Cyclotrons

Chapter 3

Cyclotron Design

- Isochronism
- Maximal energy (Bmax, R, stability)
- Simulation // tracking

Design Strategy for K=10 MeV cyclo

Design Strategy for K=250 MeV cyclo

Design Strategy for reseach facility (E/A vs I)

Isochronism : Field B=f(r)

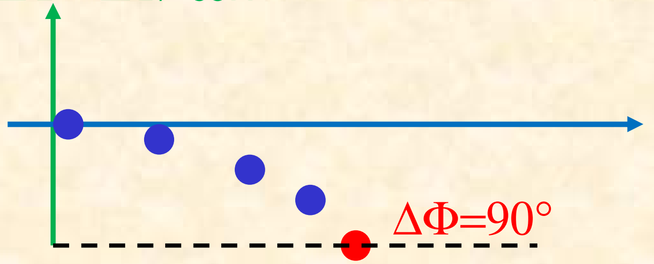
B(R) adjusted to get h ωrev = ωrf

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

$$\gamma(R) = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (R\omega_{rev})^2/c^2}}$$

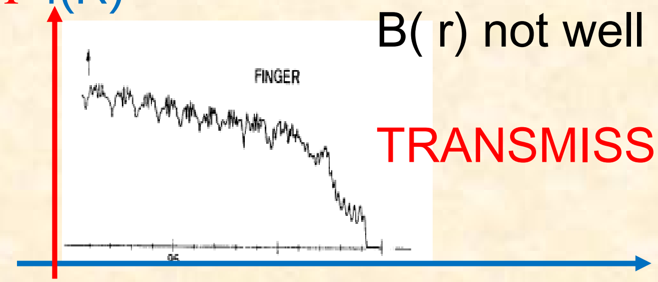
$$B_z(R) = B_{z0} / \sqrt{1 - (R\omega_{rev})^2/c^2}$$

ΔΦ= ΔT/ ωrf



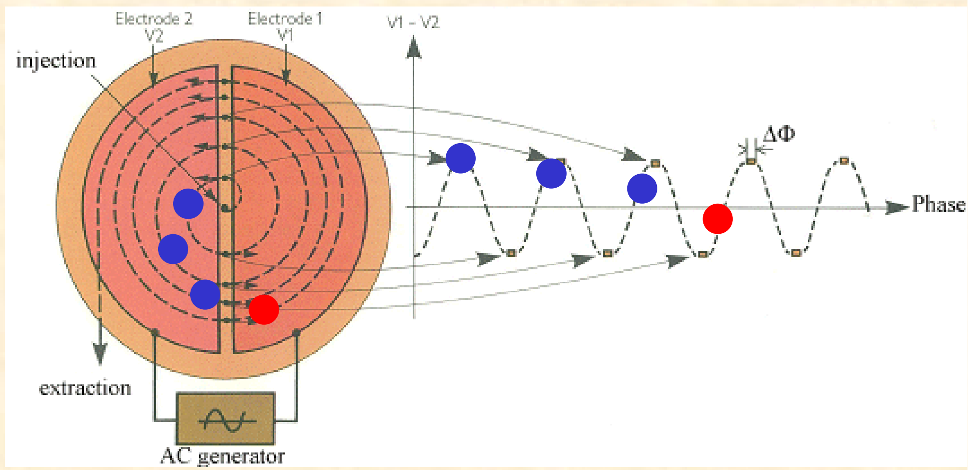
B(Radius) obtained -with correction coils
or -with pole shapes

I=f(R)



B(r) not well tuned

TRANSMISSION=0%



Magnet design

How to adjust B(r)

- Pole Gap evolution <B (r)>
- Correction coils (trim coils)

$$FLUTTER = \frac{\left(B_{hill} - B_{val}\right)^2}{8 \left\langle B \right\rangle^2}$$

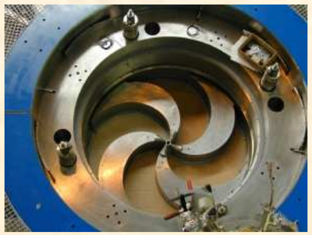
Sufficient FLUTTER F for axial stability

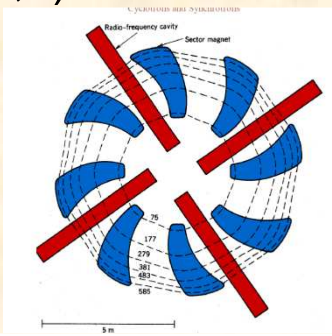
- Valley // hill B field
- sector angle
- spiral angle ε

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F \left(1 + 2 \tan^2 \varepsilon\right) > 0$$

Space for injection beam line and RF (+ vertical Stability)

- Azimutally Varying Field or Separated Sectors
- large Number of sectors N (3,4, 6,8)





Max Energy for Cyclotrons

Heavy Ion
A= nucleon number
Q= charge number

Max Kinetic Energy

$$\omega_{rev} = \frac{qB}{\gamma m} \approx \frac{qB}{m}$$

$$(\gamma-1)mc^2 \approx \frac{1}{2} m v^2$$

$$= \frac{1}{2} m (R_{extraction} . \omega_{rev})^2$$

For ions: $m = A\, m_0 = A . [1.6\ 10^{-27}\ kg]$ $q = Q\, e_0$

-

ex : $^{12}C^{4+}$ A=12 Q=4

$$-[E/A]_{max}\ (MeV/nucleon) = K_b \left\{ \frac{Q}{A} \right\}^2$$

$$\text{with} \quad K_b \approx 48.2 \left(\langle B \rangle . R_{ext} \right)^2$$

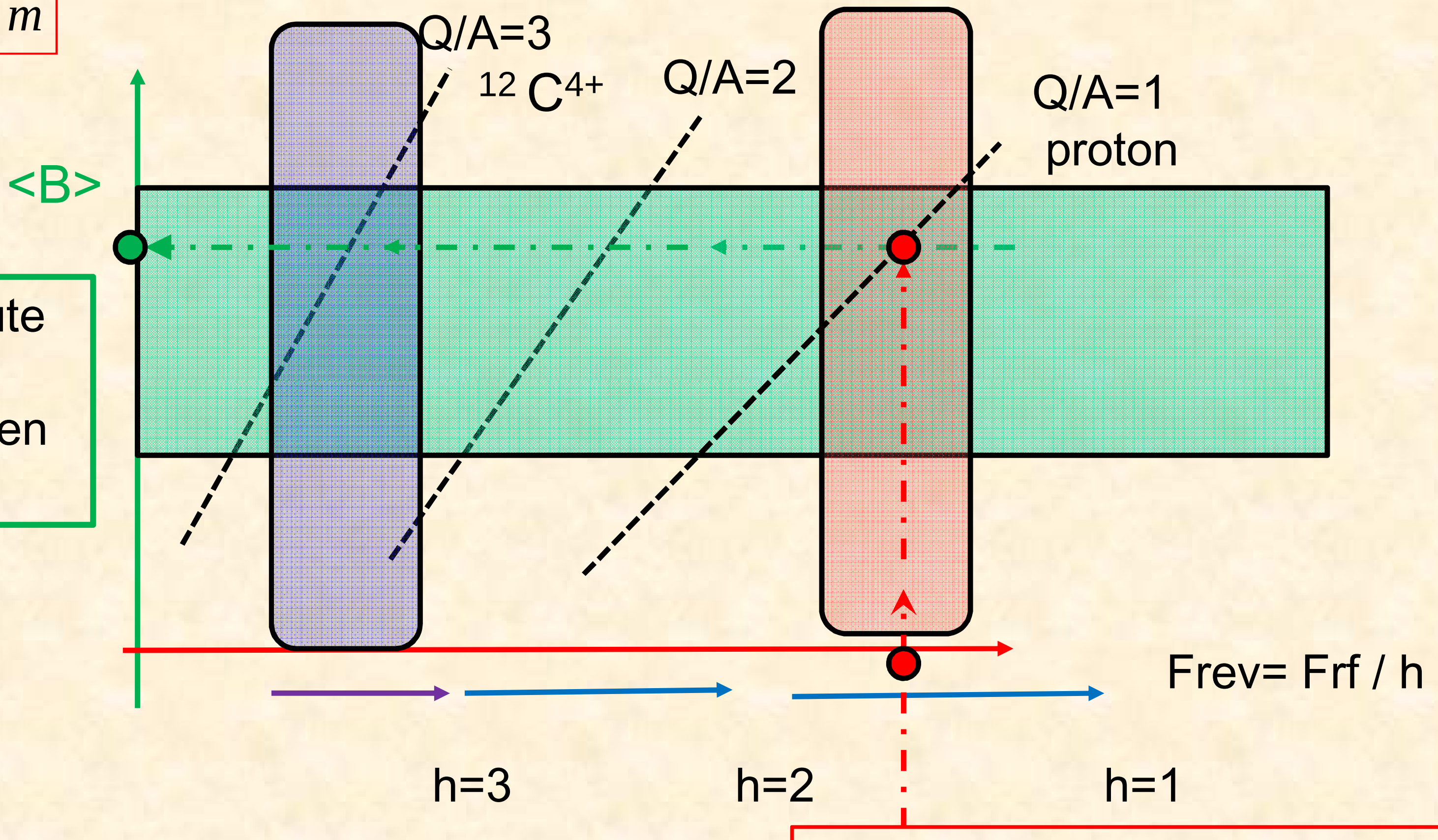
<B> limitation and size limitation (: Rextraction)

Diagram for
The variable energy cyclotrons

$$F_{rev} \propto \frac{Q.B_{cyclo}}{A} \propto h \cdot F_{RF}$$

A= nucleon number
Q= charge number

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$



2) Compute <B>
For a given (Q,A)

1) Select the energy (Frev)
Select the ions (Q,A)
Adjust <B>

Bρ # <B> Rextract

E/A (MeV/A) # K (Q/A) ^{2}

Cyclotron Design

Particle kind : proton, heavy ion ? q/A

<B Rectraction>

Max Kinetic Energy of reference ions

Bρ Injection

Bρ Extraction

with $K_b \approx 48 . \left(\langle B \rangle . R_{ext} \right)^2$

FLUTTER REQUIRED ?

AVF // ring (SCC)

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \, (1 + 2 \tan^2 \varepsilon) > 0$$

Nsector= 3,4,6

Spiral angle ε

$$F_l = \frac{\left(B_{hill} - B_{val} \right)^2}{8 \langle B \rangle^2}$$

Then, let's start the SIMULATIONs

Multi_particlelcode in « realistic » magnetic field
- compute reference orbit
- simulate injection
- simulate extraction

Analytical B Field + RF Kick
Computed field map B + RF Kick
Computed field map B + E

Cyclotron simulation :
Particle Tracking

SIMULATION : tracking ions (M,Q,v0)

Multi-particlelcode
in « realistic » magnetic field

In cylindrical coodinates

TRANSPORT (1rst order)
MAD (3rd order)
CPO

Cyclotron Magnets are too complex

$$\mathbf{r} = r \cdot \mathbf{e}_r + z \cdot \mathbf{e}_z$$

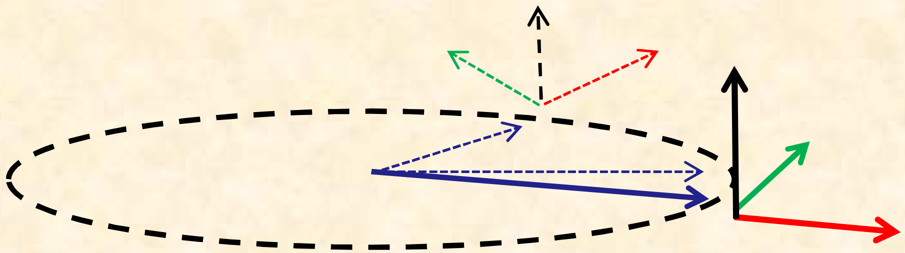
Comoving Frame : er = f(t)

$$d\mathbf{e}_r = \mathbf{e}_\theta \cdot d\theta \qquad d\mathbf{e}_z = 0 \qquad d\mathbf{e}_\theta = -\mathbf{e}_r \cdot d\theta$$

Velocity : $\dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}$?

$$\dot{\mathbf{r}} = \dot{r} \cdot \mathbf{e}_r + \dot{z} \cdot \mathbf{e}_z + r \cdot \dot{\mathbf{e}}_r + z \cdot \dot{\mathbf{e}}_z$$

$$\frac{d}{dt}\left[m\gamma \dot{\mathbf{r}} \right] = q \cdot (\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$$



Cyclotrons simulation: cylindrical equation

$$\frac{d\mathbf{p}}{dt} = q\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix} =$$

$$= (\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)\cdot\mathbf{e}_r + (r\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)\cdot\mathbf{e}_z + (\dot{r}\cdot B_z - \dot{z}\,B_r)\cdot\mathbf{e}_\theta$$

Evolution in time t is not convenient, evolution in θ is better !!!

$$\frac{d}{dt} = \frac{d\theta}{dt}\frac{d}{d\theta} = \dot{\theta}\frac{d}{d\theta}$$

$$\frac{d\mathbf{p}}{dt} = \dot{\theta}\frac{d\mathbf{p}}{d\theta} = q\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

$$\frac{d}{d\theta}\left[m\gamma\dot{r}\right] = \frac{d}{d\theta}[p_r] = m\gamma r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z}\cdot B_\theta - r\dot{\theta}\cdot B_z)$$

$$\frac{d}{d\theta}\left[m\gamma\dot{z}\right] = \frac{d}{d\theta}[pz] = \frac{q}{\dot{\theta}}(r\dot{\theta}\cdot B_x - \dot{r}\cdot B_\theta)$$

$$\frac{d}{d\theta}\left[m\gamma r\dot{\theta}\right] = \frac{d}{d\theta}[p_\theta] = \frac{q}{\dot{\theta}}\ldots.$$

$$\frac{dr}{rd\theta} = \frac{\dot{r}}{r\dot{\theta}} = \frac{p_r}{p_\theta} \qquad \frac{dz}{rd\theta} = \frac{\dot{z}}{r\dot{\theta}} = \frac{p_z}{p_\theta}$$

Cyclotrons simulation: trajectory in (r,z)=f(θ)

The integration of particle's equation can be obtained with numerical methods

The equationq to be solved is a set of Ordinary Differential Equations (ODE).

START at θ=0 (r_0,z_0,Pr,Pz,Pθ) : what are (r,z) at θ=0+Δθ ?

At first order , we can compute (r,z) and (pr,pz)

$$p_r(\theta_0 + d\theta) = p_r(\theta_0) + \frac{dp_r(\theta_0)}{d\theta} d\theta + 0(d\theta^2) + \ldots.$$

$$\frac{d}{d\theta}[pr] = m\gamma r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z} \cdot B_\theta - r\dot{\theta} \cdot B_z)$$

$$r(\theta = \theta_0 + d\theta) = r_0 + \frac{dr}{d\theta} d\theta + 0(d\theta^2)$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{pr}{p\theta}$$

$$t = t_0 + \frac{dt}{d\theta} d\theta + 0(d\theta^2)$$

This the EULER method !

Cyclotrons simulation : the algorithm

Loop j=1,Nparticle

INITIAL position and momentum : $\theta = 0$ r, z pr, pz, pθ

Loop i=1,Nstep // **step in dθ**

$$B_r = BR(r,z,\theta) \qquad B_z = BZ(r,z,\theta) \qquad B_\theta = B\theta(r,z,\theta)$$

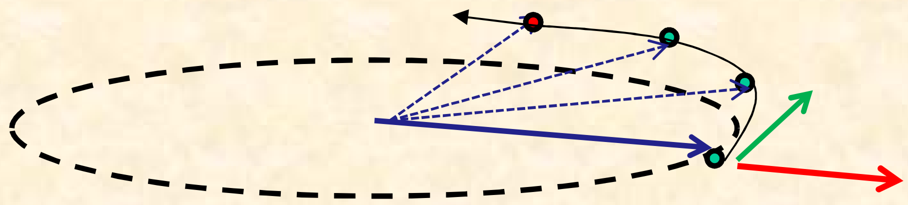
FIELD MAP

$$\theta = \theta_0 + d\theta$$

$$r(\theta = \theta + d\theta) = r + \frac{dr}{d\theta} d\theta \qquad p_r(\theta) = pr_0 + \frac{dp_r}{d\theta} dz$$

$$z(\theta = \theta + d\theta) = z + \frac{dz}{d\theta} d\theta \qquad p_z(\theta) = ...$$

$$\frac{d}{d\theta}[pr] = m\gamma r\dot{\theta} + \frac{q}{\dot{\theta}}(\dot{z} \cdot B_\theta - r\dot{\theta} \cdot B_z)$$



Endloop I // **end θ loop**

Endloop J // Nparticle **loop**

Euler algorithm (second order accurate in **dθ**)

Euler algorithm (second order accurate in $d\theta$) is not the best !!

RK4 (runge kutta order 4) is better (4th order accurate in $d\theta$)
See a Numerical analysis Lecture

SPECIAL ATTENTION to FIELD INTERPOLATION between the points of the field map

Trajectories and matching recipes

- 0) Define the basic parameters of the cyclotron

START The simulation in the middle of cycloton
With a defined magnetic structure

- 1) Find the closed orbit (1 particle) without acceleration at R=Rref

- 2) Find a matched beam in the cyclotron (multiparticles)
  backward tracking toward injection

- 3) Forward tracking (multiparticles) toward extraction
-
- 4) Extraction (multiparticles) : (deflector, precession, resonance)

Iterative process

basic parameters R,<B>,Sectors (0/4)

Ex: cyclotron design 20 MeV/A for carbon ion

What is the Max energy (MeV/A) : 20 MeV/A for carbon 4+

What are the ions (q/A) = 4+/12 ⇨ Bρextrac= 2 T.m

< Bmax>=1.5 T.m feasible with conventional magnet => <Rmax>=1.4 m

What is the ion source : Vsource =30 kVolt ⇨ Bρinj=0.04T.m

Rextraction/Rinjection~ Bρextract / Bρinj ~ 1.4m/ 0.04m

Hill and Valley : choose typical values and check νz

let's take gap hill =12 cm //gap valley =30 cm //…ε //α..

compute FLUTTER

IS IT STABLE ?

$$\nu_z^2 = n + \frac{N^2}{N^2-1} F_l (1 + 2\tan^2 \varepsilon)$$

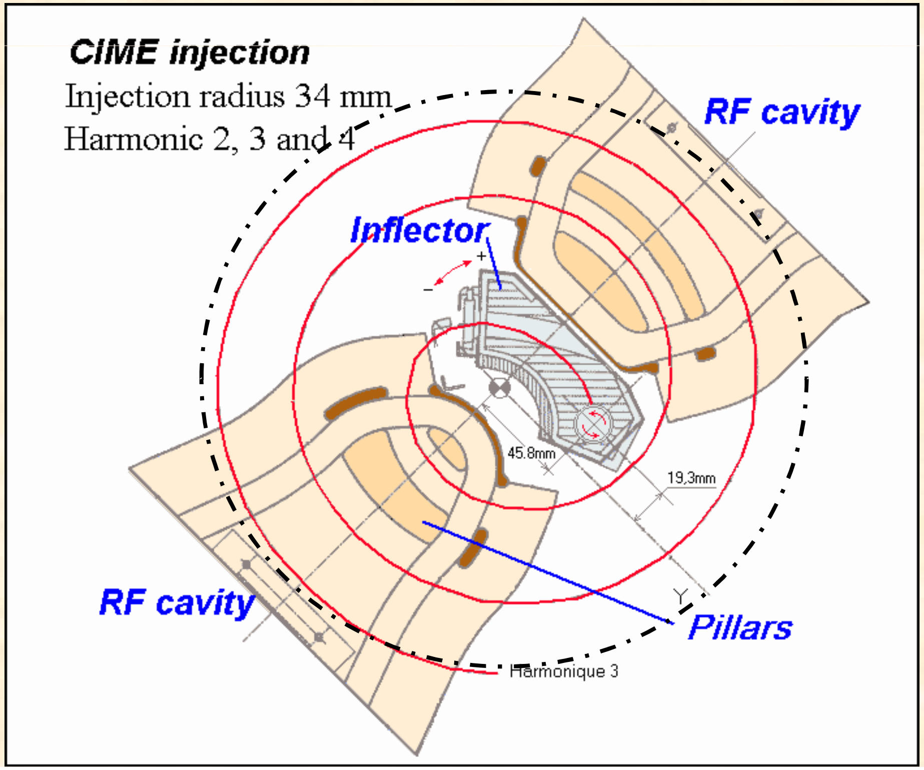
Trajectories and matching recipes (1/4)

- Find a central trajectory (1 particle) with the magnetic field map
  - For a isochronous field level and a given frequency <B>(r)

    ⇨Start from a closed orbit at large radius (no RF field)

    ⇨ Then turn on RF field to decelerate the central particle to the injection.

    ⇨ Tune the RF and the magnetic field at the injection to join the inflector output trajectory.

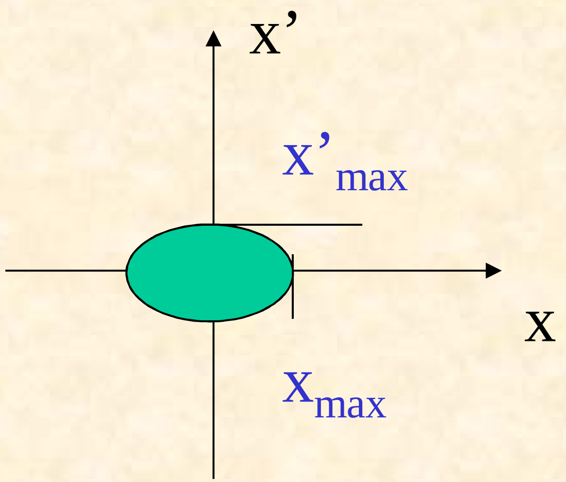


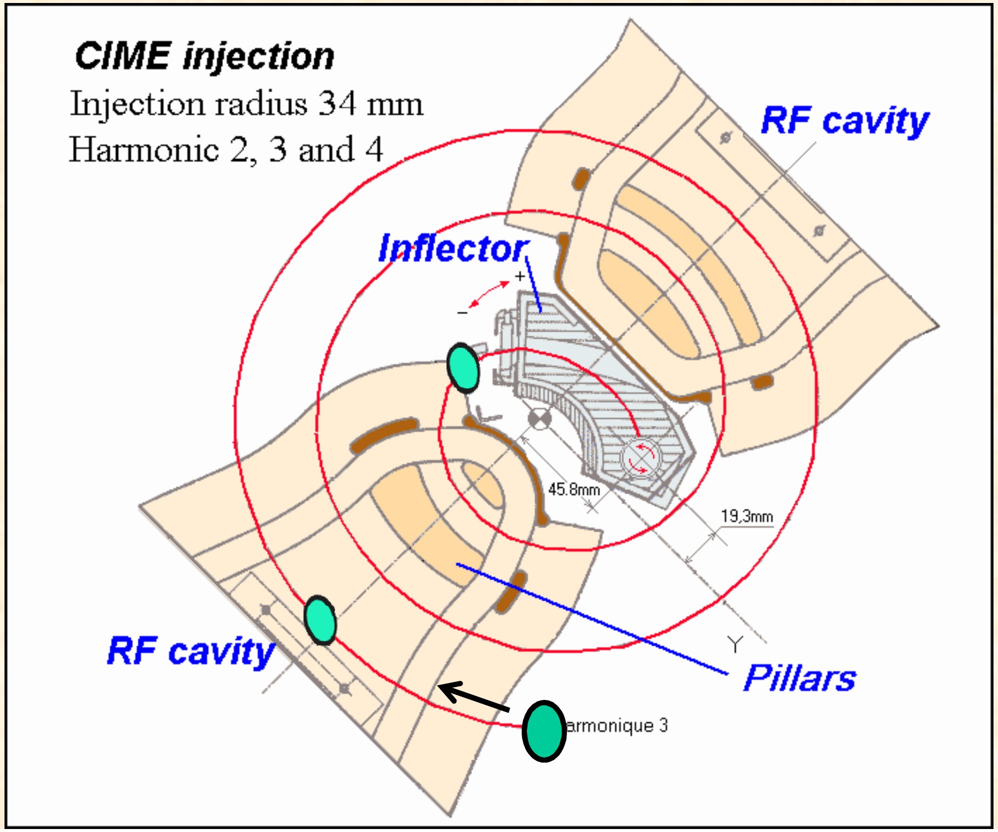
Trajectories and matching recipes (2/4)

- Find a central trajectory (1 particle)
- Find a matched beam in the cyclotron (multiparticles)

⇨ Start with a matched beam at large radius around the central trajectory (6D matching)

⇨Again in backward tracking through the field maps determine the 6D phase-space at the injection





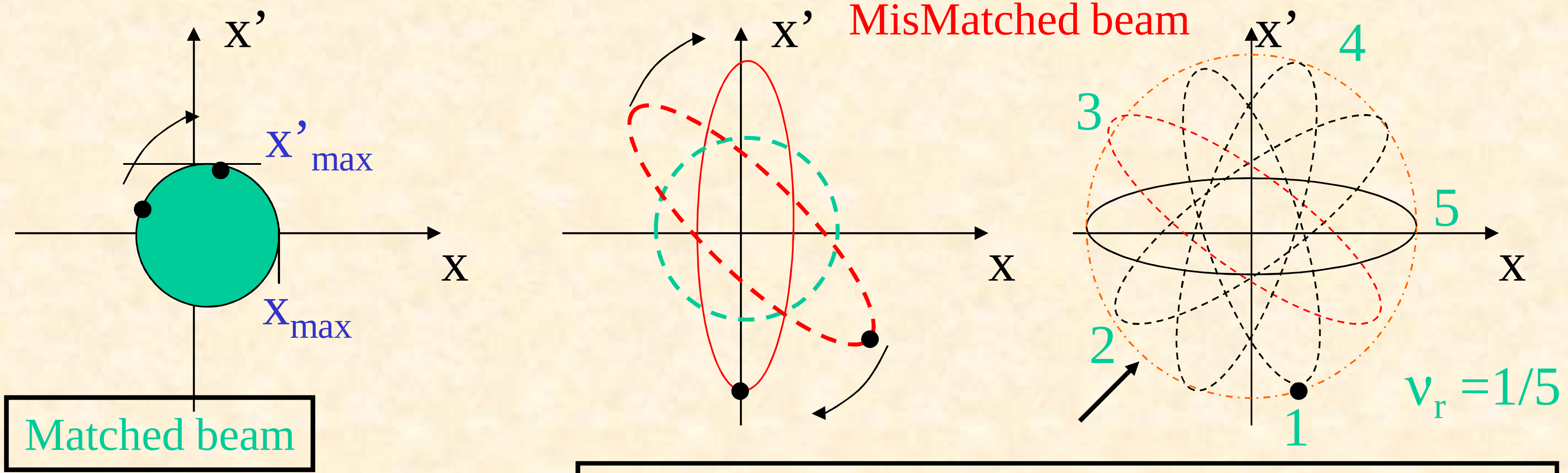
Mismatched beam recall

We define a closed orbit ⇨ without acceleration

$$\begin{cases} x(t) = x_{max}\cos(\nu_r\omega_0 t) \\ x'(t) = x'_{max}\sin(\nu_r\omega_0 t) \end{cases}$$

Emittance area : $\varepsilon = \pi\, x_{max}.x'_{max}$ (and $\varepsilon = \pi\, z_{max}.z'_{max}$)

Betatron oscillation with mismatched beam



Larger acceptance required for the cyclo : not good

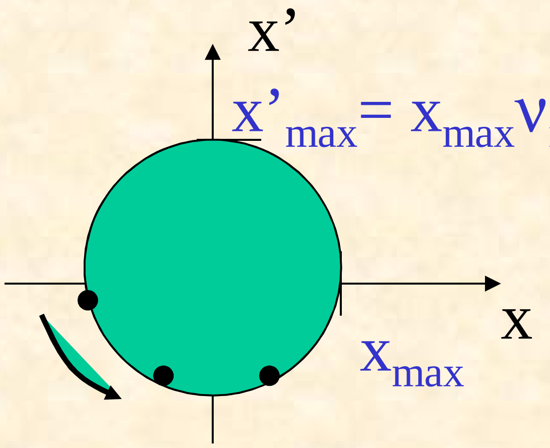
Matched beam recall

$$\begin{cases} x(t) = x_{max}\cos(\nu_r\omega_0 t) \\ x'(t) = dx/ds = dx/R\omega_0 dt = -(x_{max}\nu_r/R)\sin(\nu_r\omega_0 t) \end{cases}$$

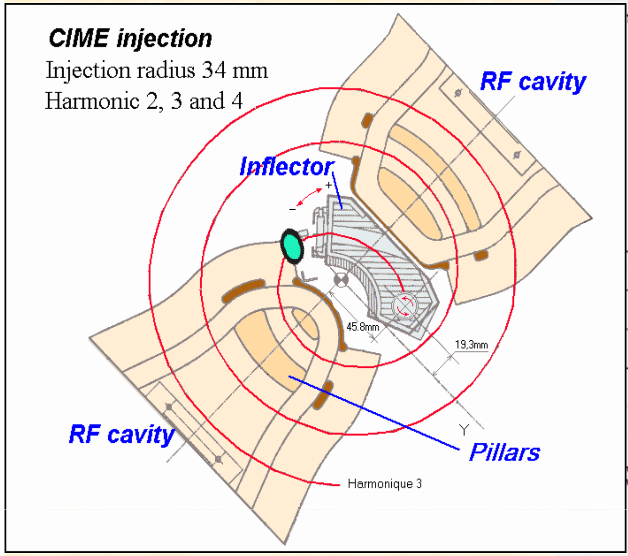
$|x'_{max}| = |x_{max}\nu_r/R|$ and $\varepsilon = \pi\, x_{max}.x'_{max} = \pi\, .x_0^2\nu_r / R$

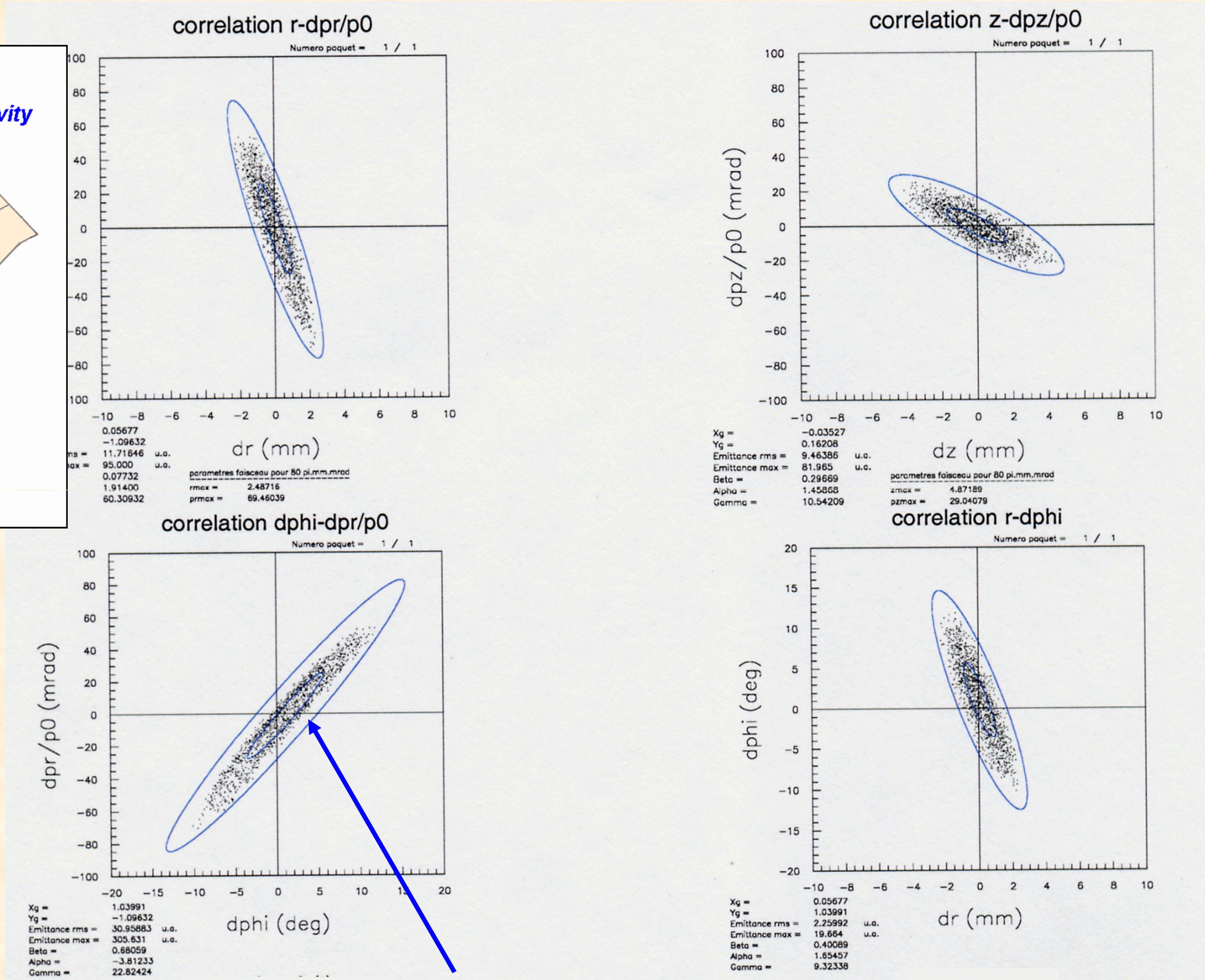
⇨ Initial beam conditions depend of the tune (ν_r) of the cyclotron at the matching point.

- ⇨ Betatron oscillation disappears
- ⇨ Matched beam
- ⇨ Minimum of acceptance



Final backward 6D matching @ injection



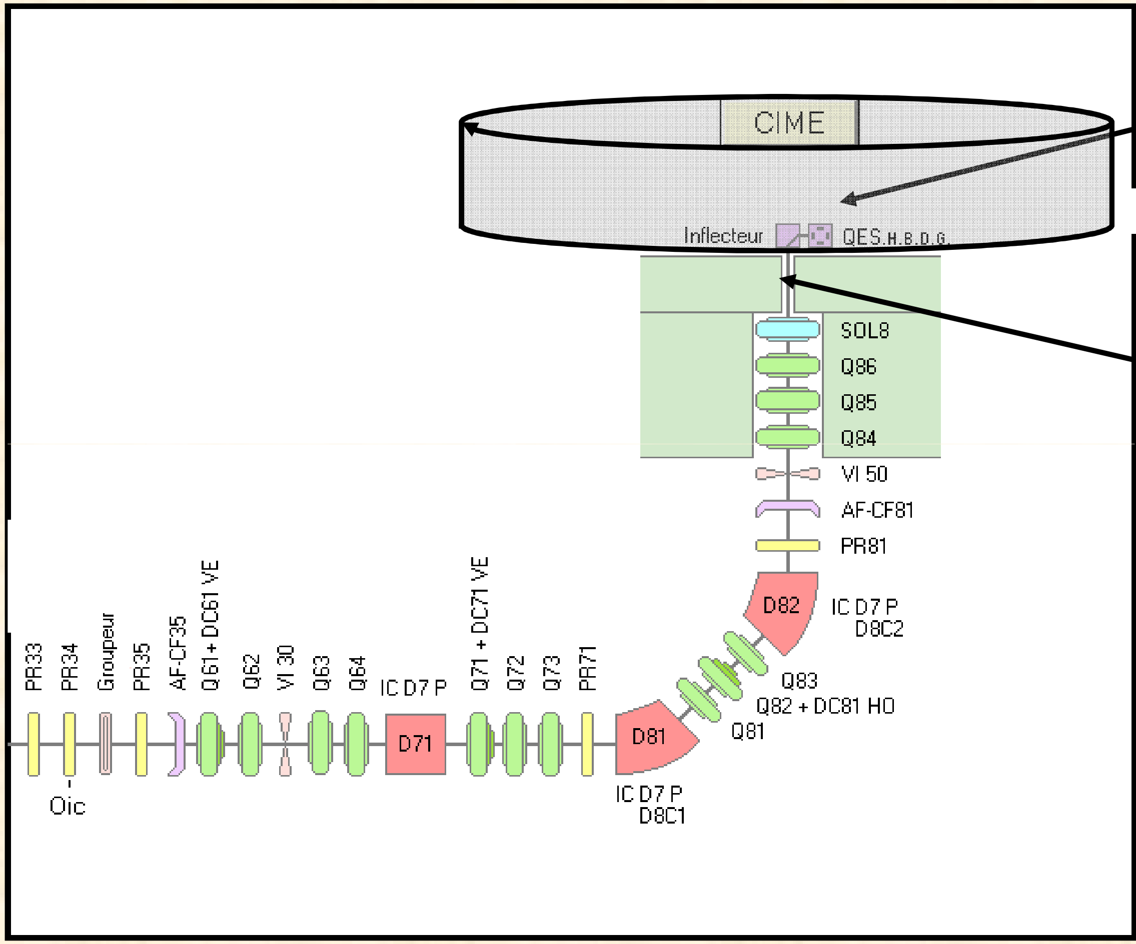


Not well represented by a gaussian beam ⇨ mismatch in forward

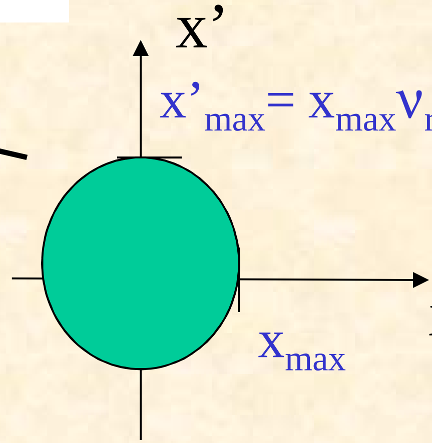
Trajectories and matching recipes (3-4/4)

- Forward tracking (multiparticles)
  - ⇨ confirm the matching to the extraction
  - ⇨ tune the isochronism
  - ⇨ and if the matching at the injection is not feasible by the injection line predict the new beam envelope and extraction
- Extraction (multiparticles)
  - design the extraction
  - turn separation (RF+precession? + magnetic bump?)

Backward 6D matching



Matching at this point



Classical transport line problems :

Adjust quad to get desired beam at injection (r,r') (z,z')

Cyclotron Design strategies

Radio-Isotopes production

cost & reliability

Medical applications : Cancer treatment

cost & reliability

Nuclear physics& Research facility

performance , intensity

Strategy for Radio-Isotopes production medical applications

10 MeV Protons / 5 MeV Deutons : @ low cost

Bρmax =0.458 T.m = <B> Rextract

Rextract = 0.34 m

<B> = 1.35Tesla [hill =1.8 T // valley =0.5 T]

AVF with 4 straight sectors (sufficient)

I~ 0.1 -0.05 mA

Rf Dees : 2 (so 4 gaps)

2 possibilities for extraction

Extraction By stripping :
external target (18F, radiotracer)

No Extraction :
internal target (in yoke)

A « low energy » industrial Cyclotron Cyclone 10/5 : H & D

Kb=10 MeV

Fixed energy ;

4 straight sector 50°

fixed Frf =42Mhz

<B> =1.35 Tesla

Harmonic h=2(p) ,4 (D)

Internal source

Rextraction=0.33m

Bρmax=0.33x 1.35=0.45 T.m

$$\left[\frac{E}{A}\right]_{max} (MeV \;/\; nucleon) = K_b \left\{\frac{Q}{A}\right\}^2$$

Eprotons=10 MeV protons= $^{1}H^{1+}$ A=1 Q=1

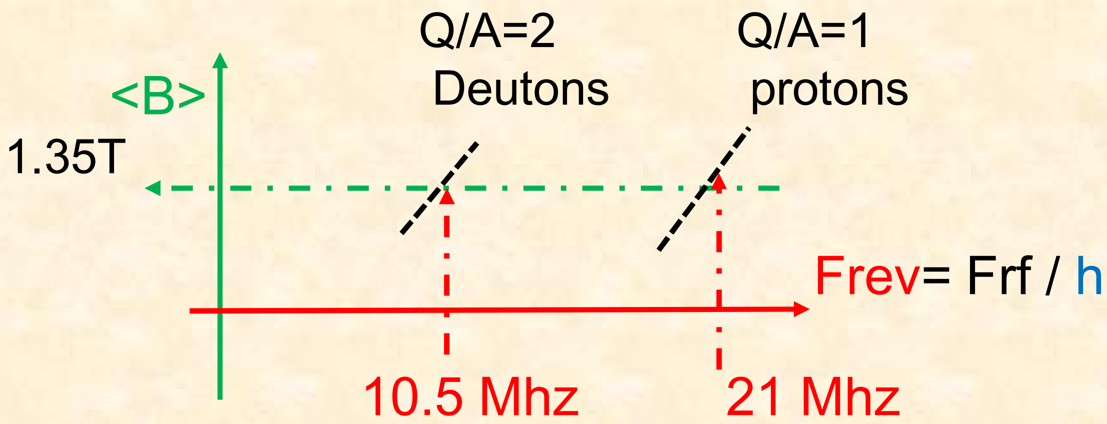
(E/A=Kb*1^2 =10MeV/A)

RF Harmonic =2 Frev=42/ h= 21 Mhz

EDeutons=5 MeV Deutons= $^{2}H^{1+}$ A=2 Q=1

(E/A=Kb*0.5^2= 2.5 MeV/A)

RF Harmonic =4



Strategy for cancer treatment
proton therapy (>80 facilities in the world)

-250 MeV Protons

Accel VARIAN Isochronous cyclo

Superconducting <B> = 2.2 Tesla

Rextrac~1.2m

-230 MeV Protons

IBA Synchro cyclotron

Superconducting <B> = 5. Tesla

Rextract ~0.6 m

Very compact

Hill/valley not needed

Strategy for a Cyclotron in a research facility

High energy

$E/A_{max} = Kb \,.(Q/A)^2$

High Kb ~ $(R.B)^2$

High ion charge Q

Large magnet (Radius)

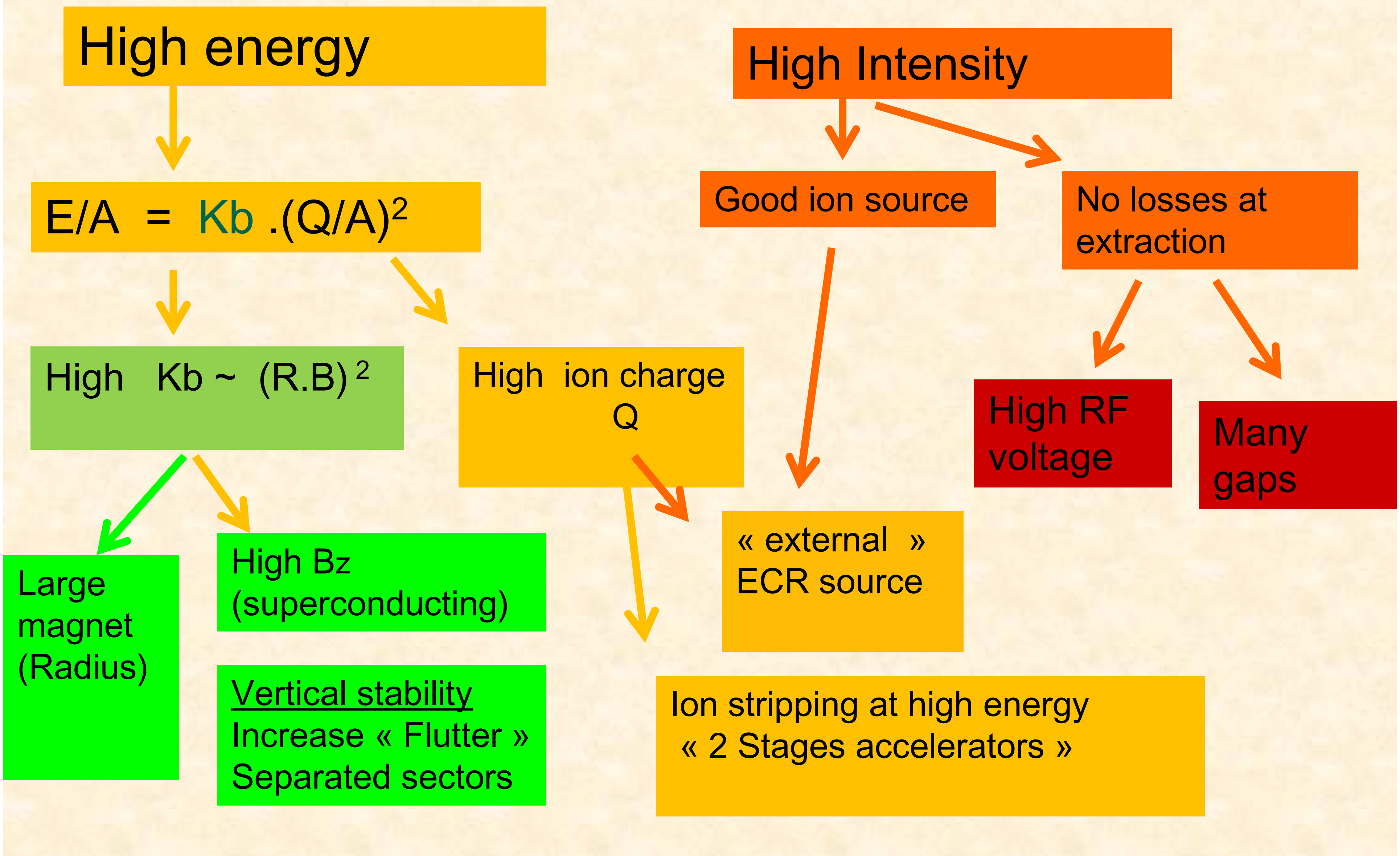
High Bz (superconducting)

Vertical stability
Increase « Flutter »
Separated sectors

« external » ECR source

Ion stripping at high energy
« 2 Stages accelerators »

Strategy for a Cyclotron in a research facility



Coupling of 2 Cyclotrons : v matching

Two cyclotrons can be used to reach higher energy :

- Harmonic/ Radius of the 2 cyclotrons have to be matched

$$\frac{v}{2\pi} = \left[\frac{F_{HF}.R_{ejec}}{h}\right]_{cycloA} = \left[\frac{F_{HF}.R_{inj}}{h}\right]_{cycloB}$$

The velocity of extraction CycloA

= velocity of injection CycloB

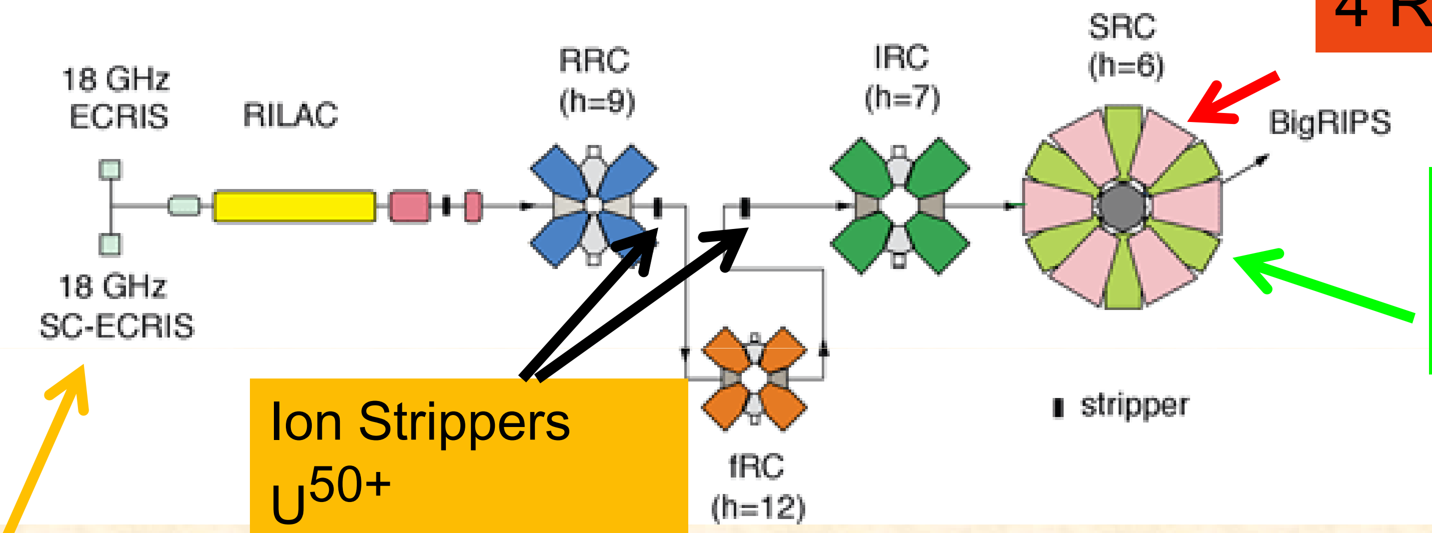
- Ion stripping can be used, to increase Q/A before injection into the second cyclo

large Q, Increase *Emax*

$$\left[\frac{E}{A}\right]_{\max} = K_b \left\{\frac{Q}{A}\right\}^2$$

RIBF (Japan) : SRC (K=2600 MeV) –the bigest cyclo
Uranium beam $^{238}U^{88+}$ @345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



4 RF cavities

Radius=6m
<B>=3.8T

Ion Strippers
U^{50+}
U^{88+}

ECR source
U^{30+}

5 Stages accelerators

1 LINAC
+ 4 Cyclotrons

Ion Stripping at high energy

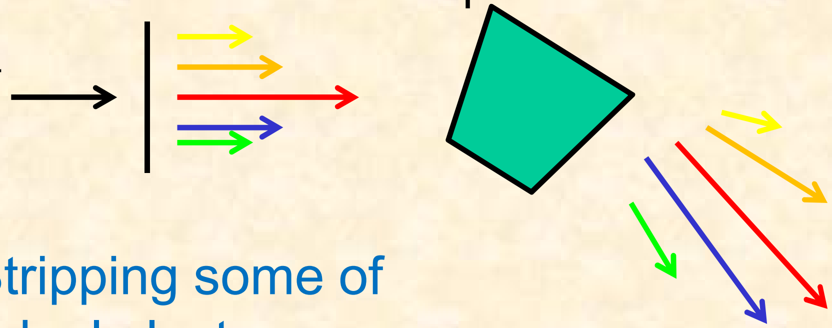
$$[E/A]\max = Kb\left[\frac{A}{Q}\right]^2$$

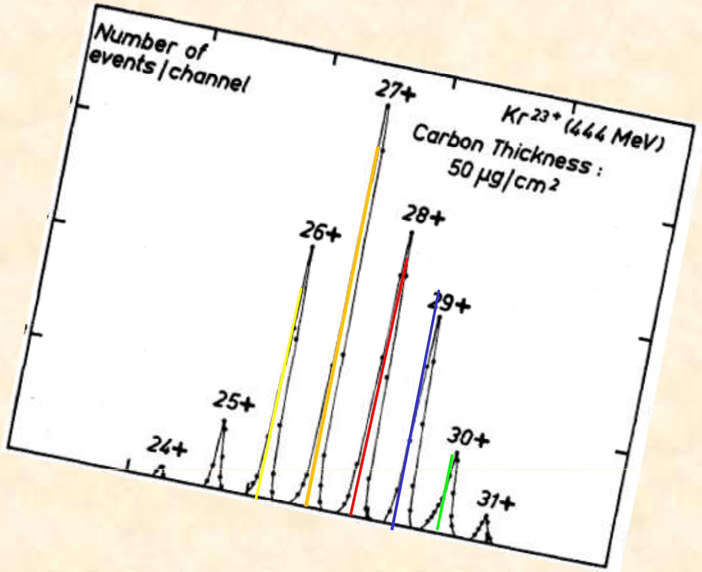
Heavy ions are not fully stripped by ion sources :

Incoming Ions

Magnetic spectrometer

A^{Q_1+}





Stripping some of residual electrons

final Ions A^{Q_2+}

$Q_2>Q_1$

$B\rho_2 < B_{\rho 1}$

$$B\rho = \frac{P}{q} = \frac{\gamma\, m.v}{q}$$

Ion Stripping help increase the maximal energy of a given cyclotron….

Cyclotrons

- End Chapter 3