

# Transverse Beam Dynamics

JUAS 2017 - tutorial 1

## 1 Exercise: Wien Filter

A Wien Filter is a device that allows to select the particles in a charged beam according to the desired velocity.

- Write down the expression of the Lorentz force.
- How is the force originating from a uniform magnetic field oriented with respect to the particle velocity and the field? How should we orient an electric field if we want to compensate that force?
- Assuming the magnetic field is 1 mT, what would be the required electric field (V/m) to select protons travelling with a velocity of  $0.1c$ ?
- [Optional] Write down the equations of motion and their solutions. What are the implications of the oscillating terms?
- [Optional] Could we use a Wien filter with a neutral beam (eg. neutron)? What other techniques may be employed in this case?

## 2 Exercise: understanding the phase space concept

1. Sketch the emittance ellipse of a particle beam in horizontal  $x-x'$  phase space (I) at the position of a transverse waist, (II) when the beam is divergent and (III) when the beam is convergent.
2. Phase Space Representation of a Particle Source:
  - Consider a source at position  $s_0$  with radius  $w$  emitting particles. Make a drawing of this setup in configuration space and in phase space. Which part of phase space can be occupied by the emitted particles?
  - Any real beam emerging from a source like the one above will be clipped by aperture limitations of the vacuum chamber. This can be modelled by assuming that a distance  $d$  away from the source there is an iris with an opening with radius  $R = w$ . Make a drawing of this setup in configuration and phase space. Which part of phase space is occupied by the beam at a location after the iris?

## 3 Stability condition

Given a periodic lattice with generic transport map  $M$ ,

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

under which condition the matrix  $M$  provides stable motion after  $N$  turns (with  $N \rightarrow \infty$ )?  
Hint: the motion is stable when all elements of  $M^N$  are finite.

## 4 Exercise: stability condition

Consider a lattice composed by a single 2 meters long quadrupole, with  $f = 1$  m

- Prove that if the quadrupole is defocusing, then a lattice is not stable
- Prove that if the quadrupole is focusing, then the lattice is stable