BEAM DYNAMICS IN CYCLOTRONS

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1. **An isochronous cyclotron uses a RF cavity at 60 MHz at the RF harmonic h=3**
   a. Compute the time needed to perform one turn for the accelerated ions.
   b. Compute the average field B needed to accelerate proton in a non relativistic approximation (2)
   Answer 2 : a; revolution freq=60/3= 20 Mhz $\Delta T = 1/20.\ 10^{-6}\ s = 50ns$
   b.

$$\omega = qB/\gamma m = \omega RF/h \quad \text{and we have } \gamma \text{ close to 1}$$

proton mass~ $1.6\ 10^{-27}$ kg // proton charge~ $1.6\ 10^{-19}$ C
Frf=60 MHz= $\omega RF/2\pi$
$B \sim m_p/q\ .\ 2\pi F_{RF}/h = 10^{-8}.\ 10^{6}\ 20\ .2\pi\ . = 1.26$ Tesla

2. **Recall, what kind of magnetic field can compensate the relativistic factor in a isochronous cyclotron (give Bz= Bz(B0, R,h, ωrf)) (2)**

   Answer

   The synchronous condition $\omega_{rf} = h\omega_p$ *with h the harmonic number. As* ω_{rf} *is constant during the whole acceleration in the cyclotron* ω_p *has to be constant too. In the relativisitic case, we replace rhe expression of the ion mass* $m=\gamma m_0$*, in the previous expression then at a radius r,* $\omega_p = \frac{f_p}{2\pi} = \frac{qB(r)}{\gamma m_0}$ *. To keep* ω_p *constant* $B(r)=B0/(1-(Rh\omega/c)^2)^{1/2}$*. The magnetic field grows with respect to the radius.*

3. **In the case of a small compact cyclotron how can be injected the particles onto the first cyclotron orbit ? (1)**
   a. *With an axial injection using "A spiral inflector"*
   b. *With an axial injection using "A hyperboloid inflector"*
   c. *A radial injection beam line*

   Answer 5 : a & b
   c is not possible : no space is available in a compact magnet for an injection beam line

4. **Demonstrate than in a uniform circular motion , the radial acceleration is $a_r = |V^2/R|$. Nota :** ***You can use parametric equations :***

   $X(t) = R\cos(\omega t)$

   $Y(t) = R\sin(\omega t)$

Then compute the velocity and the acceleration. **Demonstrate that the acceleration is radial (3)**

Answer

$V_x = -\omega R\cos(\omega t)$ $v = |\mathbf{v}| = (V_x^{\,2} + V_y^{\,2})^{1/2} = \omega R$

$V_y = +\omega R\sin(\omega t)$

$a = (a_x^{\,2} + a_y^{\,2})^{1/2} = \omega^2 R = v^2/R$

Nota : **v** perpendicular to **a** (since **v.a=0)**

5. **Compute the field index n(r) in a isochronous cyclotron using than the isochrounous average field is** $B(r) = B.\,\gamma(r)$ **(2)**

$$\text{with} \quad n = -\frac{r}{B_{0z}}\frac{\partial B_z}{\partial r}$$

Answer :

$n = 1 - \gamma^2$

6. **An given cyclotron is supposed to accelerate ions with A nucleon and a charge state Q. Demonstrate than the maximal kinetic energy E/A of a cyclotron is**

   $$E/A = Kb\,(Q/A)^2$$

Nota : Give the Kb factor in a non relativistic approximation using the extraction radius R, the maximal average magnetic field B. The mass of the ions is $m = Am_0$ & the charge of the ions is $q = Qe_0$

(2)

Answer 8:

$E = (\gamma - 1)mc^2 \sim \frac{1}{2}\, mV^2 = \frac{1}{2}\, m\,(R\omega)^2$

$= \frac{1}{2}\, m\,(R\,qB/m)^2 = \frac{1}{2}\, A\,m_0\,(R\,Q\,e_0\,B/A\,m_0)^2$

$E/A = \frac{1}{2}\,(e_0\,R\,B)^2/m_0\;(Q/A)^2$

7. **How to improve the axial stability ion beams in a isochronous cyclotron. (1)**
   a. ***With Azimuthally varying field Bz(θ)***
   b. ***With separated sectors magnets***
   c. ***With spiralled sectors***
   d. ***With magnetic field B(r) which increase with r***

   Answer : abc

8. **Why the ions do not oscillate in longitudinal plane (Energy, Phase) in a isochronous cyclotron like in a synchrotron. (2)**

   Answer : the particle in advance stay in advance because whatever the energy it get in the RF cavity, it describes one turn in fixed time interval ΔT (isochronisms). So no phase oscillation is expected...

   Frev=constant=1/ΔT

9. The axial oscillations of ion beams in a isochronous cyclotron is described by

$$\frac{d^2 z}{dt^2} + \nu_z^2 \omega_{rev}^2 = 0$$

Where **the vertical tune should respect $\nu_z^2 > 0$,**

a. Why ? (1)

b. Give a particular solution of the differential equation when $\nu_z^2 < 0$ (1)

Answer : a. Otherwise the beam is unstable (beam size increase exponentially)

b. $z(t) = Z0 \exp(-\nu_z \ \omega t)$

10. An isochronous compact cyclotron for H- beam, uses a RF cavity at Frf=44 MHz at the RF harmonic h=2 . The extraction radius $R_{extraction}$=0.55 m and the Injection radius R_{inj}=5cm.

- **a. How to extract easily such a beam (H-) from the cyclotron .**
- **b. What is the gain in momentum for the particle accelerated in such a cyclotron in a non relativistic approximation and using the magnetic rigidity concept**
- **c. What is the field B required at injection to accelerate a H- beam.**
- **d. With the same magnetic field and same RF frequency, we would like two accelerate Deuton (^{2}H-), two time heavier, what would the revolution frequency and RF harmonic h. (4)**

Answer :

a. Stripping extraction.

b. We have

$<B(R)> = B0/(1-(Rh\omega rf/c)^2)^{1/2}$

$B\rho extract/ B\rho inj = P extract/ Pinj = [<B(R_{xtraction})>/<B(R_{inj})>] . [R_{extraction} / R_{inj}]$

If $\gamma \sim 1$, then

$Pextract/ Pinj = Vextract/ Vinj = [R_{extraction} / R_{inj}]$

b.

$\omega = qB/\gamma m = \omega RF/ h$ *and We have* γinjection=1

proton mass~ $1.6\ 10^{-27}$ kg

proton charge~ $1.6\ 10^{-19}$ C

Frf=44 MHz= $2\pi\ \omega RF$

$$B = m_p/q\ .\ 2\pi FRF / h = 10^{-8}.22\ .10^6 .2\pi . = 1.4\ \text{Tesla}$$

c.proton mass~ $1.67\ 10^{-27}$ kg , deuton mass ~ $3.34\ 10^{-27}$ kg

$$\omega = qB/(\gamma\ 2m_p) = \omega RF/ h \quad = 10.5\ Mhz \quad : \quad h=4$$

Useful expressions

CONSTANTS

$c = 2{,}997925 \times 10^{8}$ m/s
1 u.m.a. = m_0= 931,478 MeV= $1{,}67.10^{-27}$ kg
$e_0 = 1{,}602 \times 10^{-19}$ C