Transverse Beam Dynamics

JUAS 2017 - tutorial 3

1 Exercise: chromaticity in a FODO cell

Consider a ring made of N_{cell} identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length l_q, but their strengths may differ.

1.1 Calculate the maximum and the minimum betatron function in the FODO cell. (*Use the thin-lens approximations*)

1.2 Calculate the natural chromaticities for this machine.

1.3 [Optional] Show that for short quadrupoles, if $f_F \simeq f_D$,

$$\xi_N \simeq -\frac{N_{cell}}{\pi} \tan \frac{\mu}{2}$$

1.4 Design the FODO cell such that it has: phase advance $\mu = 90$ degrees, a total length of 10 m, and a total bending angle of 5 degrees. What are β_{max}, β_{min}, D_{max}, D_{min}?

1.5 Add two sextupoles at appropriate locations and compute their strengths to correct the horizontal and the vertical chromaticities. (hints: use 1 sextupole for the horizontal plane and 1 for the vertical plane; do not consider geometric aberrations).

1.6 If the gradient of all focusing quadrupoles in the ring is wrong by +10%, how much is the tune-shift with and without sextupoles?

2 Exercise: Double-Bend Achromat (DBA) lattice

A DBA can be made from two dipoles with a horizontally focusing quadrupole between them. The transfer matrix through the achromat is of the form:

$$M_{\mathrm{DBA}} = M_{\mathrm{bend}} M_{\mathrm{drift}} M_{1/2\mathrm{F}} M_{1/2\mathrm{F}} M_{\mathrm{drift}} M_{\mathrm{bend}}$$

Note that this magnet configuration does not produce vertical focusing, therefore will not be enough to create a stable lattice. A full DBA typically comprises additional quadrupole doublets before and after the bending section, but for sake of simplicity we will neglect them.

2.1 Use the thin lens approximation for quadrupoles and small angle approximation for bends to find the dispersion in the middle of the quadrupole. Write the focal length in terms of the drift and bend parameters.

2.2 Show that the dispersion is again zero ($\eta = \eta' = 0$) after the bend.

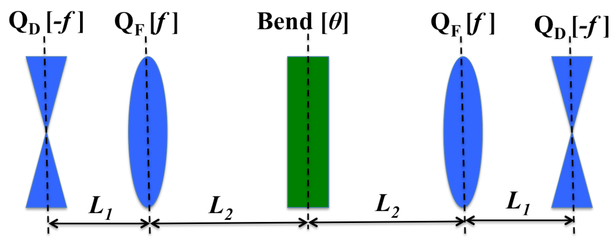
2.3 Compute the parameters L, f for a DBA 10 meters long, bending the beam by an angle of 1 radians. What is the dispersion in the centre? How much a particle with 1% energy deviation will be displaced at the centre of the cell?

3 Exercise: review of geometry, tune, optics and dispersion

Consider a proton synchrotron accumulator made of identical cells. The relevant ring parameters are given in the following table:

Proton kinetic energy	2 GeV
Cell type	Symmetric triplet$^{(*)}$
Ring circumference	960 m
Integrated quadrupole gradient ($\int G dl$)	1.5 T

$^{(*)}$Note: A thin lens symmetric triplet cell consists of a thin lens defocusing quadrupole of focal length $-f$, followed with a drift space of length L_1, a thin lens focusing quadrupole of focal length f, a drift of length L_2, a thin lens dipole of horizontal bending angle θ, a drift of length L_2, a thin lens focusing quadrupole of focal length f, a drift of length L_1, and a thin lens defocusing quadrupole of focal length $-f$ (see Figure below).



Hint:
The 3×3 horizontal transfer matrix for one symmetric triplet cell is (in the thin lens approximation):

$$M_{triplet} = \begin{pmatrix} \frac{f^3+2L_1^2L_2-2L_1f(L_1+L_2)}{f^3} & \frac{2(f-L_1)(L_1f+L_2f-L_1L_2)}{f^2} & (L_1+L_2-\frac{L_1L_2}{f})\theta \\ \frac{2L_1(L_1L_2-L_1f-f^2)}{f^4} & \frac{f^3+2L_1^2L_2-2L_1f(L_1+L_2)}{f^3} & \frac{(f^2+L_1f-L_1L_2)}{f^2}\theta \\ 0 & 0 & 1 \end{pmatrix}$$

for the transport of a vector $\begin{pmatrix} x \\ x' \\ \Delta p/p_0 \end{pmatrix}$, where $\Delta p/p_0$ is the momentum offset with respect to the design momentum p_0.

3.1 Compute the focal length f of the quadrupoles, consider that the proton rest mass is 938 MeV.

3.2 Given the numerical values $L_1 = 1.5$ m and $L_2 = 6$ m:

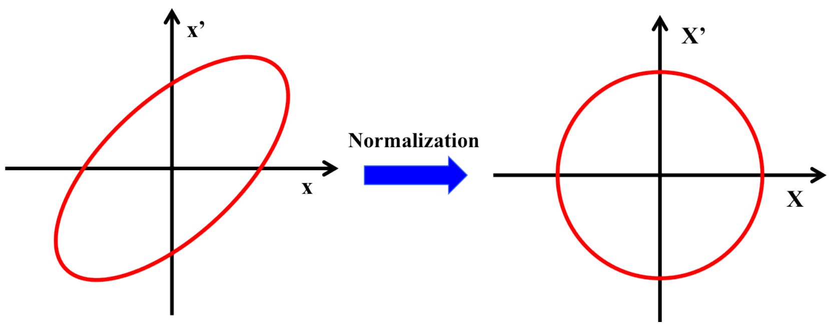
- Compute the horizontal and vertical transfer matrices of a triplet cell (take into account that the sign of the focal length changes when going from horizontal to vertical plane).
- Compute the horizontal and vertical machine tunes.
- Compute the horizontal and vertical betatron functions at the entrance of a triplet cell.
- Compute the horizontal and vertical dispersion functions at the entrance of a triplet cell.

4 Exercise: normalized phase space

Let us consider the following phase space vector: (x, x'). The transformation to a normalized phase space (X, X') is given by:

$$\begin{pmatrix} X \\ X' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta_x} & 0 \\ \alpha_x/\sqrt{\beta_x} & \sqrt{\beta_x} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

The normalization process of the phase space is illustrated in the figure below:



If we know that the transfer matrix between two points 1 and 2 (with phase advance ϕ_x between them) in the phase space (x, x') is given by:

$$M_{12} = \begin{pmatrix} \sqrt{\frac{\beta_{x2}}{\beta_{x1}}}(\cos\phi_x + \alpha_{x1}\sin\phi_x) & \sqrt{\beta_{x1}\beta_{x2}}\sin\phi_x \\ \frac{(\alpha_{x1}-\alpha_{x2})\cos\phi_x-(1+\alpha_{x1}\alpha_{x2})\sin\phi_x}{\sqrt{\beta_{x2}\beta_{x1}}} & \sqrt{\frac{\beta_{x1}}{\beta_{x2}}}(\cos\phi_x - \alpha_{x2}\sin\phi_x) \end{pmatrix}$$

Obtain the transfer matrix between two points 1 and 2 in the normalized phase space.