Exercises on Wake Fields and Instabilities

Exercise 1:

Show that the impedance of an RLC parallel circuit is that of a resonant mode and relate R, L and C to Q, R_s and ω_r

Exercise 2:

Calculate the amplitude of the resonator wake field given $R_s = 1\ k\Omega$, $\omega_r = 5$ GHz, $Q = 10^4$

Calculate the ratio $|Z(\omega_r)| / |Z(2\omega_r)|$ for $Q = 1, 10^3, 10^5$

Exercise 3: Beam Break Up (1)

Consider a beam in a linac at 1 GeV without acceleration. Obtain the growth of the oscillation amplitude after 3 km if:
N = 5e10, $w_{\perp}$(-1 mm) = 63 V/(pC m), L_w = 3.5 cm, k_y = 0.06 1/m

Exercise 4: Beam Break Up (2)

Consider the same beam of the previous exercise being now accelerated from 1 GeV with a gradient g =16.7 MeV/m. Obtain the growth of the oscillation amplitude

$$E_f = E_0 + gL_L \approx gL_L = 50\,\text{GeV}$$

$$\left(\frac{\Delta\hat{y}_2}{\hat{y}_2}\right)_{\max} = \frac{cNew_{\perp}(z)L_L}{4\omega_y(E_f/e)L_w}\ln\frac{E_f}{E_0} = ?$$

Exercise 5: Evaluate the energy lost per unit length by a charge due to the longitudinal wake field of the space charge and compare it with the longitudinal space charge force in r=0

Exercise 6: Evaluate the energy spread (U_max-U_min) of a Gaussian bunch of RMS length σ due to the longitudinal wake field of the space charge in a structure of length L

Exercise 7: Evaluate the energy lost by a charge in a uniform beam of length l_0 due to the longitudinal wake field of a pill box cavity of length g at high frequency ω>>c/b (diffraction model), with b the pipe radius

$$w(z) = \frac{Z_0 c \sqrt{2g}}{2\pi^2 b} \frac{1}{z^{1/2}}$$