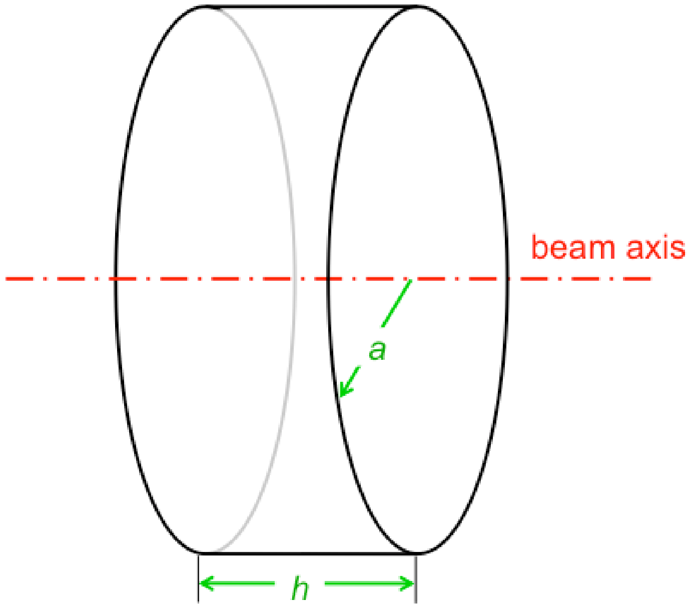
RF Tutorial 2017

A) Design of a "pillbox" cavity

Problem: Design a simple "Pillbox" cavity with the following parameters

Frequency:	f = 299.98 MHz (λ = 1.00 m)
Wall material:	Copper (equivalent skin depth δ = 3.8 µm)
Axial length:	h = 0.2 m

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



Questions:

1. Find from the analytical formulas:
    - Cavity radius a
    - Cavity quality factor Q
    - "geometry factor", also know as "characteristic impedance" R/Q

    Is the cavity completely determined?
2. Find the equivalent circuit of the intrinsic cavity.
3. Calculate the 3-dB bandwidth of the intrinsic cavity.
4. Calculate the necessary RF power for a gap voltage of V = 100 kV
5. The cavity is fed by an amplifier, designed for a load impedance of 50 Ω. Determine:
    - The peak voltage at the cavity input.
    - The necessary transformer ratio k of the input coupler.

B) Impedances in the complex plane (1)

Questions:

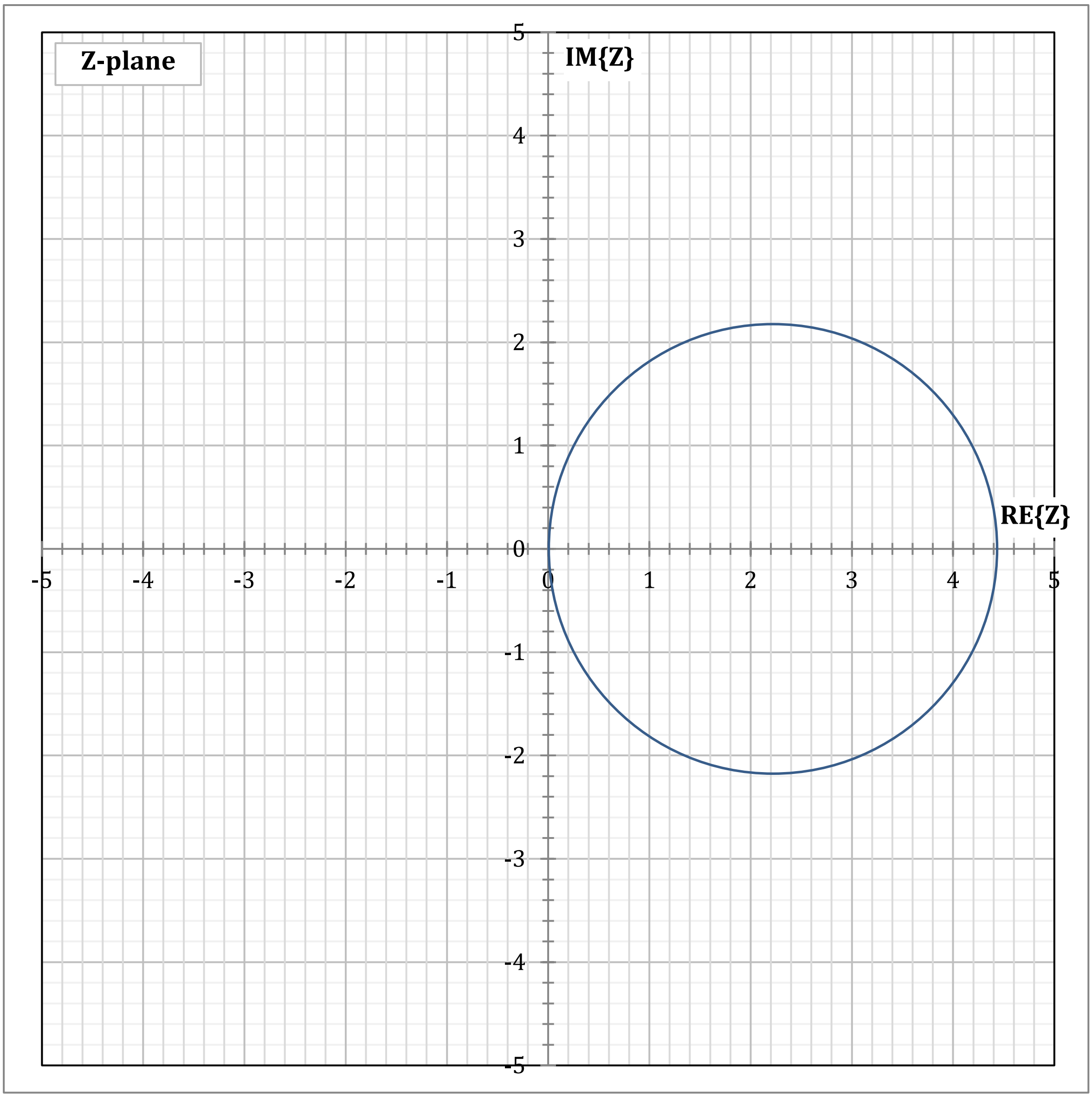
1. Plot the following impedances in the Z-plane, use the plot axes on the next page:

$Z = (3 + 4\,j)\ \Omega$	$\lvert Z\rvert = 2$, $\arg(Z) = \pi/4$	Z = short circuit
$Z = 2\ \Omega$	$\lvert Z\rvert = 1$, $\arg(Z) = -\pi/2$	$Y = Z^{-1} = (0.16 + 0.12j)\ \Omega^{-1}$
$Z = (1 - 4\,j)\ \Omega$	$\lvert Z\rvert = 5$, $\arg(Z) = 53°$	

2. Qualitatively, how would an inductor look like, plotted from DC to some arbitrary frequency, in the Z-plane?

   Hint: $Z_L = j\omega L$
3. How would a capacitor look like? Hint: $Z_C = 1/(j\omega C)$
4. The input impedance of a RLC circuit has been plotted in the Z-plane (blue circle).

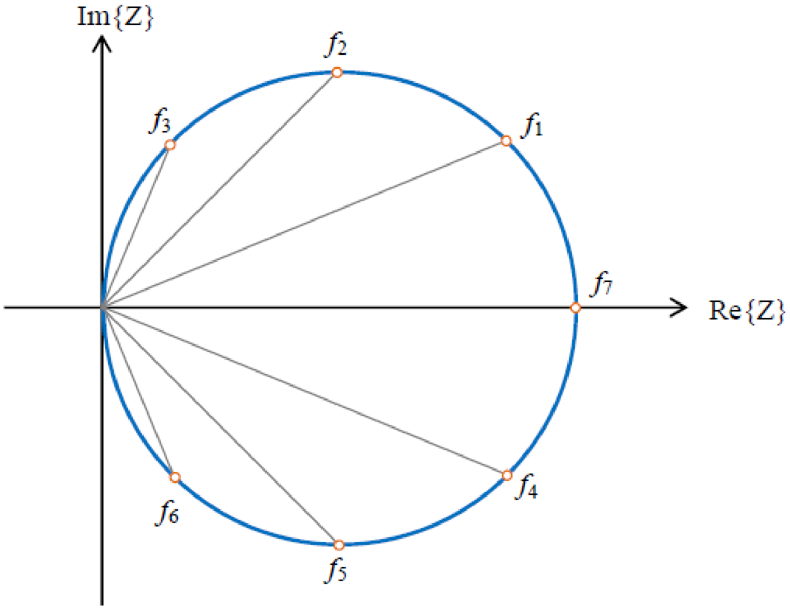
   Mark the points in the diagram describing:
   a. Impedance at the resonant frequency
   b. DC impedance
   c. 3-dB bandwidth
   d. Impedance at $f \to \infty$



Z-plane
IM{Z}
RE{Z}
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5

D) Impedances in the complex plane (2)

The impedance of a resonant circuit is a function of frequency. For a given resonator the impedance was measured at 7 different frequencies, $f_1 \ldots f_7$. The result is shown in the complex Z-plane:



	f_1	f_2	f_3	f_4	f_5	f_6	f_7
f / MHz	105.11	105.05	104.94	105.29	105.35	105.46	105.20
Z / kΩ	$200\ e^{j30°}$	$162.6\ e^{j45°}$	$115\ e^{j60°}$	$200\ e^{-j30°}$	$162.6\ e^{-j45°}$	$115\ e^{-j60°}$	$230\ e^{j0°}$

Questions:

1. Determine the resonant frequency.
2. Determine the 3-dB bandwidth (*BW*) of this resonator.
   (Hint: The bandwidth of a resonator is defined as the frequency difference between the upper and lower 3-dB frequency points.)

In order to evaluate the properties of a resonator,
it is common to model it as equivalent circuit with lumped RLC elements.

3. Sketch the equivalent circuit for the measured resonator.
4. Determine *R*.
5. Draw the locus of admittance of this circuit in the *Y*-plane,
   and indicate lower and upper 3-dB points.
6. Determine the Q-value, as well as *L* and *C* for this circuit.

C) “Pillbox” cavity characteristics

The following data was measured on a “pillbox” cavity”:

Inductance:	L = 15.915 nH
Capacitance:	C = 1.5915 pF
3-dB bandwidth:	BW = 50 kHz

Questions:

Determine

- the frequency at resonance
- the characteristic impedance R/Q
- the quality factor Q
- the time constant τ
- the peak induced voltage immediately after the passage of a short particle bunch with charge q = 15.916e-9 As
- the remnant cavity voltage 10 μs after the passage of the bunch

D) An accelerator cavity heats up under high RF power load

A cavity is constructed from a material with”
C_2/C_1

thermal expansion coefficient:	$\Delta l/l$ = 20e-6/^{0}C (per degree Centigrade)
thermal resistivity coefficient:	$\Delta \rho/\rho$ = 4e-3/^{0}C (per degree Centigrade)

At room temperature the cavities resonance frequency is f_1 = 100 MHz, and has a 3-dB bandwidth of BW_1 = 100 kHz.
Under RF power the cavity temperature increases by 100 ^{0}C (subscripts 2 apply).