- decreasing the frequency of the generator (Synchrocyclotron). However, apart for energy limitation, this solution can accelerate only one bunch and the next one can be accelerated only when the last has been ejected.

# Cyclotron's limits

The cyclotron continues to be used for its versatility, simplicity and robustness. It is not a Ferrari but an overpowering bulldozer! It has a very wide range application in fields such as cancer treatment and the production of radionuclides employed in diagnostic imaging.



# The Guinness Record Cyclotron at Gatchina, Russia

Its weight exceeds of 25% the weight of Eiffel Tower (10,100 tons)!



# What we have learned

Exploring the infinitely small requires more and more energetic projectiles. This may be done only with charged particles beams with quite demanding features: strong collimation; high intensity; high energy.

1. Horizontal focusing competes with the vertical one: weak focusing
2. DC accelerators suffer limitation in energy and current.
3. Cyclotrons are limited in energy
4. Synchronization and focusing are important issues to overcome.

# The new generation: the Synchrotron

In order to meet the new requirements a new generation of accelerators, the synchrotrons, was born. They are based on the idea that the apparatus should spatially separate guiding devices (magnets of various types) from the one that accelerates (resonant cavities capable of generating high fields with variable frequency,  $f$ ). The latter apparatus must be capable to be synchronous with particle crossing (**remember Wideroe linac in slide 14**), which implies that the frequency is proportional to the particle angular frequency  $\omega$ . The beams of charged particles traveling circular closed trajectories are bent and collimated by the strength of the magnetic fields and accelerated by the electric field forces.

# The 1<sup>st</sup> synchrotron: the Cosmotron

<http://www.lns.cornell.edu/~dugan/USPAS/>

- The basic principles of synchrotron design (**phase stability principle**) were proposed independently by **Vladimir Veksler** in the Soviet Union (1944) and **Edwin McMillan** in the United States (1945). According to this principle, it is possible to accelerate bunches of charged particles of finite dimensions. **Those dimensions are as smaller as higher is the acceleration.**

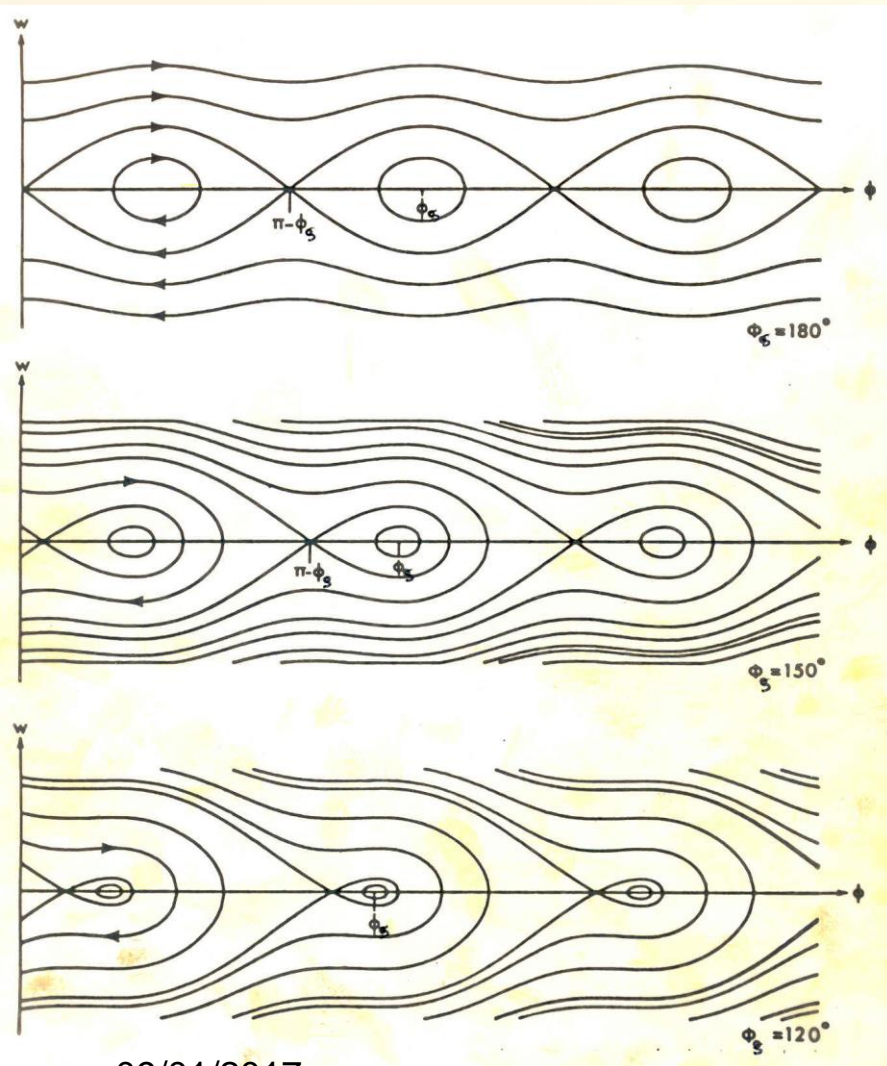


09/01/2017

<http://www.lns.cornell.edu/~dugan/USPAS/>

38

# The phase stability principle



The areas of stable motion (closed trajectories) are called “Buckets”. As the synchronous phase gets closer to  $90^\circ$  the buckets get smaller.

The number of circulating buckets is equal to  $2\pi f / \omega$  ( $f = \text{radio frequency}$  ;  $\omega = \text{particle angular frequency}$ ). The phase extension of the bucket is maximum for  $\phi_s = 180^\circ$  (or  $0^\circ$  ) which correspond to no acceleration ..

# A glance to the future: the great leap forward

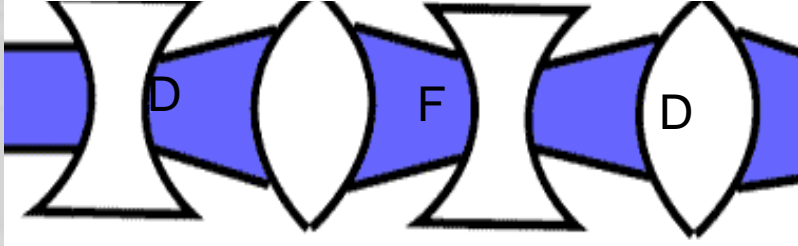
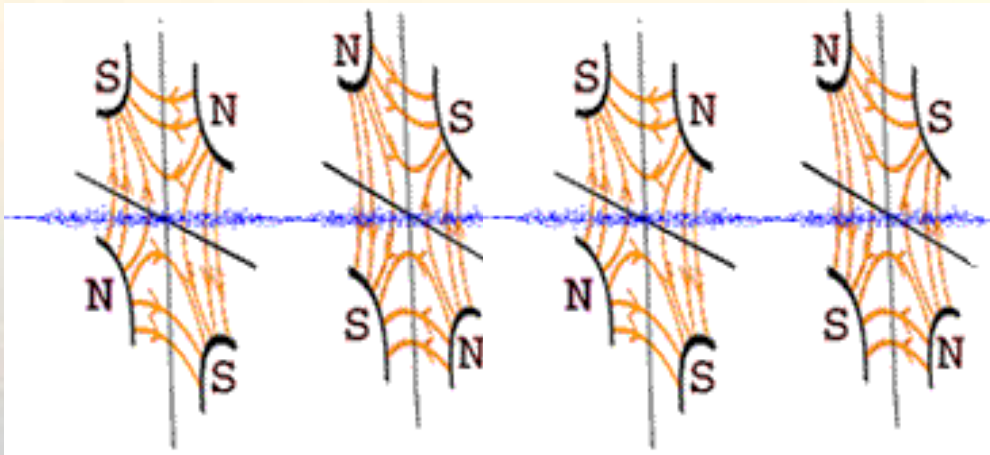
Since when Brookhaven's Cosmotron went into operation in the early 1950's, scientists knew that achieving the higher energies needed for future research was going to be a difficult problem. Calculations showed that, using existing technology, building a proton accelerator **10 times more powerful** than the 3.3GeV Cosmotron, would require **100 times as much steel**. Such a machine would weigh an astronomical 200,000 tons.



# the great leap forward: the strong focusing

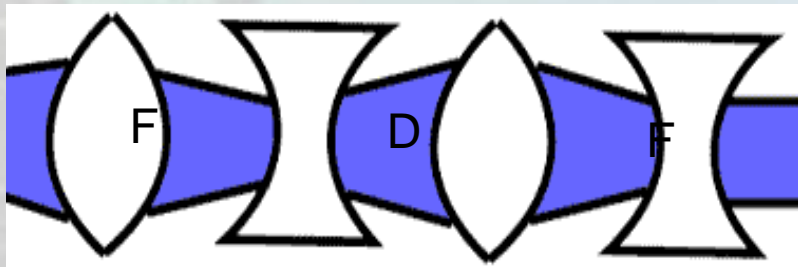
While the first synchrotrons and storage rings strictly used the toroid shape, **the strong focusing principle** independently discovered by Ernest Courant and Nicholas Christofilos allowed the complete separation of the accelerator into components with specialized functions along the particle path, shaping the path into a round-cornered polygon. Some important components are given by radio frequency cavities for direct acceleration, dipole magnets (bending magnets) for deflection of particles (to close the path), and quadrupole / sextupole magnets for beam focusing.

# the great leap forward: the strong focusing



H F

Alternating the focusing and defocusing actions, one gets focusing in both planes.



V  
D

**Drawback: High sensitivity on magnets imperfections!!!**

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{\Delta}{f_1 f_2}$$

if  $f_1 = -f_2$   
and  $\Delta = |f_1|$

$$\frac{1}{f} = \frac{1}{|f_1|}$$

# AGS

Alternating Gradient Synchrotron  
33 GeV on July 29, 1960



# The Already 'Passed by' Future

Even before the successful achievements of AGS, the scientific community was aware of another step needed to revolution the accelerator techniques. Indeed, the impact of particles against fixed targets is very inefficient from the point of view of the energy actually available to create new particles: much more effective could be the head on collisions between high-energy particles.

# BUT THERE IS A REAL NEED OF COLLIDERS

- With increasing energy, the energy available in the centre of mass with fixed targets is incomparably smaller than in the head-on collision, as Wideroe thought some decades before. That is, if we want the same energy in the centre of mass, using fixed targets one should build gigantic accelerators that no Country or supranational organization could afford. From relativistic dynamics, for an LHC-equivalent fixed target accelerator, energy and circumference should be:

$$E_{FT} = 2\gamma_{LHC} E_{LHC} \cong 15 \times 10^3 E_{LHC}$$
$$C_{FT} \cong 15 \times 10^3 C_{LHC} = 405,000 \text{ km}$$

- The price to pay consist in producing intense and high collimated beams in order to increase the luminosity

# The Contest: Frascati vs Princeton

- A contest between Princeton and the Frascati Laboratories started. Princeton chose a eight-shaped structure: two circular rings in which electrons and positron were circulating with the same orientation, meeting at the collision point. Frascati was even more audacious: they used a single ring with "counter-rotating" beams of electrons and positrons. Although it started for last, it came first: it was 1960, when the era of the colliders has begun. This success had great prominence internationally..

# The Contest: Frascati vs Princeton

The enterprise began March 7, 1960. When Bruno Touschek held a seminar at Frascati Laboratories where he proposed to build an electron-positron storage ring. According to Wideroe's visionary ideas concerning storage rings and colliders, on March 14, a preliminary study demonstrated the feasibility of the proposal. The storage ring was called ADA ( Anello Di Accumulazione = storage ring).

The total cost of the project was around 4000 € at present currency.

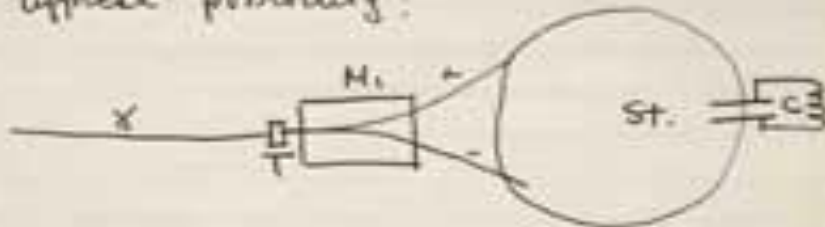
# The Contest: Frascati vs Princeton

A page from Touschek's notebook

ADA

18-2-60

State of affairs - Discussed plan with  
Guiso - Decided for "subtle" storage  
& proposed use of  $\gamma$ -beam also  
for electrons  
Typical possibility:



$\gamma$  =  $\gamma$ -beam, T = target, H<sub>1</sub> = separating magnet, St. = Storage magnet, C = Arc circuit.

Basic formula

$$q = N^2 (v\tau)^2 \frac{\sigma}{q} \cdot \frac{c}{\pi R}$$

N = number of particles accepted per pulse  
v = repetition rate of the Synch (v = 10)





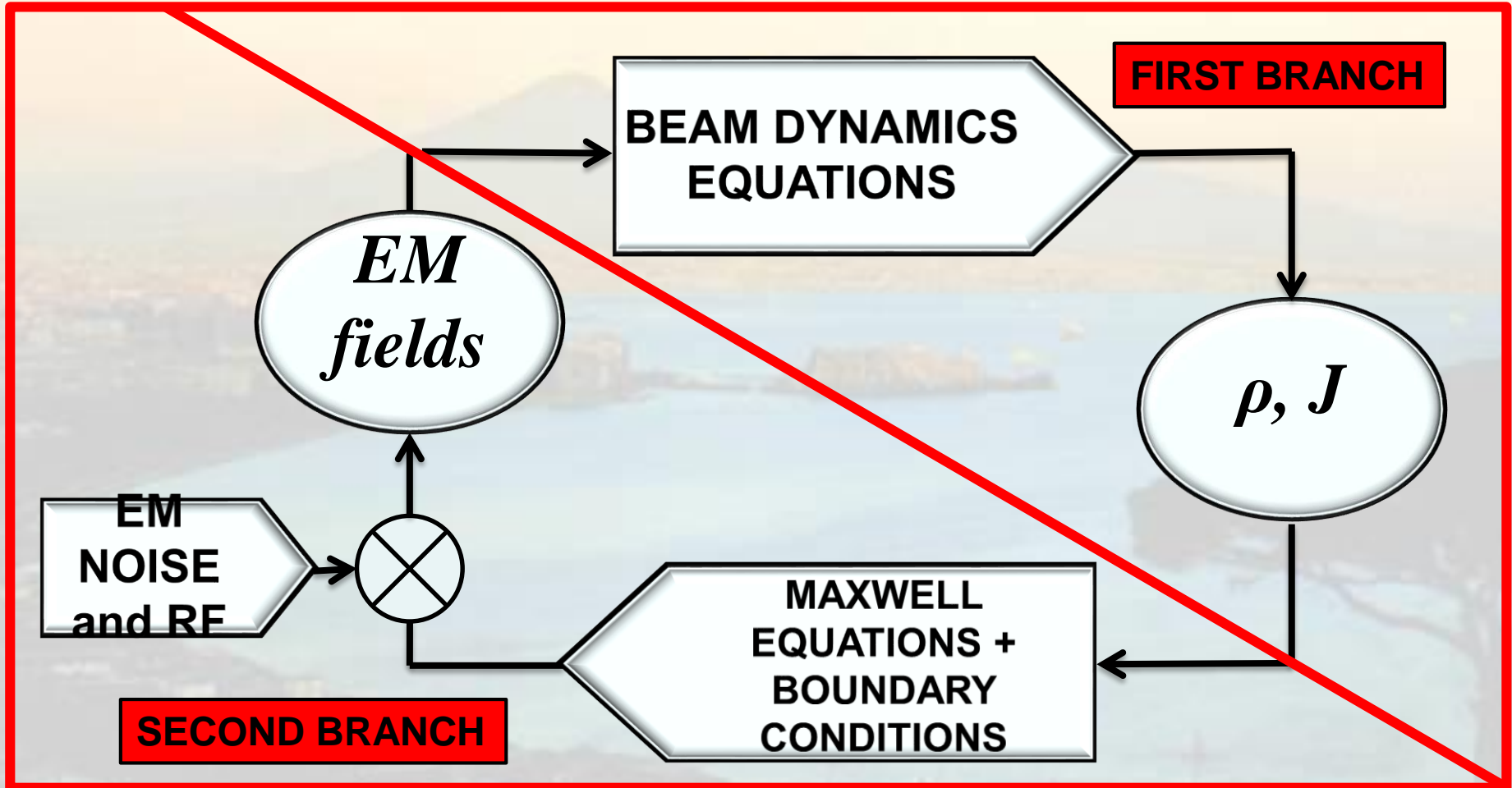
# Beware

- Needs: beams with much larger intensity, much smaller energy spread , higher and higher collimation.
- Early '60s: MURA 50 MeV ( as well as other accelerators) exhibited instabilities consisting in a coherent self bunching and vertical coherent oscillation during particle storing. **The phenomenon seemed to be related to the amount of current stored. The accumulated current could not go beyond a certain limit.** Above this threshold there was evidence of coherent instabilities leading to beam loss. **These were interpreted as due to the presence of some special equipment inside the vacuum tank (clearing electrodes).**

# Somebody leads the way

- A number of scientists tackled this problem resorting to Vlasov equation. They showed that the beam could undergo to modulational instabilities if the the electromagnetic interaction is taken into account.

# Beam-equipment e-m interaction and Dynamics: the Flow Chart (circular accelerators)

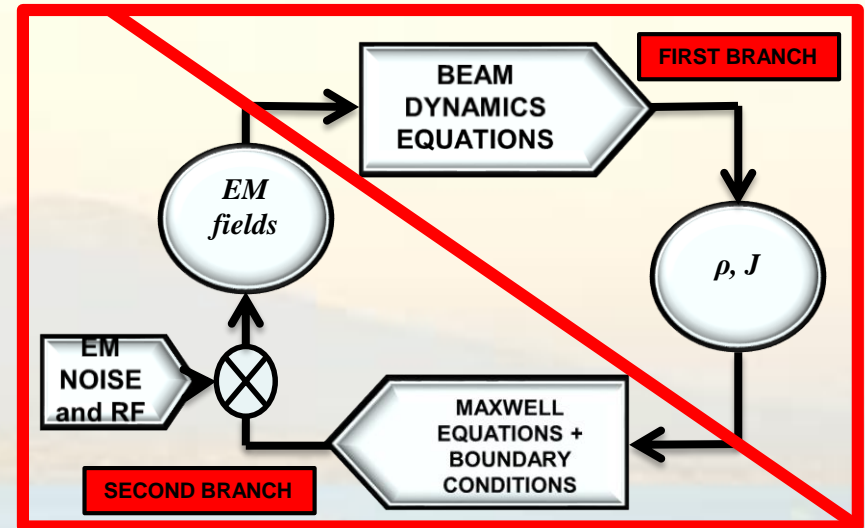


# Instabilities for pedestrians: **gedanken experiment**

## STEPWISE. **THE FIRST BRANCH ONLY BEAM DYNAMICS**

The first turn:  $\Delta E(\Omega)$  a small longitudinal field perturbation. The integrated field gives a voltage  $\Delta V(\Omega)$ , produces a density perturbations in the current  $I$  at a frequency  $\Omega$

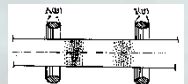
$$I = I_0 + \Delta I(\Omega)$$



measure the perturbation by means of a reciprocal device which may define the perturbed current  $\Delta I(\Omega)$

$$\Delta I(\Omega) = Y_B(\Omega) \Delta V(\Omega)$$

where the **BEAM ADMITTANCE**  $Y_B(\Omega)$  depends on the beam properties: charge/mass, focusing properties etc. Of course the perturbed current is proportional to the d.c. current  $I_0$



# Instabilities for pedestrians **gedanken experiment**

Taking into account the e.m. interaction on one turn of the beam with the environment, this interaction will produce e.m. forces having the same space and time distribution as the perturbation. The work done by these forces can be represented by an equivalent voltage  $\Delta V_1'(\Omega)$  which can be meant as produced by the current  $\Delta I(\Omega)$ , loading an **IMPEDANCE**. This impedance,  $Z_M(\Omega)$ , represents the overall interaction with surrounding equipment.

$$\Delta V_1'(\Omega) = Z_M(\Omega) \Delta I(\Omega)$$

$$\Delta V_1'(\Omega) = Z_M(\Omega) Y_B(\Omega) \Delta V(\Omega)$$

This voltage now acts back again on the beam, producing an additional perturbation, and so on. After  $m$  turns we have:

$$\Delta V_m'(\Omega) = [Z_M(\Omega) Y_B(\Omega)]^m \Delta V(\Omega)$$

# The universal maps

The picture yields the results of the mapping relevant to the stability boundaries for different distribution functions.

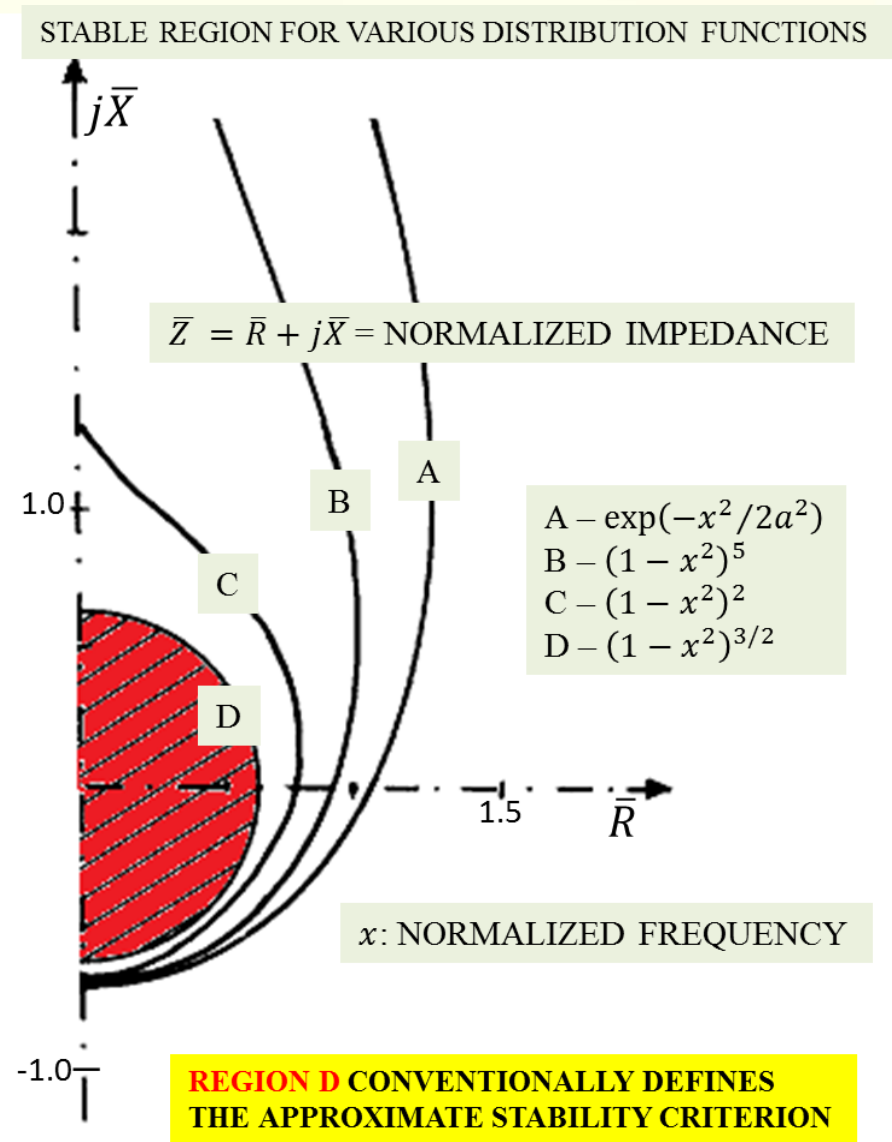
The domain in red represents the most conservative approximation of the calculations from realistic distribution functions.

## IMPEDANCE BUDGET

$$|Z_M(\Omega) Y_B(\Omega)| \leq 1$$

$$|Z_M(\Omega)| \leq |Y_B(\Omega)|^{-1}$$

09/01/2017





- TOUSCHEK
- Not just a scientist...



Thank you, and keep up the good work



09/01/2017



In 1969, when Wilson was in the hot seat testifying before the Congressional Joint Committee on Atomic Energy, Sen. John Pastore demanded to know how a multimillion-dollar particle accelerator (the Fermilab accelerator) improved the security of the Country.

Senator: "Is there anything connected with the hopes of this accelerator that in anyway involves the security of the Country?"

Wilson: "No, Sir. I don't believe so."

Senator: "Nothing at all?"

Wilson: "Nothing at all."

Senator: "It has no value in that respect?"

Wilson: "It has only to do with the respect with which we regard one another, the dignity of man, our love of culture. It has to do with: Are we good painters, good sculptors, great poets? I mean all the things we really venerate in our Country and are patriotic about. **It has nothing to do directly with defending our Country except to make it worth defending.**"

# Somebody leads the way

- Sessler et alii tackled this problem resorting to Vlasov equation. They showed that the beam could undergo to modulational instabilities if the finite conductivity of the pipe is taken into account.

# Somebody leads the way

- In the absence of frequency spread the growth rate is proportional to  $(N/\sigma)^{1/2}$  in the longitudinal case and to  $N/\sigma^{1/2}$  in the transverse case.
- A criterion of stability was established which seemingly does not depend on  $\sigma$ . The authors preconized that it could depend on certain geometrical properties of the accelerator.
- These studies, even though the authors cautiously claim to be possibly related to the observed phenomena, represent an epochal breakthrough in coherent instabilities understanding.

# Concern

- This evidence produced in the scientific community the conviction that in the beam dynamics the interaction with the surrounding equipment should not be neglected (**e.m. field produced by image currents**).
- On 1966 the design of a large Intersecting proton Storage Ring (ISR) started. It was supposed to accumulate at 31GeV the largest current ever reached. This is the equivalent of a 2000 GeV beam hitting a stationary target.
- **Among other, it was foreseen a large number of clearing electrodes, pick-ups and variety of lumped components.**

# Concern and Call for Help

- Inside the designers community there was very much concern about the successful achievement of the design.
- On August 1966, on leave of absence from Berkeley, Andy Sessler (2013 Fermi award) joined CERN for one year.
- Claudio Pellegrini (2014 Fermi award), Fernando Amman (director of Laboratori di Frascati), Ernest Courant (1986 Fermi award), Krinskii, Dikanski, Auslander were paying visits at CERN for discussions on ISR design.

# The Fluke of my Life

- On June 1966 I was hired from CERN with a three year contract.
- On August, when Andy came, I was committed to him. I discovered new horizons.
- I was already acquainted with his work.

## An Impedance is in the Air

- I was assigned the task to introduce in Vlasov equation the contribution of a lumped element, e. g. cavity of impedance  $Z_c$  (and/or pickups).

# An Impedance is in the Air

## The Banality of Invention

- Vlasov:  $\frac{d\psi}{dt} + \dot{\theta} \frac{d\psi}{d\theta} + e \langle 2\pi R E_{\theta} \rangle \frac{d\psi}{dW} = 0$

$\psi(W, \theta, t)$  is the particle distribution function

- Perturbative solution

- $\psi(W, \theta, t) = \psi_0(W) + \psi_1(W) \exp[-i(\omega t - n\theta)]$

- $I_1 = -i\beta c e^2 \langle 2\pi R E_{\theta} \rangle \int \frac{d\psi_0}{dW} \frac{dW}{(\omega - n\omega_0)}$

- $I_1 = e\beta c \int \psi_1(W) dW$  is the perturbed current;  $\omega_0 = \dot{\theta}$

# An Impedance is in the Air

## The Banality of Invention

- The term  $\langle 2\pi R E_\theta \rangle$  was calculated for a **smooth** vacuum chamber with **constant** resistivity. Its explicit expression had an awkward feature.
- The idea was to expand the impressed field distribution at the cavity gaps in waves along the circumference and to take the one which was riding along with the particle.
- In this way the factor

$\langle 2\pi R E_\theta \rangle$  is replaced by the factor

$$\langle 2\pi R E_\theta \rangle - I_1 Z_c$$



# An Impedance is in the Air

## The Banality of Invention

- I felt uneasy, as an engineer, to tackle a problem expounded in that unfamiliar feature: the imaginary unit  $i$  was quite misleading.
- free space impedance  $4\pi/c$ . NO!!!.

# An Impedance is in the Air

## The Banality of Invention

- I discussed with Andy to try to express the beam interaction with the vacuum chamber ( $\langle 2\pi R E_\theta \rangle - I_1 Z_c$ ) in MKS units in order to compare it to lumped element impedance. It was a choice to make my life easier.
- The Andy idea was than to generalize this concept also for more complicated circuits.

# An Impedance is in the Air

## The Banality of Invention

- “It was emphasized that  $Z$  described the impedance of the wall elements and was, thus, amenable to computation--or measurement--by means of all the standard techniques employed in electrical engineering. This "engineering technique" was applied to a number of problems - such as helical insert - and allowed complicated structures to be readily analysed, such as pickup electrodes.” Sessler PAC71

# An Impedance is in the Air The Banality of Invention

Now, I have the impression that the coupling impedance was a concept which was just waiting to be introduced:

I was at the right moment in the right place with the right people.

**Shortly she will be fifty,  
but she does not look it**

09/01/2017

Sophie

Marceau

La Boom - The Party – II





09/01/2017

70

- ...and that we had to produce a more general paper.

CERN 67-2

ISR-Division

February 6, 1967

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE  
**CERN** EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

LONGITUDINAL INSTABILITIES OF AZIMUTHALLY UNIFORM BEAMS IN  
CIRCULAR VACUUM CHAMBERS WITH WALLS OF ARBITRARY  
ELECTRICAL PROPERTIES

by

A. Sessler and V. Vaccaro

# Further developments

- A collaboration starts with Sandro Ruggiero and, later on, with Kurt Huebner, which last for almost two years.
- More than twenty paper written on coupling Impedance an Dispersion Relation.



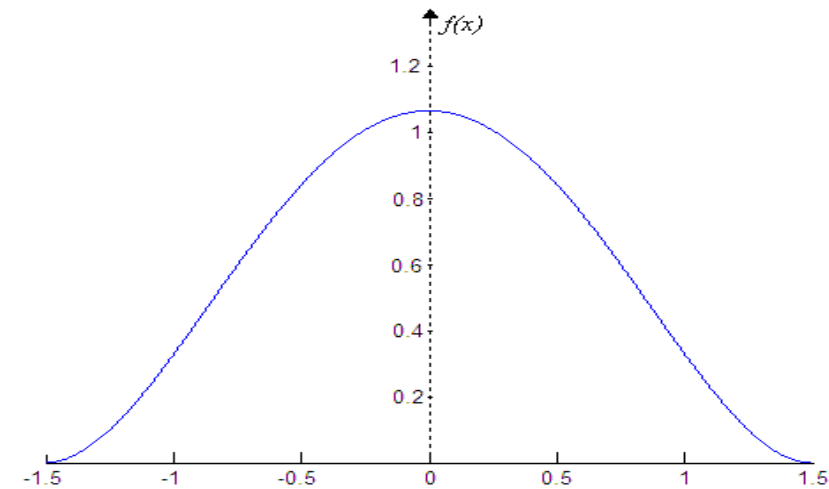
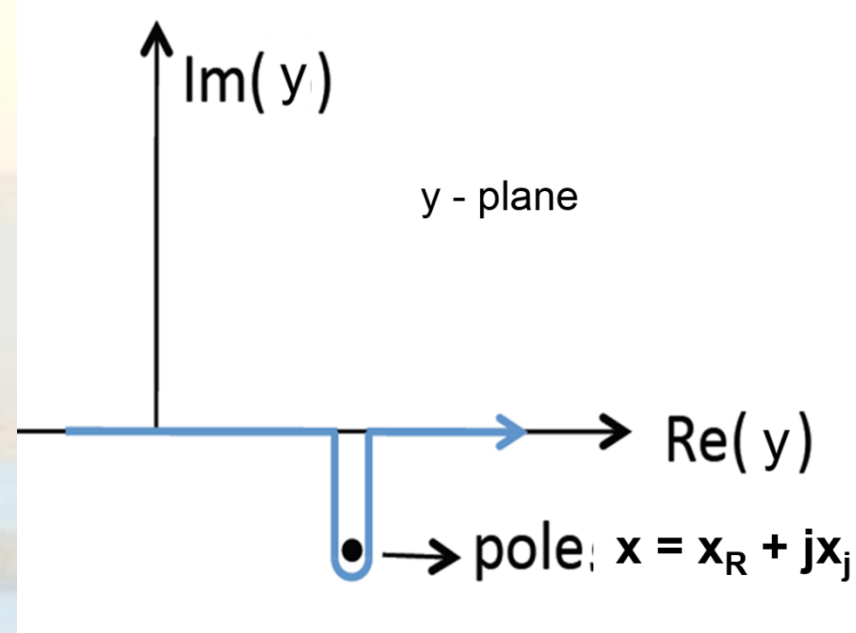
# My preferred child: The Universal Maps of Stability

- Given a certain value of coupling impedance it was (at that time) almost impossible to solve the dispersion relation.
- We (Sandro and I) thought that it could be interesting to draw maps of stability, **better to say "maps of instability"**.

# The Universal Map of stability

The variables in the dispersion relation (DR) can be modified in order to let them be dimensionless. Assuming  $x$  as a complex variable, DR is a conformal transformation from  $x$ -plane to  $y$ -plane.

$$Z(x) = 1/Y_b \int \frac{df_0}{dy} \frac{1}{y-x} dy$$



# The universal maps

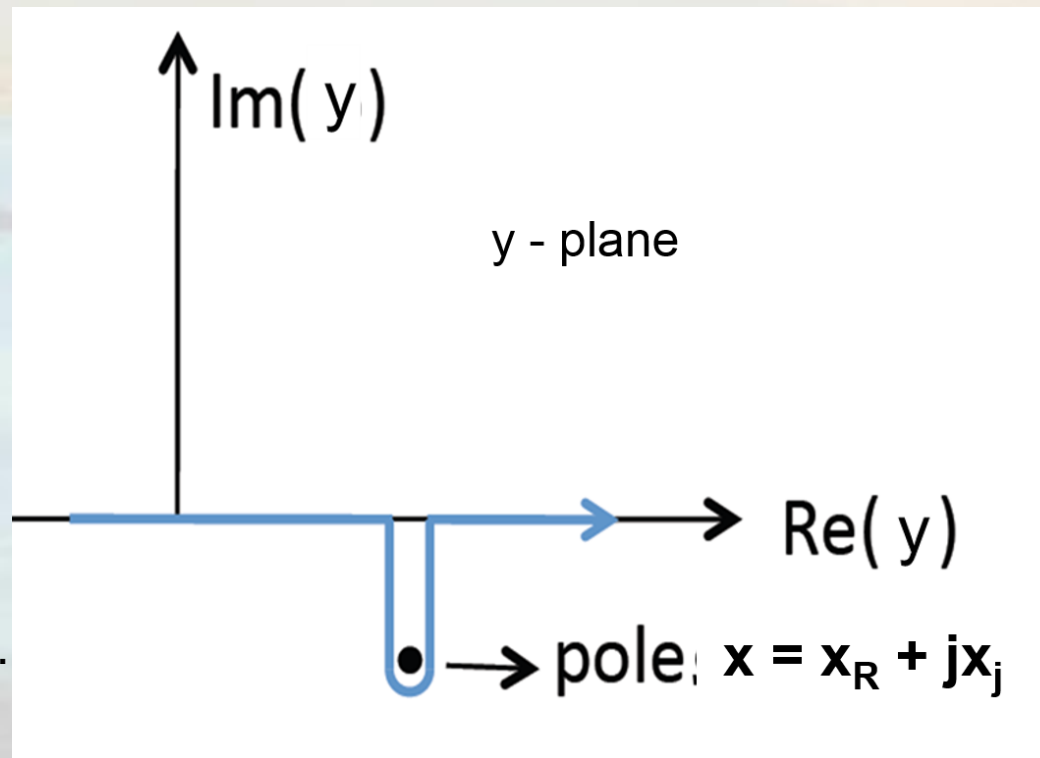
Let us consider



$$Z(x) = \frac{-Y_B^{-1}}{\int_{-\infty}^{+\infty} \frac{df_0(y)}{dy} \frac{1}{x-y} dy}$$

This explicit form allow us to calculate the value of  $Z(x)$  for any given value of  $x$ . The procedure consists in assign a value of  $x$ , the imaginary part of which is negative (a diverging instability). In order to calculate the integral, the integration path may be supposed to approach the real  $y$ -axis from lower half plane. This integral is then equal to the principal values plus the residual in the pole.

GO TO THE NEXT SLIDE.

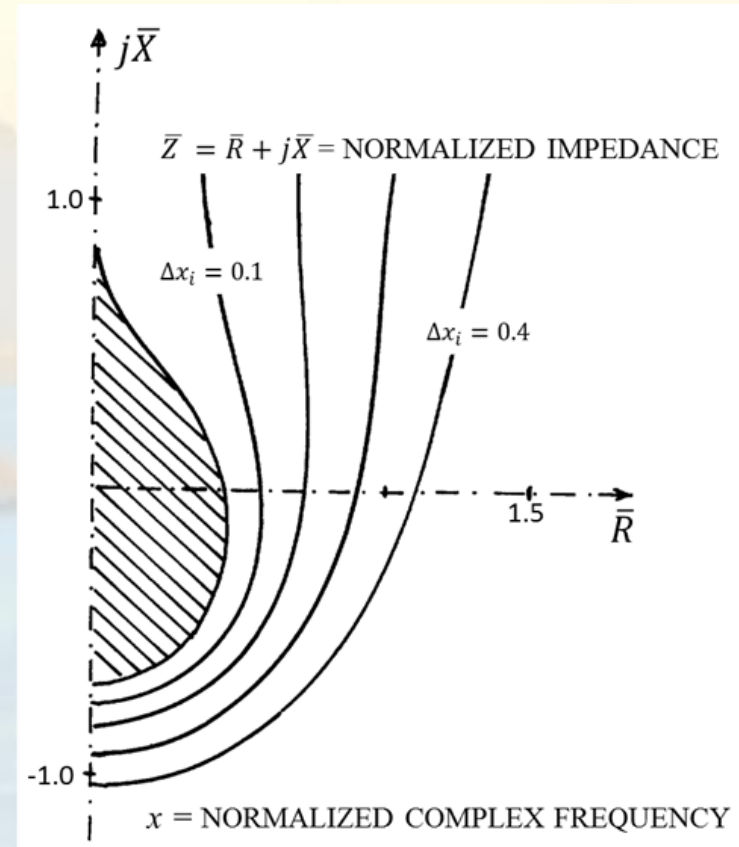


# The universal maps

$$\bar{Z}(\Omega) = \bar{R} + j\bar{X} = \frac{-Y_B^{-1}}{\left( 2\pi j \frac{df}{dy} \Big|_{y=x} + \int_{-\infty}^{+\infty} \dots dy \right)}$$

The picture on the right tells us that the mapping of lower half  $y$  - plane into the  $Z$  - plane covers this plane almost entirely. Only a small portion (dashed) is left. It is worth of note that this domain is not covered by an analogous mapping of lower half plane. This domain is interpreted as a region of stability.

If the accelerator has an impedance which is for any frequency inside this domain, the beam is longitudinally stable



# The universal maps

The picture yields the results of the mapping relevant to the stability boundaries for different distribution functions.

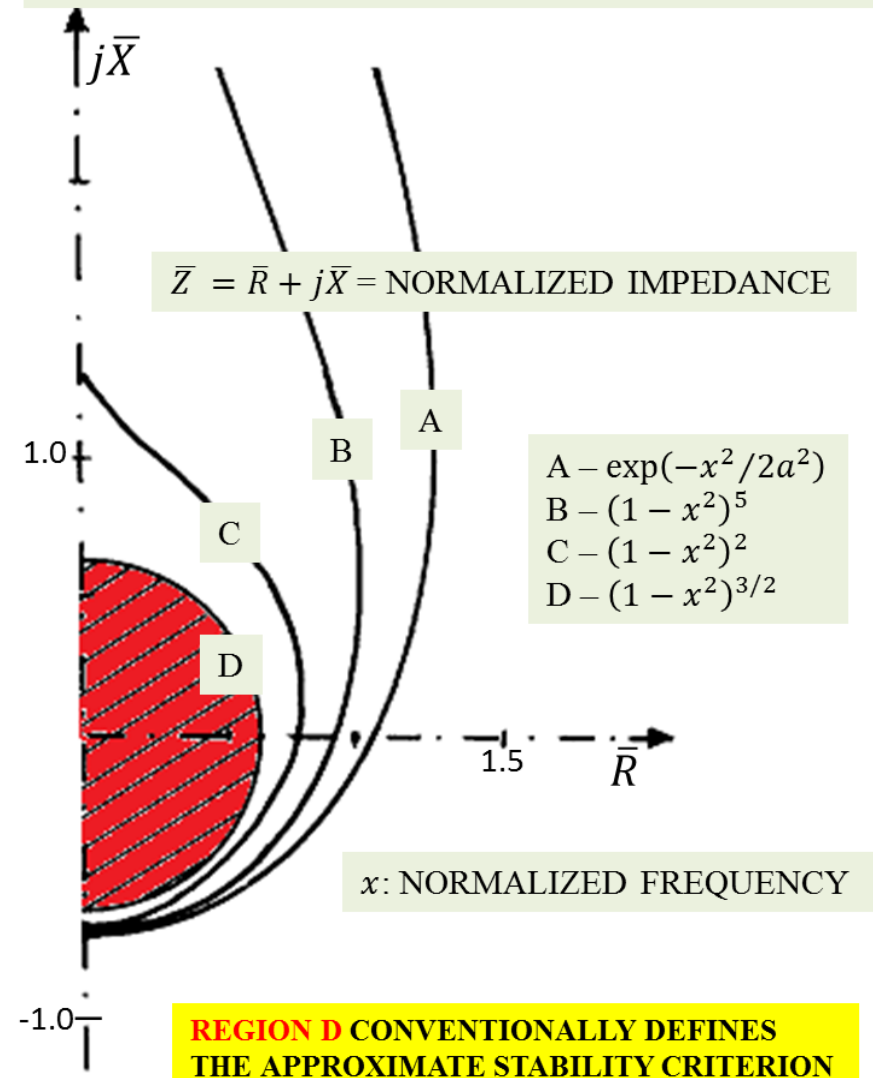
The domain in red represents the most conservative approximation of the calculations from realistic distribution functions.

## IMPEDANCE BUDGET

$$\left| \frac{Z}{n} \right| \leq F \frac{m_0 c^2 |\eta|}{e \gamma} \left( \frac{(\Delta \beta \gamma)^2}{I_0} \right)$$

$$\left| \frac{Z}{n} \right| \leq F \frac{m_0 c^2 \beta^2 \gamma |\eta|}{e I_0} \left( \frac{\Delta p}{p} \right)^2$$

STABLE REGION FOR VARIOUS DISTRIBUTION FUNCTIONS



- Andy Sessler proposed that I could write a paper concerning the cases we have already discussed...



ISR-RF/66-35  
November 18, 1966

LONGITUDINAL INSTABILITY OF A COASTING BEAM ABOVE TRANSITION, DUE TO  
THE ACTION OF LUMPED DISCONTINUITIES.

by V.G. Vaccaro

1. Generalities

We assume that the electrical action on an ion beam, of a discontinuity in a tank is that of an impedance. We still consider the case in which this discontinuity is sufficiently small compared with the wavelength of the perturbation, to be considered as concentrated.

## REFERENCES

1) V.K. Neil and A.M. Sessler

Longitudinal Resistive Instabilities of Intense Coasting  
Beams in Particle Accelerator  
Rev. Sci. Instr. 36, 429 (1965)

1) A.M. Sessler and V.G. Vaccaro

Longitudinal Instabilities of Azimuthally Uniform Beams in  
Circular Vacuum Chambers of Arbitrary Electrical Properties  
(in preparation).

Distribution: (closed) AR and ISR Scientific Staff.