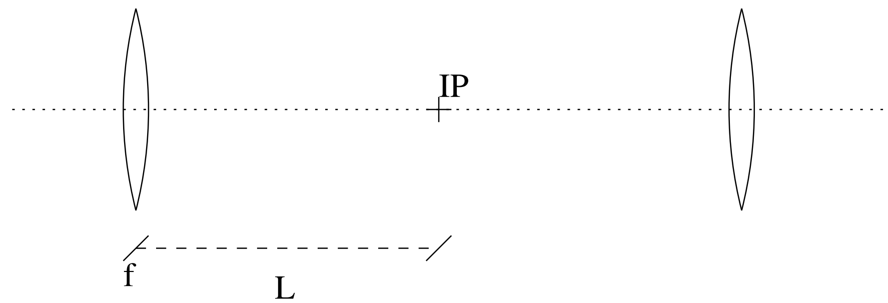
# Transverse Beam Dynamics

JUAS 2017 - tutorial 4

## 1 Exercise: Low-Beta Insertion

Consider the following low-beta insertion around an interaction point (IP). The quadrupoles are placed with mirror-symmetry with respect to the IP:




The beam enters the quadrupole with Twiss parameters $\beta_0 = 20$ m and $\alpha_0 = 0$. The drift space has length $L = 10$ m.

(i) Determine the focal length of the quadrupole in order to locate the waist at the IP.

(ii) What is the value of $\beta^\star$?

(iii) What is the phase advance between the quadrupole and the IP?

## 2 Exercise: beam size and luminosity

An $e^+e^-$ collider has an interaction Point (IP) with $\beta_x^* = 0.5$ m and $\beta_y^* = 0.1$ cm. The peak luminosity available by a $e^+e^-$ collider can be written as:

$$L = \frac{N_\mathrm{b} N_{e^-} N_{e^+} f_\mathrm{rev}}{4\pi\sigma_x^*\sigma_y^*}\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]$$

where $N_\mathrm{b} = 80$ is the number of bunches per beam (we assume the same number of bunches for both the $e^-$ and the $e^+$ beams), $N_{e^-} = N_{e^+} = 5 \times 10^{11}$ is the number of particles per bunch (we assume the same number for both $e^-$ and $e^+$ bunches), and $f_\mathrm{rev}$ is the revolution frequency. The horizontal and vertical normalised beam emittances are respectively: $\epsilon_{x,N} = 2.2$ mm and $\epsilon_{y,N} = 4.7$ $\mu$m.

- Compute the revolution frequency $f_\mathrm{rev}$, knowing that the circumference is 80 km and that the beam moves nearly at the speed of light
- Calculate the beam transverse beam sizes $\sigma_x^*$ and $\sigma_y^*$ at the IP, and the luminosity $L$ for two different beam energies: 45 GeV and 120 GeV
- What are the beam divergences (horizontal and vertical) at the IP for the 45 GeV case?
- What is the value of the betatron function at position $s = 0.5$ m from the IP?

## 3 Exercise: Bump and Orbit Control

Given two kickers located at the two ends of a FODO cell with phase advance 45 degrees (the two kickers are located at $L_\mathrm{cell}$ distance from each other), compute the strengths of such kickers (in radians) in order to give the beam, initially at $(x_i,\, x_i') = (0,\, 0)$, an arbitrary offset at the end of the cell while preserving its angle, $\left(x_f,\, x_f'\right) = (x_\mathrm{arbitrary},\, 0)$.

# 4 Exercise: measurement of Twiss parameters

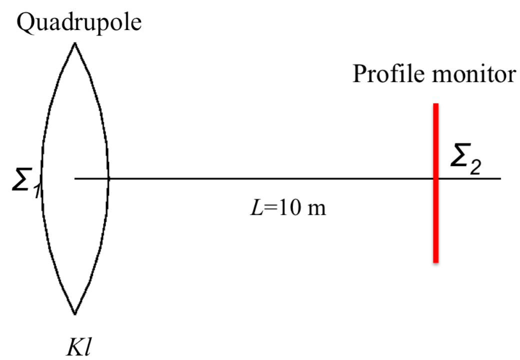
One of the possible ways to determine experimentally the Twiss parameters at a given point makes use of a so-called quadrupole scan. One can measure the transverse size of the beam in a profile monitor, called Wire Beam Scanner (WBS), located at a distance $L$ downstream a focusing quadrupole, as a function of the normalized gradient in this quadrupole. This allows to compute the emittance of the beam, as well as the $\beta$ and the $\alpha$ functions at the entrance of the quadrupole.

Let's consider a quadrupole $Q$ with a length of $l = 20$ cm. This quadrupole is installed in an electron transport line where the particle momentum is 300 MeV$/c$. At a distance $L = 10$ m from the quadrupole the transverse beam size is measured with a WBS, for various values of the current $I_Q$. The maximum value of the quadrupole gradient $G$ is obtained for a current of 100 A, and is $G = 1$ T/m. $G$ is proportional to the current.

**Advice:** use thin-lens approximation.

## 4.1 How does the normalized focusing strength $K$ vary with $I_Q$?

## 4.2 Let $\Sigma_1$ and $\Sigma_2$ be the $2 \times 2$ matrices with the Twiss parameters, $\Sigma = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$, at the quadrupole entrance and at the wire scanner, respectively.




It is worth explaining that the matrix $\Sigma$ multiplied by the emittance $\epsilon$ is the covariance matrix of the beam distribution:

$$\Sigma\epsilon = \begin{pmatrix} \beta\epsilon & -\alpha\epsilon \\ -\alpha\epsilon & \gamma\epsilon \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle x x' \rangle \\ \langle x' x \rangle & \langle x'^2 \rangle \end{pmatrix}$$

The transverse beam size of the beam is given by $\sigma_x = \sqrt{\langle x^2 \rangle} = \sqrt{\beta_x \epsilon_x}$ (horizontal beam size), and $\sigma_y = \sqrt{\langle y^2 \rangle} = \sqrt{\beta_y \epsilon_y}$ (vertical beam size). Here we will simply use the following notation: $\sigma_1 = \sqrt{\beta_1 \epsilon}$ for the beam size (horizontal or vertical) at position 1, and $\sigma_2 = \sqrt{\beta_2 \epsilon}$ for the beam size (horizontal or vertical) at position 2.

- Give the expression $\Sigma_2$ as function of $\alpha_1$, $\beta_1$, and $\gamma_1$
- Show that $\beta_2$ can be written in the form: $\beta_2 = A_2 (Kl)^2 + A_1 (Kl) + A_0$
- Express $A_0$, $A_1$, and $A_2$ as a function of $L$, $\alpha_1$, $\beta_1$, and $\gamma_1$

**Hint** for the next questions: show that if you express $\beta_2$ as

$$\beta_2 = B_0 + B_1 (Kl - B_2)^2$$

you have:

$B_0 = A_0 - A_1^2/4A_2^2 = L^2/\beta_1$

$B_1 = A_2 = L^2\beta_1$

$B_2 = -A_1/A_2 = 1/L - \alpha_1/\beta_1$

**4.3** **The transverse beam r.m.s. beam size is $\sigma = \sqrt{\epsilon\beta}$, where $\epsilon$ is the transverse emittance. Express $\sigma_2$ as a function of $Kl$ and find its minimum, $(Kl)_{\mathbf{min}}$. Give the expression for $\frac{\mathrm{d}\sigma_2}{\mathrm{d}(Kl)}$.**

**4.4** **How does $\sigma_2$ vary with $Kl$ when $|Kl - (Kl)_{\mathbf{min}}| \gg 1/\beta_1$ ?**

**4.5** **Deduce the values of $\alpha_1$, $\beta_1$, and $\gamma_1$ from the measurement $\sigma_2$, as a function of the quadrupole current $I_Q$.**