JUAS 2017 – RF Exercises

$\mu = \mu_0 \, \mu_r$

$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)

$\varepsilon = \varepsilon_0 \, \varepsilon_r$

$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)

$c_0 = 2.998 \cdot 10^{8}$ m/s

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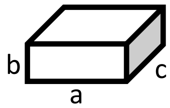
1.) Cavities



1.1) Analyze a rectangular cavity

A rectangular TE_{101} (=H_{101}) mode cavity has the dimensions $a = c = 100$ mm, $b = 50$ mm.

Questions:

1. Determine the resonant frequency f_{res}
2. Determine the unloaded Q-factor Q_0 for copper walls ($\sigma = 58 \cdot 10^6$ S/m, $\mu_r = 1$)
   S/m = Siemens/meter = 1/(Ωhm · meter)
3. A coupler, connecting the cavity to the outside world is adjusted for critical coupling, i.e. $Q_0 = Q_{EXT}$, which allows maximum power transfer into or out of the resonator. What is the loaded Q_L value of the cavity?
4. What is the 3-dB bandwidth of the loaded cavity for the TE_{101} mode?
5. The critically coupled cavity is driven by $P_{IN} = 50$ W of input power on its resonant frequency. How much power is thermally dissipated in the cavity?
6. How much energy is stored in the cavity?

1.2) Design a pillbox cavity (1)

Design a copper pillbox cavity for f_{res} = 500 MHz. The cavity should use the E_{010} (TM_{010}) mode. Keep the ratio *h/a* = 1 for good mode separation.
Copper conductivity $\sigma_{Cu} = 58 \cdot 10^6$ S/m, $\mu_r = 1$.

Questions:

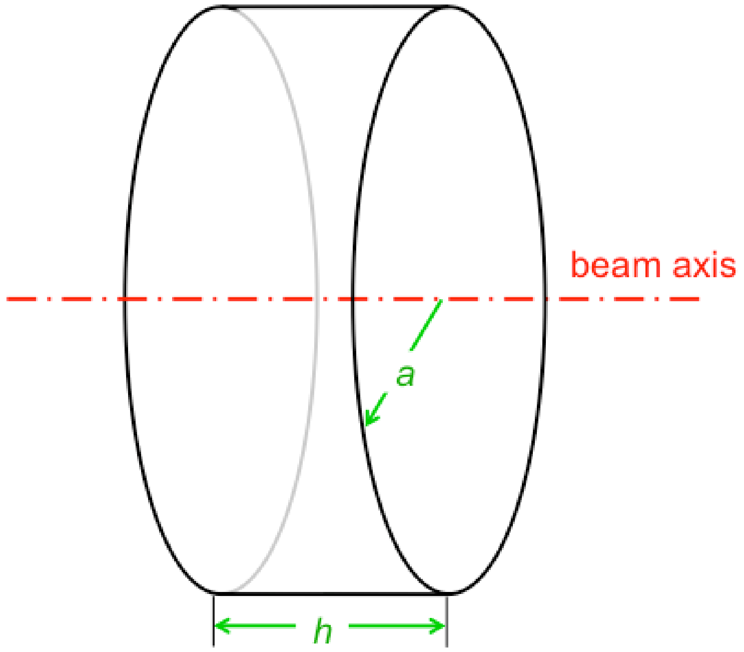
1. Calculate radius *a* of the cavity?
2. Determine the height *h* of the cavity?
3. What would be the first higher order mode (see the pill box cavity mode chart)?
4. Determine the *Q* factor of this cavity.
5. Determine the *R/Q* of this cavity.
6. Determine the values of the lumped elements (parallel R-L-C circuit model) of the cavity.
7. Calculate the peak gap voltage, if the cavity is driven by a 300 kW_{RMS} transmitter. Assume critical coupling, i.e. all transmitter power is transferred into the cavity. (RMS = Root Mean Square = effective values)
8. What would be the Q factor and the gap voltage if the cavity would be made out of stainless steel? $\sigma_{SS} = 1.45 \cdot 10^6$ S/m

1.3) Design of a pillbox cavity (2)

Problem: Design a simple “Pillbox” cavity with the following parameters

Frequency:	f = 299.98 MHz (λ = 1.00 m)
Wall material:	Copper (equivalent skin depth δ = 3.8 µm)
Axial length:	h = 0.2 m

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



Questions:

1. Find from the analytical formulas:
    - Cavity radius *a*
    - Cavity quality factor *Q*
    - “geometry factor”, also known as “characteristic impedance” *R/Q*

    Is the cavity completely determined?

2. Find the equivalent circuit of the cavity.

3. Calculate the 3-dB bandwidth of the intrinsic (not connector to any generator) cavity.

4. Calculate the necessary RF power (RMS) for a gap peak voltage of V = 100 kV, assuming critical coupling.

5. The cavity is fed by an amplifier, designed for a load impedance of 50 Ω.
    Determine:

    - The peak voltage at the cavity input.
    - The necessary transformer ratio k of the input coupler.

2.) Multiple choice questions

1. How will the resonant frequency f_{res} of the E_{010} (TM_{010}) mode of a pill box cavity change if height of the cavity is doubled? (check 1)

- ○ The f_{res} decreases by a factor 2.
- ○ The f_{res} decreases by a factor $\sqrt{2}$.
- ○ The f_{res} increases by a factor 2.
- ○ The f_{res} increases by a factor $\sqrt{2}$.
- ○ The f_{res} will not change.

2. A critically coupled aluminum pill-box cavity is driven by an RF generator with an output power of 100 kW. How much power would be dissipated by the cavity if it were made of silver? $\sigma_{Aluminium} = 38*10^6$ S/m, $\sigma_{Silver} = 63*10^6$ S/m. Note: the silver cavity would also be critically coupled, and should achieve the same gap voltage. (check 1)

- ○ The power dissipation decreases by a factor $\sigma_{Aluminium}/\sigma_{Silver}$
- ○ The power dissipation increases by a factor $\sigma_{Aluminium}/\sigma_{Silver}$
- ○ The power dissipation will not change

3. Calculate the thickness of a copper wall of 5 times the penetrations depth for 50 Hz signals. . $\sigma_{Copper} = 58*10^6$ S/m, $\mu = \mu_0 \mu_r$, $\mu_0 = 4\pi*10^{-7}$ Vs/Am (check 1)

- ○ 46.7 mm
- ○ 4.67 mm
- ○ 0.46 mm
- ○ 0.046 mm

4. A rectangular waveguide has a width (long side!) of $a = 10$ cm. (check 2)

- ○ The mode TE_{10} or H_{10} has a cutoff frequency of 3 GHz.
- ○ The mode TE_{10} or H_{10} has a cutoff frequency of 1.5 GHz.
- ○ The electric field is parallel to the side with the larger dimension.
- ○ The electric field is orthogonal to the side with the larger dimension.

6. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor? (check 1)

- ○ TE
- ○ TEM
- ○ TM

7. Which mode is the fundamental mode in a cylindrical waveguide *with* inner conductor (coaxial line)? (check 1)

- ○ TE
- ○ TEM
- ○ TM

8. Adding capacitive loading to a cavity (check 1)

  - ○ lowers the resonance frequency
  - ○ does not affect the resonance frequency
  - ○ increases the resonance frequency

9. When doing numerical simulations, geometrical symmetries are exploited in order to (check 1)

  - ○ ensure convergence of the simulation algorithms for resonant structures
  - ○ reduce calculation time
  - ○ account for the transit time factor
  - ○ rule out the impact of conductivity

10. When you cover the antenna of your mobile with your hand, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the hand? (check 1)

  - ○ 9
  - ○ 99
  - ○ 99.9
  - ○ 99.99

3.) Impedances

3.1) Impedances in the complex plane and in the Smith chart

Plot the following impedances as points (marks)

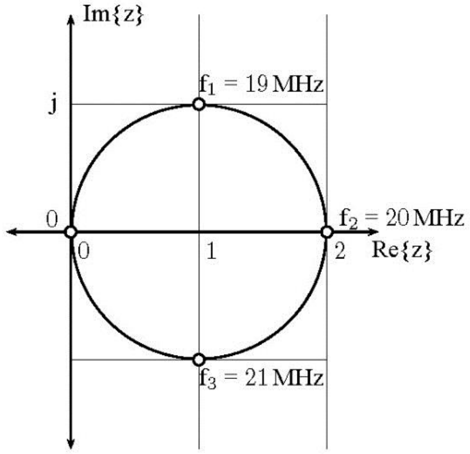
- in a "normal" Cartesian coordinate system (complex plane)
- as reflection factors in the *Smith* chart (the reflection factor coordinates are given for convenience)

X_{rect}	X_{polar}	Γ_{rect}	Γ_{polar}
0.05	$0.05 \angle 0^0$	-0.904	$0.904 \angle 180^0$
0.5	$0.5 \angle 0^0$	-0.333	$0.333 \angle 180^0$
1	$1.0 \angle 0^0$	0	0
2	$2.0 \angle 0^0$	0.333	$0.333 \angle 0^0$
20	$20 \angle 0^0$	0.904	$0.904 \angle 0^0$
0.8	$0.8 \angle 0^0$	-0.111	$0.111 \angle 180^0$
0.8 + j0.6	$1.00 \angle 36.9^0$	0 + j0.333	$0.333 \angle 90^0$
0.8 + j1.0	$1.28 \angle 51.3^0$	0.159 + j0.472	$0.459 \angle 72.3^0$
0.8 + j1.5	$1.70 \angle 61.9^0$	0.344 + j0.546	$0.645 \angle 57.8^0$
0.8 + j2.0	$2.15 \angle 68.2^0$	0.502 + j0.552	$0.747 \angle 47.7^0$
0.8 – j0.6	$1.00 \angle -36.9^0$	0 - j0.333	$0.333 \angle -90^0$

Convince yourself with a few examples that $\Gamma(1/X) = -\Gamma(X)$

3.2) Smith Chart (1)

The locus of impedance of a parallel RLC resonant circuit is given in the complex *z*-plane (*z*-pane = normalized *Z*-plane, normalization to 50 Ω; *z* = *Z* / 50 Ω).



Questions:

1. Transform this locus of impedance into the Smith Chart
2. Mark the resonance frequency, both, in the *z*-plane and in the Smith Chart.
3. Mark the 3-dB points (for the unloaded Q), both, in the *z*-plane and in the Smith Chart.

3.3) Smith Chart (2)

1. Plot the following **normalized** impedances *z* into the Smith Chart:

Point	A	B	C	D
z	0.6 + j0	0.6 - j0.6	0.6 - j0.8	0.6 - j1.0

2. Plot the following impedances *Z* into the Smith Chart

Point	A	B	C	D
Z	50 + j0	20 – j15	10 + j25	0 – j50

3.4) Smith Chart (3)

1. Mark the reflection factors *Γ* of points A to F in the Smith Chart and find approximate values for the corresponding (normalized) impedances z:

Point	Reflection factor *Γ*	Normalized impedance *z*
A	1 ∠0°	
B	1 ∠45°	
C	1 ∠90°	
C	1 ∠180°	
E	1 ∠-90°	
F	0.5	

4.) S-Parameters

Match the following S-Matrices to the corresponding components

$$S_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \qquad S_2 = \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} \qquad S_3 = \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix} \qquad S_4 = \begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}$$

Component	**Isolator**	**Circulator**	**Transmission line, length = λ/2**	**3-dB attenuator**
S-matrix				

5.) Scaling laws

A cavity shall be scaled from existing designs for a frequency f_x = 318.32 MHz and C_x = 10 pF. There are three test designs, with the following parameters:

Cavity	**f_{res} / MHz**	***C* / pF**	***Q***	**Diameter / mm**
A	100	7.957	10000	600
B	500	3.18	5000	200
C	3000	1.061	2000	25

Questions:

1. Which cavity is suitable as reference design?
2. Calculate the diameter of the new design.
3. Calculate the expected *Q* factor of the new design, provided it will be build out of the same material as the reference design.

6.) Various questions

1. What is the difference between a *Stripline* and a *Microstripline*?
2. Name 3 disadvantages of *Microstriplines* compared to *Striplines*.
3. A RF signal needs to be guided from a power amplifier on the surface to the cavity of a particle accelerator in the tunnel below ground.
   The distance is *l* = 100 m. The signal has parameters: *f* = 50 MHz, *P* = 100 kW
   a. Would you use waveguide or coaxial transmission line? Why?
   b. What would you use if the signal would have a frequency of 500 MHz? Why?
4. Why are some accelerator cavities (for frequencies in the MHz range) loaded with ferrite? Explain how the resonant frequency of those cavities can be tuned without moving parts.
5. After deploying a new accelerating cavity, a RF-engineer starts pounding on it with a hammer. What is he doing?