Linear imperfections and correction

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References

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Reminder of transverse beam dynamics

Equation reminder

Lorentz equation $$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

E : total energy

T : kinetic energy $$E = \sqrt{p^2c^2 + m_0^2c^4} = T + m_0c^2 = T + E_0$$

p : momentum

β : reduced velocity $$\beta = \frac{v}{c}$$

γ : reduced energy $$\gamma = \frac{E}{m_0c^2}$$

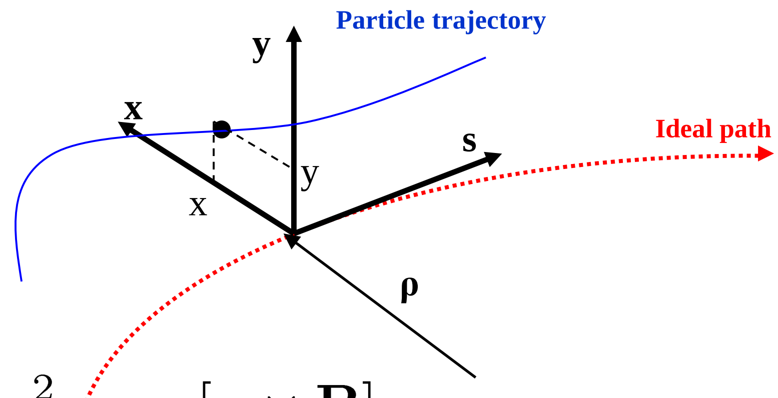
$\beta\gamma$: reduced momentum $$\beta\gamma = \frac{p}{m_0c^2}$$

Reference trajectory

- Cartesian coordinates not useful to describe motion in a circular accelerator (not true for linacs)

- A system following an ideal path along the accelerator is used (**Frenet** reference system) $(\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_z}) \rightarrow (\mathbf{u_x}, \mathbf{u_y}, \mathbf{u_s})$

  - The curvature vector is $\boldsymbol{\kappa} = -\frac{d^2\mathbf{s}}{ds^2}$



  - From Lorentz equation

$$\frac{d\mathbf{p}}{dt} = m_0\gamma\frac{d^2\mathbf{s}}{dt^2} = m_0\gamma v_s^2\frac{d^2\mathbf{s}}{ds^2} = -m_0\gamma v_s^2\boldsymbol{\kappa} = q[\mathbf{v}\times\mathbf{B}]$$

  where we used the curvature vector definition and $\frac{d^2}{dt^2} = v_s^2\frac{d^2}{ds^2}$

- Using $m_0\gamma v_s = p_s = (p^2 - p_x^2 - p_y^2)^{1/2} \approx p$, the ideal path of the reference trajectory is defined by

$$\boldsymbol{\kappa}_0 = -\frac{q}{p}\left[\frac{\mathbf{v}}{v_s}\times\mathbf{B_0}\right]$$

Beam guidance

- Consider uniform magnetic field $\mathbf{B} = \{0, B_y, 0\}$ in a direction perpendicular to particle motion. From the reference trajectory equation, after developing the cross product and considering that the transverse velocities $v_x\ , v_y \ll v_s$, the radius of curvature is

$$\frac{1}{\rho} = |k| = |\frac{q}{p}B|$$

- We define the **magnetic rigidity** $|B\rho| = \frac{p}{q}$

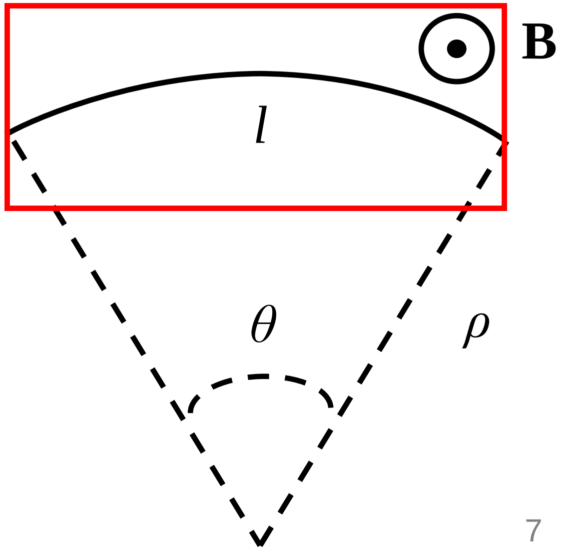
- In more practical units $\boxed{\beta E[GeV] = 0.2998|B\rho|[Tm]}$

- For ions with charge multiplicity *n* and atomic number *A*, the energy per nucleon is

$$\beta\bar{E}[GeV/u] = 0.2998\frac{n}{A}|B\rho|[Tm]$$

Dipoles

- Consider ring for particles with energy E with N dipoles of length L (or effective length l, i.e. measured on beam path)
  - **Bending angle** $\theta = \frac{2\pi}{N}$
  - **Bending radius** $\rho = \frac{l}{\theta}$
  - **Integrated dipole strength** $Bl = \frac{2\pi}{N}\frac{p}{q}$
- Note:
  - By choosing a dipole field, the dipole length is imposed and vice versa
  - The higher the field, the shorter or smaller number of dipoles can be used
  - The ring circumference (cost) is influenced by the field choice

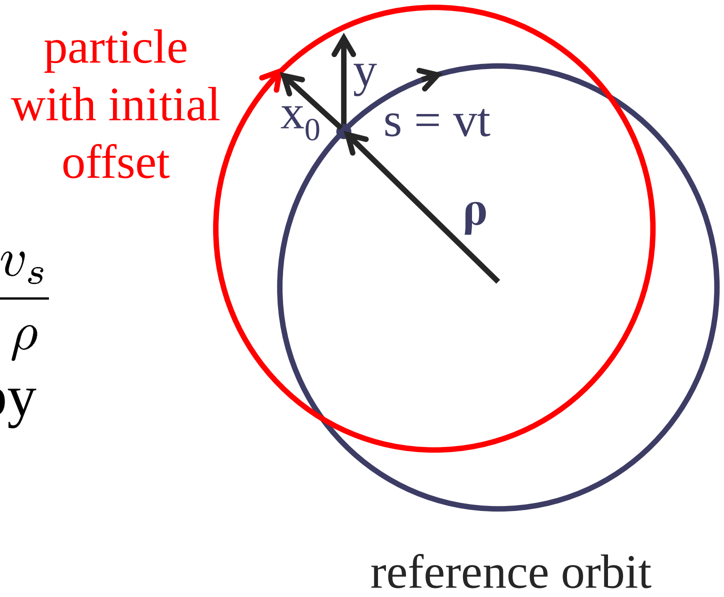


Beam focusing

- Consider a particle in a dipole field
- In the **horizontal plane**
  - it performs harmonic oscillations $x = x_0 \cos(\omega t + \phi)$ with frequency $\omega = \frac{v_s}{\rho}$
  - the horizontal acceleration is described by

$$\frac{d^2x}{ds^2} = \frac{1}{v_s^2}\frac{d^2x}{dt^2} = -\frac{1}{\rho^2}x$$

  - there is a **week focusing** effect in the horizontal plane



- In the **vertical plane**, the only force present is gravitation
  - Particles are displaced vertically following the usual law $\Delta y = \frac{1}{2} a_g \Delta t^2$
  - With $a_g \approx 10$ m/s^2, the particle is displaced by **18 mm** (LHC dipole aperture) in **60 ms** (few hundred turns in LHC) ➔ **need focusing!**

Quadrupoles

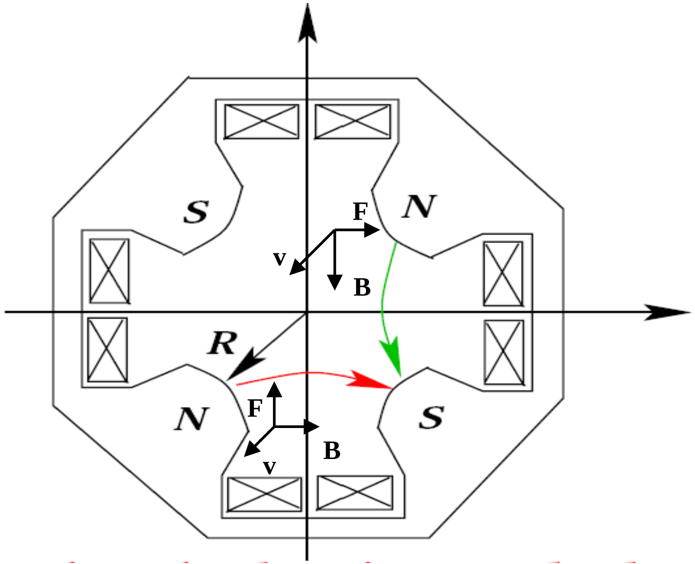
- Quadrupoles are focusing in one plane and defocusing in the other
- The field is $(B_x, B_y) = G(y, x)$
- The resulting force $(F_x, F_y) = k(y, -x)$ with the normalised gradient defined as

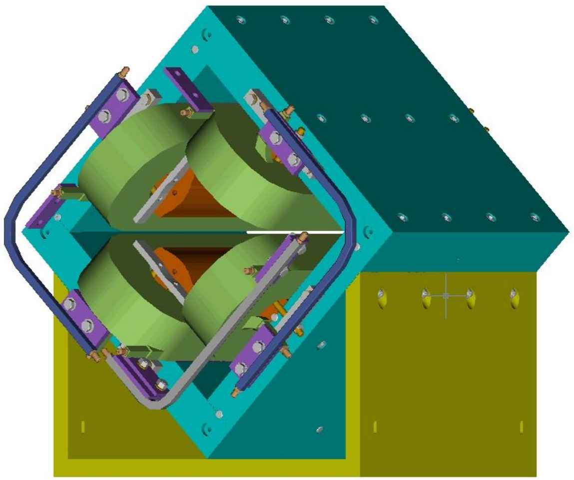
$$k = \frac{qG}{\beta E}$$

- In more practical units:

$$k[m^{-2}] = 0.2998 \frac{G[T/m]}{\beta E[GeV]}$$

- Need to alternate focusing and defocusing to control the beam, i.e. **alternating gradient focusing**





Equations of motion – Linear fields

- Consider s-dependent fields from dipoles and normal quadrupoles

$$B_y = B_0(s) - G(s)x \ , \quad B_x = -G(s)y$$

- The total momentum can be written $p = p_0(1 + \frac{\Delta p}{p})$
- With magnetic rigidity $B_0\rho = \frac{p_0}{q}$ and normalized gradient $k(s) = \frac{G(s)}{B_0\rho}$ the equations of motion are

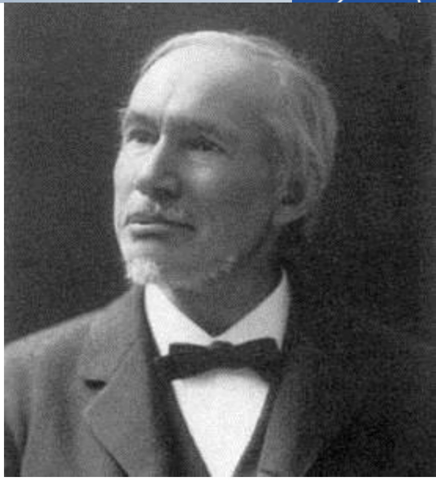
$$\boxed{\begin{aligned} x'' - \left(k(s) - \frac{1}{\rho(s)^2}\right) x &= \frac{1}{\rho(s)} \frac{\Delta p}{p} \\ y'' + k(s)\, y &= 0 \end{aligned}}$$

  - Inhomogeneous equations with s-dependent coefficients
  - The term $\frac{1}{\rho^2}$ corresponds to the dipole **week focusing** and $\frac{1}{\rho}\frac{\Delta p}{p}$ represents **off-momentum** particles

Hill's equations

- Solutions are combination of the homogeneous and inhomogeneous equations' solutions

- Consider particles with the design momentum. Equations of motion become



George Hill

$$\boxed{\begin{array}{rcl} x'' + K_x(s)\ x & = & 0 \\ y'' + K_y(s)\ y & = & 0 \end{array}}$$

with $K_x(s) = -\left(k(s) - \dfrac{1}{\rho(s)^2}\right)\ ,\quad K_y(s) = k(s)$

- **Hill's equations of linear transverse particle motion**
  - Linear equations with s-dependent coefficients (harmonic oscillator with time dependent frequency)
  - In a ring (or in transport line with symmetries), the **coefficients are periodic** $K_x(s) = K_x(s+C)\ ,\quad K_y(s) = K_y(s+C)$
  - Not straightforward to derive analytical solutions for whole accelerator

- The on-momentum linear betatron motion of a particle in both planes, is described by

$$\boxed{u(s) = \sqrt{\epsilon\beta(s)}\cos(\psi(s) + \psi_0)} \qquad u \mapsto \{x, y\}$$

with $\alpha,\ \beta,\ \gamma$ the twiss functions $\alpha(s) = -\frac{\beta(s)'}{2},\ \ \gamma = \frac{1+\alpha(s)^2}{\beta(s)}$

ψ the **betatron phase** $\psi(s) = \int \frac{ds}{\beta(s)}$

and the **beta function** β is defined by the **envelope equation**

$$\boxed{2\beta\beta'' - \beta'^2 + 4\beta^2 K = 4}$$

- By differentiation, we have that the **angle** is

$$\boxed{u'(s) = \sqrt{\frac{\epsilon}{\beta(s)}}\left(\sin(\psi(s) + \psi_0) + \alpha(s)\cos(\psi(s) + \psi_0)\right)}$$

General transfer matrix

- From the position and angle equations it follows that

$$\cos(\psi(s)+\psi_0) = \frac{u}{\sqrt{\epsilon\beta(s)}}\,, \quad \sin(\psi(s)+\psi_0) = \sqrt{\frac{\beta(s)}{\epsilon}}u' + \frac{\alpha(s)}{\sqrt{\epsilon\beta(s)}}u$$

- Expand the trigonometric formulas and set $\psi(0)=0$ to get the transfer matrix from location 0 to s

$$\begin{pmatrix} u(s) \\ u'(s) \end{pmatrix} = \mathcal{M}_{0\to s} \begin{pmatrix} u_0 \\ u'_0 \end{pmatrix}$$

with

$$\mathcal{M}_{0\to s} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}}(\cos\Delta\psi + \alpha_0 \sin\Delta\psi) & \sqrt{\beta(s)\beta_0}\sin\Delta\psi \\ \frac{(a_0 - a(s))\cos\Delta\psi - (1+\alpha_0\alpha(s))\sin\Delta\psi}{\sqrt{\beta(s)\beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}}(\cos\Delta\psi - \alpha_0\sin\Delta\psi) \end{pmatrix}$$

and $\mu(s) = \Delta\psi = \int_0^s \frac{ds}{\beta(s)}$ the **phase advance**

Periodic transfer matrix

- Consider a periodic cell of length C
- The optics functions are $\beta_0 = \beta(C) = \beta\ , \quad \alpha_0 = \alpha(C) = \alpha$

  and the phase advance $\mu = \int_0^C \frac{ds}{\beta(s)}$

- The transfer matrix is $\mathcal{M}_C = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$

- The cell matrix can be also written as

$$\mathcal{M}_C = \mathcal{I}\cos\mu + \mathcal{J}\sin\mu$$

with $\mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and the **Twiss matrix** $\boxed{\mathcal{J} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}}$

Tune and working point

- In a ring, the **tune** is defined from the 1-turn phase advance

$$Q_{x,y} = \frac{1}{2\pi} \oint \frac{ds}{\beta_{x,y}(s)} = \frac{\nu_{x,y}}{2\pi}$$

  i.e. number of betatron oscillations per turn

- Taking the average of the betatron tune around the ring we have in **smooth approximation**

$$\nu = 2\pi Q = \frac{C}{\langle\beta\rangle} \to Q = \frac{R}{\langle\beta\rangle}$$

  - Extremely useful formula for deriving scaling laws!

- The position of the tunes in a diagram of horizontal versus vertical tune is called a **working point**
- The tunes are imposed by the choice of the quadrupole strengths
- One should try to avoid **resonance conditions**

Transverse linear imperfections and correction

Outline

- Closed orbit distortion (steering error)
  - Beam orbit stability importance
  - Imperfections leading to closed orbit distortion
  - Interlude: dispersion and chromatic orbit
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
- Optics function distortion (gradient error)
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- Coupling error
  - Coupling errors and their effect
  - Coupling correction
- Chromaticity

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Beam orbit stability

- Beam orbit stability is very critical
  - Injection and extraction efficiency of synchrotrons
  - Stability of collision point in colliders
  - Stability of the synchrotron light spot in the beam lines of light sources
- Consequences of orbit distortion
  - Miss-steering of beams, modification of dispersion function, resonance excitation, aperture limitations, lifetime reduction, coupling, modulation of lattice functions, poor injection/extraction efficiency
- Causes
  - **Long term (years - months)**: ground settling, season changes
  - **Medium term (days - hours)**: sun and moon, day-night variations (thermal), rivers, rain, wind, refills and start-up, sensor motion, drift of electronics, local machinery, filling patterns
  - **Short term (minutes - seconds)**: ground vibrations, power supplies, experimental magnets, air conditioning, refrigerators/compressors

Imperfections distorting closed orbit

- Magnetic imperfections distorting the orbit
  - Dipole field errors (or energy errors)
  - Dipole rolls
  - Quadrupole misalignments
    - Consider the displacement of a particle δx from the ideal orbit. The vertical field in the quadrupole is

$$B_y = G\bar{x} = G(x+\delta x) = \underbrace{Gx}_{\text{quadrupole}} + \underbrace{G\delta x}_{\text{dipole}}$$

- Remark: Dispersion can be interpreted as closed orbit distortion for off-momentum particles with $\delta x = D(s)\dfrac{\delta p}{p}$
- Effect of orbit errors in any multi-pole magnet

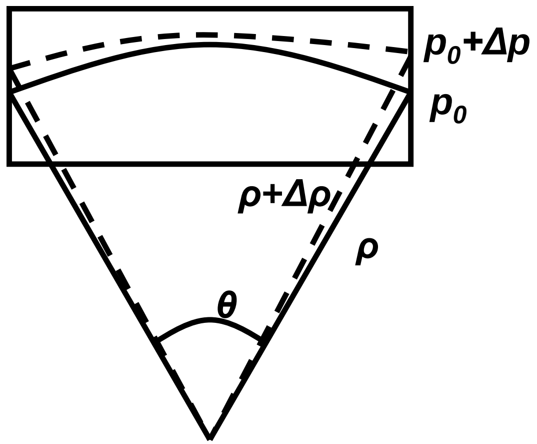
$$B_y = b_n \bar{x}^n = b_n(x+\delta x)^n = b_n(\underbrace{x^n}_{\textbf{2(n+1)-pole}} + \underbrace{n\delta x x^{n-1}}_{\textbf{2n-pole}} + \underbrace{\frac{n(n-1)}{2}(\delta x)^2 x^{n-2}}_{\textbf{2(n-1)-pole}} + \cdots + \underbrace{(\delta x)^n}_{\textbf{dipole}})$$

  - Feed-down:

Off-momentum particles in a dipole

- Up to now all particles had the same momentum p_0
- What happens for off-momentum particles, i.e. particles with momentum $p_0+\Delta p$?
  - Consider a dipole with field B and bending radius ρ
  - Recall that the magnetic rigidity is $B\rho = \frac{p_0}{q}$ and for off-momentum particles

$$B(\rho+\Delta\rho) = \frac{p_0+\Delta p}{q} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta p}{p_0}$$



  - Considering the effective length of the dipole unchanged

$$\theta\rho = l = \text{const.} \Rightarrow \rho\Delta\theta + \theta\Delta\rho = 0 \Rightarrow \frac{\Delta\theta}{\theta} = -\frac{\Delta\rho}{\rho} = -\frac{\Delta p}{p_0}$$

- Off-momentum particles get different deflection (different orbit)

$$\boxed{\Delta\theta = -\theta\frac{\Delta p}{p_0}}$$

Dispersion equation

- Consider the equations of motion for off-momentum particles

$$x'' + K_x(s)x = \frac{1}{\rho(s)}\frac{\Delta p}{p}$$

  - The solution is a sum of the **homogeneous** (on-momentum) and the **inhomogeneous** (off-momentum) equation solutions

$$x(s) = x_H(s) + x_I(s)$$

  - In that way, the equations of motion are split in two parts

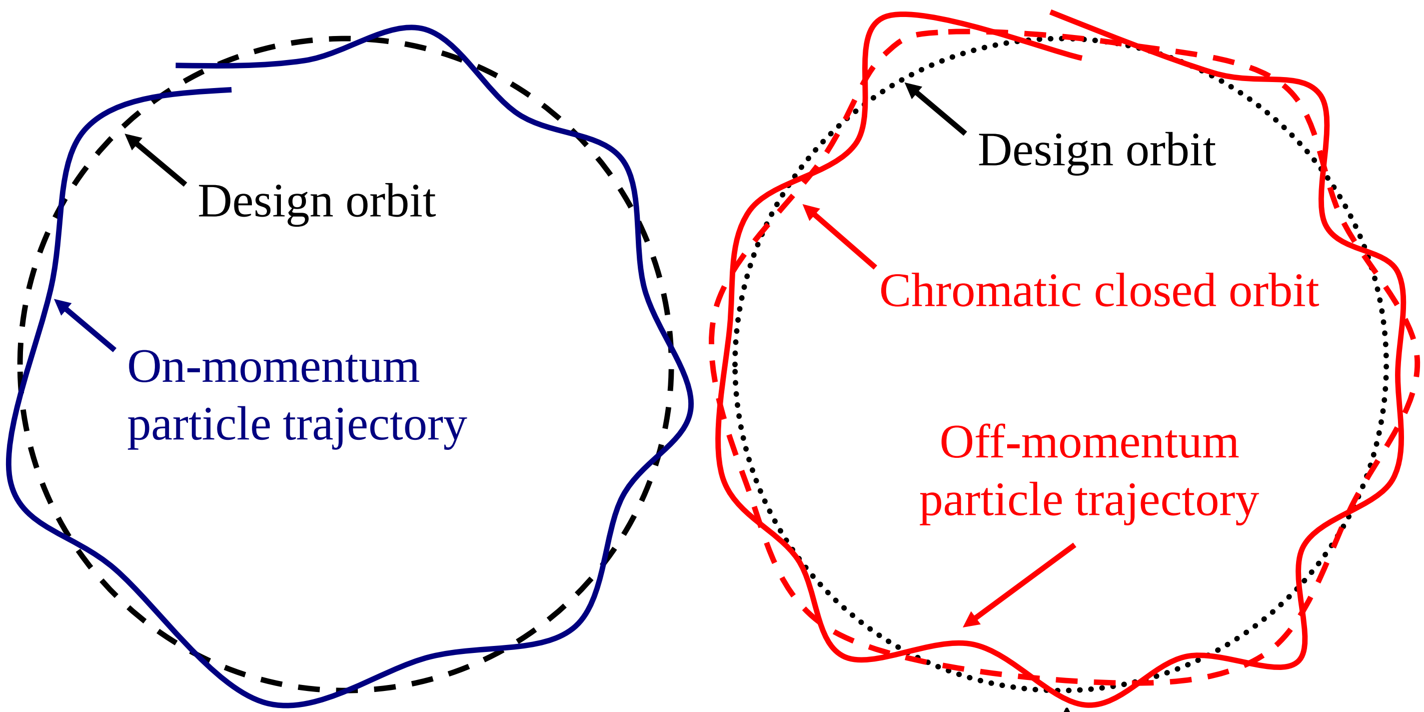
$$\begin{aligned} x_H'' + K_x(s)x_H &= 0 \\ x_I'' + K_x(s)x_I &= \frac{1}{\rho(s)}\frac{\Delta p}{p} \end{aligned}$$

- The **dispersion function** can be defined as $D(s) = \dfrac{x_I(s)}{\Delta p/p}$
- The dispersion equation is

$$\boxed{D''(s) + K_x(s)\ D(s) = \frac{1}{\rho(s)}}$$

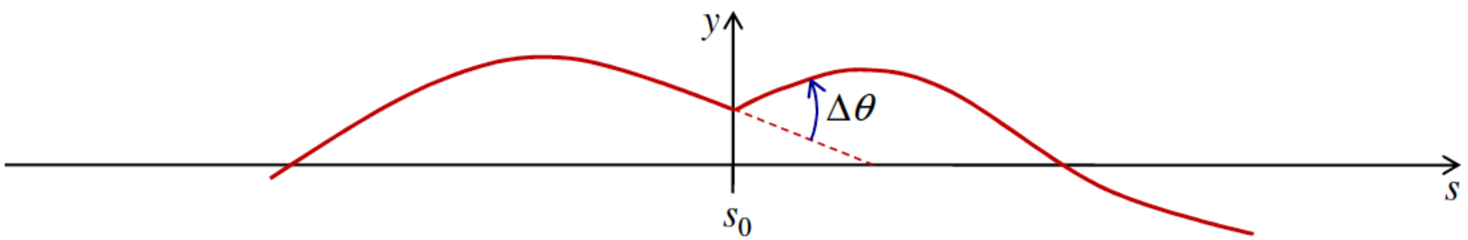
Closed orbit

- Design orbit defined by main dipole field
- On-momentum particles oscillate around design orbit (w/o errors)
- Off-momentum particles are not oscillating around design orbit, but around "chromatic" closed orbit, defined by the dispersion function times the momentum offset



$$x = x_{CO}(s) + x_\beta(s) + D(s)\frac{\Delta p}{p}$$

Effect of single dipole kick



- Consider a single dipole kick $\theta = \delta u_0' = \delta u'(s_0) = \dfrac{\delta(Bl)}{B\rho}$ at $s=s_0$
- The coordinates before and after the kick are

$$\begin{pmatrix} u_0 \\ u_0' - \theta \end{pmatrix} = \mathcal{M} \begin{pmatrix} u_0 \\ u_0' \end{pmatrix}$$

with the 1-turn transfer matrix

$$\mathcal{M} = \begin{pmatrix} \cos 2\pi Q + \alpha_0 \sin 2\pi Q & \beta_0 \sin 2\pi Q \\ -\gamma_0 \sin 2\pi Q & \cos 2\pi Q - \alpha_0 \sin 2\pi Q \end{pmatrix}$$

- The final coordinates are

$$u_0 = \theta \frac{\beta_0}{2 \tan \pi Q} \qquad \text{and} \qquad u_0' = \frac{\theta}{2}\left(1 - \frac{\alpha_0}{\tan \pi Q}\right)$$

Closed orbit from single dipole kick

- Taking the solutions of Hill's equations at the location of the kick, the orbit will close to itself only if

$$\sqrt{\epsilon\beta_0}\cos(\phi_0) = \sqrt{\epsilon\beta_0}\cos(\phi_0 + 2\pi Q)$$

$$-\sqrt{\frac{\epsilon}{\beta_0}}\left(\sin(\phi_0) + \alpha_0\cos(\phi_0)\right) = -\sqrt{\frac{\epsilon}{\beta_0}}\left(\sin(\phi_0 + 2\pi Q)) + \alpha_0\cos(\phi_0 + 2\pi Q)\right)) - \theta$$

  - This yields the following relations for the invariant and phase (this can be also derived by the equations in the previous slide)

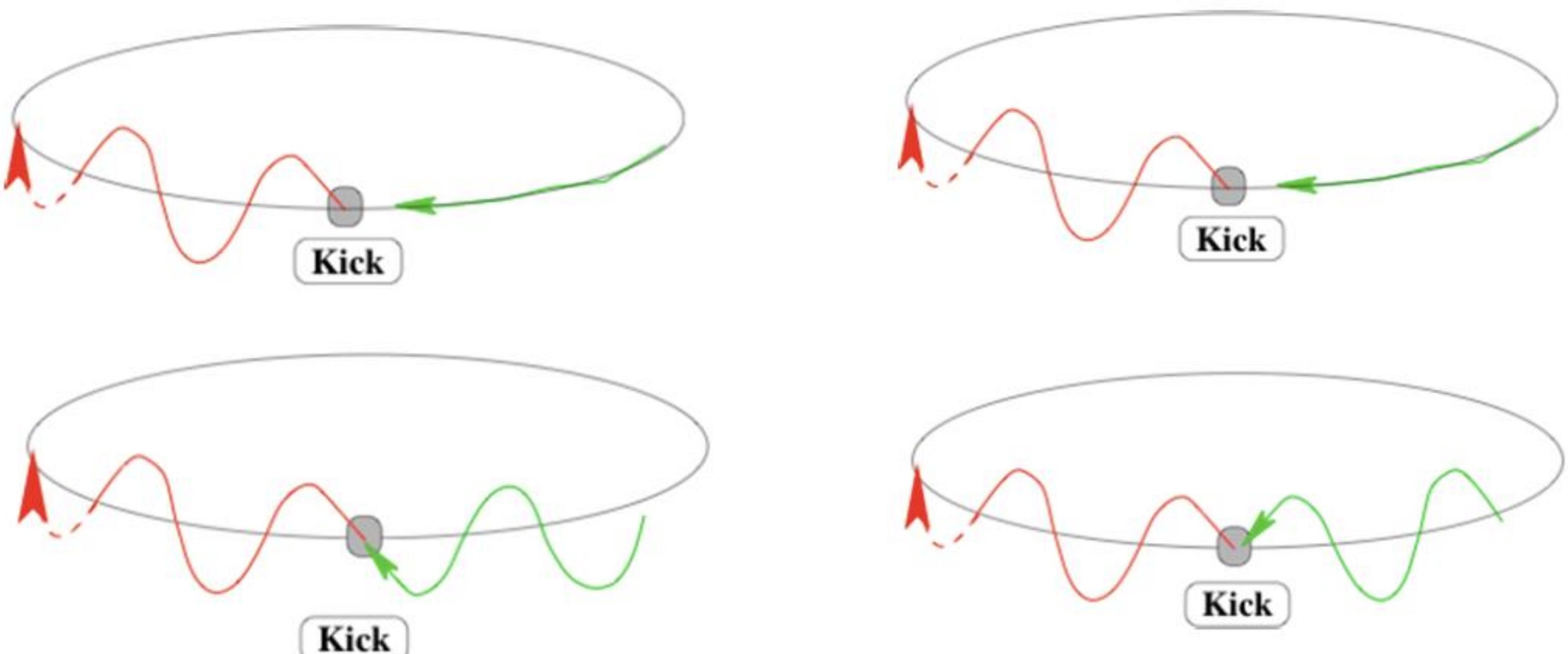
$$\epsilon = \frac{\beta_0\theta^2}{4\sin^2(\pi Q)}, \quad \phi_0 = -\pi Q$$

- For any location around the ring, the orbit distortion is written as

$$u(s) = \theta \underbrace{\frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)}}_{\textbf{Maximum distortion amplitude}} \cos(\pi Q - |\psi(s) - \psi_0|)$$

$$u(s) = \theta \frac{\sqrt{\beta(s)\beta_0}}{2\sin(\pi Q)} \cos(\pi Q - |\psi(s) - \psi_0|)$$

- Dipole perturbations add-up in consecutive turns for $Q = n$
- Integer tune excites orbit oscillations (resonance)
- Dipole kicks get cancelled in consecutive turns for $Q = n/2$
- Half-integer tune cancels orbit oscillations



Transport of closed orbit distortion

- Consider a transport matrix between positions 1 and 2

$$\mathcal{M}_{1\to 2} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

- The transport of transverse coordinates is written as

$$\begin{aligned} u_2 &= m_{11}u_1 + m_{12}u'_1 \\ u'_2 &= m_{21}u_1 + m_{22}u'_1 \end{aligned}$$

- Consider a single dipole kick at position 1: $\theta_1 = \dfrac{\delta(Bl)}{B\rho}$
- Then, the first equation may be rewritten

$$u_2 + \delta u_2 = m_{11}u_1 + m_{12}(u'_1 + \theta_1) \rightarrow \delta u_2 = m_{12}\theta_1$$

- Replacing the coefficient from the general betatron matrix

$$\delta u_2 = \sqrt{\beta_1\beta_2}\sin(\psi_{12})\theta_1$$

$$\delta u'_2 = \sqrt{\frac{\beta_1}{\beta_2}}\left[\cos(\psi_{12})\theta_1 - \alpha_2\sin(\psi_{12})\right]$$

Global orbit distortion

- Orbit distortion due to many errors

Courant and Snyder, 1957

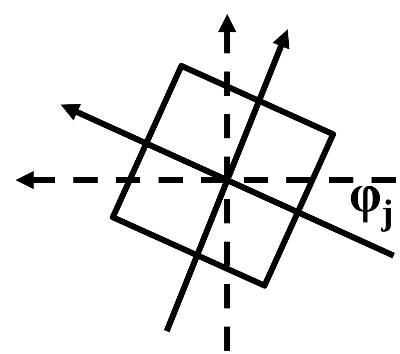
$$u(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \int_s^{s+C} \theta(\tau)\sqrt{\beta(\tau)}\cos(\pi Q - |\psi(s) - \psi(\tau)|)d\tau$$

- By approximating the errors as delta functions in *n* locations, the distortion at *i* observation points (Beam Position Monitors) is

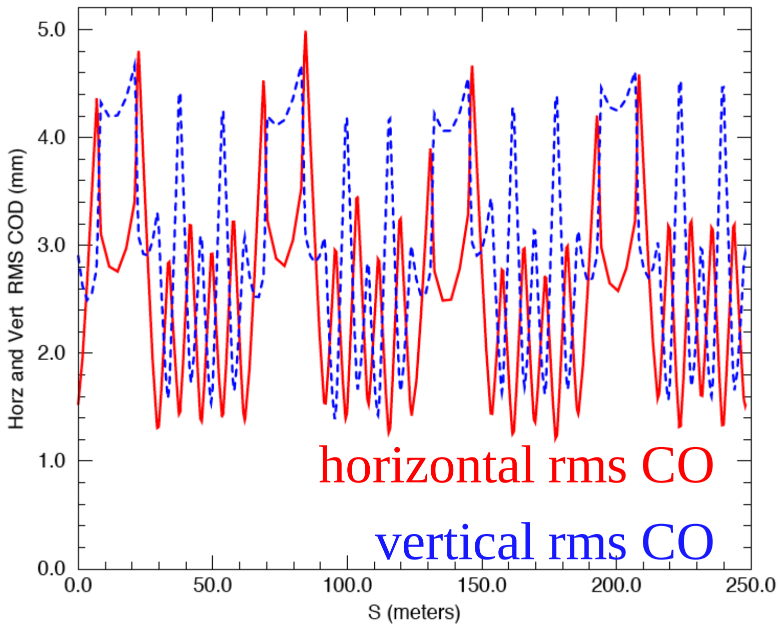
$$u_i = \frac{\sqrt{\beta_i}}{2\sin(\pi Q)} \sum_{j=i+1}^{i+n} \theta_j \sqrt{\beta_j}\cos(\pi Q - |\psi_i - \psi_j|)$$

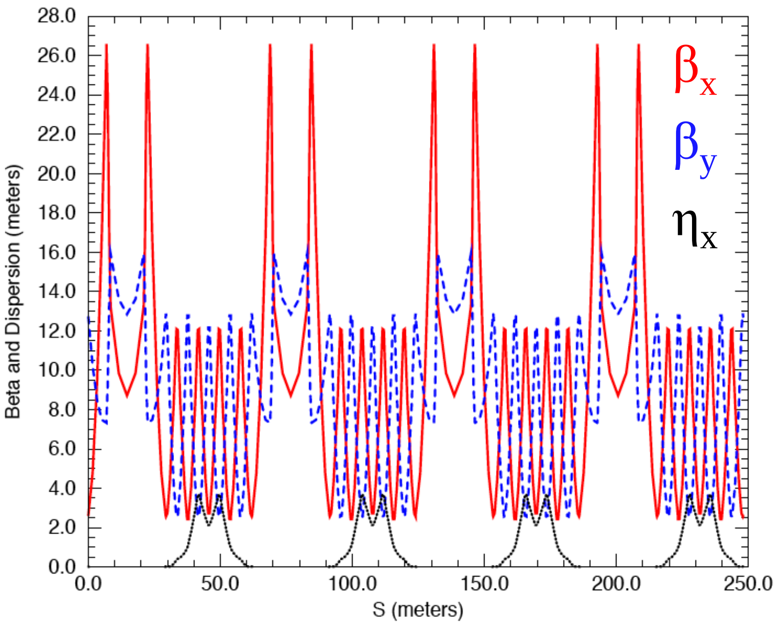
with the kick produced by the j^{th} error

- Integrated dipole field error $\theta_j = \frac{\delta(B_j l_j)}{B\rho}$
- Dipole roll $\theta_j = \frac{B_j l_j \sin\phi_j}{B\rho}$
- Quadrupole displacement $\theta_j = \frac{G_j l_j \delta u_j}{B\rho}$



Example: Orbit distortion in SNS





- In the SNS accumulator ring, the beta function is **6m** in the dipoles and **30m** in the quadrupoles, the tune is **6.2**
- Consider dipole error of **1mrad**
- The maximum orbit distortion in dipoles is $u_0 = \frac{\sqrt{6 \cdot 6}}{2\sin(6.2\pi)} \cdot 10^{-3} \approx 5\text{mm}$
- For quadrupole displacement giving the same **1mrad** kick (and betas of 30m) the maximum orbit distortion is 25mm, to be compared to magnet radius of 105mm

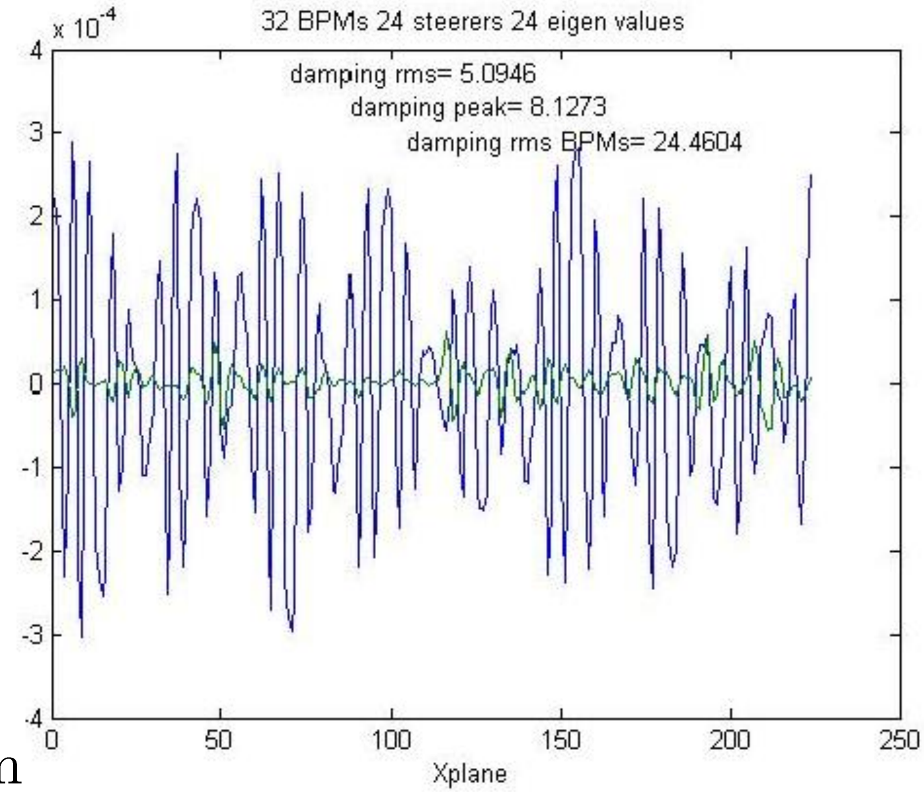
Example: Orbit distortion in ESRF

- In the ESRF storage ring, the beta function is **1.5m** in the dipoles and **30m** in the quadrupoles, the horizontal tune is **36.44**
- Consider dipole error of **1mrad**
- Maximum orbit distortion in dipoles

$$u_0 = \frac{\sqrt{1.5 \cdot 1.5}}{2\sin(36.44\pi)} \cdot 10^{-3} \approx 1\text{mm}$$

- For quadrupole displacement with **1mm**, the distortion is $u_0 \approx 8\text{mm}$!!!
- Magnet alignment is critical

Vertical orbit correction



Statistical estimation of orbit errors

- Consider random distribution of errors in N magnets
  - By squaring the orbit distortion expression and averaging over the angles (considering uncorrelated errors), the expectation (rms) value is given by

$$u_{\rm rms}(s) = \frac{\sqrt{\beta(s)}}{2\sqrt{2}|\sin(\pi Q)|}\Big(\sum_i \sqrt{\beta_i}\theta_i\Big)_{\rm rms} = \frac{\sqrt{N\beta(s)\beta_{\rm rms}}}{2\sqrt{2}|\sin(\pi Q)|}\theta_{\rm rms}$$

- Example:
  - In the SNS ring, there are **32** dipoles and **54** quadrupoles
  - The rms value of the orbit distortion in the dipoles

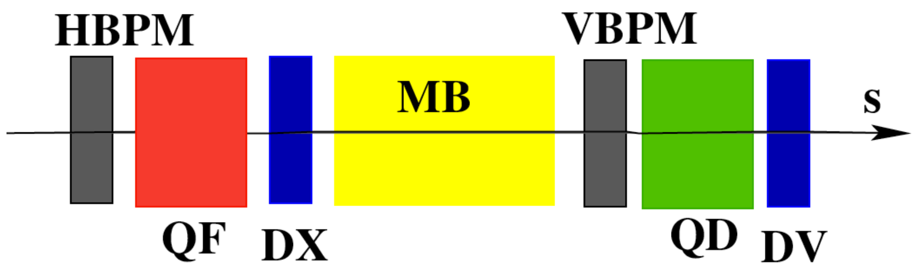
$$u_{\rm rms}^{\rm dip} = \frac{\sqrt{6\cdot 6}\sqrt{32}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 2{\rm cm}$$

  - In the quadrupoles, for equivalent kick

$$u_{\rm rms}^{\rm quad} = \frac{\sqrt{30\cdot 30}\sqrt{54}}{2\sqrt{2}\sin(6.2\pi)}\cdot 10^{-3} \approx 13{\rm cm}$$

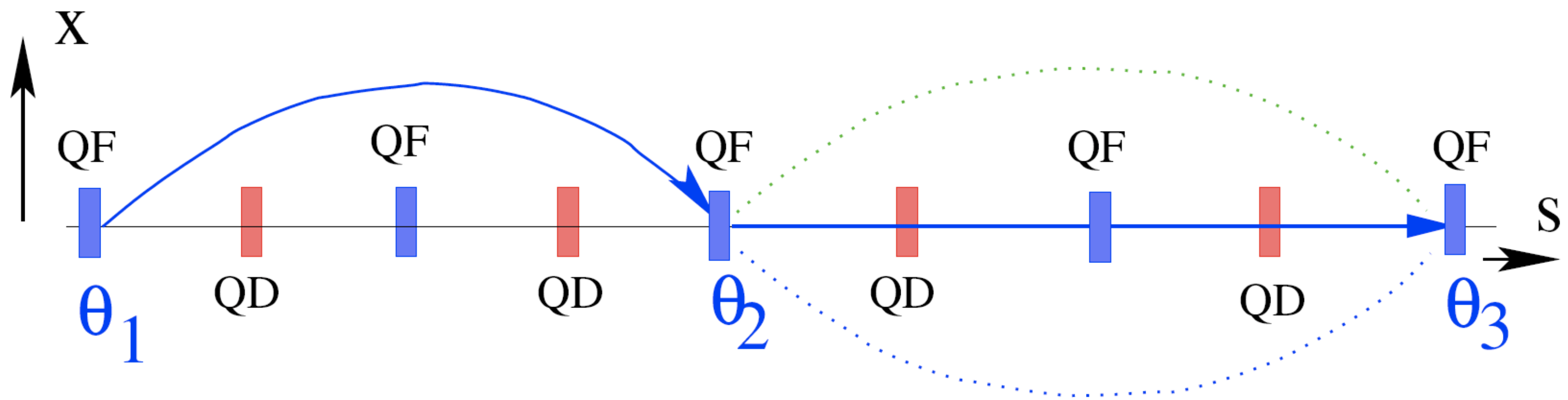
Correcting the orbit distortion

- Place horizontal and vertical dipole correctors and **b**eam **p**osition **m**onitors close to focusing and defocusing quads, respectively



- Measure orbit in BPMs and minimize orbit distortion
  - Globally
    - Harmonic, minimizing components of the orbit frequency response after a Fourier analysis
    - Most efficient corrector (MICADO), finding the most efficient corrector for minimizing the rms orbit
    - Least square minimization using the orbit response matrix of the correctors
  - Locally
    - Sliding Bumps
    - Singular Value Decomposition (SVD)

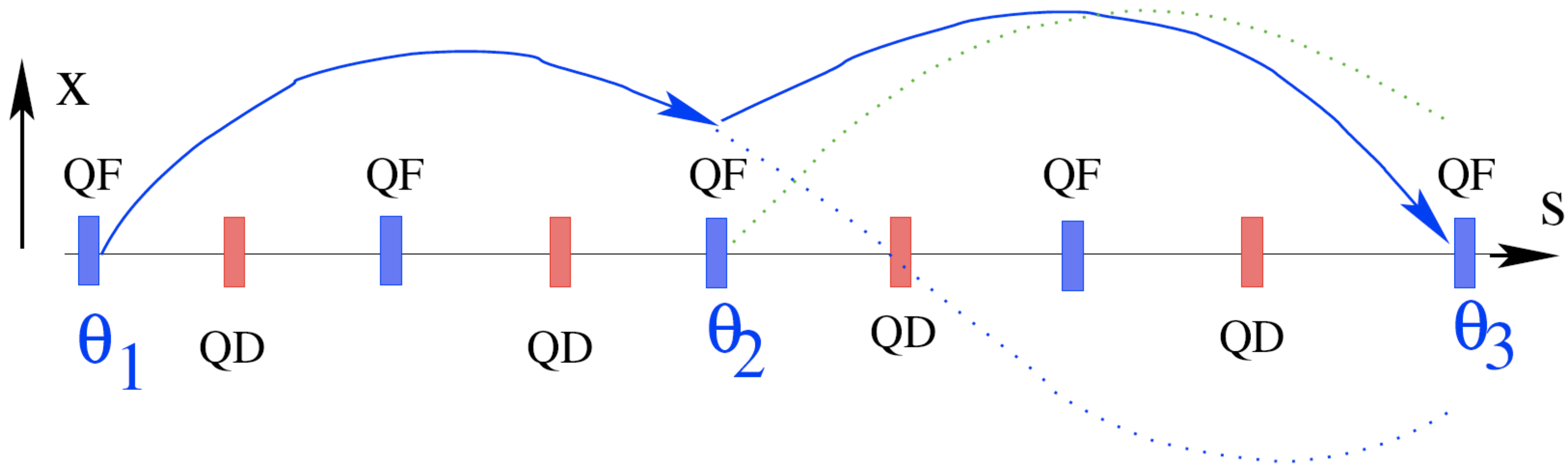
Orbit bumps: 2-bump



- Consider a cell, where correctors are placed close to the focusing quads
- The orbit shift at the 2nd corrector is $\delta u_2 = \sqrt{\beta_1 \beta_2}\sin(\psi_{12})\,\theta_1$
- This orbit error can be eliminated by choosing a phase advance equal to $\boldsymbol{\pi}$ between correctors
- The angle should satisfy the following equation

$$\theta_2 = \delta u_2' = \sqrt{\frac{\beta_1}{\beta_2}}\left[\cos(\psi_{12})\,\theta_1 - \alpha_2 \sin(\psi_{12})\right] = \sqrt{\frac{\beta_1}{\beta_2}}\theta_1$$

Orbit bumps: 3-bump

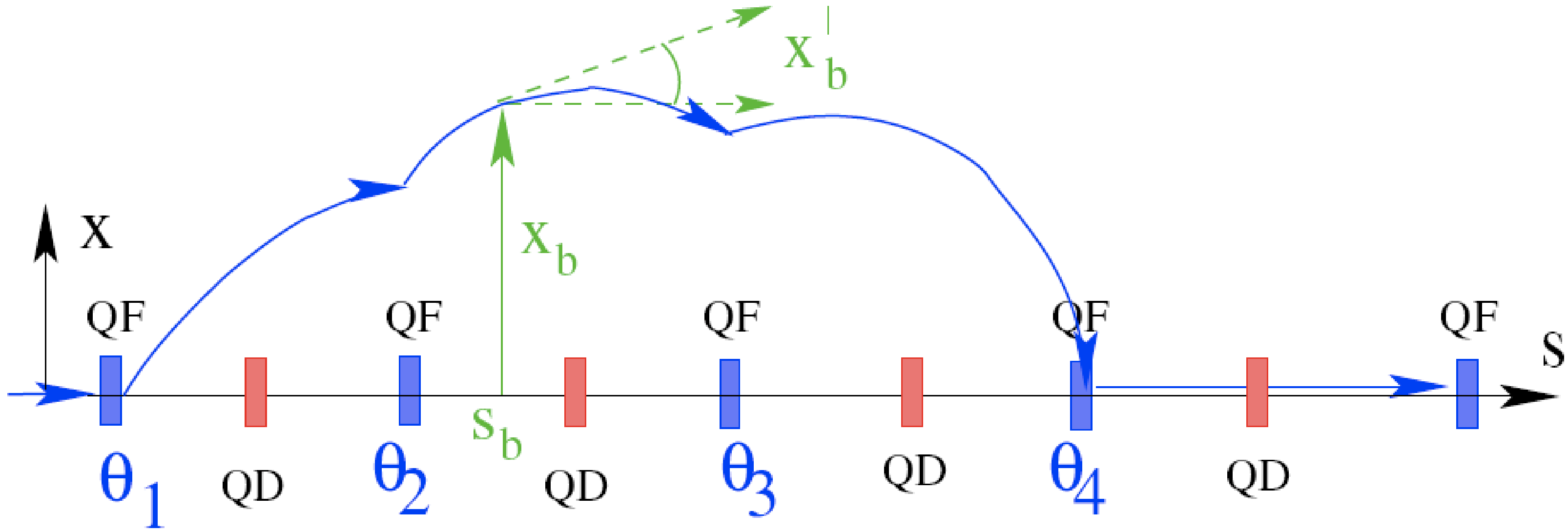


- **3-bump**: works for any phase advance if the three correctors satisfy

$$\frac{\sqrt{\beta_1}}{\sin\psi_{23}}\theta_1 = \frac{\sqrt{\beta_2}}{\sin\psi_{31}}\theta_2 = \frac{\sqrt{\beta_3}}{\sin\psi_{12}}\theta_3$$

- Need large number of correctors
- No control of the angles at the entrance and exit of the bump

Orbit bumps: 4-bump



$$\theta_1 = \frac{1}{\sqrt{\beta_1\beta_s}}\frac{\cos\psi_{2s} - \alpha_s\sin\psi_{2s}}{\sin\psi_{12}}x_s - \sqrt{\frac{\beta_s}{\beta_1}}\frac{\sin\psi_{2s}}{\sin\psi_{12}}x'_s$$

$$\theta_2 = \frac{1}{\sqrt{\beta_2\beta_s}}\frac{\cos\psi_{1s} - \alpha_s\sin\psi_{1s}}{\sin\psi_{12}}x_s + \sqrt{\frac{\beta_s}{\beta_2}}\frac{\sin\psi_{1s}}{\sin\psi_{12}}x'_s$$

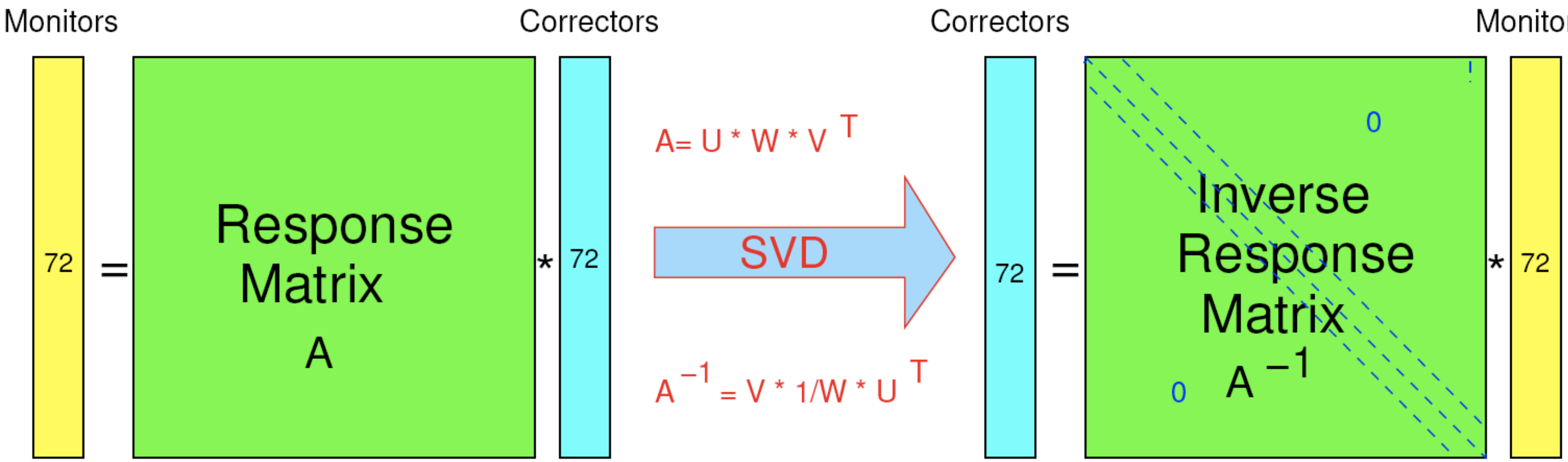
$$\theta_3 = \frac{1}{\sqrt{\beta_3\beta_s}}\frac{\cos\psi_{s4} - \alpha_s\sin\psi_{s4}}{\sin\psi_{34}}x_s - \sqrt{\frac{\beta_s}{\beta_3}}\frac{\sin\psi_{s4}}{\sin\psi_{34}}x'_s$$

$$\theta_4 = \frac{1}{\sqrt{\beta_4\beta_s}}\frac{\cos\psi_{s3} - \alpha_s\sin\psi_{s3}}{\sin\psi_{34}}x_s + \sqrt{\frac{\beta_s}{\beta_4}}\frac{\sin\psi_{s3}}{\sin\psi_{34}}x'_s$$

- **4-bump**: works for any phase advance
- Cancels position and angle outside of the bump
- Can be used for aperture scanning, extraction bumps, …

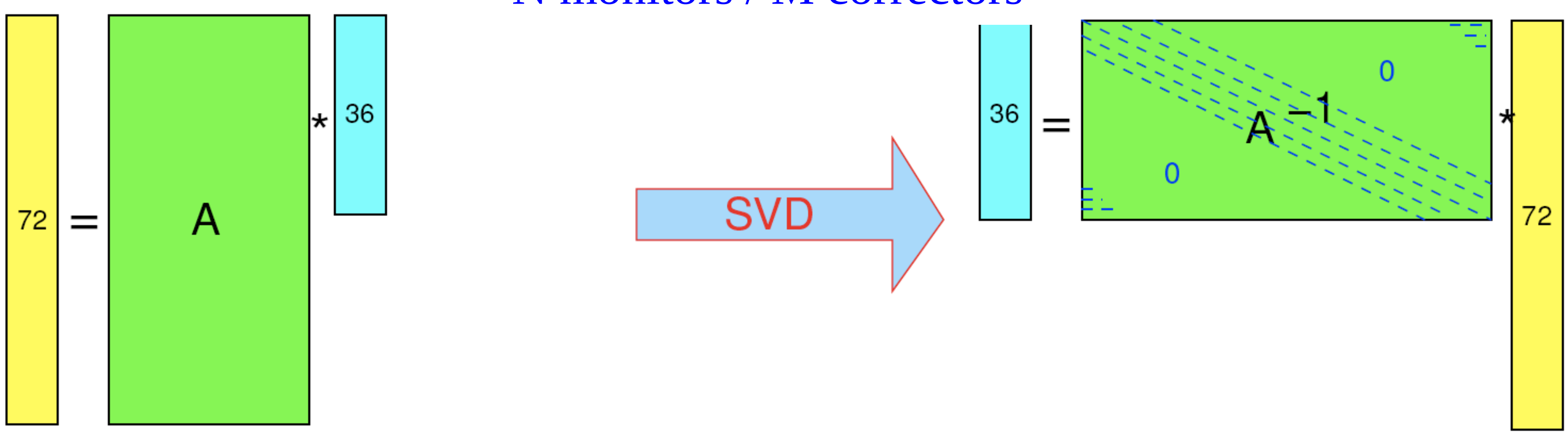
Singular Value Decomposition

N monitors / N correctors



=> Minimization of the RMS orbit (=0 in case of "Matrix Inversion" using all Eigenvalues)

N monitors / M correctors



=> Minimization of the RMS orbit (monitor averaging)

Orbit feedback

- Closed orbit stabilization performed using slow and fast orbit feedback system.
- Slow feedback operates every few seconds and uses complete set of BPMs for both planes
- Efficient in correcting distortion due to current decay in magnets or other slow processes
- Fast orbit correction system operates in a wide frequency range
  - correcting distortions induced by quadrupole and girder vibrations (up to 10kHz for the ESRF)
- Local feedback systems used to damp oscillations in areas where beam stabilization is critical (interaction points, insertion devices)

Feedback performance

Summary of integrated rms beam motion (1-100 Hz) with FOFB
and comparison with 10% beam stability target

	FOFB BW	**Horizontal**	**Vertical**
ALS	40 Hz	< 2 μm in H (30 μm)*	< 1 μm in V (2.3 μm)*
APS	60 Hz	< 3.2 μm in H (6 μm)**	< 1.8 μm in V (0.8 μm)**
Diamond	100 Hz	< 0.9 μm in H (12 μm)	< 0.1 μm in V (0.6 μm)
ESRF	100 Hz	< 1.5 μm in H (40 μm)	~ 0.7 μm in V (0.8 μm)
ELETTRA	100 Hz	< 1.1 μm in H (24 μm)	< 0.7 μm in V (1.5 μm)
SLS	100 Hz	< 0.5 μm in H (9.7 μm)	< 0.25 μm in V (0.3 μm)
SPEAR3	60Hz	~ 1 μm in H (30 μm)	~ 1 μm in V (0.8 μm)

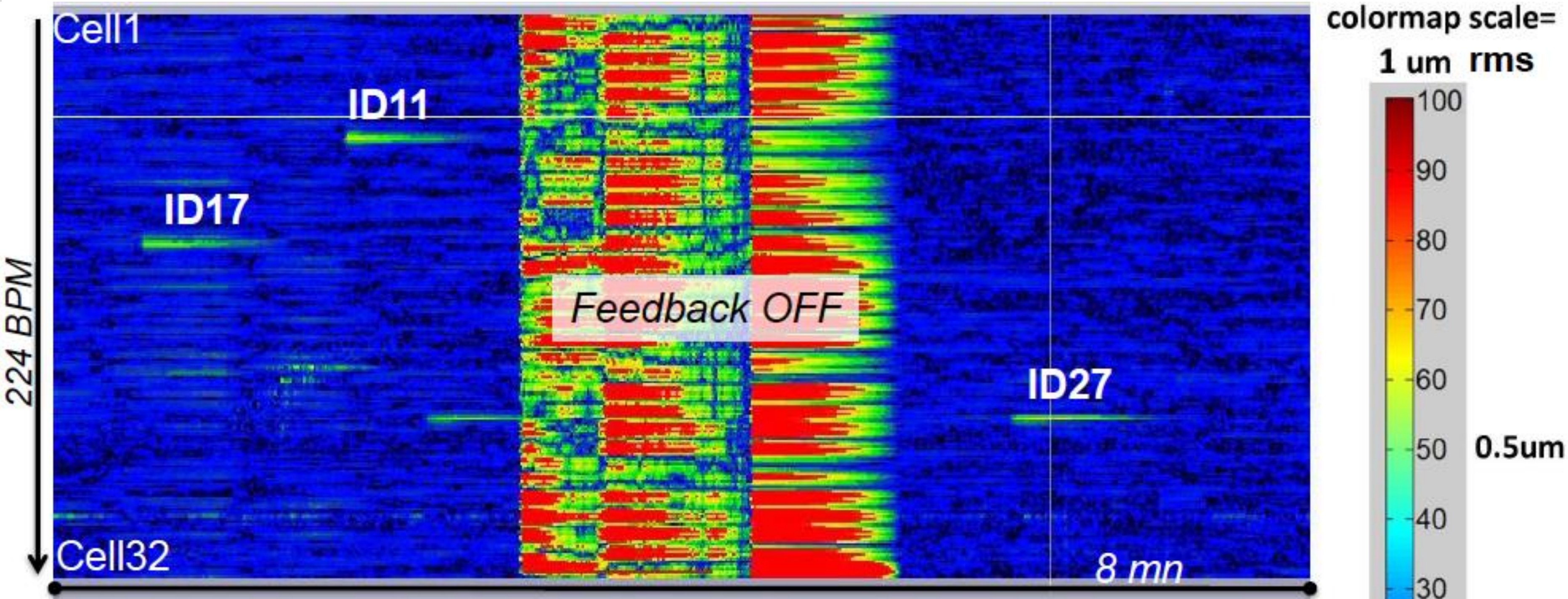
* up to 500 Hz

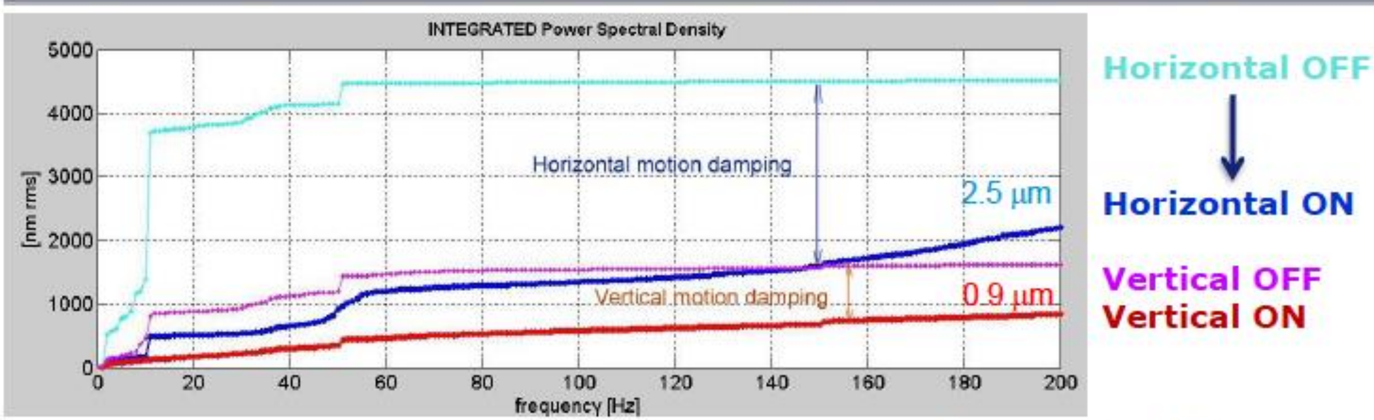
** up to 200 Hz

- Trends on Orbit Feedback
  - restriction of tolerances w.r.t. to beam size and divergence
  - higher frequencies ranges
  - integration of XBPMs
  - feedback on beamlines components

R. Bartolini, LER2010

Example: Orbit Feedback at ESRF





~0.1µm stability routinely achieved in V

~1.0µm stability routinely achieved in H

P. Raimondi 2014

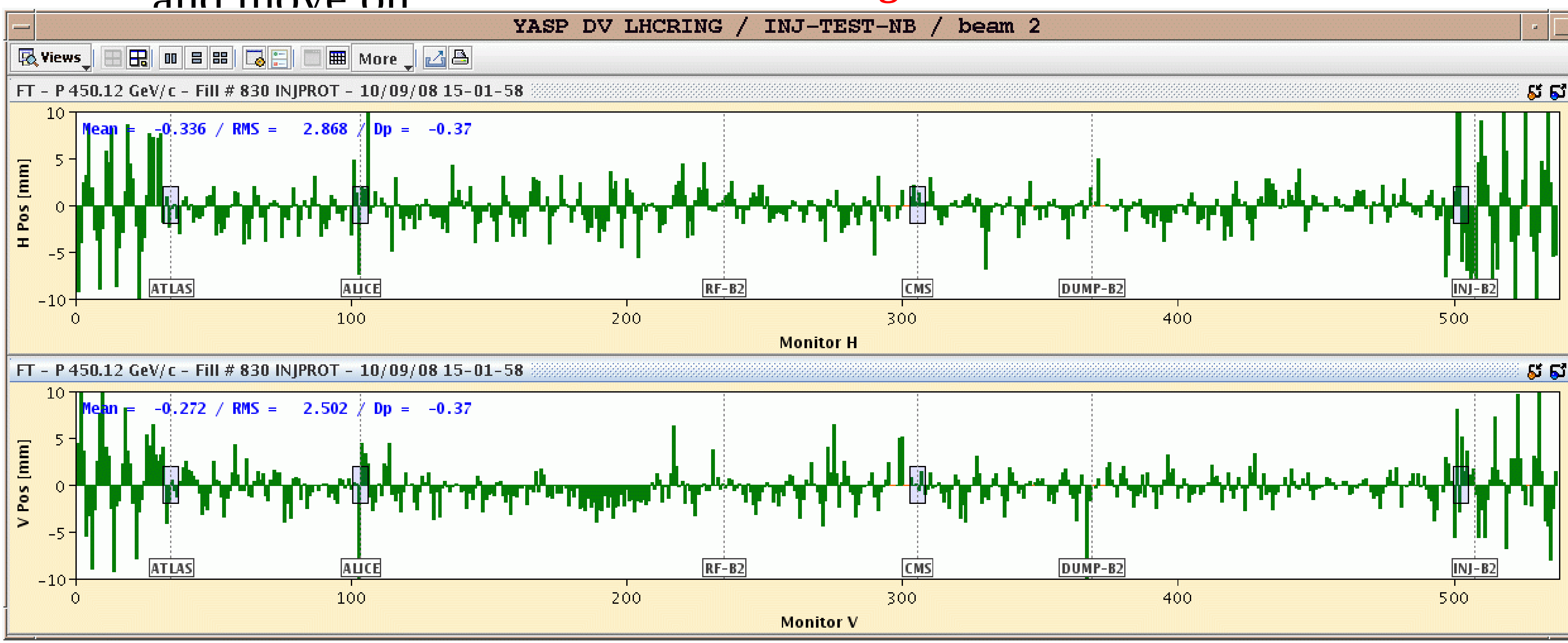
The European Synchrotron | ESRF

Beam threading

- Threading the beam round the LHC ring (very first commissioning)
  - One beam at a time, one hour per beam.
  - Collimators were used to intercept the beam (1 bunch, 2 × 109 protons)
  - Beam through 1 sector (1/8 ring) → correct trajectory, open collimator and move on

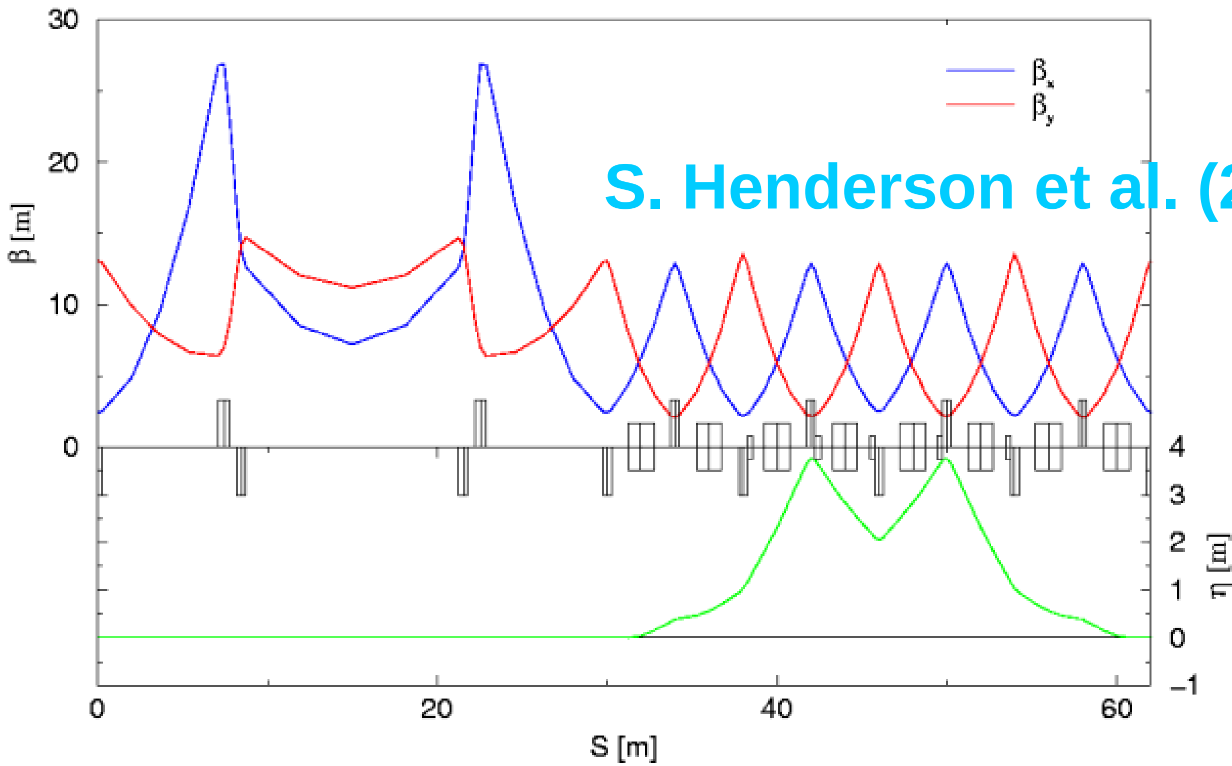
Beam 2 threading

BPM availability ~ 99%



Problem 1

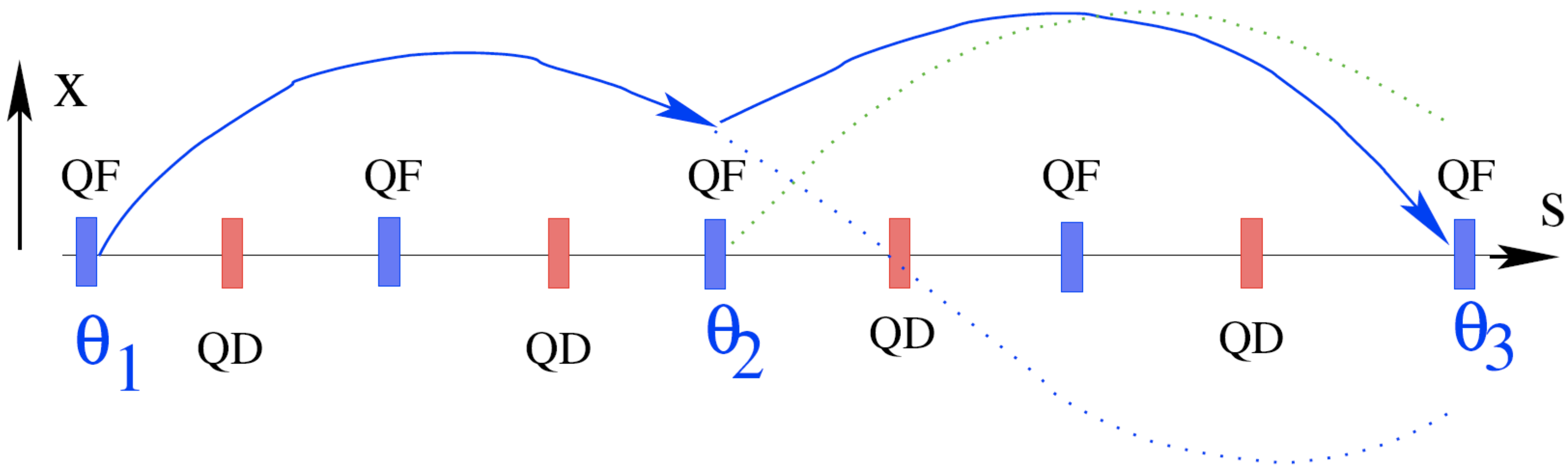
SNS: A **proton** ring with kinetic energy of **1GeV** and a **circumference** of **248m** has **18, 1m-long** focusing quads with **gradient** of **5T/m**. In one of the quads, the horizontal and vertical **beta** function is of **12m** and **2m** respectively. The **rms beta** function in both planes on the focusing quads is **8m**. With a **horizontal tune** of **6.23** and a vertical of **6.2**, compute the expected horizontal and vertical orbit distortions on the single focusing quad given by **horizontal** and by **vertical** misalignments of **1mm** in all the quads. What happens to the horizontal and vertical orbit distortions if the horizontal tune drops to **6.1** and **6.01**?



S. Henderson et al. (2005)

Problem 2

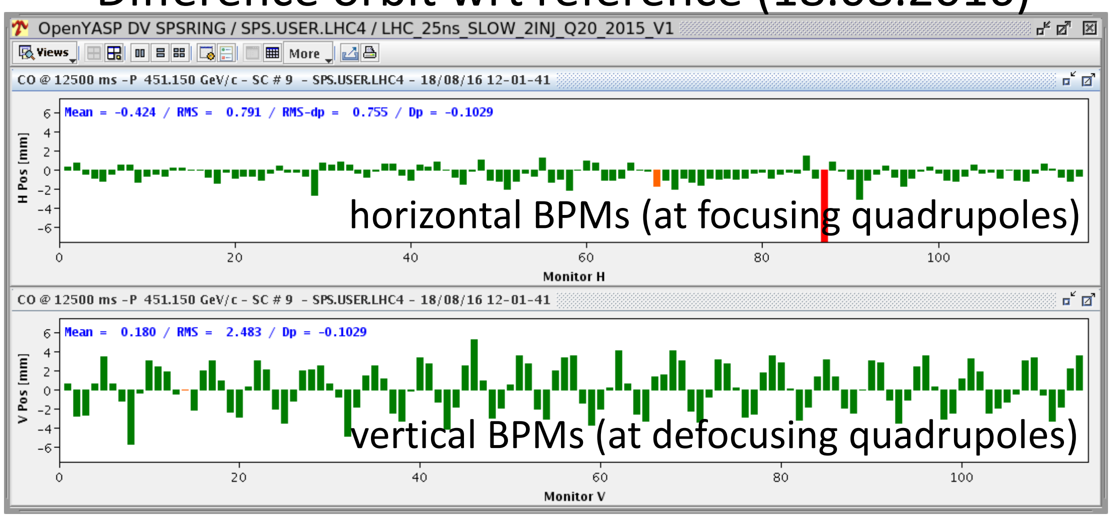
Three correctors are placed at locations with phase advance of **π/4** between them and beta functions of **12**, **2** and **12m**. How are the corrector kicks related to each other in order to achieve a closed 3-bump.

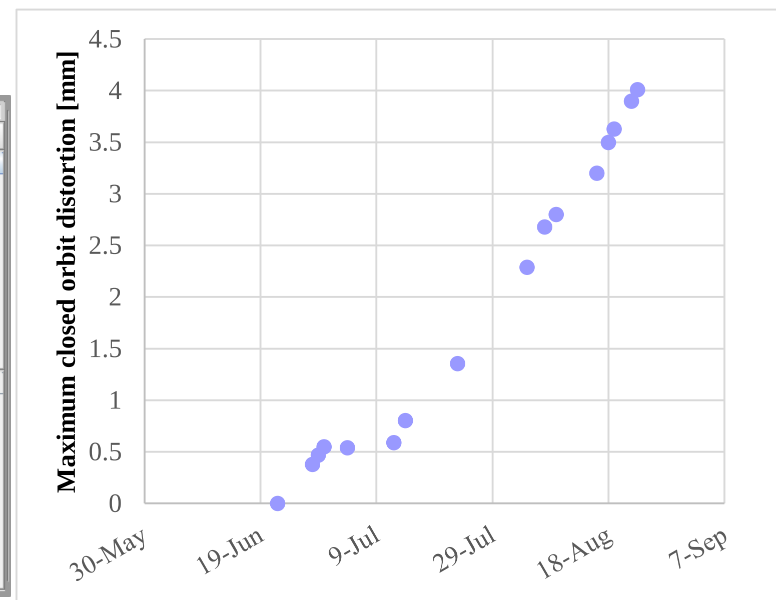


<u>SPS:</u> Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **15 T/m**, with a horizontal and vertical **beta** of **108m** and **30m** in the focusing quads which are **30m** and **108m** for the defocusing ones. The tunes of the machine are **Qx=20.13** and **Qy=20.18**. Due to a mechanical problem, one **<u>focusing quadrupole</u>** was **slowly sinking down** in 2016, resulting in an increasing closed orbit distortion wrt a reference taken in the beginning of the year.

- By how much the quadrupole had shifted down when the maximum vertical closed orbit distortion amplitude in <u>defocusing</u> quadrupoles reached 4 mm?
- Why was there no change of the horizontal orbit measured?
- How big would have been the maximum closed orbit distortion amplitude if it would have been a defocusing quadrupole?

Difference orbit wrt reference (18.08.2016)





Outline

- Closed orbit distortion (steering error)
  - Beam orbit stability importance
  - Imperfections leading to closed orbit distortion
  - Interlude: dispersion and chromatic orbit
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
- Optics function distortion (gradient error)
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- Coupling error
  - Coupling errors and their effect
  - Coupling correction
- Chromaticity

Gradient error and optics distortion

- Optics functions perturbation can induce aperture restrictions
- Tune perturbation can lead to dynamic aperture loss
- Broken super-periodicity → excitation of all resonances
  - In a ring made out of *N* identical cells, only resonances that are integer multiples of *N* can be excited
- Causes
  - Errors in quadrupole strengths (random and systematic)
  - Injection elements
  - Higher-order multi-pole magnets and errors
- Observables
  - Tune-shift
  - Beta-beating
  - Excitation of integer and half integer resonances

Gradient error

- Consider the transfer matrix for 1-turn

$$\mathcal{M}_0 = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ -\gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - \alpha_0 \sin(2\pi Q) \end{pmatrix}$$

- Consider a gradient error in a quad. In thin element approximation the quad matrix with and without error are

$$m_0 = \begin{pmatrix} 1 & 0 \\ -K_0(s)ds & 1 \end{pmatrix} \text{ and } m = \begin{pmatrix} 1 & 0 \\ -(K_0(s) + \delta K)ds & 1 \end{pmatrix}$$

- The new 1-turn matrix is $\mathcal{M} = m m_0^{-1} \mathcal{M}_0 = \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} \mathcal{M}_0$

  which yields

$$\mathcal{M} = \begin{pmatrix} \cos(2\pi Q) + \alpha_0 \sin(2\pi Q) & \beta_0 \sin(2\pi Q) \\ \delta K ds(\cos(2\pi Q) - \alpha_0 \sin(2\pi Q)) - \gamma_0 \sin(2\pi Q) & \cos(2\pi Q) - (\delta K ds \beta_0 + \alpha_0) \sin(2\pi Q) \end{pmatrix}$$

Gradient error and tune-shift

- Consider a new matrix after 1 turn with a new tune $\chi = 2\pi(Q+\delta Q)$

$$\mathcal{M}^\star = \begin{pmatrix} \cos(\chi) + \alpha_0 \sin(\chi) & \beta_0 \sin(\chi) \\ -\gamma_0 \sin(\chi) & \cos(\chi) - \alpha_0 \sin(\chi) \end{pmatrix}$$

- The traces of the two matrices describing the 1-turn should be equal

$$\mathrm{Tra}(\mathcal{M}^\star) = \mathrm{Tra}(\mathcal{M})$$

which gives $2\cos(2\pi Q) - \delta K ds \beta_0 \sin(2\pi Q) = 2\cos(2\pi(Q+\delta Q))$

- Developing the right hand side

$$\cos(2\pi(Q+\delta Q)) = \cos(2\pi Q)\underbrace{\cos(2\pi\delta Q)}_{1} - \sin(2\pi Q)\underbrace{\sin(2\pi\delta Q)}_{2\pi\delta Q}$$

and finally $4\pi\delta Q = \delta K ds \beta_0$

- For a quadrupole of finite length, we have $\boxed{\delta Q = \frac{1}{4\pi}\int_{s_0}^{s_0+l} \delta K \beta_0 ds}$

- Consider the unperturbed transfer matrix for one turn

$$M_0 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = B \cdot A \quad \text{with} \qquad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- Introduce a gradient perturbation between the two matrices

$$\mathcal{M}_0^\star = \begin{pmatrix} m_{11}^\star & m_{12}^\star \\ m_{21}^\star & m_{22}^\star \end{pmatrix} = B \begin{pmatrix} 1 & 0 \\ -\delta K ds & 1 \end{pmatrix} A$$

- Recall that $m_{12} = \beta_0 \sin(2\pi Q)$ and write the perturbed term as

$$m_{12}^\star = (\beta_0 + \delta\beta)\sin(2\pi(Q + \delta Q)) = m_{12} + \delta\beta \sin(2\pi Q) + 2\pi\delta Q \beta_0 \cos(2\pi Q)$$

where we used $\sin(2\pi\delta Q) \approx 2\pi\delta Q$ and $\cos(2\pi\delta Q) \approx 1$

- On the other hand

$$a_{12} = \sqrt{\beta_0 \beta(s_1)}\sin\psi, \;\; b_{12} = \sqrt{\beta_0 \beta(s_1)}\sin(2\pi Q - \psi)$$

and $m_{12}^{\star} = \underbrace{b_{11}a_{12} + b_{12}a_{22}}_{m_{12}} - a_{12}b_{12}\delta K ds = m_{12} - a_{12}b_{12}\delta K ds$

- Equating the two terms

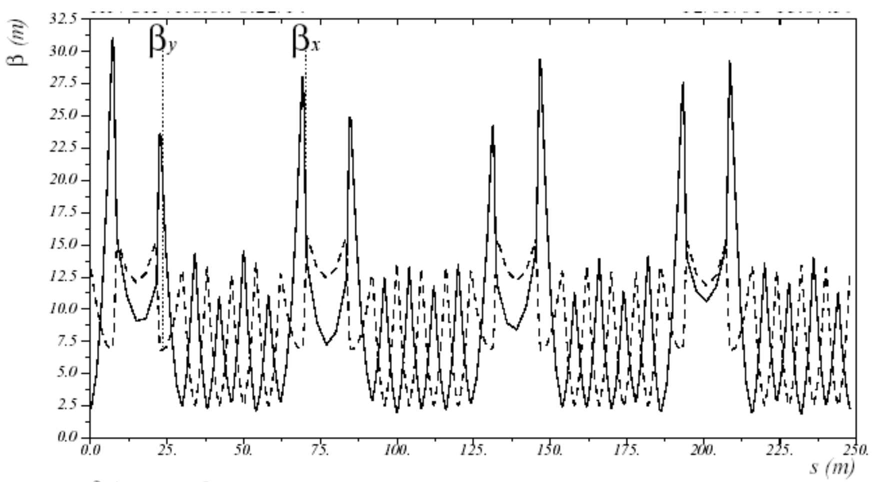
$$\delta\beta\sin(2\pi Q) + 2\pi\delta Q\beta_0\cos(2\pi Q) = -a_{12}b_{12}\delta K ds$$

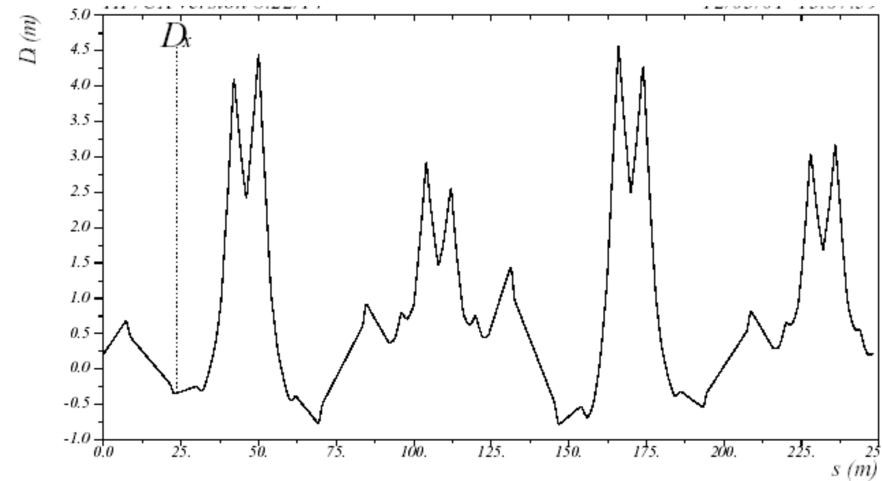
- Integrating through the quad

$$\frac{\delta\beta}{\beta_0} = -\frac{1}{2\sin(2\pi Q)}\int_{s_1}^{s_1+l}\beta(s)\delta K(s)\cos(2\psi - 2\pi Q)ds$$

- There is also an equivalent effect on dispersion

Example: Gradient error in SNS





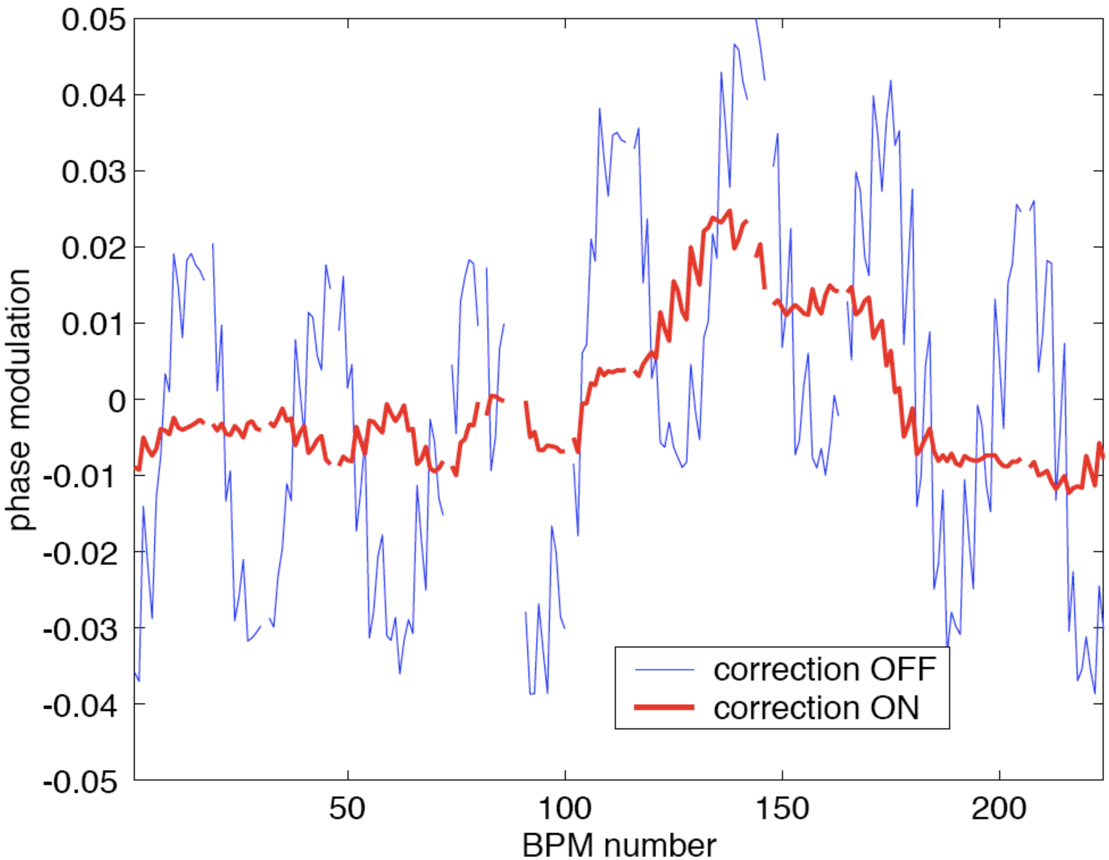
- Consider **18** focusing quads in the SNS ring with **0.01T/m** gradient error. In this location β=**12m**. The length of the quads is **0.5m** and the magnetic rigidity is **5.6567Tm**
  - The tune-shift is $\delta Q = \frac{1}{4\pi} 18 \cdot 12 \frac{0.01}{5.6567} 0.5 = 0.015$
- For a random distribution of errors the beta beating is

$$\frac{\delta\beta}{\beta_0}_{\mathrm{rms}} = -\frac{1}{2\sqrt{2}|\sin(2\pi Q)|} \left(\sum_i \delta k_i^2 \beta_i^2\right)^{1/2}$$

  - Optics functions beating > **20%** by random errors (1% of gradient) in high dispersion quads of the SNS ring … justifies correctors strengths

Example: Gradient error in ESRF

- Consider **128** focusing arc quads in the ESRF storage ring with **0.001T/m** gradient error. In this location β=**30m**. The length of the quads is around **1m.** The magnetic rigidity of the ESRF is **20Tm.**



- The tune-shift is

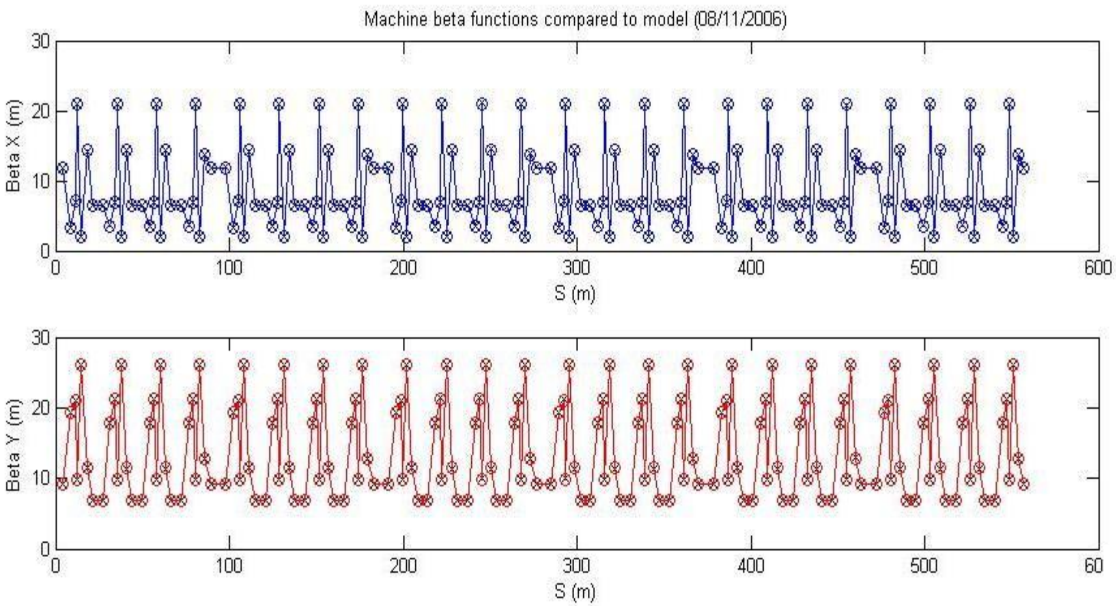
$$\delta Q = \frac{1}{4\pi} 128 \cdot 30 \frac{0.001}{20} 1 = 0.014$$

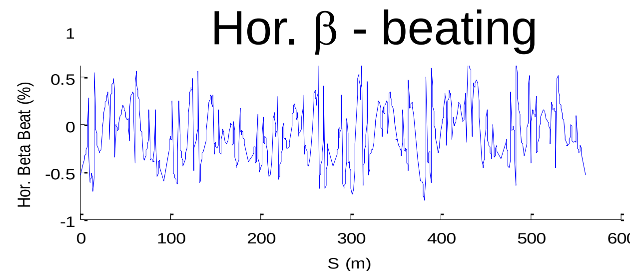
Gradient error correction

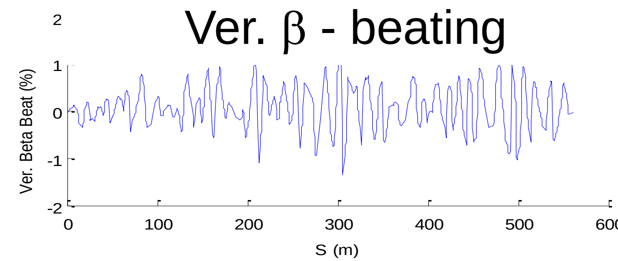
- Windings on the core of the quadrupoles or individual correction magnets (trim windings or quadrupoles)
- Compute tune-shift and optics function beta distortion
- Move working point close to integer and half integer resonance
- Minimize beta wave or quadrupole resonance width with trim windings
- Individual powering of trim windings can provide flexibility and beam based alignment of BPM
- Modern methods of response matrix analysis (LOCO) can fit optics model to real machine and correct optics distortion

J. Safranek et al.

R. Bartolini, LER2010





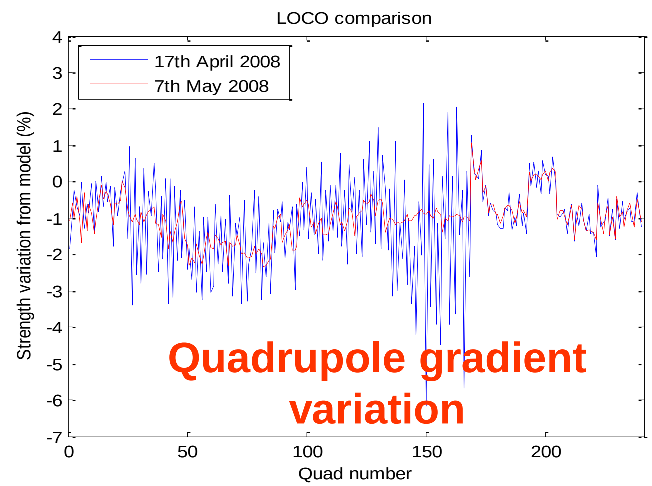


Modified version of LOCO with constraints on gradient variations **(see [ICFA Newsl, Dec' 07])**

β - beating reduced to 0.4% rms

Quadrupole variation reduced to 2%

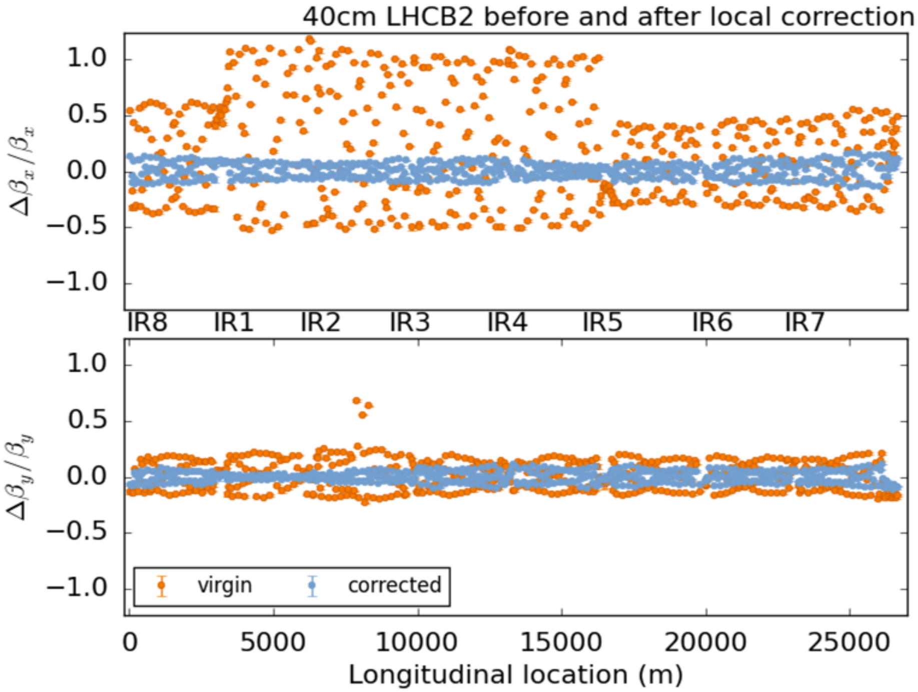
Results compatible with mag. meas. and calibrations

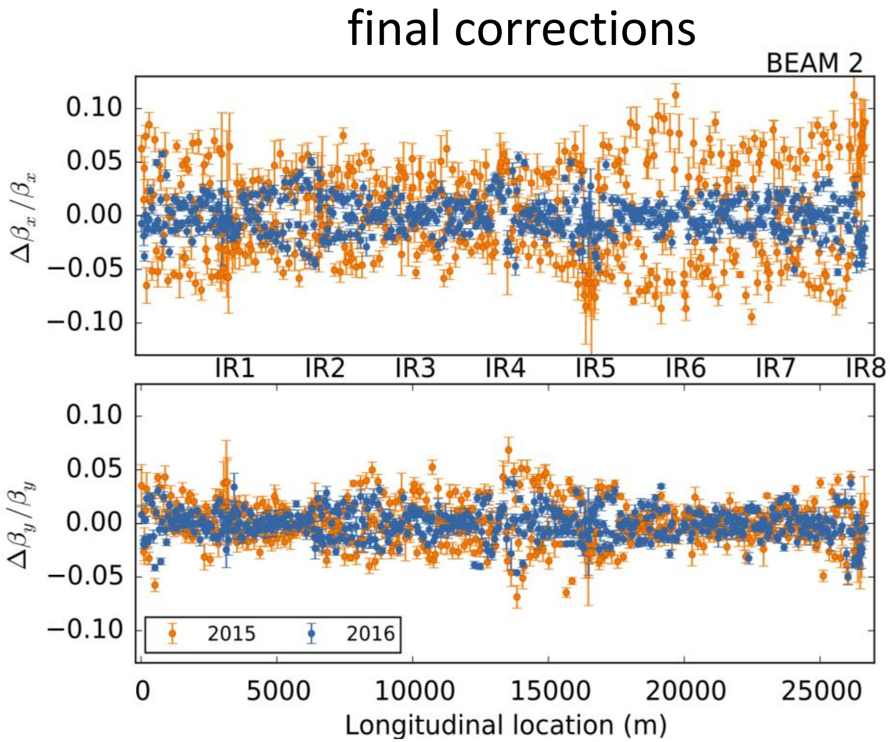


LOCO allowed remarkable progress with the correct implementation of the linear optics

Example: LHC optics corrections

- At β*=40cm, the bare machine has a beta-beat of more than 100%
- After global and local corrections, β-beating was reduced to few %





R. Tomas et al. 2016

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4x4 Matrices

- Combine the matrices for each plane

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) \\ C'_x(s) & S'_x(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_y(s) & S_y(s) \\ C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

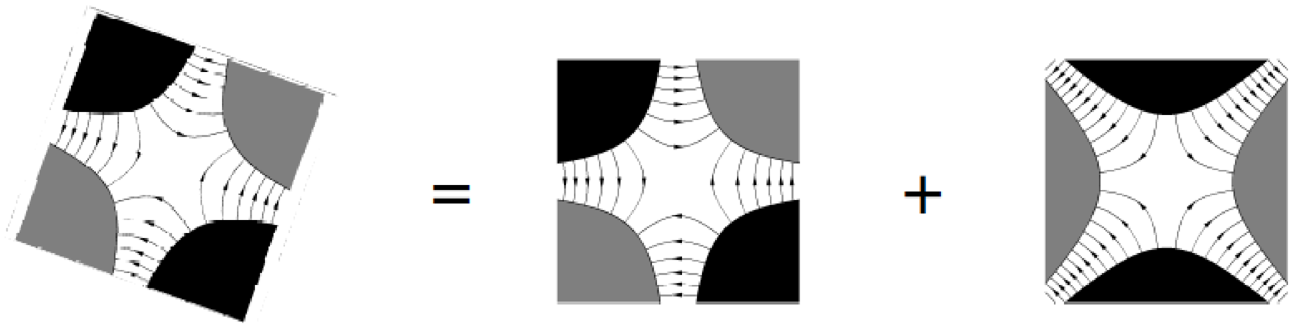
to get a total 4x4 matrix

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix}$$

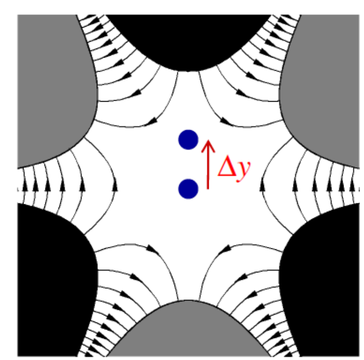
Uncoupled motion

Coupling error

- Coupling errors lead to transfer of horizontal betatron motion and dispersion into the vertical plane
- Coupling may result from rotation of a quadrupole, so that the field contains a skew component



- A vertical beam offset in a sextupole has the same effect as a skew quadrupole. The sextupole field for the displacement of a particle ***δy*** becomes



$$B_x = k_2 x \bar{y} = k_2 x y + \underbrace{k_2 x \delta y}$$

skew quadrupole

$$B_y = \frac{1}{2} k_2 (x^2 - \bar{y}^2) = \overbrace{-k_2 y \delta y} + \frac{1}{2} k_2 (x^2 - y^2) - \frac{1}{2} k_2 \delta y^2$$

Effect of coupling

- Betatron motion is coupled in the presence of skew quadrupoles
- The field is $(B_x, B_y) = k_s(x, y)$ and Hill's equations are coupled
- Motion still linear with two new eigen-mode tunes, which are always split. In the case of a thin skew quad:

$$\delta Q \propto |k_s|\sqrt{\beta_x\beta_y}$$

- Coupling coefficients represent the degree of coupling

$$|C_\pm| = \left|\frac{1}{2\pi}\oint ds k_s(s)\sqrt{\beta_x(s)\beta_y(s)}e^{i(\psi_x\pm\psi_y-(Q_x\pm Q_y-q_\pm)2\pi s/C)}\right|$$

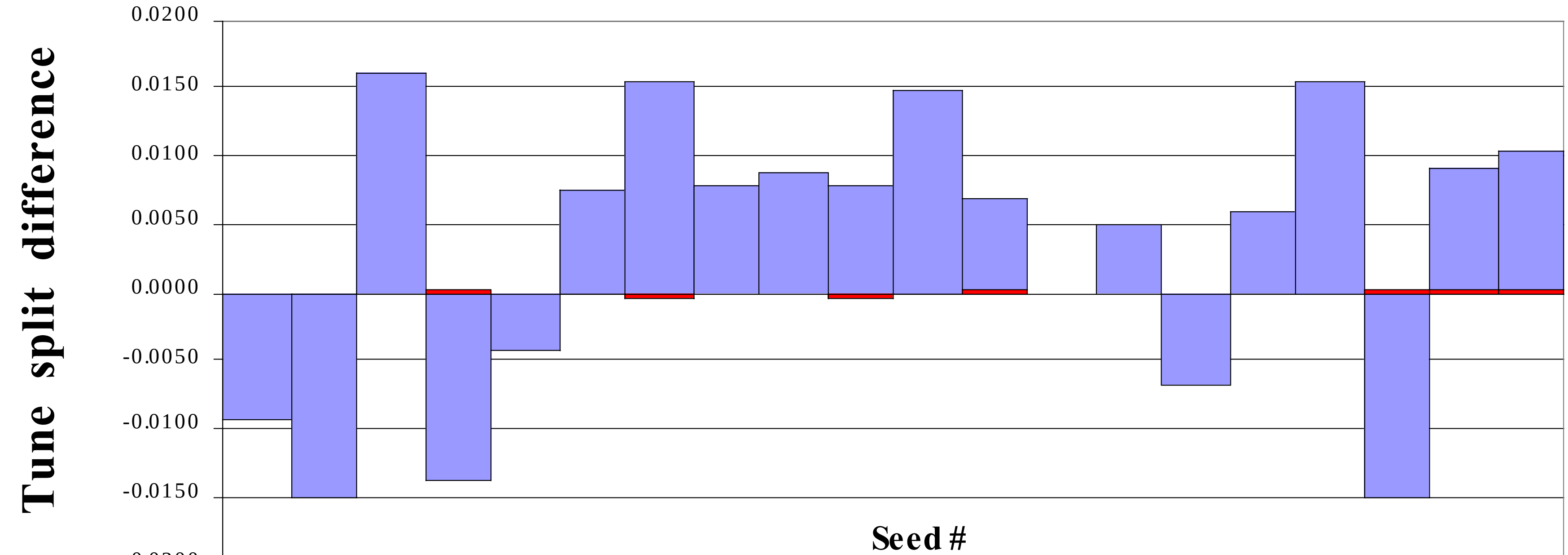
- As motion is coupled, vertical dispersion and optics function distortion appears

Linear coupling correction

- Introduce skew quadrupole correctors
- Correct globally/locally coupling coefficient (or resonance driving term)
- Correct optics distortion (especially vertical dispersion)
- Move working point close to coupling resonances and repeat
- Correction especially important for flat beams
- Note that (vertical) orbit correction may be critical for reducing coupling

Example: SNS coupling correction

- Local decoupling by super period using 16 skew quadrupole correctors
- Results of Q_x=6.23 Q_y=6.20 after a **2mrad** quad roll
- Additional 8 correctors used to compensate vertical dispersion



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Before Correction	-0.009	-0.014	0.016	-0.013	-0.004	0.007	0.015	0.008	0.008	0.007	0.014	0.006	0.000	0.005	-0.006	0.006	0.015	-0.015	0.009	0.010
After correction	0.000	0.000	0.000	0.000	-0.000	0.000	-0.000	0.000	-0.000	-0.000	0.000	0.000	0.000	-0.000	-0.000	0.000	-0.000	0.000	0.000	0.000

Vertical dispersion

- The equation of motion for a particle with momentum *p* is

$$\frac{d^2y}{ds^2} = \frac{e}{p} B_x$$

  - For small energy deviation δ, *p* is related to the reference momentum

$$p \approx (1+\delta) p_0$$

  - We can write for the horizontal field (to first order in the derivatives)

$$B_x \approx B_{0x} + y\frac{\partial B_x}{\partial y} + x\frac{\partial B_x}{\partial x}$$

  - If we consider a particle following an off-momentum closed orbit

$$y = \eta_y \delta, \quad \text{and} \quad x = \eta_x \delta$$

- Combining the above equations, we find to first order in

$$\frac{d^2\eta_y}{ds^2} - k_1\eta_y \approx -k_{0s} + k_{1s}\eta_x$$

Vertical dispersion from errors

- The previous equation is similar to the equation of the closed orbit

$$\frac{d^2 y_{co}}{ds^2} - k_1 y_{co} \approx -k_{0s} + k_{1s} x_{co}$$

- It is thus reasonable to generalize the relationship between the closed orbit and the quadrupole misalignments, to find

$$\left\langle \frac{\eta_y^2}{\beta_y} \right\rangle = \frac{\Delta y_Q^2}{8 \sin^2 \pi Q_y} \sum_{\text{quads}} \beta_y (k_1 L)^2 + \frac{\langle \Delta \theta_Q^2 \rangle}{8 \sin^2 \pi Q_y} \sum_{\text{quads}} \eta_x^2 \beta_y (k_1 L)^2 +$$
$$\frac{\langle \Delta y_s^2 \rangle}{8 \sin^2 \pi Q_y} \sum_{\text{sexts}} \eta_x^2 \beta_y (k_2 L)^2$$

  - Skew dipole terms assumed arise from vertical misalignments of quadrupoles
  - Skew quadrupoles assumed to come from tilts on the quads and vertical misalignments of sextupoles
  - All alignment errors are considered uncorrelated.

Impact on vertical emittance

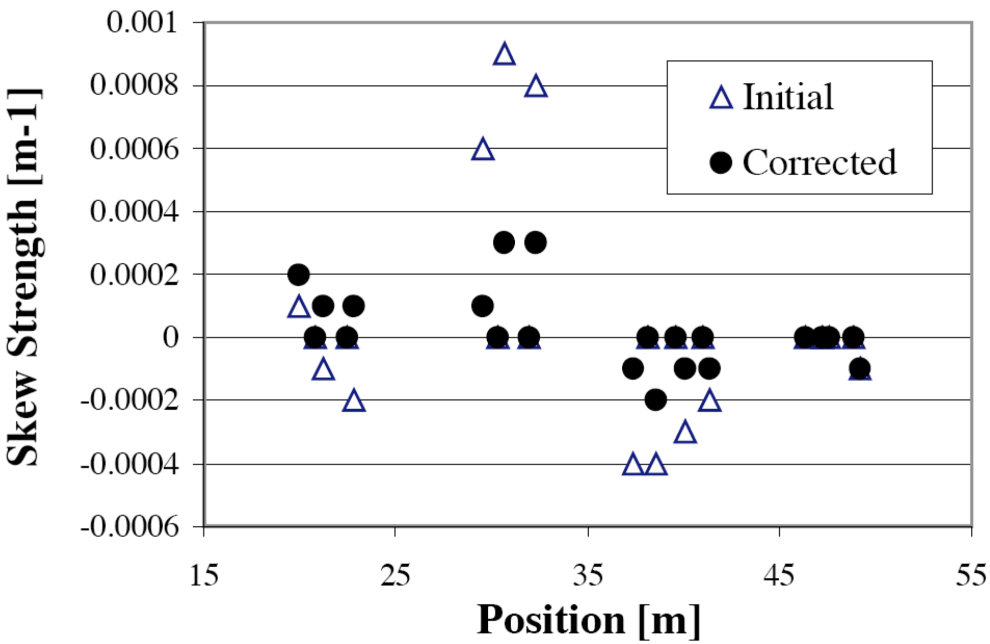
- The natural emittance in the vertical plane can be written as the horizontal one
$$\varepsilon_y = C_q \gamma^2 \frac{I_{5y}}{j_y I_2}$$
- The synchrotron radiation integrals are given by
$$I_{5y} = \oint \frac{\mathcal{H}_y}{|\rho|^3} ds \approx \langle \mathcal{H}_y \rangle \oint \frac{1}{|\rho|^3} ds = \langle \mathcal{H}_y \rangle I_3 \quad \text{and} \quad I_2 = \oint \frac{1}{\rho^2} ds$$
with the dispersion invariant $\mathcal{H}_y = \gamma_y \eta_y^2 + 2\alpha_y \eta_y \eta_{py} + \beta_y \eta_{py}^2$
- Then the vertical emittance is $\varepsilon_y \approx C_q \gamma^2 \langle \mathcal{H}_y \rangle \dfrac{I_3}{j_y I_2}$
or in terms of energy spread $\varepsilon_y \approx \dfrac{j_z}{j_y} \langle \mathcal{H}_y \rangle \sigma_\delta^2$ with $\sigma_\delta^2 = C_q \gamma^2 \dfrac{I_3}{j_z I_2}$
- Note that $\left\langle \dfrac{\eta_y^2}{\beta_y} \right\rangle = \dfrac{1}{2} \langle \mathcal{H}_y \rangle$ so that finally
$$\boxed{\varepsilon_y \approx 2 \frac{j_z}{j_y} \left\langle \frac{\eta_y^2}{\beta_y} \right\rangle \sigma_\delta^2}$$

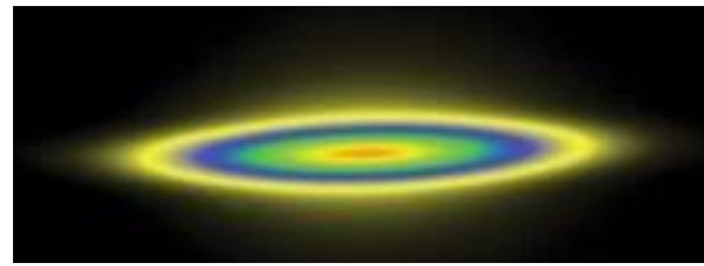
Methods for coupling control

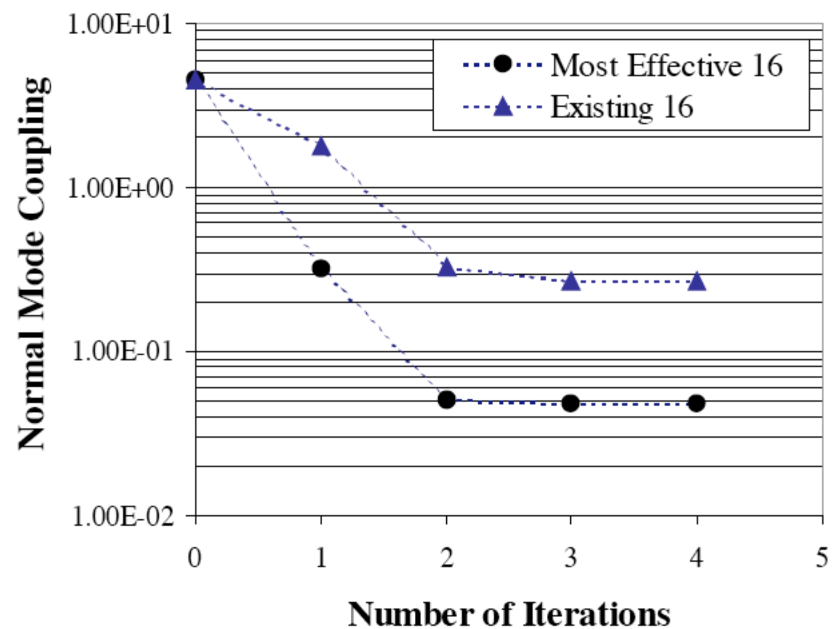
- Measurement or estimation of BPM roll errors to avoid “fake” vertical dispersion measurement.
- Realignment of girders / magnets to remove sources of coupling and vertical dispersion.
- Model based corrections:
  - Establish lattice model: multi-parameter fit to orbit response matrix (using LOCO or related methods) to obtain a calibrated model.
  - Use calibrated model to perform correction or to minimize derived lattice parameters (e.g. vertical emittance) in simulation and apply to machine.
  - Application to coupling control: correction of vertical dispersion, coupled response matrix, resonance drive terms using skew quads and orbit bumps, or direct minimization of vertical emittance in model.
- Model independent corrections:
  - empirical optimization of observable quantities related to coupling (e.g. beam size, beam life time).
- Coupling control in operation: on-line iteration of correction

Example: ESRF coupling correction

- Local decoupling using 16 skew quadrupole correctors and coupled response matrix reconstruction
- Achieved correction of below 0.25% reaching vertical emittance of below 4pm







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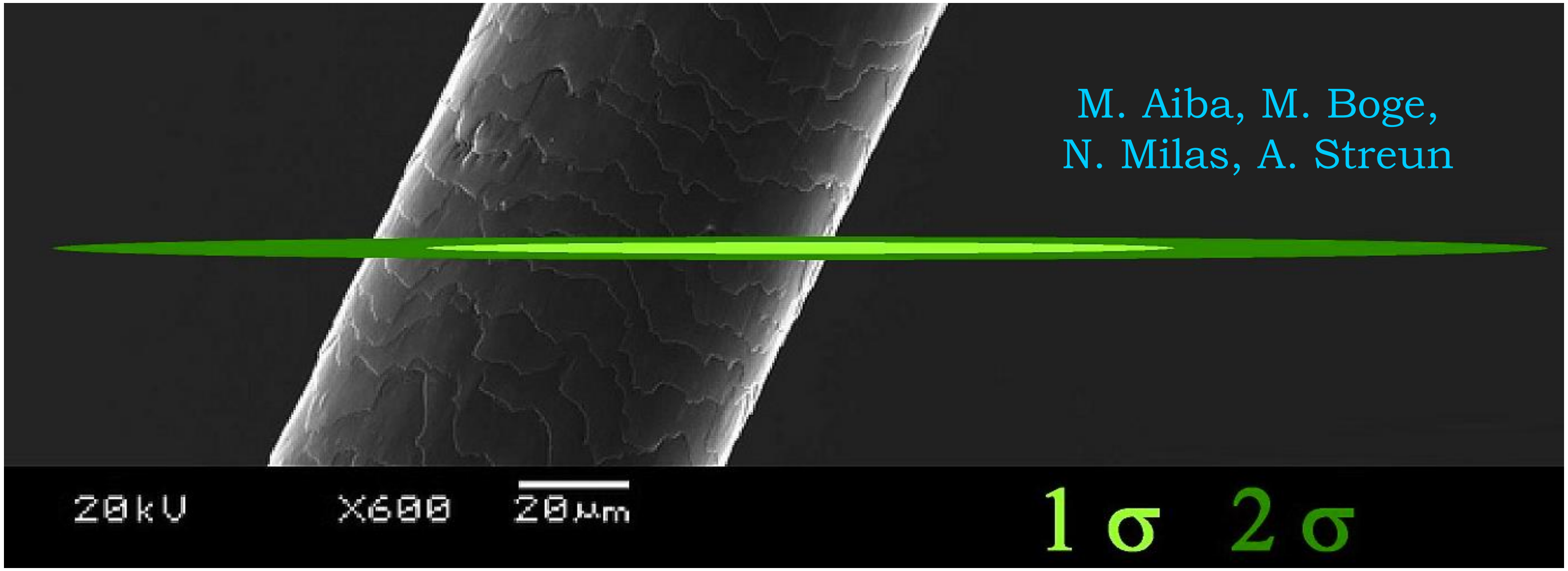
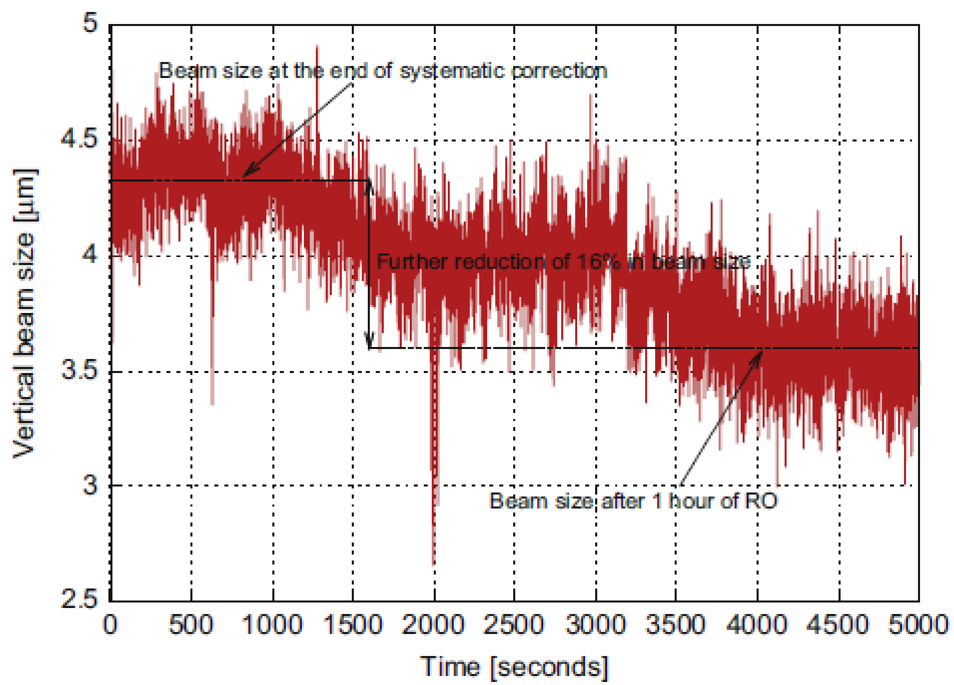
Vertical emittance at PSI



- Vertical emittance reduced to a minimum value of **0.9±0.4pm**
- Achieved by careful re-alignment campaign and different methods of coupling suppression using 36 skew quadrupoles (combination of response matrix based correction and random walk optimisation)

Random walk optimisation

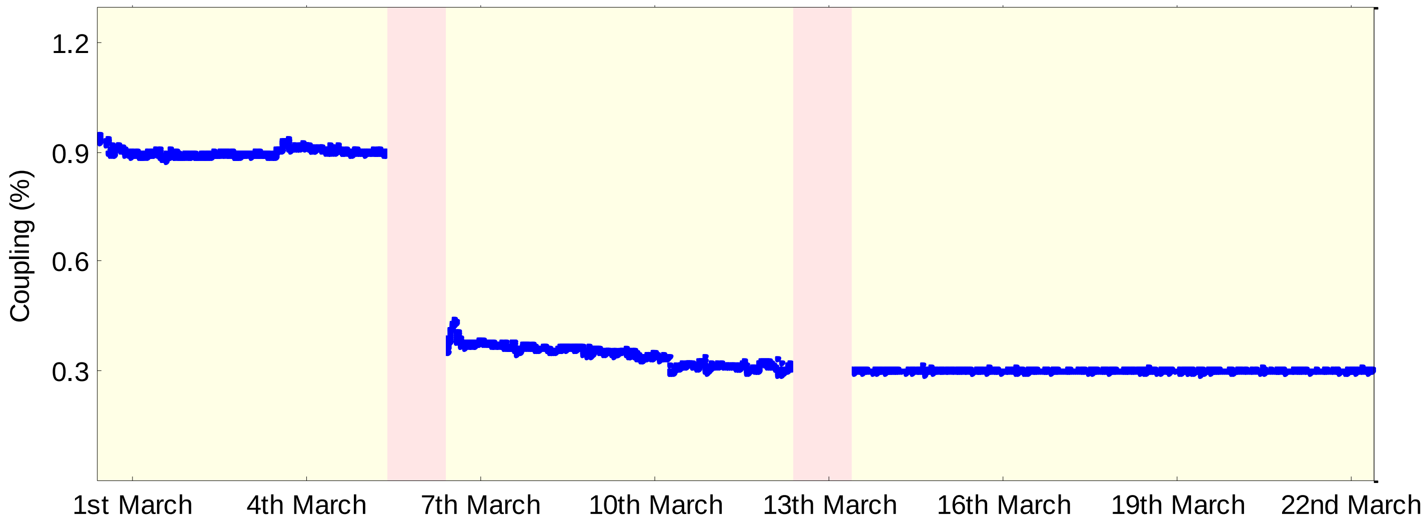
- Coupling minimization at SLS observable: vertical beam size from monitor
- Knobs: 24 skew quadrupoles
- Random optimization: trial & error (small steps)
- Start: model based correction: $\boldsymbol{\varepsilon_y}$ **= 1.3 pm**
- 1 hour of random optimization $\boldsymbol{\varepsilon_y \rightarrow}$ **0.9±0.4 pm**



- Measured coupled response matrix off-diagonal terms were reduced after optimization
- Model based correction limited by model deficiencies rather than measurement errors.

Coupling control in operation

- Keep vertical emittance constant during insertion device gap changes
- Example from DIAMOND
  - Offset δSQ to ALL skew quads generates dispersion wave and increases vert. emittance without coupling.
  - Skew quads from LOCO for low vert .emit. of ~ 3pm
  - Increase vertical emit to 8 pm by increasing the offset δSQ
  - Use the relation between vertical emittance and δSQ in a slow feedback loop (5 Hz)



Outline

- Closed orbit distortion (steering error)
  - Beam orbit stability importance
  - Imperfections leading to closed orbit distortion
  - Interlude: dispersion and chromatic orbit
  - Effect of single and multiple dipole kicks
  - Closed orbit correction methods
- Optics function distortion (gradient error)
  - Imperfections leading to optics distortion
  - Tune-shift and beta distortion due to gradient errors
  - Gradient error correction
- Coupling error
  - Coupling errors and their effect
  - Coupling correction
- Chromaticity

Chromaticity

- Linear equations of motion depend on the energy (term proportional to dispersion)
- Chromaticity is defined as: $\xi_{x,y} = \frac{\delta Q_{x,y}}{\delta p/p}$
- Recall that the gradient is $k = \frac{G}{B\rho} = \frac{eG}{p} \rightarrow \frac{\delta k}{k} = \mp \frac{\delta p}{p}$
- This leads to dependence of tunes and optics function on the particle's momentum
- For a linear lattice the tune shift is:

$$\delta Q_{x,y} = \frac{1}{4\pi} \oint \beta_{x,y} \delta k(s) ds = -\frac{1}{4\pi} \frac{\delta p}{p} \oint \beta_{x,y} k(s) ds$$

- So the **natural** chromaticity is:

$$\xi_{x,y} = -\frac{1}{4\pi} \oint \beta_{x,y} k(s) ds$$

- Sometimes the chromaticity is quoted as $\overline{\xi_{x,y}} = \frac{\xi_{x,y}}{Q_{x,y}}$

Example: Chromaticity in SNS

- In the SNS ring, the **natural chromaticity is –7**
- Consider that momentum spread ***δp/p* = ±1%**
- The tune-shift for off-momentum particles is

$$\delta Q_{x,y} = \xi_{x,y} \delta p/p = \pm 0.07$$

- In order to correct chromaticity introduce particles which can focus off-momentum particle

↓

Sextupoles

Chromaticity from sextupoles

- The sextupole field component in the *x*-plane is: $B_y = \frac{S}{2}x^2$
- In an area with non-zero dispersion $x = x_0 + D\frac{\delta P}{P}$
- Then the field is $B_y = \frac{S}{2}x_0^2 + \underbrace{SD\frac{\delta P}{P}x_0}_{\text{quadrupole}} + \underbrace{\frac{S}{2}D^2\frac{\delta P}{P}^2}_{\text{dipole}}$
- Sextupoles introduce an equivalent focusing correction $\delta k = SD\frac{\delta P}{P}$
- The sextupole induced chromaticity is

$$\xi^S_{x,y} = -\frac{1}{4\pi}\oint \mp\beta_{x,y}(s)S(s)D_x(s)ds$$

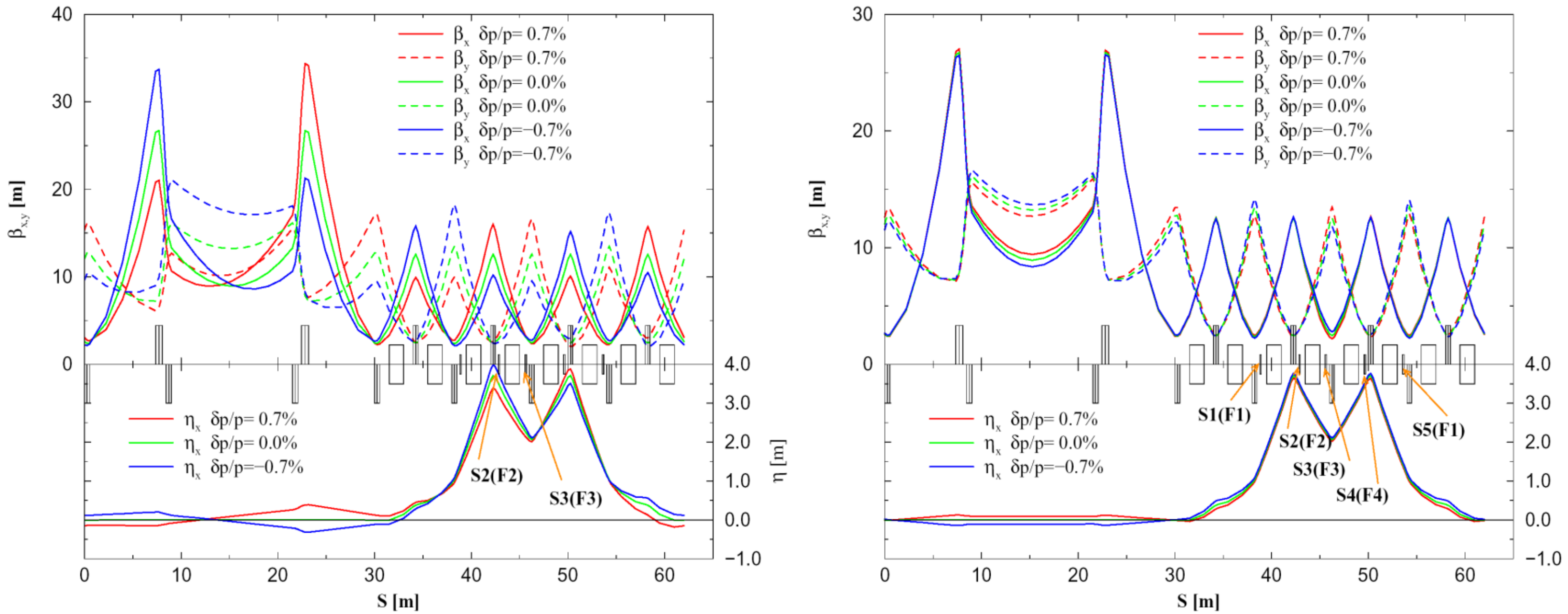
- The total chromaticity is the sum of the natural and sextupole induced chromaticity

$$\xi^{\rm tot}_{x,y} = -\frac{1}{4\pi}\oint \beta_{x,y}(s)\left(k(s) \mp S(s)D_x(s)\right)ds$$

Chromaticity correction

- Introduce sextupoles in high-dispersion areas
- Tune them to achieve desired chromaticity
- Two families are able to control horizontal and vertical chromaticity
- Sextupoles introduce non-linear fields (chaotic motion)
- Sextupoles introduce tune-shift with amplitude
- Example:
  - The SNS ring has natural chromaticity of **–7**
  - Placing two sextupoles of length **0.3m** in locations where β=**12m**, and the dispersion D=**4m**
  - For getting **0** chromaticity, their strength should be $S = \frac{7 \cdot 4\pi}{12 \cdot 4 \cdot 2 \cdot 0.3} \approx 3\mathrm{m}^{-3}$ or a gradient of **17.3 T/m²**

Two vs. four sextupole families

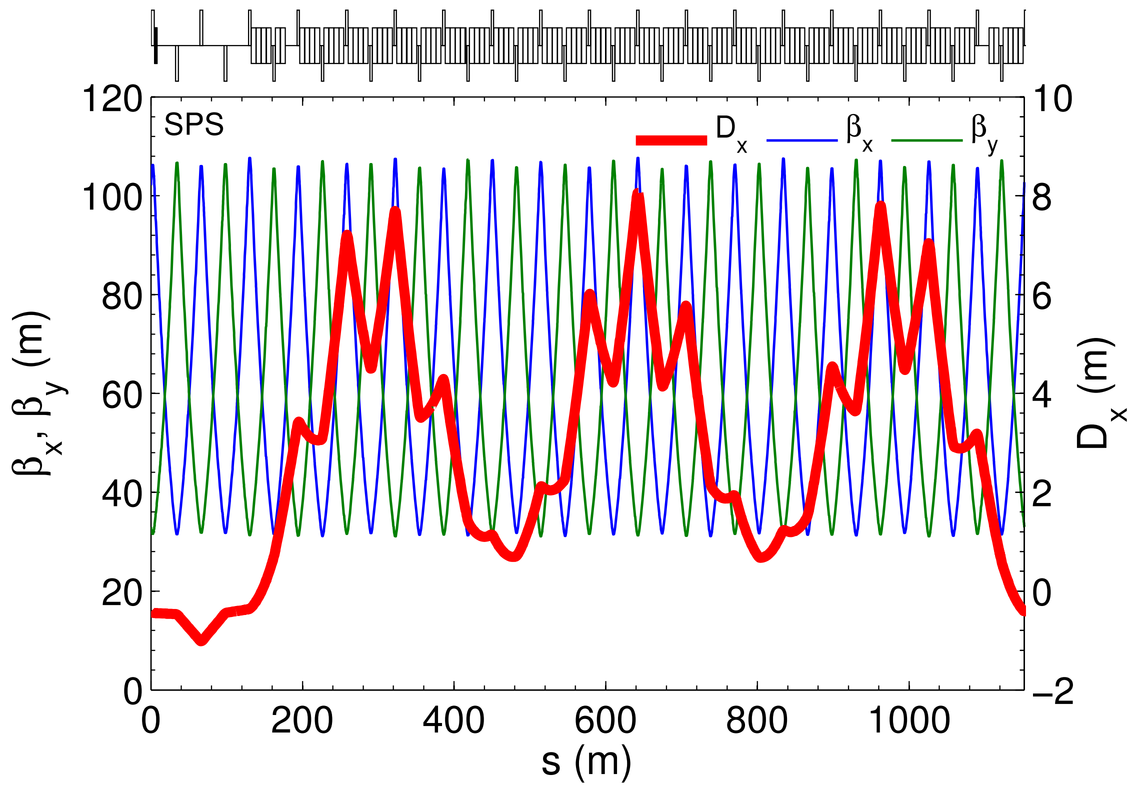


- Two families of sextupoles not enough for correcting off-momentum optics functions’ distortion and second order chromaticity
- Possible solutions:
  - Place sextupoles accordingly to eliminate second order effects (difficult)
  - Use more families (4 in the case of the SNS ring)
- Large optics function distortion for momentum spreads of ±0.7%,when using only two families of sextupoles; Correction of off-momentum optics beating with four families

Problem 4

<u>SPS:</u> Consider a **400GeV** proton synchrotron with **108 3.22m-long** focusing and defocusing quads of **15 T/m**, with a horizontal and vertical **beta** of **108m** and **30m** in the focusing quads, and horizontal and vertical **beta** of **30m** and **108m** for the defocusing ones.

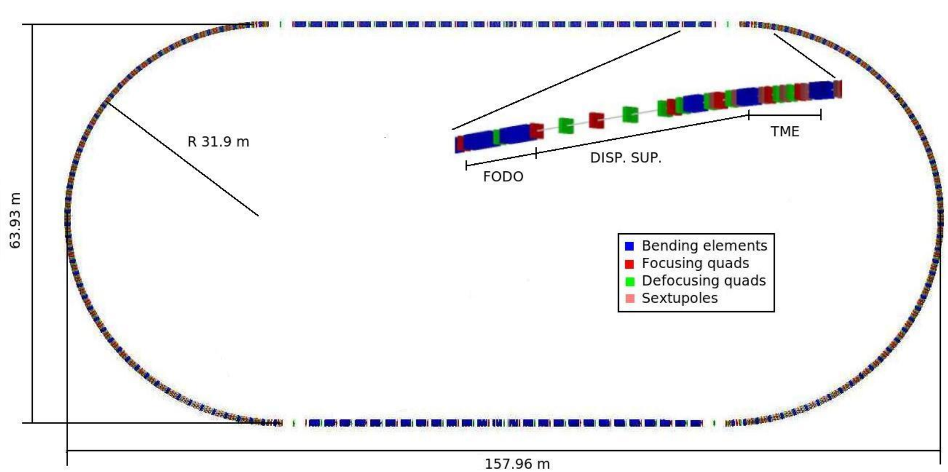
- Find the tune change for systematic gradient errors of **1%** in the focusing and **0.5%** in the defocusing quads.
- What is the chromaticity of the machine?

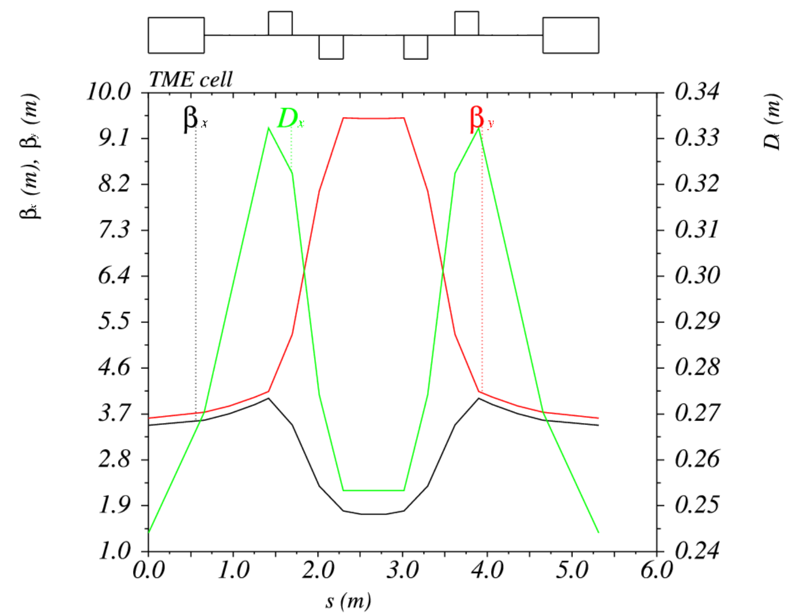


Problem 5

<u>CLIC pre-damping rings:</u> Consider a 2.86 GeV electron storage ring with a racetrack shape of 389 m circumference. Each arc is composed of **17 regular "TME" cells**, each consisting of 2 dipoles, 2 focusing and 2 defocusing quadrupoles. The beta functions are around **β_x=4m (2m)** and **β_y=4.2m (9m)** in the **focusing (defocusing) quadrupoles** and the normalized quadrupole gradients are **2.49/m^2 (2.07/m^2)**. The **quadrupoles** have a **length** of **0.28m**. The **natural chromaticity** of the machine is about **-19** and **-23** in the horizontal and vertical plane, respectively.

- How big is the chromaticity contribution from the arcs?
- Where would you install sextupole magnets for correcting chromaticity?
- Can you give an estimation for the required sextupole gradient assuming the sextupoles have the same length as the quadrupoles?





Problem 6

Derive an expression for the resulting magnetic field when a normal sextupole with field $\mathbf{B = S/2\ x^2}$ is displaced by $\boldsymbol{\delta x}$ from its center position. At what type of fields correspond the resulting components? Do the same for an octupole with field $\mathbf{B = O/3\ x^3}$. What is the leading order multi-pole field error when displacing a general **2n-pole** magnet?