# Transverse Beam Dynamics

JUAS 2017 - tutorial 2

## 1 Exercise: local radius, rigidity

We wish to design an electron ring with a radius of R=200 m. Let us assume that only 50% of the circumference is occupied by bending magnets:

- What will be the local radius of bend $\rho$ in these magnets if they all have the same strength?
- If the momentum of the electrons is 12 GeV/c, calculate the rigidity $B\rho$ and the field in the dipoles.

## 2 Exercise: particle momentum, geometry of a storage ring and thin lenses

The LHC storage ring at CERN will collide proton beams with a maximum momentum of $p = 7$ TeV/c per beam. The main parameters of this machine are:

| | | |
|---|---|---|
| Circumference | $C_0 = 26658.9$ m | |
| Particle momentum | $p = 7$ TeV/c | |
| Main dipoles | $B = 8.392$ T | $l_B = 14.2$ m |
| Main quadrupoles | $G = 235$ T/m | $l_q = 5.5$ m |

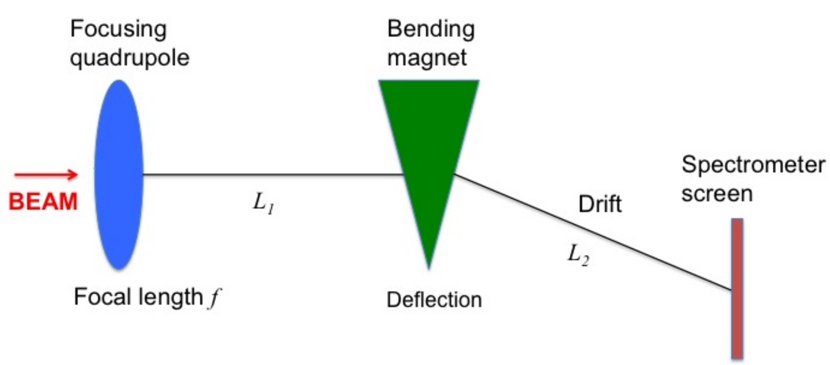
- Calculate the magnetic rigidity of the design beam, the bending radius of the main dipole magnets in the arc and determine the number of dipoles that is needed in the machine.
- Calculate the k-strength of the quadrupole magnets and compare its focal length to the length of the magnet. Can this magnet be treated as a thin lens?
- [Optional] Compute and compare the matrices for the quadrupole for the thin (drift-kick-drift) and thick cases.

## 3 Exercise: A spectrometer line in CTF3

The CTF3 (CLIC Test Facility 3) experiment at CERN consists of a linac that injects very short electron bunches into an isochronous ring.

A spectrometer line made of one quadrupole and one bending magnet is located at the end of the linac where the particle momentum is 350 MeV/c. The goal of the spectrometer is to measure the energy before injecting the electrons in the ring.

The spectrometer line is sketched on the figure below. It is made of a focusing quadrupole of focal length $f$, a drift space of length $L_1$, a bending magnet of deflection angle $\theta$ in the horizontal plane, and a drift space of length $L_2$. We assume that the spectrometer line starts at the quadrupole and ends at the end of the second drift. We neglect the focusing effect of the dipole.



**3.1 If the effective length of the dipole is $l_B = 0.43$ m, what should be the magnetic field (in Tesla) inside the dipole to deflect the electrons by an angle of 35 degrees?**

**3.2 Starting from the general horizontal $3 \times 3$ transfer matrix of a sector dipole of deflection angle $\theta$, show that the transfer matrix of a dipole in the thin lens approximation is**

$$M_{dipole} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

Which approximations are done?
**Hint:** A sector dipole has the following $3 \times 3$ transfer matrix:

$$M_{\text{dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{\sin\theta}{\rho} & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

**3.3 In the thin lens approximation, derive the horizontal extended $3 \times 3$ transfer matrix of the spectrometer line. Show that it is:**

$$M_{spectro} = \begin{pmatrix} \frac{f-L_1-L_2}{f} & L_1+L_2 & L_2\theta \\ -\frac{1}{f} & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

**3.4 Assuming $D = D' = 0$ at the entrance of the quadrupole, what is the dispersion and its derivative at the end of the spectrometer line? Give the numerical value of $D'$ at the end of the spectrometer line for the angle of 35 degrees.**

**3.5 What is the difference between a periodic lattice and a beam transport lattice (or transfer line) as concerns the betatron function ?**

**3.6 The Courant-Snyder invariant allows to trace the Twiss parameters $\alpha$, $\beta$, and $\gamma$ through a transfer line.**

Remember from the lecture:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_s = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC'+S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

An alternative way to transport the Twiss parameters is through the $\sigma$ matrix:

$$\sigma_i = \begin{pmatrix} \beta_i & -\alpha_i \\ -\alpha_i & \gamma_i \end{pmatrix}$$

This matrix multiplied by the emittance $\epsilon$ gives the so-called beam matrix (which has already been introduced during the lecture):

$$\Sigma_i = \begin{pmatrix} \beta_i\epsilon & -\alpha_i\epsilon \\ -\alpha_i\epsilon & \gamma_i\epsilon \end{pmatrix}$$

If $\sigma_1$ is the matrix at the entrance of the transfer line, the matrix $\sigma_2$ at the exit of the transfer line is given by

$$\sigma_2 = M\sigma_1 M^T$$

where $M$ is the $2 \times 2$ transfer matrix of the line extracted from the extended $3 \times 3$ transfer matrix (see question 1.3), and $M^T$ the transpose matrix of $M$.

**Assuming $\alpha_1 = 0$, derive the betatron function $\beta_2$ at the end of the spectrometer line in terms of $L_1$, $L_2$, $f$ and $\beta_1$.**

**Hint:** For the calculations, write $M$ as $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ and replace the values of the matrix elements only at the end.

**3.7** **Given the numerical values $L_1 = 1$ m, $L_2 = 2$ m, $\beta_1 = 10$ m, $\alpha_1 = 0$, compute the betatron function $\beta_2$ at the end of the spectrometer line as a function of the focal length $f$ of the quadrupole.**

**3.8** **Find the value of the focal length $f$ such that the betatron function $\beta_2$ at the end of the spectrometer line is minimum. Give the minimum value of $\beta_2$.**

**3.9** **In the presence of dispersion, what is the particle deviation from the design orbit due to the different particle momentum $p \neq p_0$ ($p_0$ is the design momentum)? Why is it important to minimize the $\beta$ function in the spectrometer?**