



2017 Joint Universities Accelerator School

Superconducting Magnets

Section I

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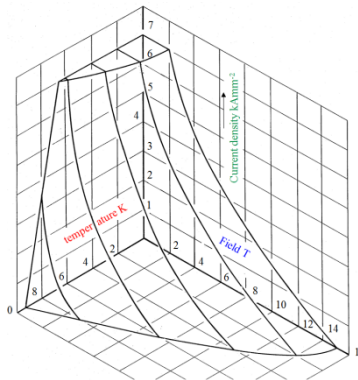
European Organization for Nuclear Research (CERN)

Introduction

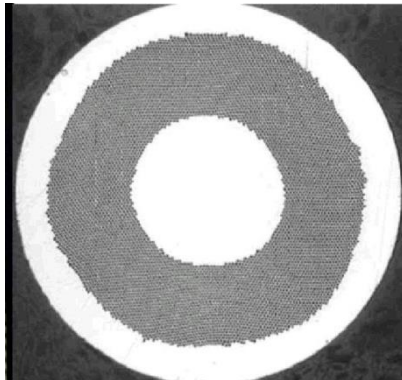
Goal of the course

- Overview of **superconducting magnets** for particle accelerators (dipoles and quadrupoles)
 - Description of the components and their function
- From the superconducting material to the full magnet

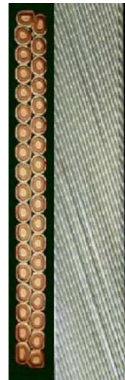
Superconducting material



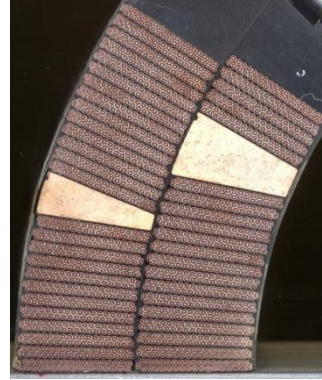
Superconducting strand



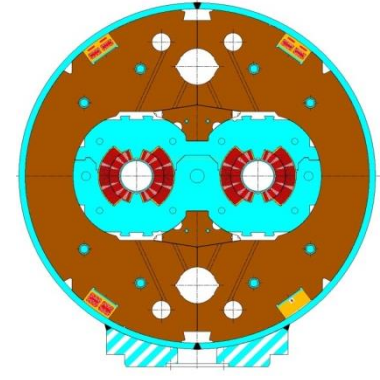
Superconducting cable



Superconducting coil



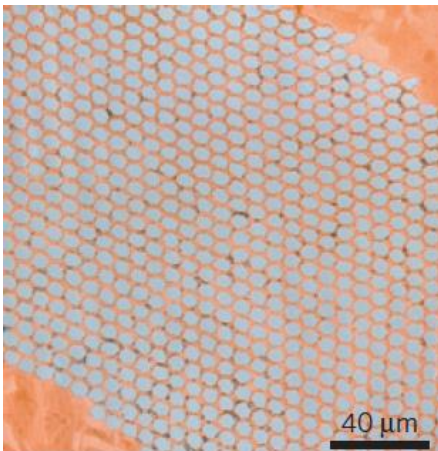
Superconducting magnet



Introduction

Superconducting magnet technology

- Multidisciplinary field: mixture of
 - Chemistry and material science: **superconducting materials**
 - Quantum physics: the key mechanisms of **superconductivity**
 - Classical electrodynamics: **magnet design**
 - Mechanical engineering: **support structures**
 - Electrical engineering: powering of the magnets
 - Cryogenics: keep them **cool** ...
- Very different order of magnitudes





Outline

- **Section I**
 - Particle accelerators and magnets
 - Superconductivity and practical superconductors
 - Magnetic design
- **Section II**
 - Coil fabrication
 - Forces, stress, pre-stress
 - Support structures
- **Section III**
 - Quench, training, protection



References

- Particle accelerators and magnets
- Superconductivity and practical superconductors
 - K.-H. Mess, P. Schmuser, S. Wolff, "*Superconducting accelerator magnets*", Singapore: World Scientific, 1996.
 - Martin N. Wilson, "*Superconducting Magnets*", 1983.
 - Fred M. Asner, "*High Field Superconducting Magnets*", 1999.
 - P. Ferracin, E. Todesco, S. Prestemon, "*Superconducting accelerator magnets*", US Particle Accelerator School, www.uspas.fnal.gov.
 - Units 2 by E. Todesco
 - A. Devred, "*Practical low-temperature superconductors for electromagnets*", CERN-2004-006, 2006.
 - Presentations from Luca Bottura and Martin Wilson

● Magnetic design

- K.-H. Mess, P. Schmuser, S. Wolff, "*Superconducting accelerator magnets*", Singapore: World Scientific, 1996.
- Martin N. Wilson, "*Superconducting Magnets*", 1983.
- Fred M. Asner, "*High Field Superconducting Magnets*", 1999.
- S. Russenschuck, "*Field computation for accelerator magnets*", J. Wiley & Sons (2010).

- P. Ferracin, E. Todesco, S. Prestemon, "*Superconducting accelerator magnets*", US Particle Accelerator School, www.uspas.fnal.gov.
 - Units 5, 8, 9 by E. Todesco
- A. Jain, "*Basic theory of magnets*", CERN 98-05 (1998) 1-26

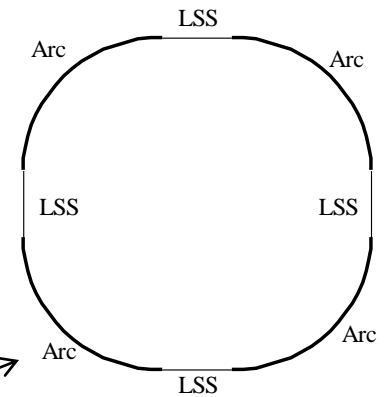
- L. Rossi, E. Todesco, "*Electromagnetic design of superconducting quadrupoles*", Phys. Rev. ST Accel. Beams 10 (2007) 112401.
- L. Rossi and Ezio Todesco, "*Electromagnetic design of superconducting dipoles based on sector coils*", Phys. Rev. ST Accel. Beams 9 (2006) 102401.

- Principle of synchrotrons
 - Driving particles in the same accelerating structure several times
- **Electro-magnetic field** accelerates particles

$$\vec{F} = e\vec{E} \longrightarrow$$

- **Magnetic field steers** the particles in a ~circular orbit

$$\vec{F} = e\vec{v} \times \vec{B} \nearrow$$



- Particle accelerated \rightarrow energy increased \rightarrow magnetic field increased (“**synchro**”) to keep the particles on the same orbit of curvature ρ

$$p = eB\rho$$

Constant

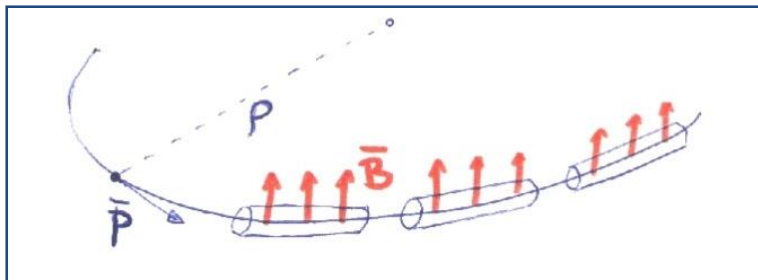
Particle accelerators and magnets

Dipoles

- Main field components is B_y
 - Perpendicular to the axis of the magnet z
- Electro-magnets: field produced by a current (or current density)

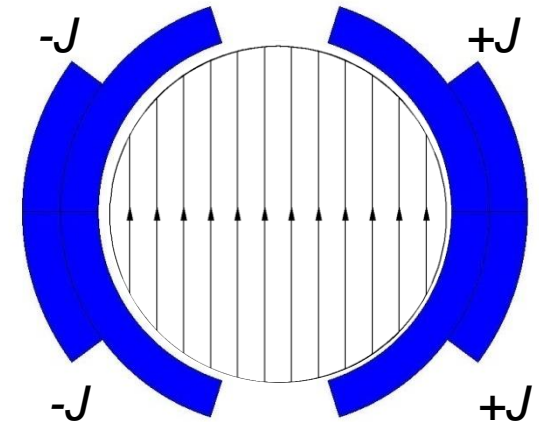
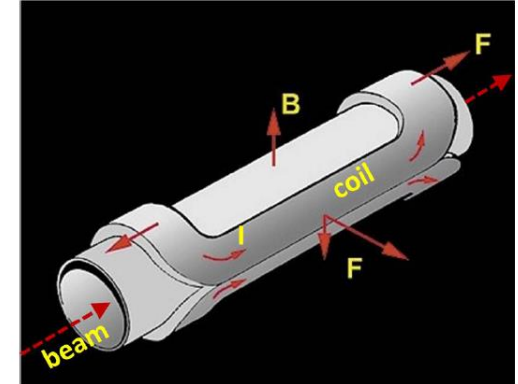
$$B_y = -\frac{\mu_0 J_0}{2} (r_{out} - r_{in})$$

- **Magnetic field steers (bends) the particles in a ~circular orbit**



by E. Todesco

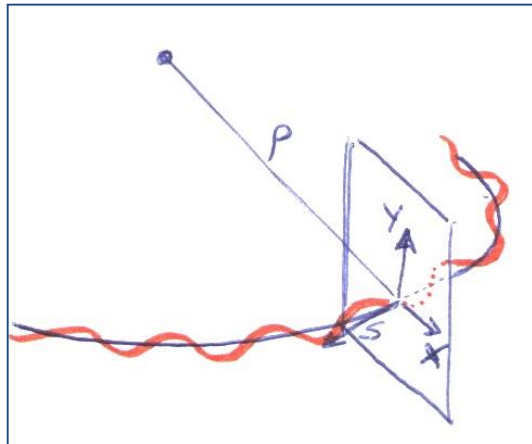
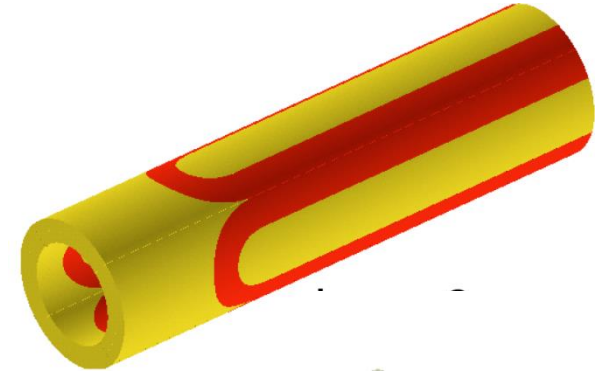
$$p = eB\rho$$



Particle accelerators and magnets

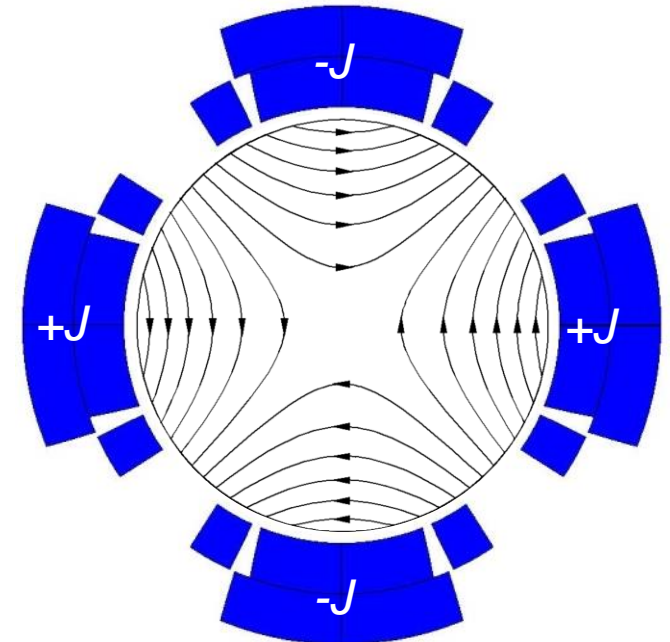
Quadrupoles

- The force necessary to stabilize linear motion is provided by the quadrupoles
 - They provide a field
 - equal to zero in the center
 - increasing linearly with the radius
- They act as a spring: **focus the beam**
- Prevent protons from **falling** to the bottom of the aperture due to the **gravitational force**
 - it would happen in less than 60 ms



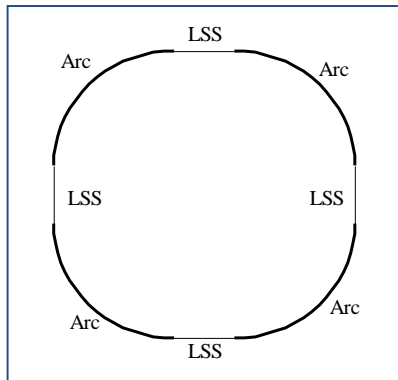
by E. Todesco

$$G = \frac{B_y}{r} = -\frac{\mu_0 J_0}{2} \ln \frac{r_{out}}{r_{in}}$$

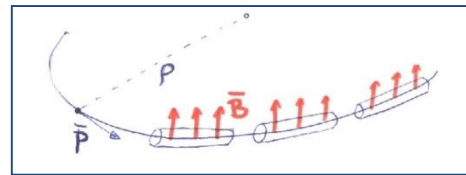


Particle accelerators and magnets

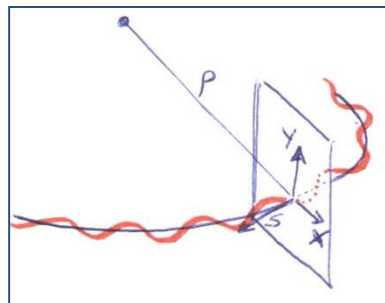
- Dipoles: the larger **B**, the larger the **energy**
- Quadrupoles: the larger **B**, the larger the **focusing** strength
- For an electro-magnet, the larger **B**, the larger must be **J**



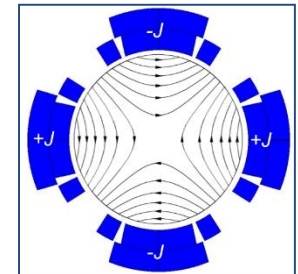
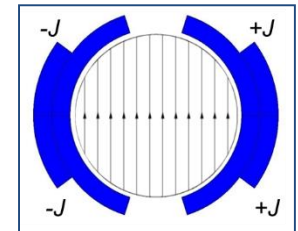
$$p = eB\rho$$



$$B_y = -\frac{\mu_0 J_0}{2} (r_{out} - r_{in})$$



$$G = \frac{B_y}{r} = -\frac{\mu_0 J_0}{2} \ln \frac{r_{out}}{r_{in}}$$

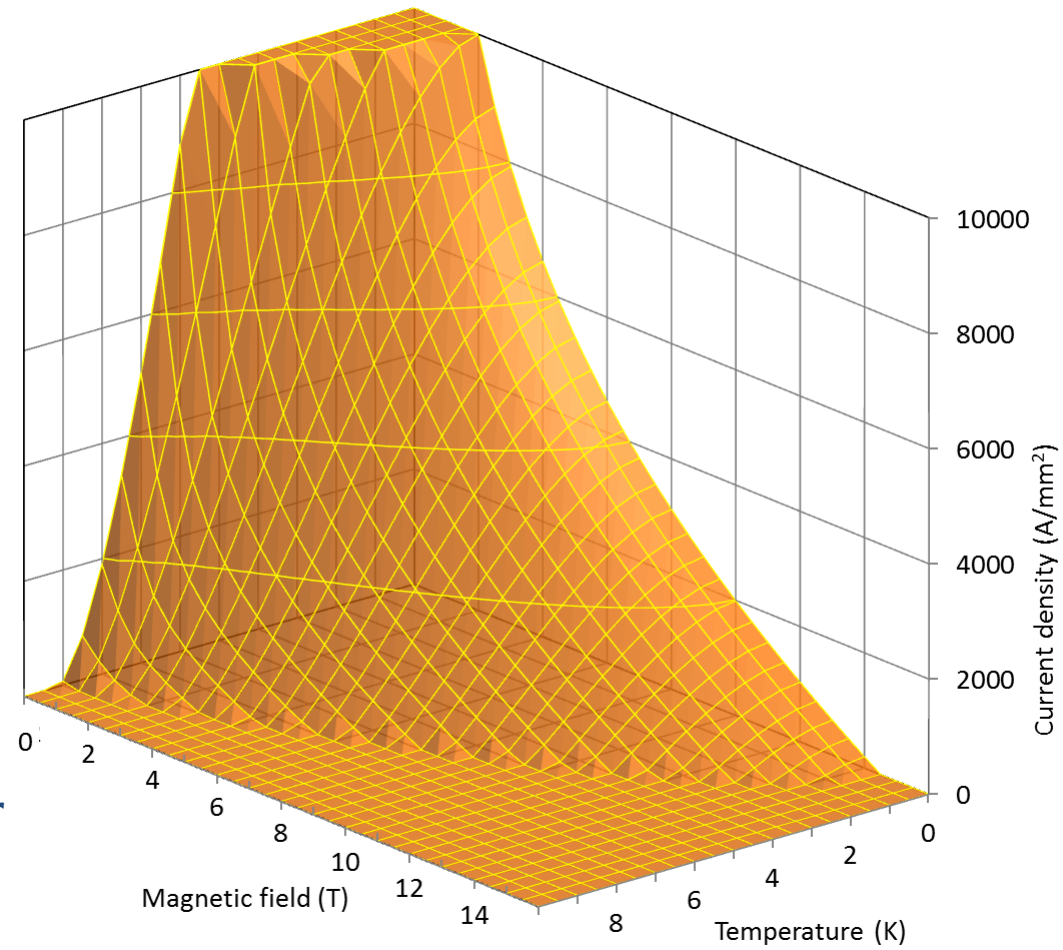


- In **normal** conducting magnets, $J \sim 5 \text{ A/mm}^2$
- In **superconducting** magnets, $J_e \sim 600-700 \text{ A/mm}^2$

Superconductivity

Critical surface

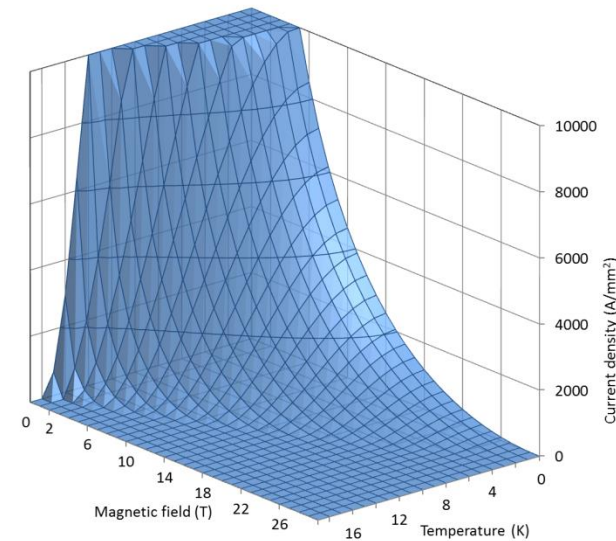
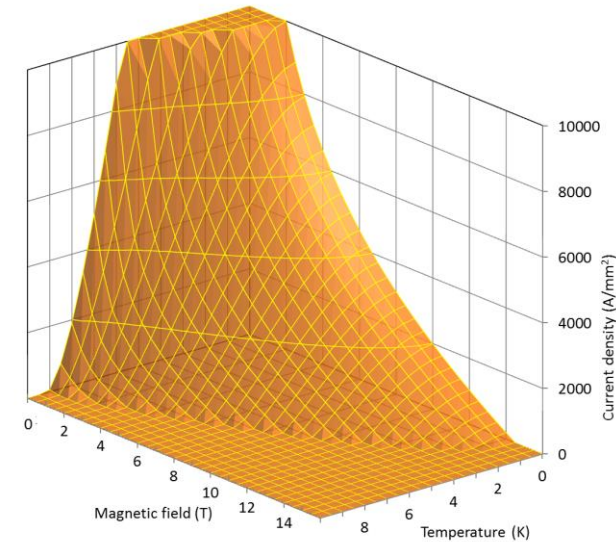
- A type II material is supercond. below the **critical surface** defined by
 - Critical temperature T_c
 - Property of the material
 - Upper critical field B_{c2}
 - Property of the material
 - Critical current density J_c
 - Hard work by the producer



Superconductivity

Nb-Ti (1961) and Nb₃Sn (1954)

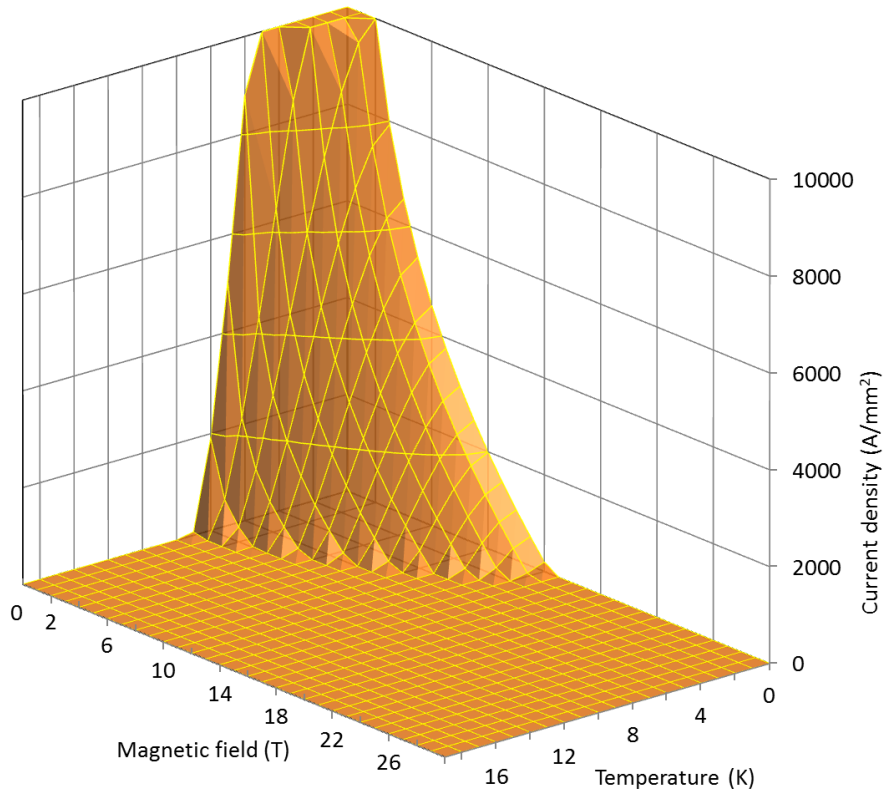
- Nb and Ti → ductile alloy
 - Extrusion + drawing
 - T_c is ~**9.2 K** at 0 T
 - B_{C2} is ~**14.5 T** at 0 K
 - Firstly in **Tevatron** (80s), then all the other
 - ~50-200 US\$ per kg of wire (1 euro per m)
- Nb and Sn → intermetallic compound
 - Brittle, strain sensitive, formed at ~650-700°C
 - T_c is ~**18 K** at 0 T
 - B_{C2} is ~**28 T** at 0 K
 - Used in **NMR, ITER**
 - ~700-1500 US\$ per kg of wire (5 euro per m)



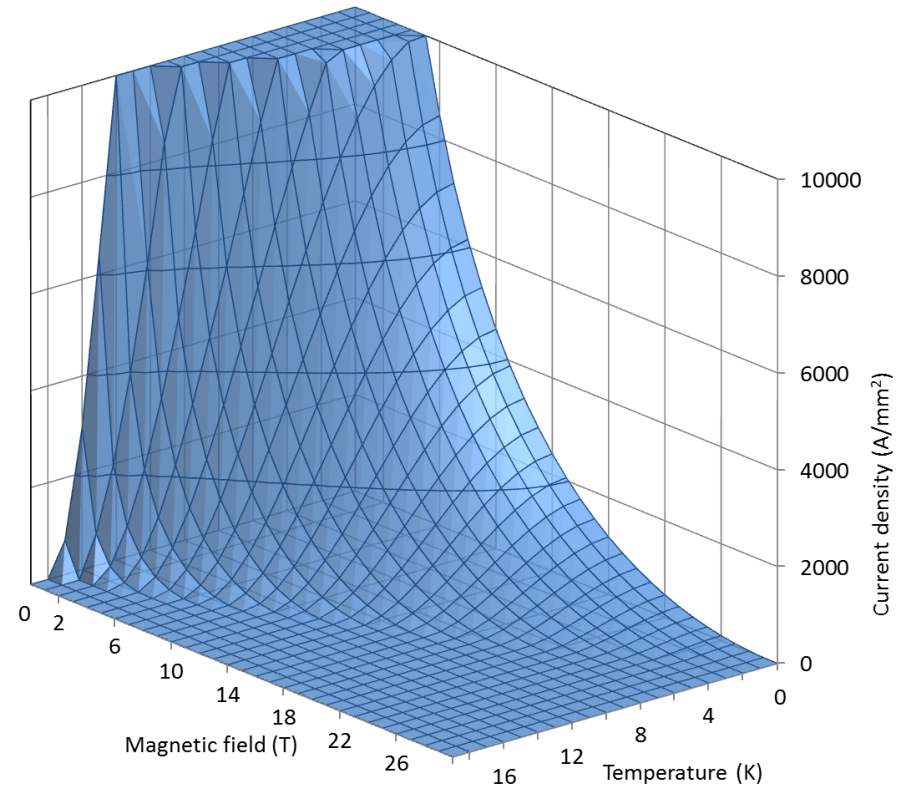
Superconductivity

Nb-Ti vs. Nb₃Sn

Nb-Ti



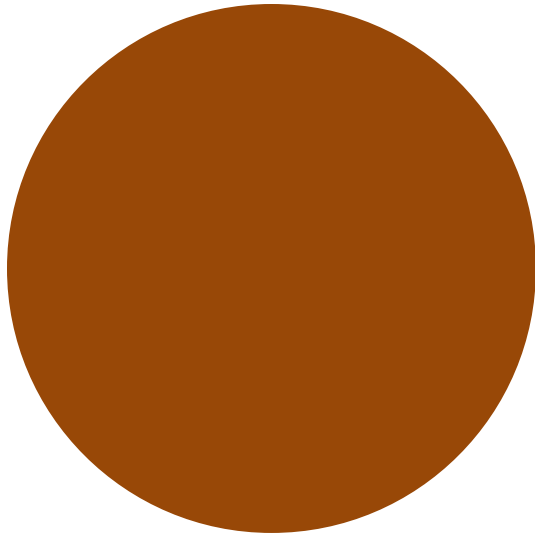
Nb₃Sn



Superconductivity from Cu to Nb₃Sn

- Typical operational conditions (0.85 mm diameter strand)

Cu

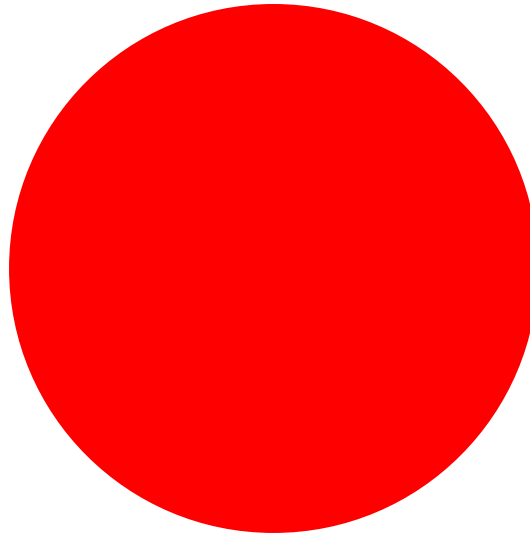


$$J_e \sim 5 \text{ A/mm}^2$$

$$I \sim 3 \text{ A}$$

$$B = 2 \text{ T}$$

Nb-Ti

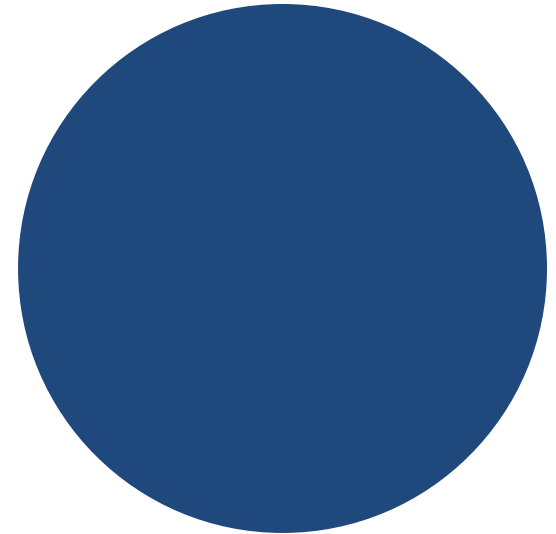


$$J_e \sim 600\text{-}700 \text{ A/mm}^2$$

$$I \sim 300\text{-}400 \text{ A}$$

$$B = 8\text{-}9 \text{ T}$$

Nb₃Sn



$$J_e \sim 600\text{-}700 \text{ A/mm}^2$$

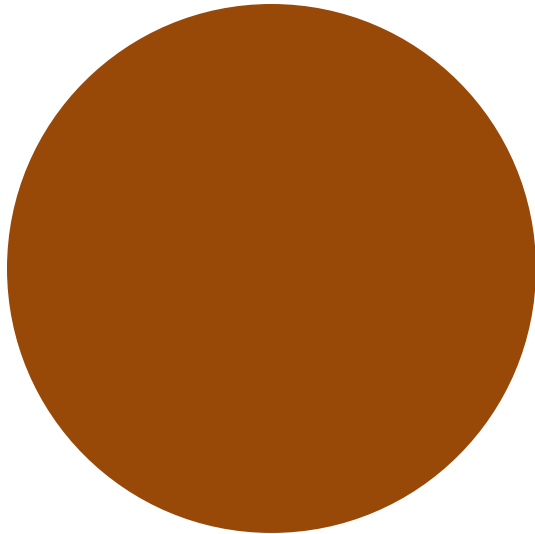
$$I \sim 300\text{-}400 \text{ A}$$

$$B = 12\text{-}13 \text{ T}$$

Practical superconductors

- Typical operational conditions (0.85 mm diameter strand)

Cu

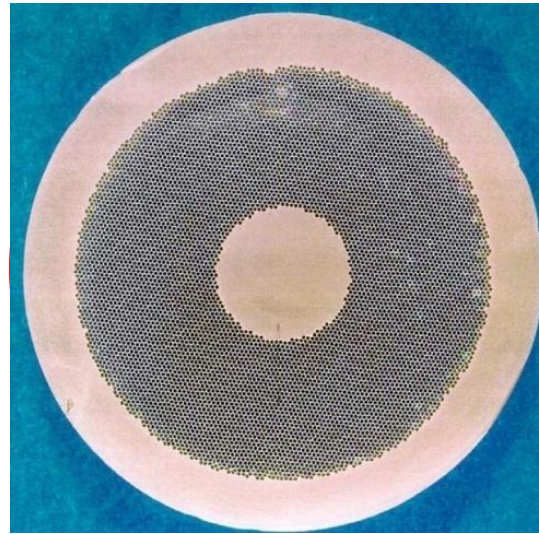


$$J_e \sim 5 \text{ A/mm}^2$$

$$I \sim 3 \text{ A}$$

$$B = 2 \text{ T}$$

Nb-Ti

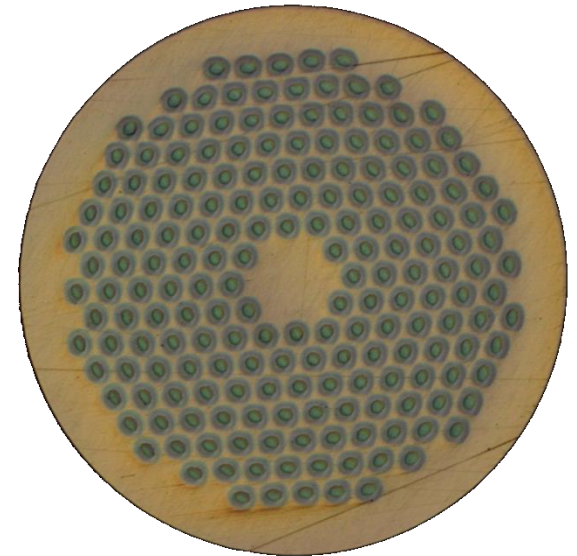


$$J_e \sim 600\text{-}700 \text{ A/mm}^2$$

$$I \sim 300\text{-}400 \text{ A}$$

$$B = 8\text{-}9 \text{ T}$$

Nb₃Sn



$$J_e \sim 600\text{-}700 \text{ A/mm}^2$$

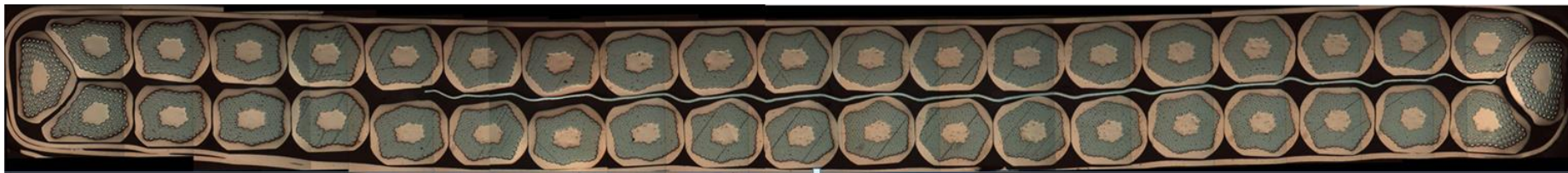
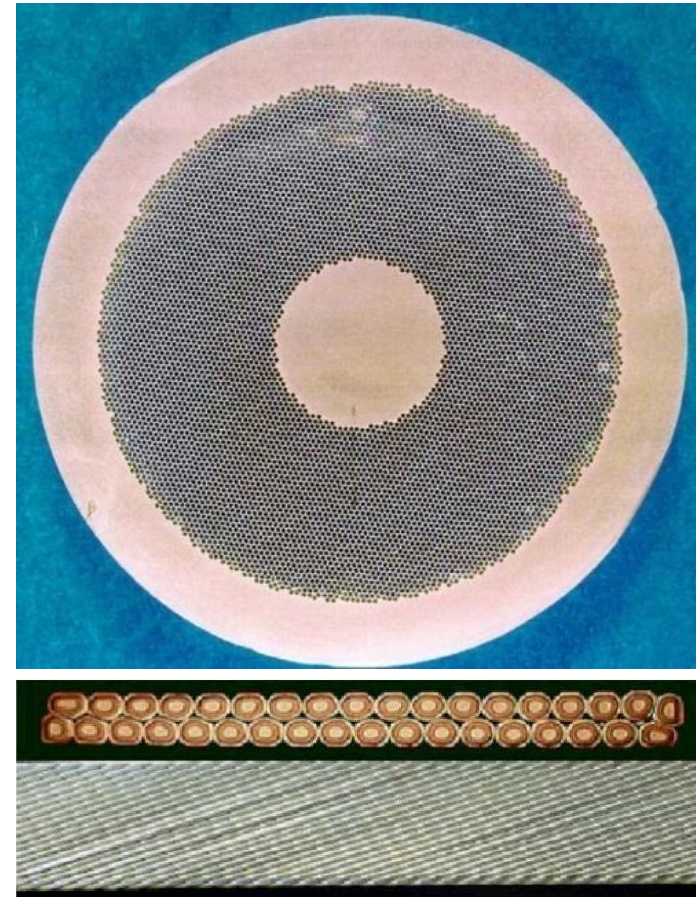
$$I \sim 300\text{-}400 \text{ A}$$

$$B = 12\text{-}13 \text{ T}$$

Practical superconductors

Introduction

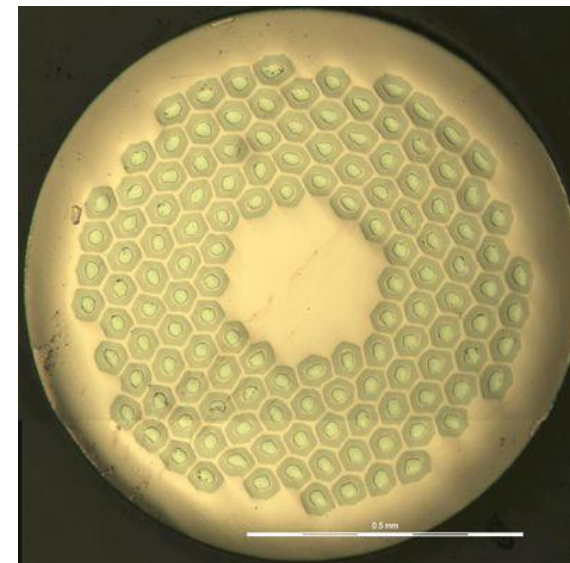
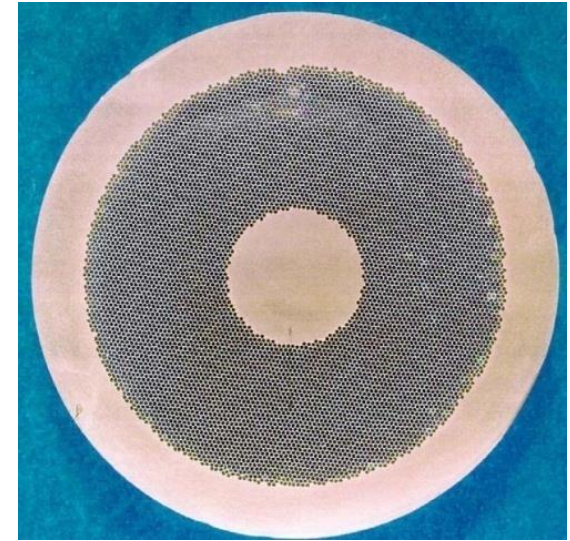
- Superconducting materials are produced in small filaments and surrounded by a stabilizer (typically copper) to form a *multi-filament wire* or *strand*.
- A superconducting cable is composed by several wires: *multi-strand cable*.



Practical superconductors

Multi-filament wires motivations

- The superconducting materials used in accelerator magnets are
 - subdivided in filaments of small diameters
 - to reduce magnetic instabilities called **flux jumps**
 - to minimize field distortions due to superconductor **magnetization**
 - twisted together
 - to reduce interfilament coupling and **AC losses**
 - embedded in a copper matrix
 - to **protect** the superconductor **after a quench**
 - to reduce magnetic instabilities called flux jumps



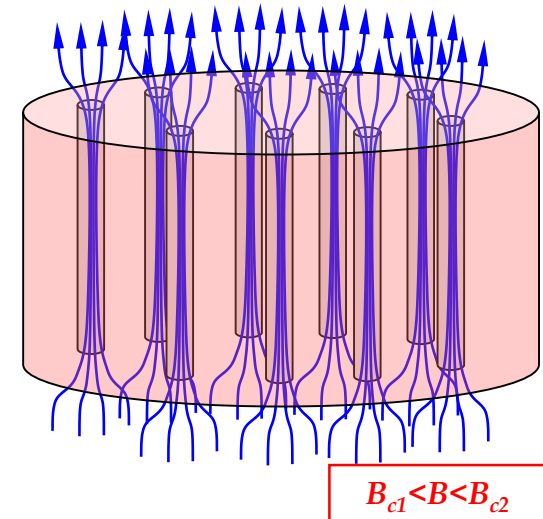
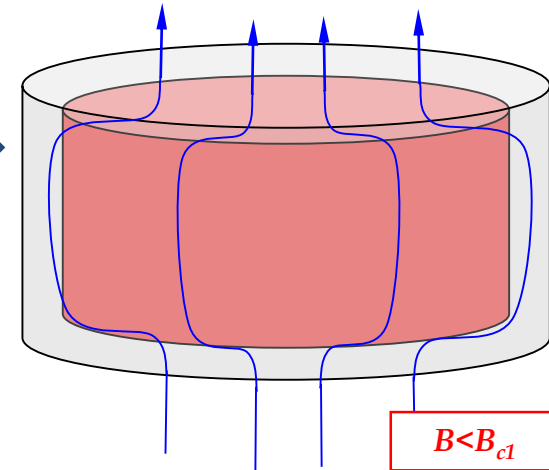
Practical superconductors

Multi-filament wires motivations

- Fluxoid distribution depends on the applied B and on J_c .
- Thermal disturbance \rightarrow the local change in $J_c \rightarrow$ motion or “**flux jump**” \rightarrow power dissipation
- Stability criteria for a slab (adiabatic condition)

$$a \leq \sqrt{\frac{3\gamma C(\theta_c - \theta_0)}{\mu_0 j_c^2}}$$

- a is the half-thickness of the slab
 - j_c is the critical current density [A m^{-2}]
 - γ is the density [kg m^{-3}]
 - C is the specific heat [J kg^{-1}]
 - θ_c is the critical temperature.
- Nb-Ti filament diameters usually $< 50 \mu\text{m}$



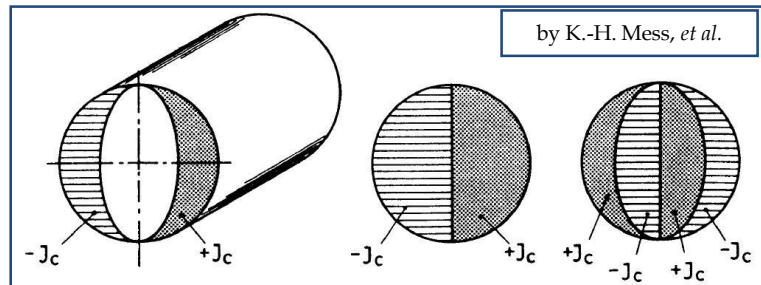
by L. Bottura

Practical superconductors

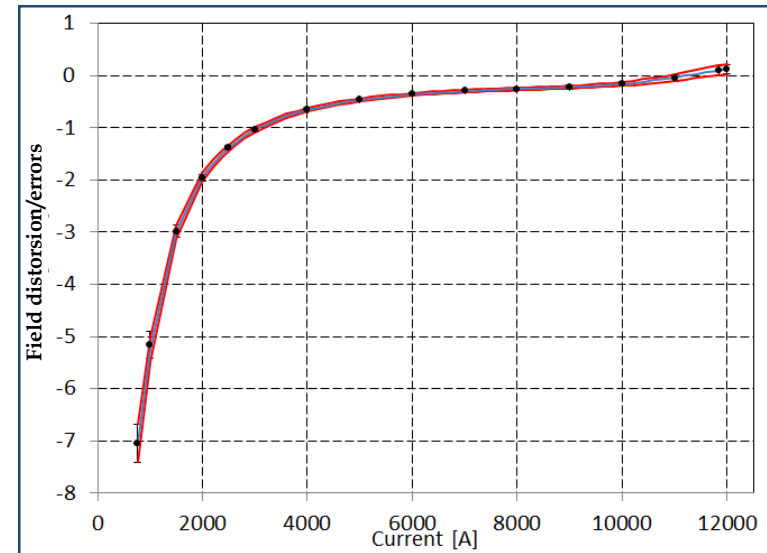
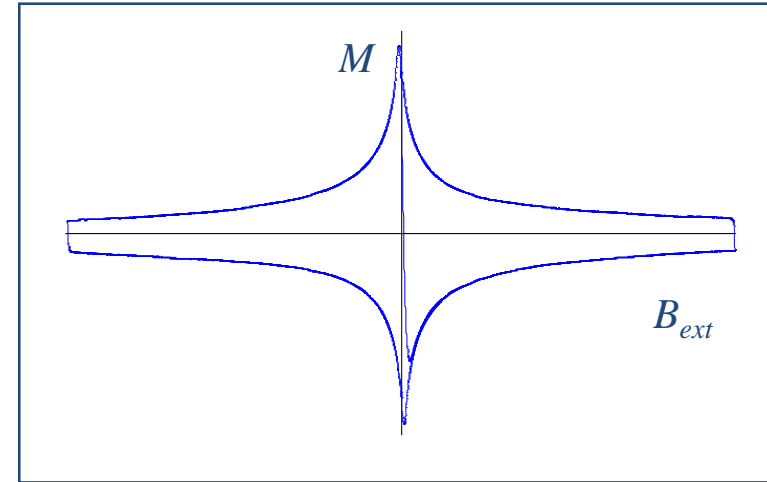
Multi-filament wires motivations

Superconductor magnetization

- When a filament is in a varying B_{ext} , its inner part is shielded by currents distribution in the filament periphery
 - They **do not decay** when B_{ex} is held constant \rightarrow **persistent currents**



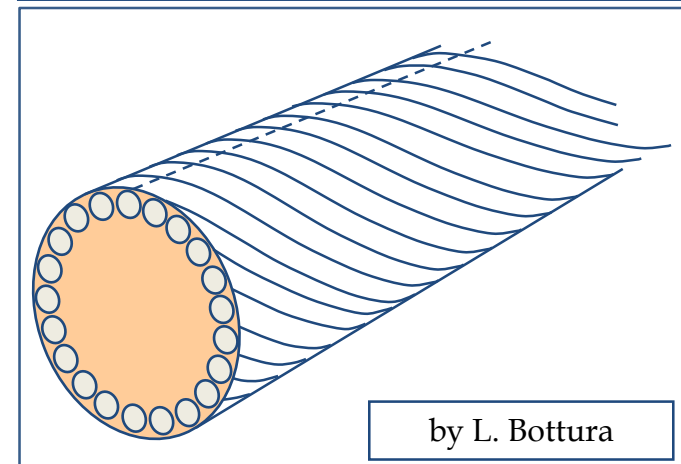
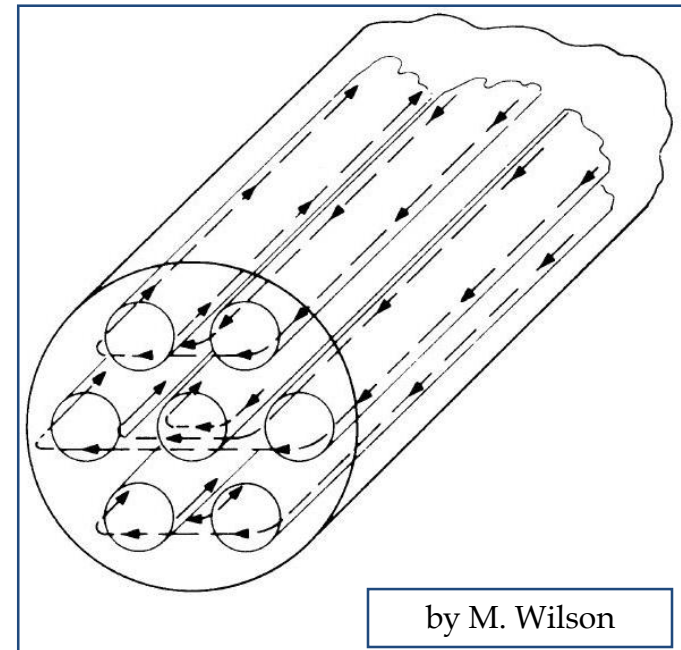
- These currents produce **field errors** and **ac losses** proportional to $J_c r_f$
 - LHC filament diameter 6-7 μm .
 - HERA filament diameter 14 μm .



Practical superconductors

Multi-filament wires motivations

- **Inter-filament coupling**
 - When a multi-filamentary wire is subjected to a time varying magnetic field, **current loops** are generated between filaments.
 - If filaments are straight, large loops with large currents → **ac losses**
 - If the strands are magnetically coupled the effective filament size is larger → **flux jumps**
- To reduce these effects, filaments are **twisted**
 - twist pitch of the order of 20-30 times of the wire diameter.

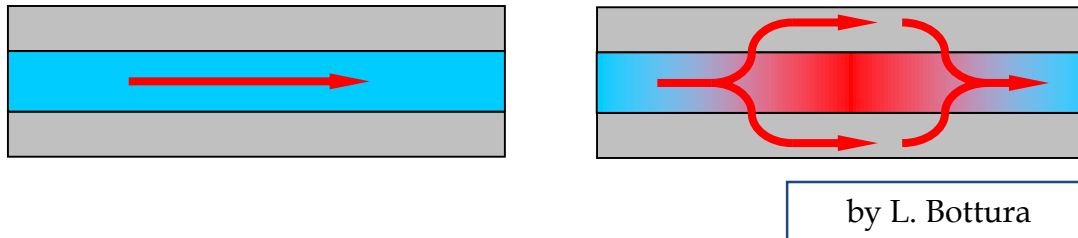


Practical superconductors

Multi-filament wires motivations

- **Quench protection**

- Superconductors have a very high normal state resistivity
 - If quenched, could reach very high temperatures in few ms.
- If embedded in a **copper matrix**, when a quench occurs, current redistributes in the low-resistivity matrix → **lower peak temperature**

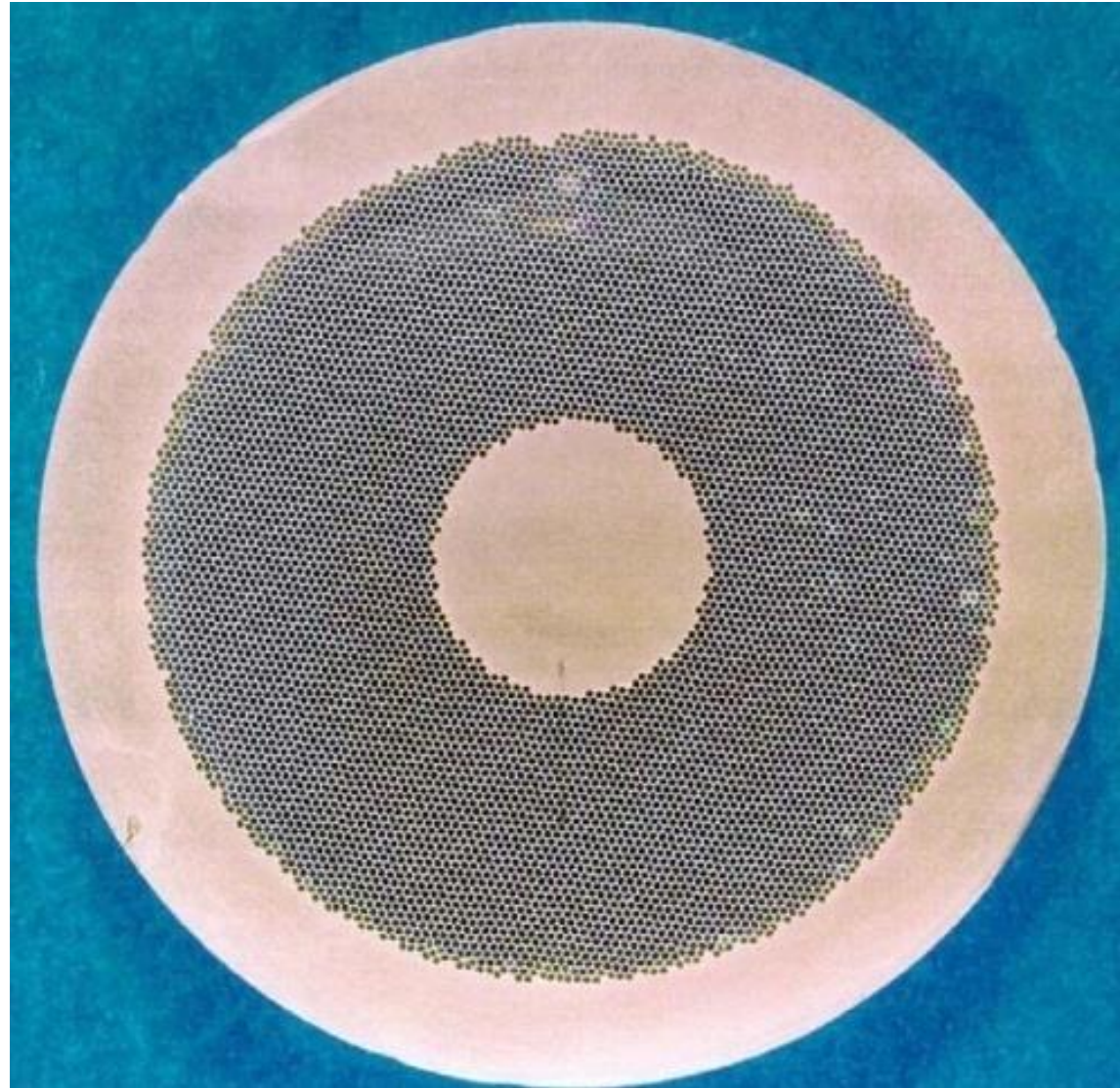


- The copper matrix provides **time to act** on the power circuit
- In the case of a small volume of superconductor heated beyond the critical temperature the current can flow in the copper for a short moment, allowing the filament to **cool-down and recover** supercond.
- The matrix also helps stabilizing the conductor against **flux jumps**

Practical superconductors

Multi-filament wires motivations

- **Flux jumps**
- **Persistent currents**
- **AC losses**
- **Quench protection**



Practical superconductors

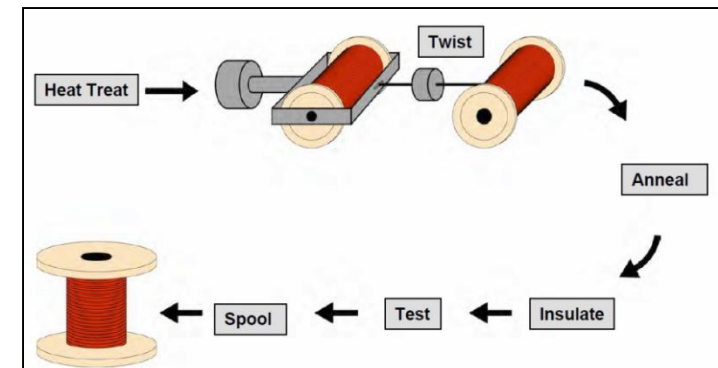
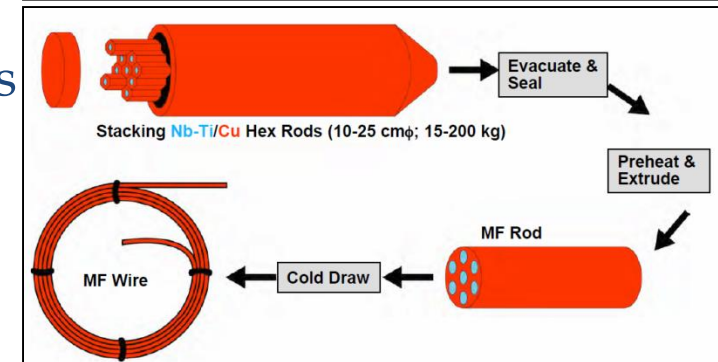
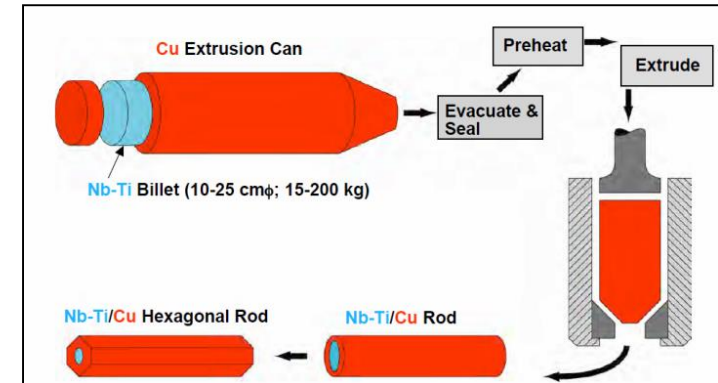
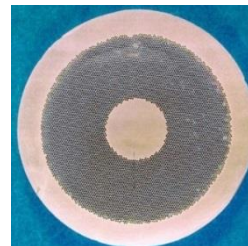
Fabrication of Nb-Ti multifilament wires

Nb-Ti ingots

- 200 mm ϕ , 750 mm long

Monofilament rods are stacked to form a multifilament billet

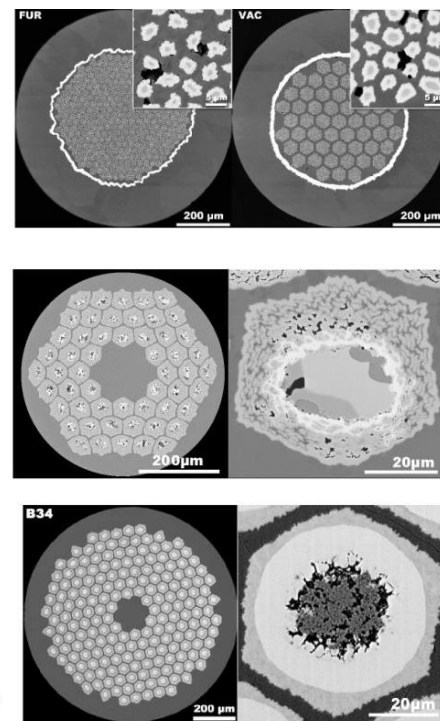
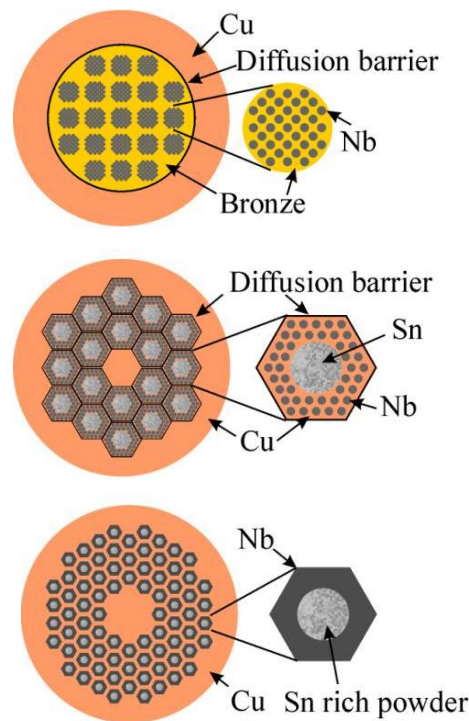
- then extruded and drawn down
- can be re-stacked: double-stacking process



Multifilament wires

Fabrication of Nb₃Sn multifilament wires

- Since Nb₃Sn is brittle
 - it cannot be extruded and drawn like Nb-Ti.
- Process in several steps
 - Assembly multifilament billets from with **Nb and Sn separated**
 - Fabrication of the wire through extrusion-drawing
 - Fabrication of the cable
 - Fabrication of the coil
 - **“Reaction”**
 - Sn and Nb are heated to 600-700 C
 - Sn diffuses in Nb and reacts to form Nb₃Sn

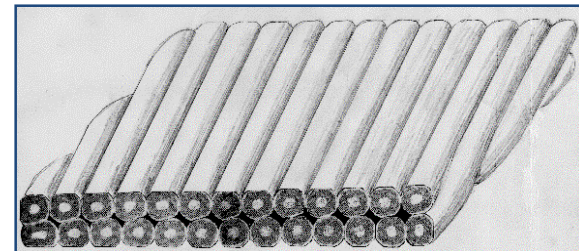
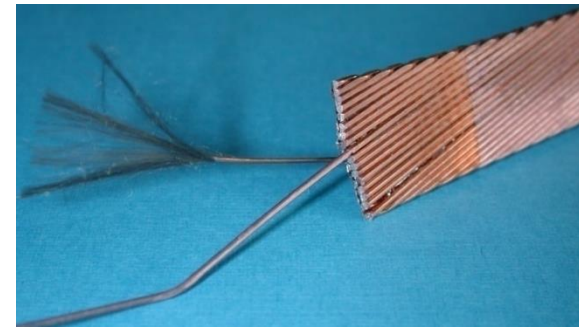
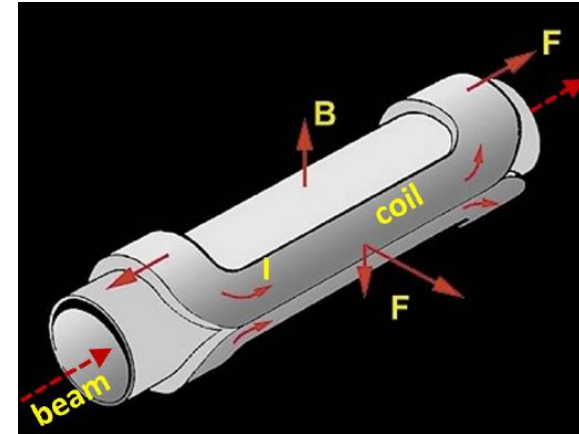


by A. Godeke

Practical superconductors

Multi-strand cables motivations

- Most of the superconducting coils for particle accelerators wound from a multi-strand cable (**Rutherford cable**)
 - Reduction of strand **piece length**
 - reduction of **number of turns**
 - easy winding
 - smaller coil inductance
 - less V for power supply during ramp-up;
 - after a quench, faster discharge and V
 - **current redistribution** in case of a defect or a quench in one strand
- The strands are **twisted** to
 - Reduce **inter-strand coupling currents**
 - Losses and field distortions
 - Provide more **mechanical stability**

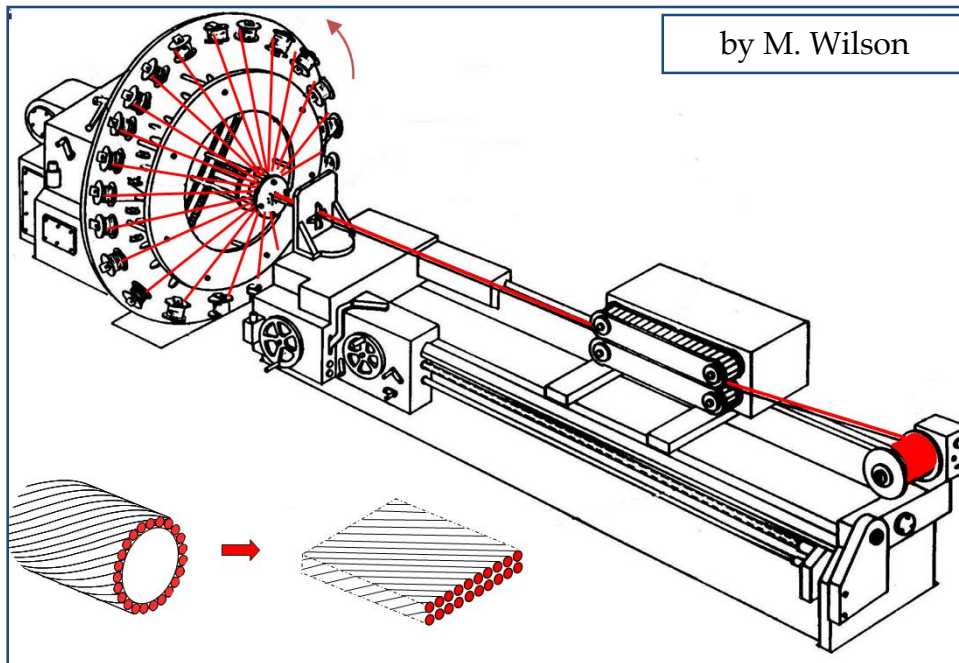
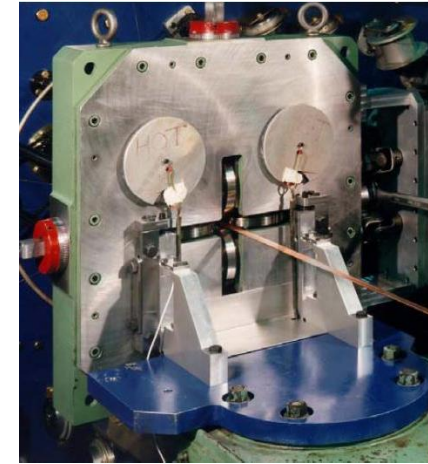


by M. Wilson

Practical superconductors

Multi-strand cables motivations

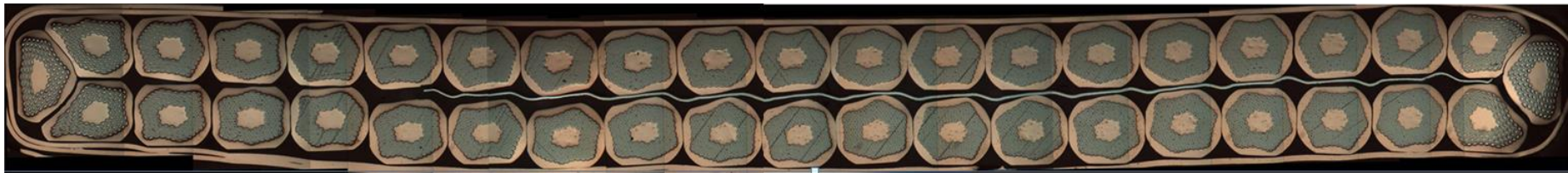
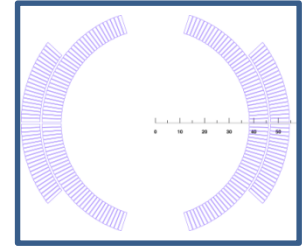
- Rutherford cables fabricated by **cabling machine**
 - Strands wound on spools mounted on a rotating drum
 - Strands twisted around a conical mandrel into rolls (Turk's head)
 - The rolls compact the cable and provide the final shape



Practical superconductors

Multi-strand cables

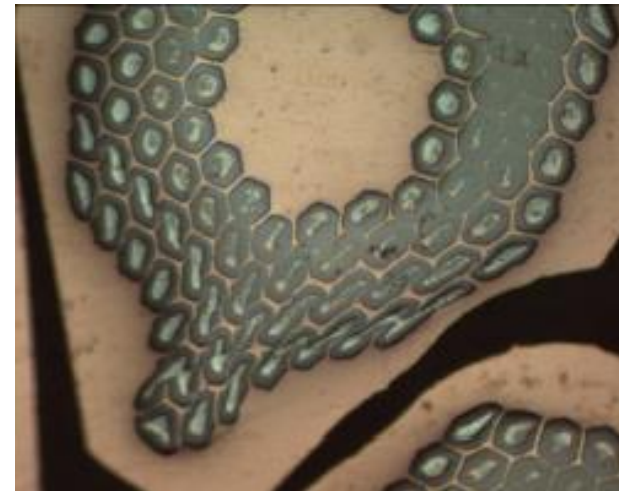
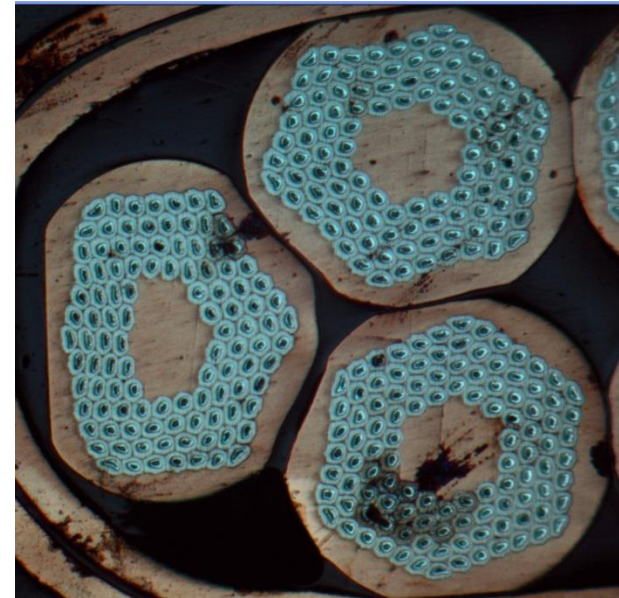
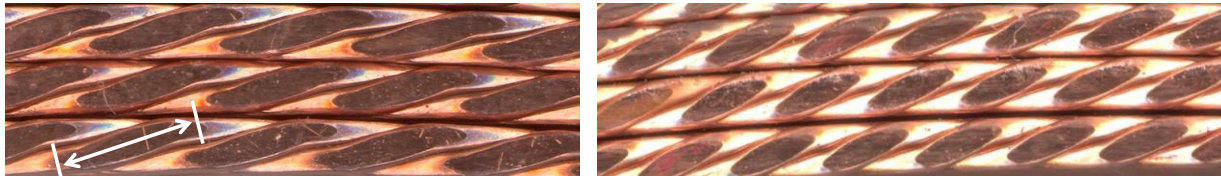
- A Rutherford cable can be **rectangular** or **trapezoidal**
 - To stacking cables in an arc-shaped coil around the beam pipe
- **Cable compaction**
 - Ratio of the sum of the cross-sectional area of the strands (direction parallel to the cable axis) to the cross-sectional area of the cable
 - 88% (Tevatron) to 92.3% (HERA).
 - Chosen to provide good mechanical stability + high current capability + enough space for helium cooling or epoxy impregnation.
- **Cables degradation**
 - Critical current density of a virgin wire before cabling is higher than the one of a wire after cabling



Practical superconductors

Multi-strand cables

- **Edge deformation** may cause
 - reduction of the filament cross-sectional area (Nb-Ti)
 - breakage of reaction barrier with incomplete tin reaction (Nb_3Sn)
- In order to **avoid degradation**
 - strand cross-section investigated
 - Edge facets are measured
 - General rule: no overlapping of facets

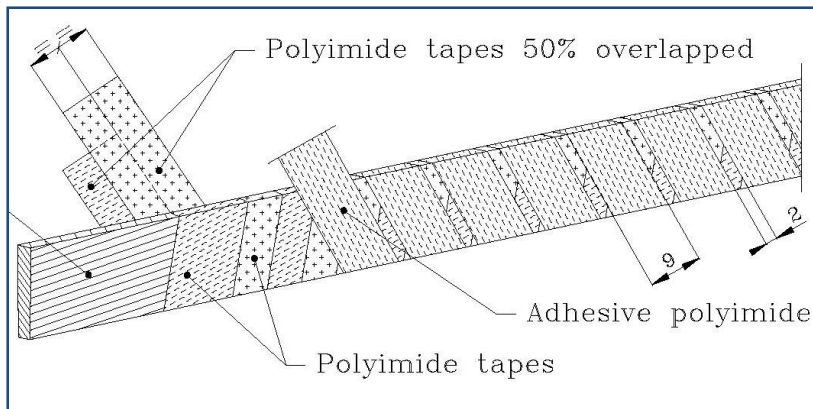


- **Keystone angle** is usually of $\sim 1^\circ$ to 2°

Practical superconductors

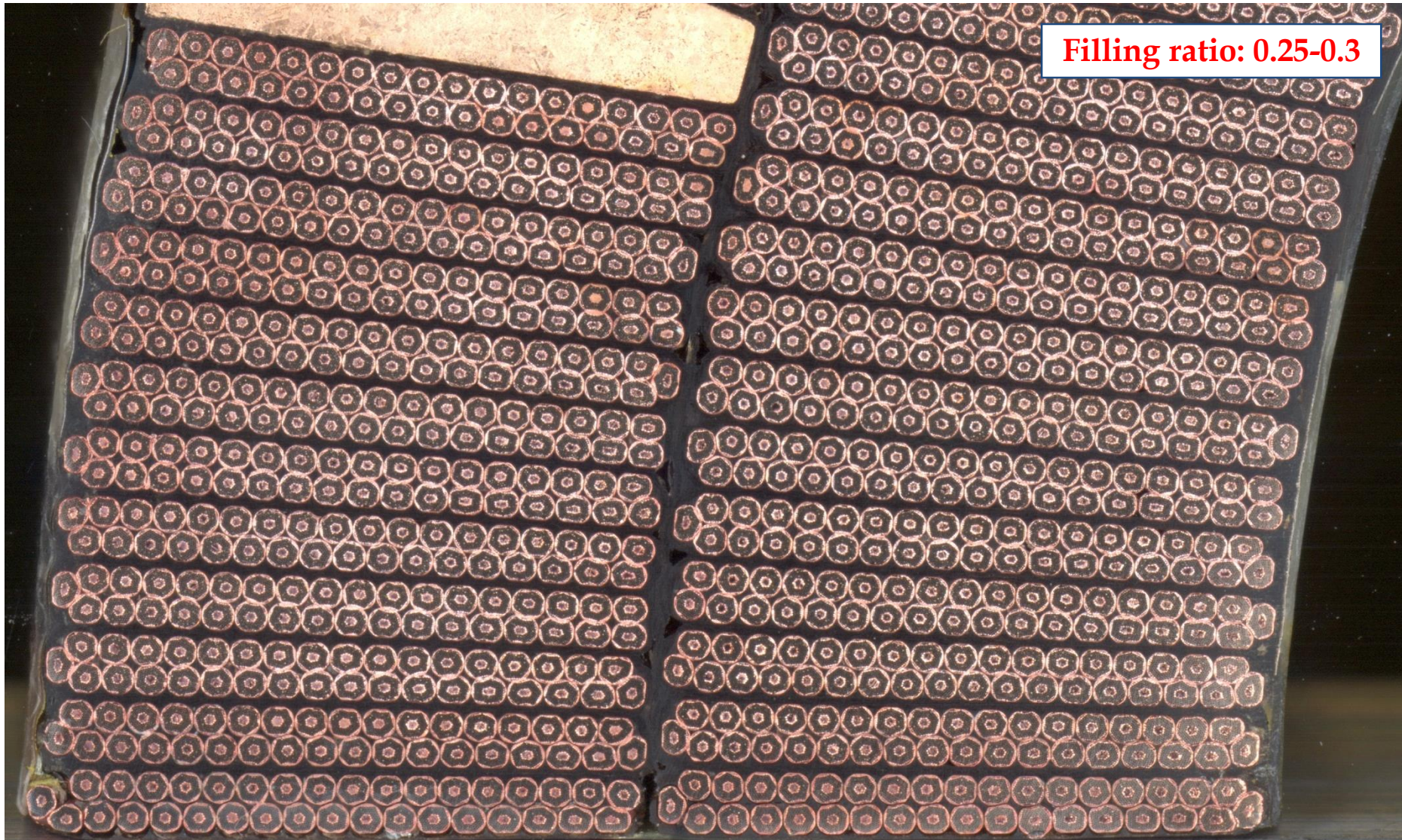
Cable insulation

- The cable insulation must feature
 - Good **electrical properties** to withstand turn-to-turn V after a quench
 - Good **mechanical properties** to withstand high pressure conditions
 - **Porosity** to allow penetration of helium (or epoxy)
 - **Radiation hardness**
- In Nb-Ti magnets overlapped layers of **polyimide**
- In Nb₃Sn magnets, **fiber-glass** braided or as tape/sleeve.
- Typically the insulation thickness: 100 and 200 μm .



Practical superconductors

Superconducting cables



Filling ratio: 0.25-0.3



Outline

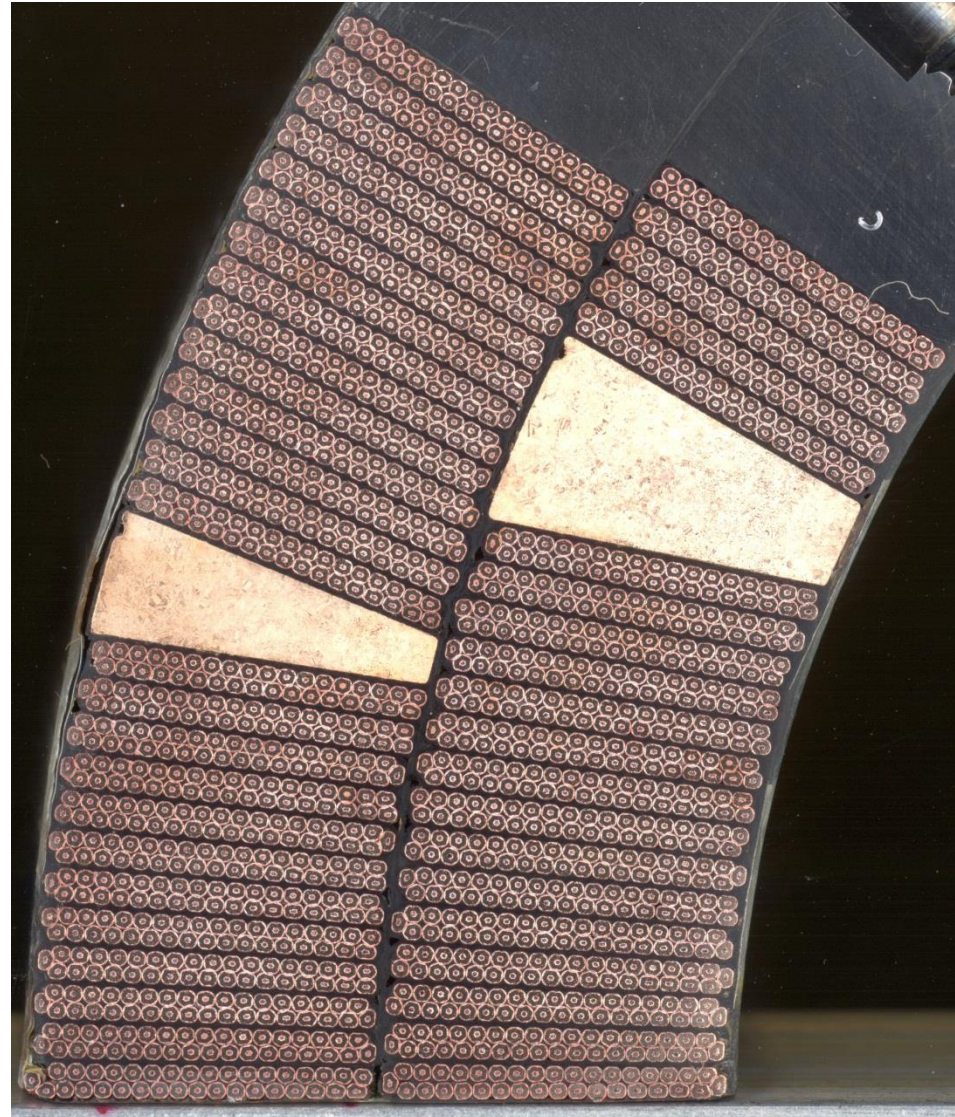
- **Section I**
 - Particle accelerators and magnets
 - Superconductivity and practical superconductors
 - **Magnetic design**
- **Section II**
 - Coil fabrication
 - Forces, stress, pre-stress
 - Support structures
- **Section III**
 - Quench, training, protection



Magnetic design Introduction

- The magnetic design is one of the first steps in the a superconducting magnet development
- It starts from the **requirements** (from accelerator physicists, researchers, medical doctors...others)
 - A field “**shape**”
 - Dipole, quadrupole, etc
 - A field **magnitude**
 - Usually with low T superconductors from 5 to 20 T
 - A field **homogeneity**
 - Uniformity inside a solenoid, harmonics in a accelerator magnet
 - A given **aperture** (and **volume**)
 - Some cm diameter for accelerator magnets, much more for detectors and fusion magnets

- **How much** conductor do we need to meet the requirements?
- And in **which configuration**?
- **Outline**
 - How do we create a **perfect field**?
 - How do we express the field and its “**imperfections**”?
 - How do we **design a coil** to minimize field errors?
 - **Overview** of different designs

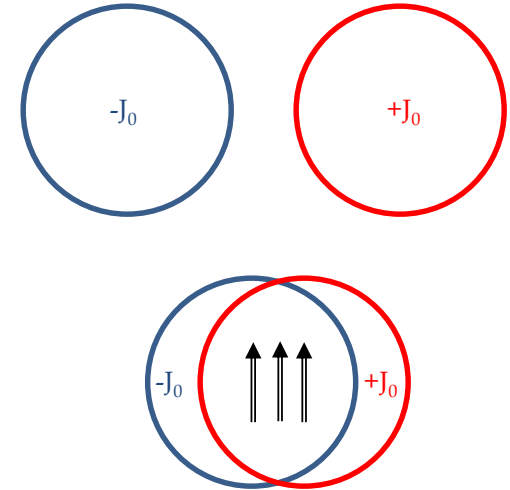


Perfect dipole field

Intercepting circles (or ellipses)

- Within a cylinder carrying j_0 , the field is perpendicular to the radial direction and proportional to the distance to the centre r :

$$B = -\frac{\mu_0 j_0 r}{2}$$

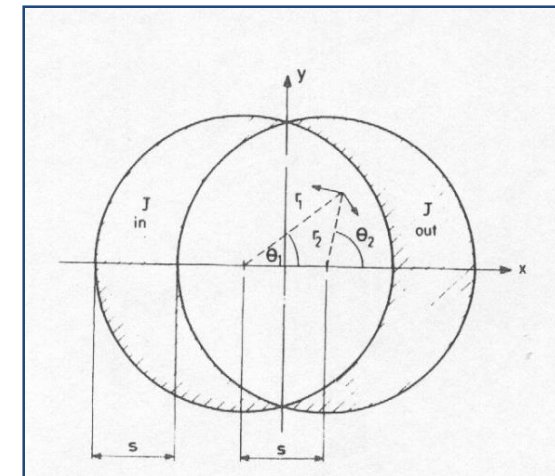


- Combining the effect of two intersecting cylinders

$$B_x = \frac{\mu_0 j_0 r}{2} \{-r_1 \sin \theta_1 + r_2 \sin \theta_2\} = 0$$

$$B_y = \frac{\mu_0 j_0 r}{2} \{-r_1 \cos \theta_1 + r_2 \cos \theta_2\} = -\frac{\mu_0 j_0}{2} s$$

- A uniform current density in the area of two **intersecting circles** produces a pure dipole
 - The aperture is not circular
 - Not easy to simulate with a flat cable
- Similar proof for **intersecting ellipses**



by M. Wilson

Perfect dipole field

Thick shell with $\cos\theta$ current distribution

- Thick shell

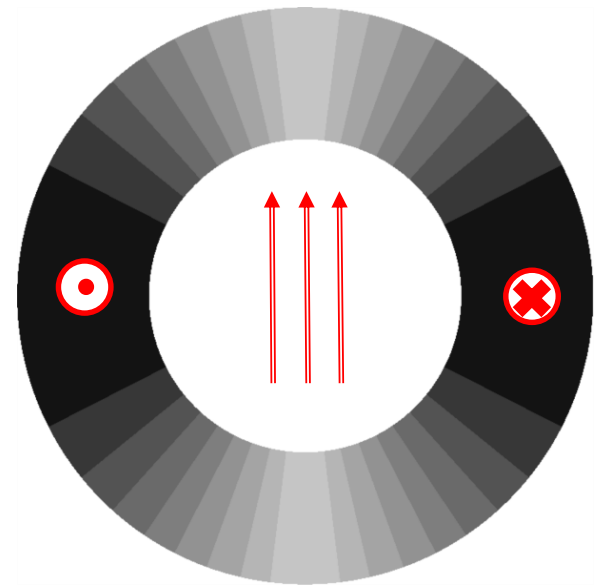
- Current density $J = J_0 \cos\theta$ on a shell with a finite thickness

$$B_{\text{bore}} = -\frac{J_0 \mu_0}{2} w$$

- Where, B_{bore} is the bore field, J_0 is overall current density and w is the coil width
- Ideal case
 - Conductor peak field $B_{\text{peak}} = B_{\text{bore}}$**
 - Perfect field quality

- Comparison:

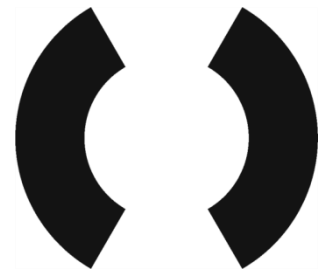
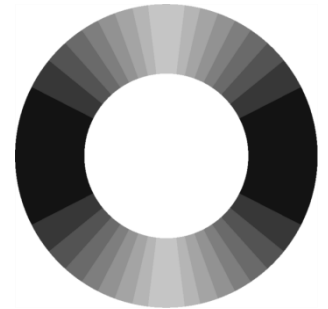
- For solenoid
 - $B_1 = -J_0 \mu_0 w$
 - Twice more efficient than a dipole



From ideal to practical configuration

- How can I reproduce **thick shell with a $\cos\theta$** distribution with a cable?
 - Rectangular cross-section and constant J

- First “rough” approximation
 - **Sector dipole**



Computation of the load line

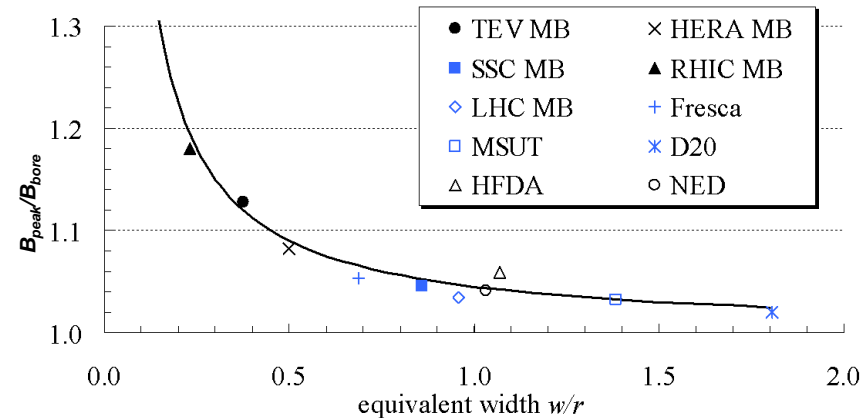
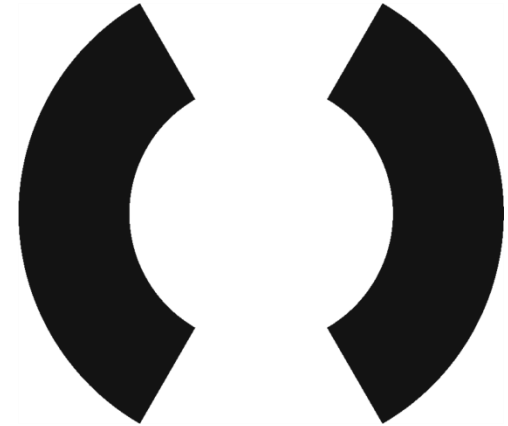
Approximations of practical winding cross-sections

- Sector coil

- Current density $J = J_0$ (A per unit area) on a sector with a maximum angle α

$$B_{\text{bore}} = -\frac{2J_0\mu_0}{\pi} w \sin(\alpha)$$

- Where, B_{bore} is the bore field, J_0 is overall current density and w is the coil width
- “Less ideal” case
 - “Not so perfect” field quality
 - Best with $\alpha = 60$ degrees
 - $B_{\text{peak}} = B_{\text{bore}} \cdot \sim 1.04$

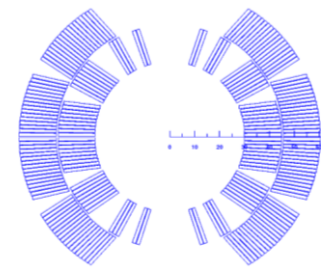
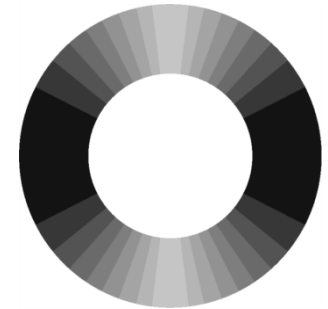


L. Rossi, E. Todesco, “Electromagnetic design of superconducting quadrupoles”, Phys. Rev. ST Accel. Beams 9 (2006) 102401.

- With a w/r of $30/30 = 1 \rightarrow 1.04$

From ideal to practical configuration

- How can I reproduce **thick shell with a $\cos\theta$** distribution with a cable?
 - Rectangular cross-section and constant J
- First “rough” approximation
 - **Sector dipole**
- Better ones
 - More **layers** and **wedges** to reduce J towards 90°
- As a result, the field is **not perfect** anymore
 - How can I express in improve the “imperfect” field inside the aperture?



Field representation

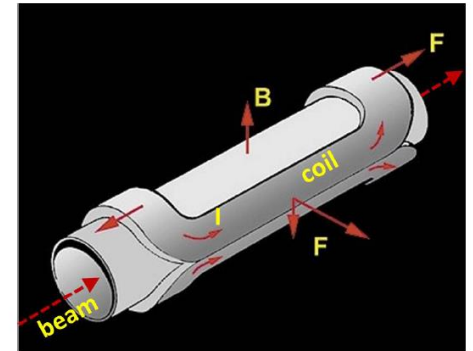
Maxwell equations

- **Maxwell equations** for magnetic field

$$\nabla \cdot \mathbf{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

- In absence of charge and magnetized material

$$\nabla \times \mathbf{B} = \left(\frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y}, \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}, \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} \right) = 0$$



- If $\frac{\partial B_z}{\partial z} = 0$ (constant longitudinal field), then

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \quad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

Field representation

Analytic functions

- If $\frac{\partial B_z}{\partial z} = 0$

Maxwell gives

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = 0$$

$$\frac{\partial B_y}{\partial y} + \frac{\partial B_x}{\partial x} = 0$$

$$\begin{cases} \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} = 0 \\ \frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} = 0 \end{cases}$$

Cauchy-Riemann conditions

and therefore the function $B_y + iB_x$ is analytic

$$B_y(x, y) + iB_x(x, y) = \sum_{n=1}^{\infty} C_n (x + iy)^{n-1}$$

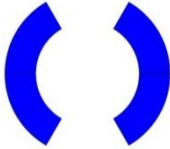
where C_n are **complex coefficients**

$$B_y(x, y) + iB_x(x, y) = \sum_{n=1}^{\infty} C_n (x + iy)^{n-1} = \sum_{n=1}^{\infty} (B_n + iA_n) (x + iy)^{n-1}$$

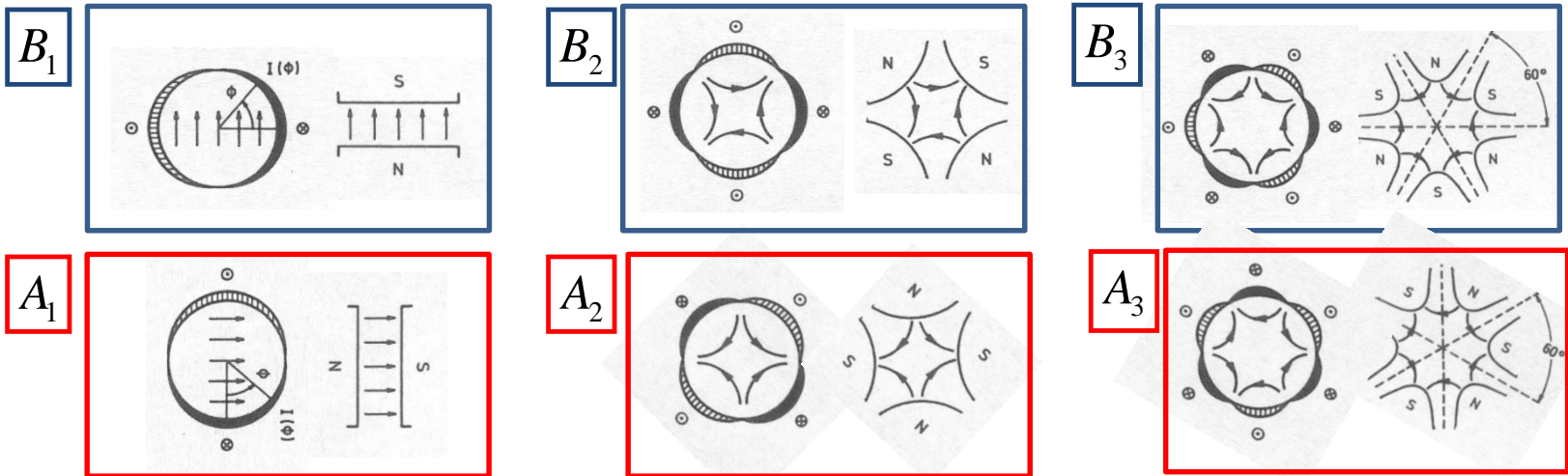
- Advantage: we reduce the description of the field to a (simple) series of complex coefficients

Magnetic design Harmonics

- The field can be expressed as (simple) series of coefficients
- So, each coefficient corresponds to a “pure” multipolar field



$$B_y(x, y) + iB_x(x, y) = \sum_{n=1}^{\infty} C_n(x + iy) = \sum_{n=1}^{\infty} (B_n + iA_n)(x + iy)$$



- The field harmonics are rewritten as

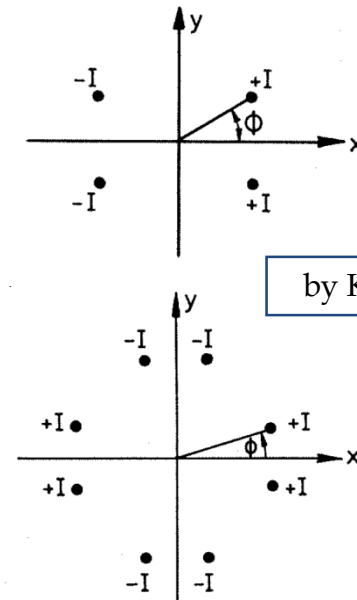
by K.-H. Mess, *et al.*

$$B_y + iB_x = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x + iy}{R_{ref}} \right)^{n-1}$$

- The coefficients b_n, a_n are called **normalized multipoles**
 - b_n are the **normal**, a_n are the **skew** (adimensional)

Field representation Harmonics

- One can demonstrate that with line currents with a **dipole** or a **quadrupole symmetry**, most of the **multipoles cancelled**
- For $n=1 \rightarrow$ *dipole*
 - Only b_3, b_5, b_7, \dots are present
- For $n=2 \rightarrow$ *quadrupole*
 - Only $b_6, b_{10}, b_{14}, \dots$ are present
- ...and so on
- These multipoles are called ***allowed multipoles***
- The field quality optimization of a coil lay-out concerns only a **few** quantities
 - For a dipole, usually b_3, b_5, b_7 , and possibly b_9, b_{11}

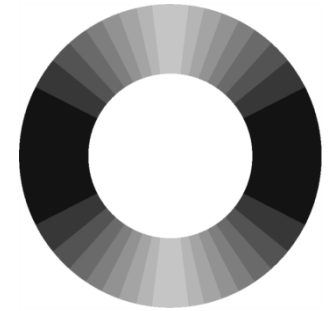


by K.-H. Mess, *et al.*

Back to the original issue: From ideal to practical configuration

- How can I reproduce **thick shell with a $\cos\theta$** distribution with a cable?

- Rectangular cross-section and constant J



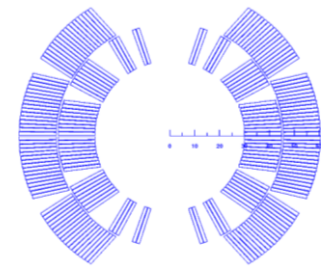
- First “rough” approximation

- **Sector dipole**



- Better ones

- More **layers** and **wedges** to reduce J towards 90°



- Now, I can use the multipolar expansion to **optimize** my “practical” **cross-section**

A “good” field quality dipole Sector dipole

- We compute the central field given by a **sector dipole with 2 blocks**

- Equations to set to zero B_3, B_5 and B_7

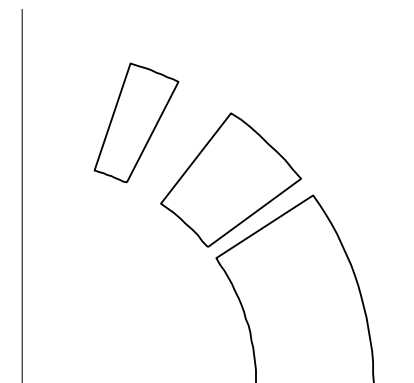
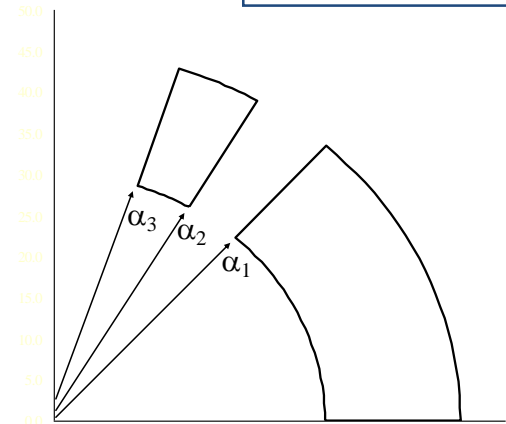
$$\begin{cases} \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) = 0 \\ \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) = 0 \end{cases}$$

- And the one given by a **3 blocks**

- Equations to set to zero B_3, B_5, B_7, B_9 and B_{11}

$$\begin{aligned} \sin(3\alpha_5) - \sin(3\alpha_4) + \sin(3\alpha_3) - \sin(3\alpha_2) + \sin(3\alpha_1) &= 0 \\ \sin(5\alpha_5) - \sin(5\alpha_4) + \sin(5\alpha_3) - \sin(5\alpha_2) + \sin(5\alpha_1) &= 0 \\ \sin(7\alpha_5) - \sin(7\alpha_4) + \sin(7\alpha_3) - \sin(7\alpha_2) + \sin(7\alpha_1) &= 0 \\ \sin(9\alpha_5) - \sin(9\alpha_4) + \sin(9\alpha_3) - \sin(9\alpha_2) + \sin(9\alpha_1) &= 0 \\ \sin(11\alpha_5) - \sin(11\alpha_4) + \sin(11\alpha_3) - \sin(11\alpha_2) + \sin(11\alpha_1) &= 0 \end{aligned}$$

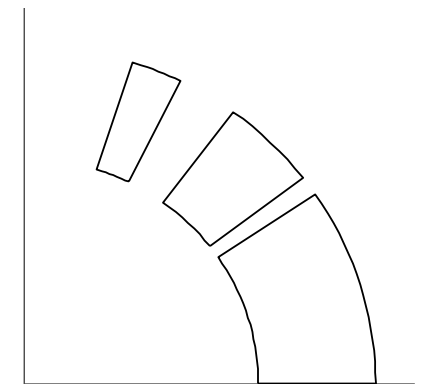
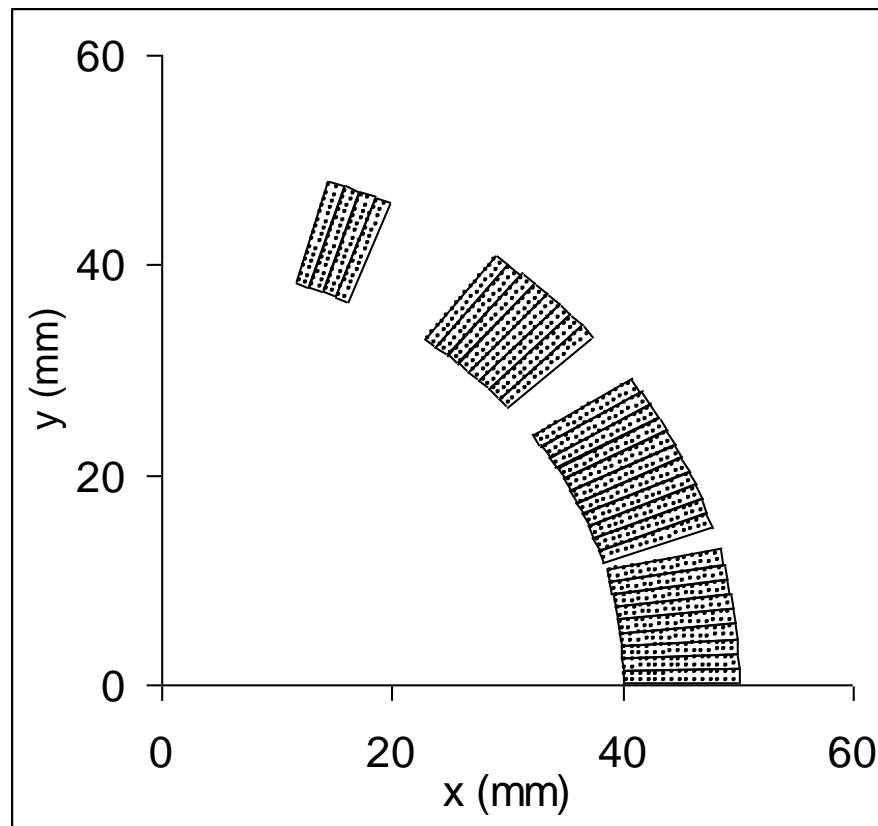
by E. Todesco



Two wedges, $b_3=b_5=b_7=b_9=b_{11}=0$
[0°-33.3°, 37.1°-53.1°, 63.4°- 71.8°]

A “good” field quality dipole Sector dipole

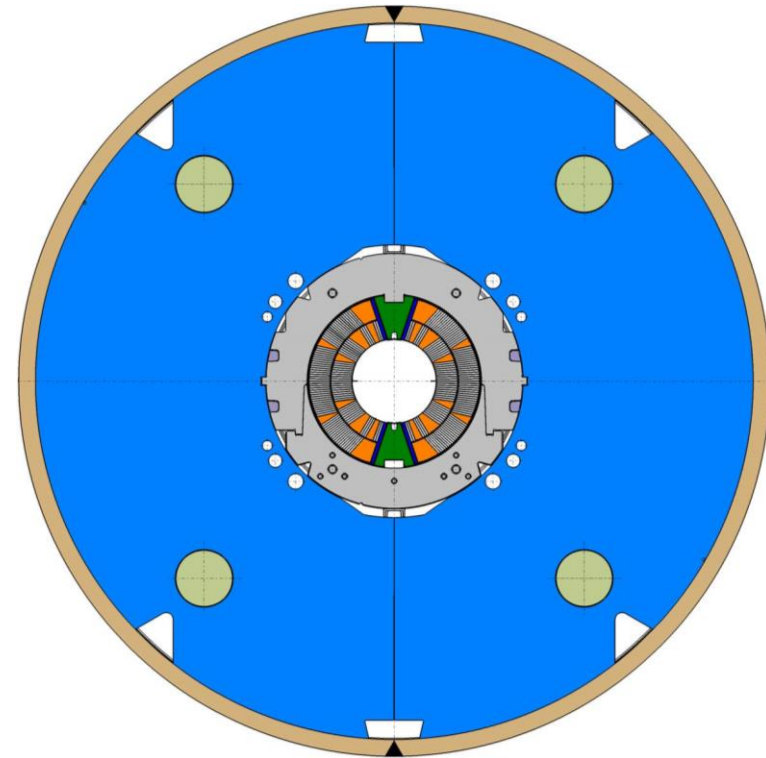
- Let us see two coil lay-outs of real magnets
 - The **RHIC dipole** has **four blocks**



Two wedges, $b_3=b_5=b_7=b_9=b_{11}=0$
 $[0^\circ-33.3^\circ, 37.1^\circ-53.1^\circ, 63.4^\circ-71.8^\circ]$

Iron yoke

- Keep the **return magnetic flux** close to the coils, thus avoiding fringe fields
- In some cases the iron is partially or totally contributing to the **mechanical structure**
- Considerably **enhance the field** for a given current density
 - The increase is relevant (10-30%), getting higher for thin coils
 - This allows using lower currents, easing the protection



Iron yoke

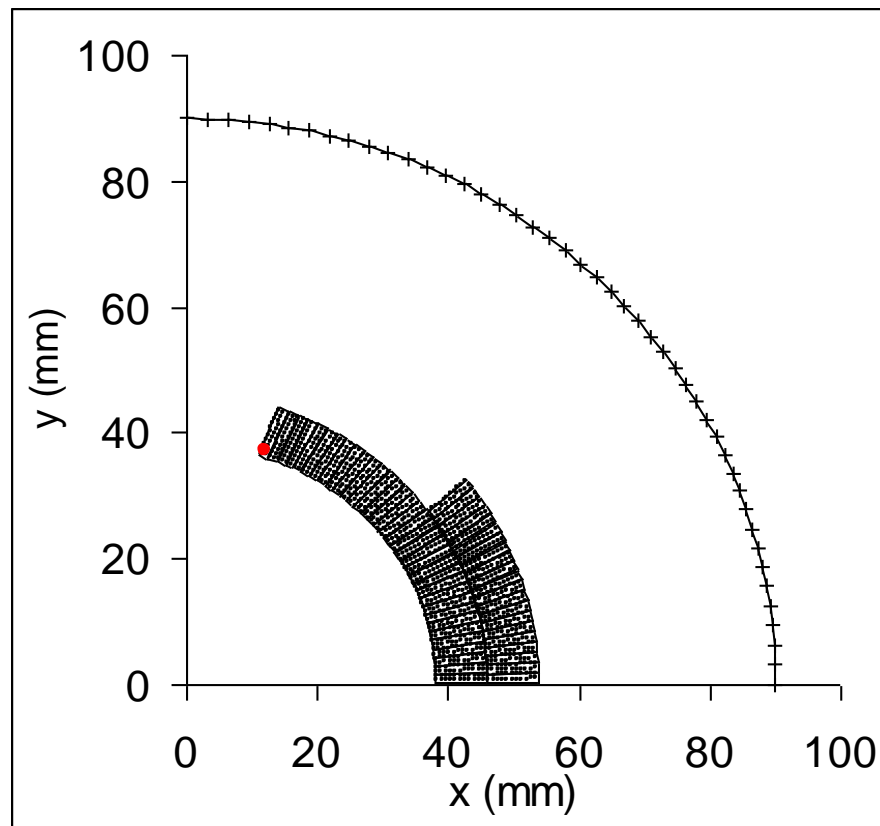
- A **rough estimate** of the **iron thickness** necessary to avoid fields outside the magnet
 - The iron cannot withstand more than 2 T
 - **Shielding condition** for dipoles: $rB \sim t_{iron} B_{sat}$
 - i.e., the iron thickness times 2 T is equal to the central field times the magnet aperture – One assumes that all the field lines in the aperture go through the iron (and not for instance through the collars)
 - Example: in the LHC main dipole **the iron thickness is 150 mm**

$$t_{iron} \sim \frac{rB}{B_{sat}} = \frac{28 \cdot 9}{2} \sim 130 \text{ mm}$$

- Shielding condition for quadrupoles: $\frac{r^2 G}{2} \sim t_{iron} B_{sat}$

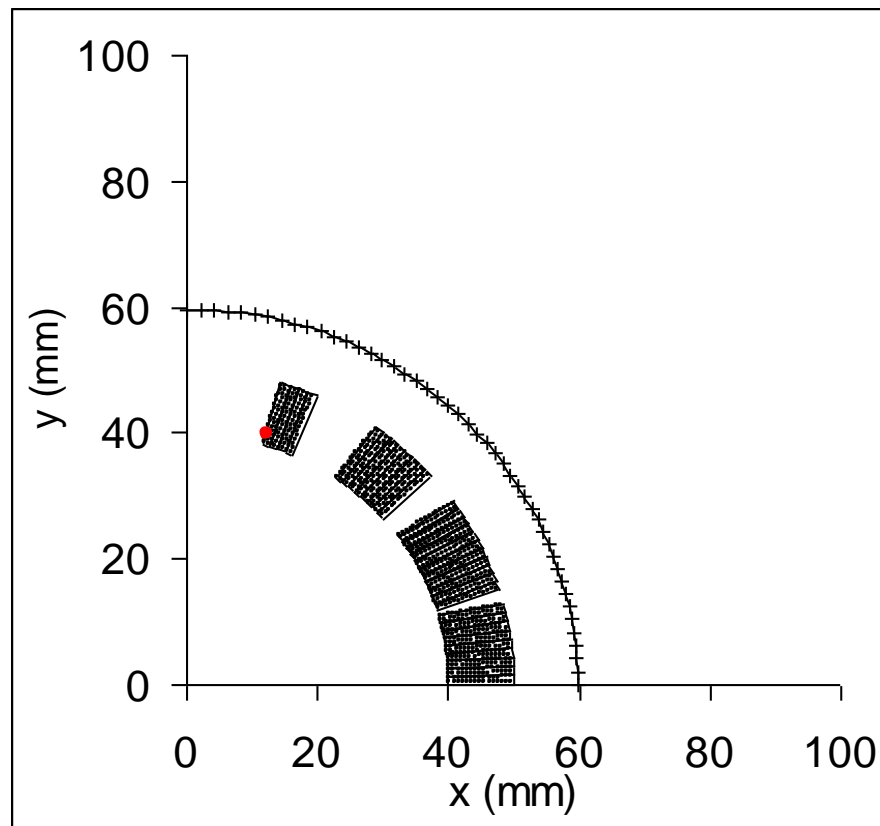
A review of dipole lay-outs

- Tevatron MB



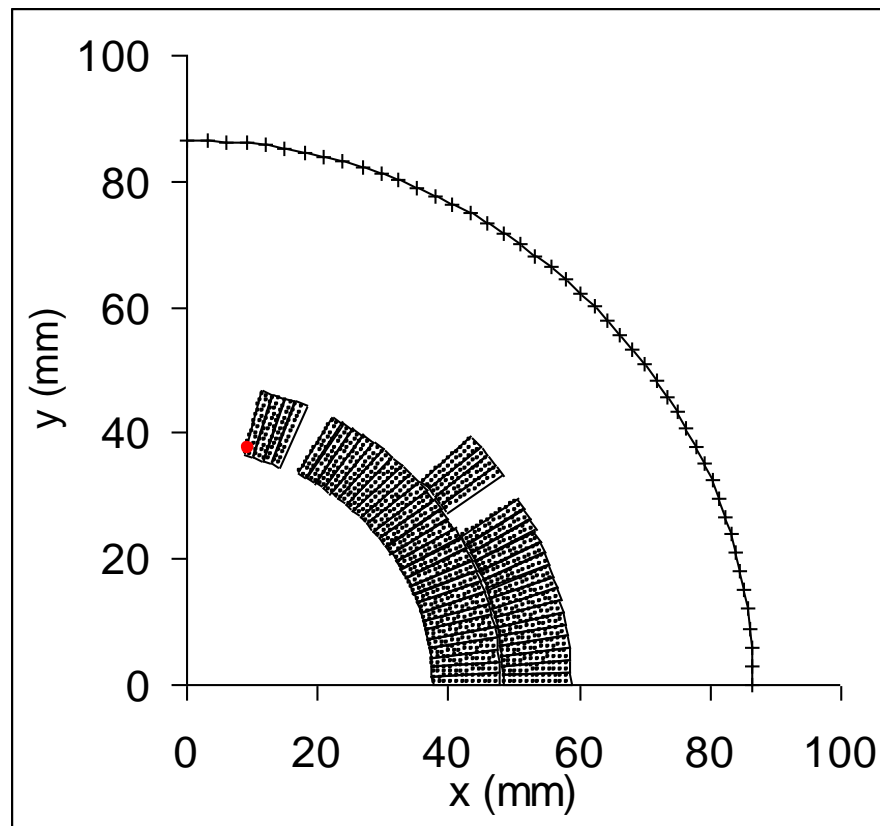
A review of dipole lay-outs

- RHIC MB



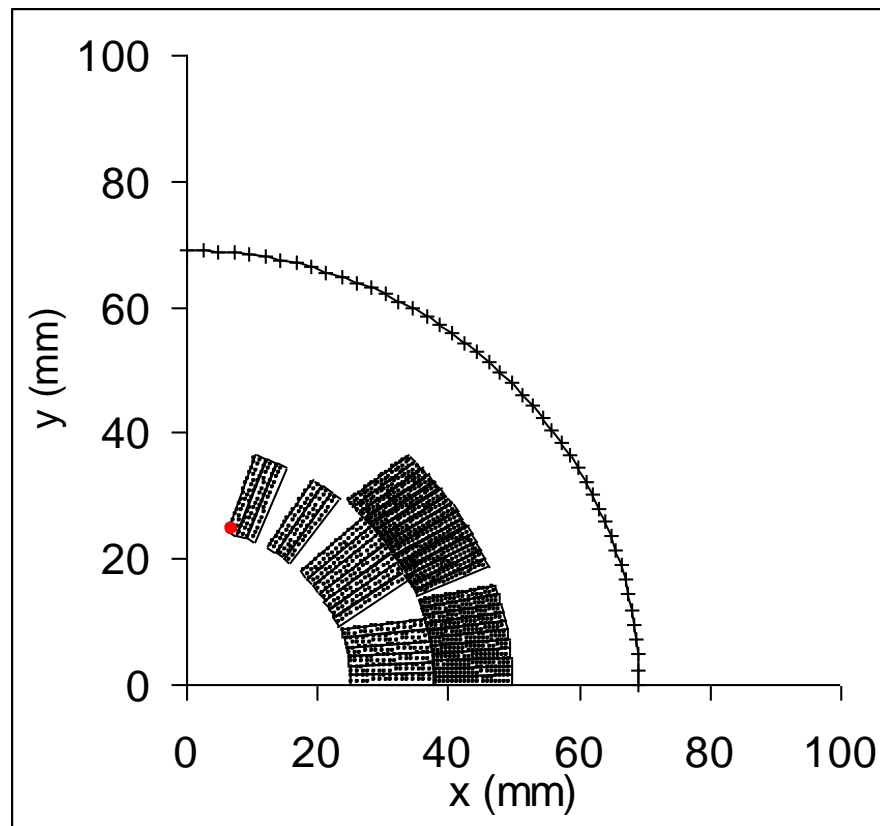
A review of dipole lay-outs

- HERA MB



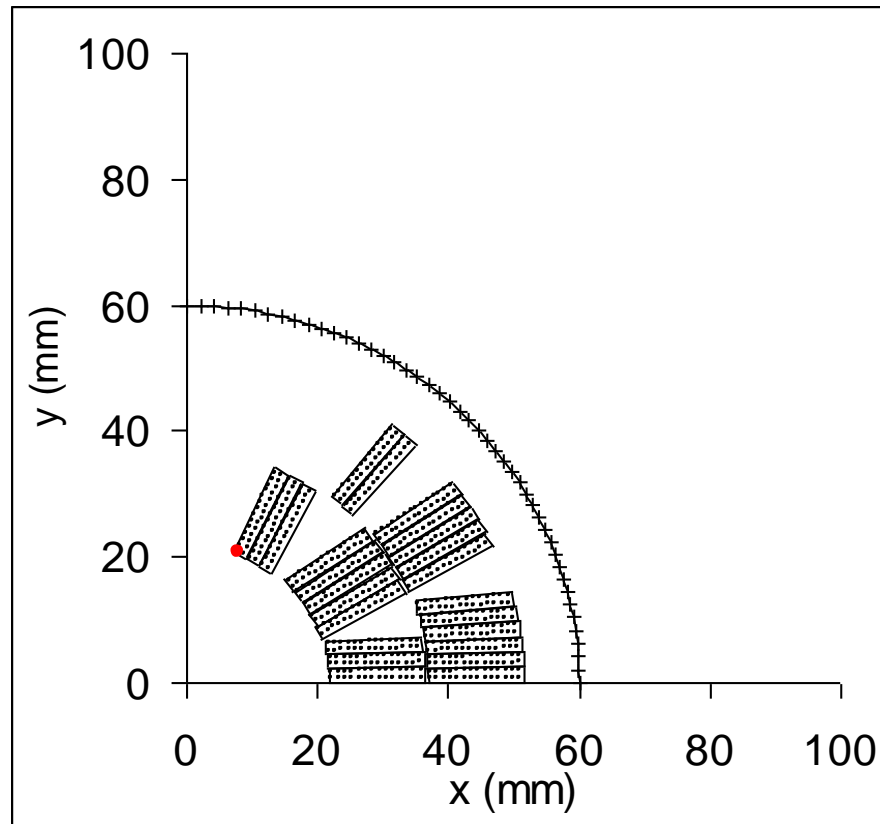
A review of dipole lay-outs

- SSC MB



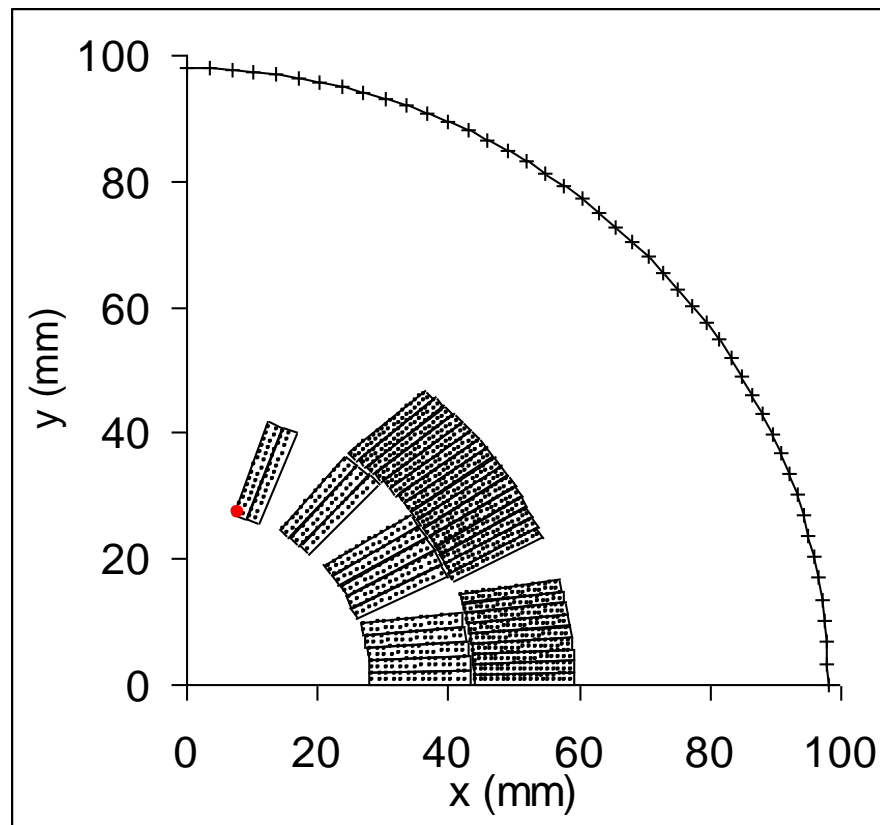
A review of dipole lay-outs

- HFDA dipole



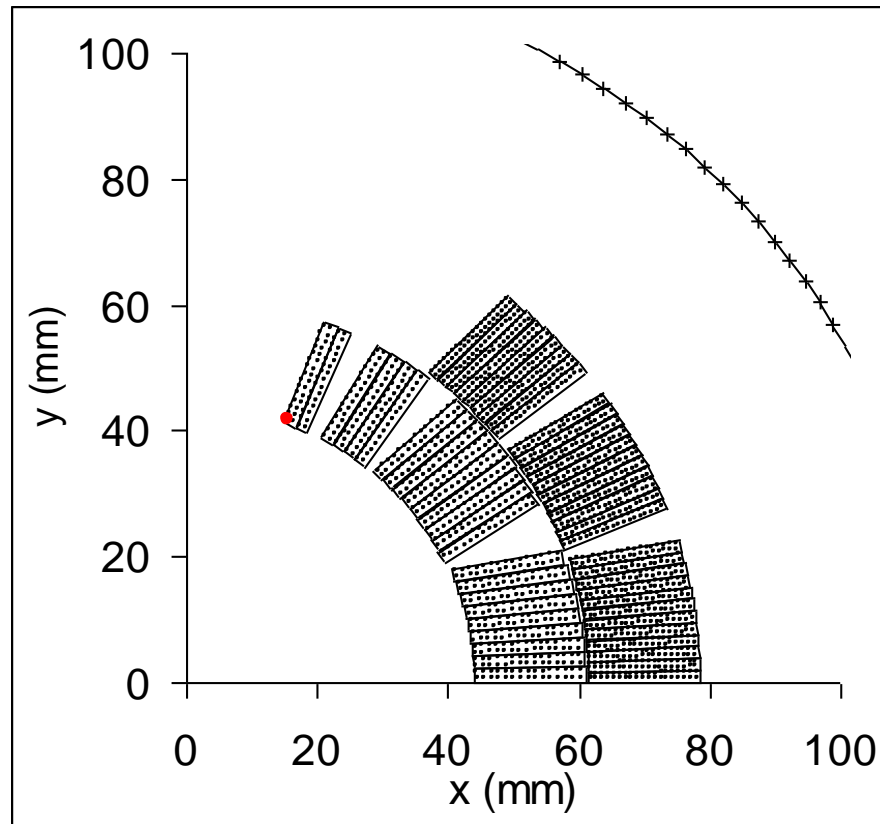
A review of dipole lay-outs

- LHC MB



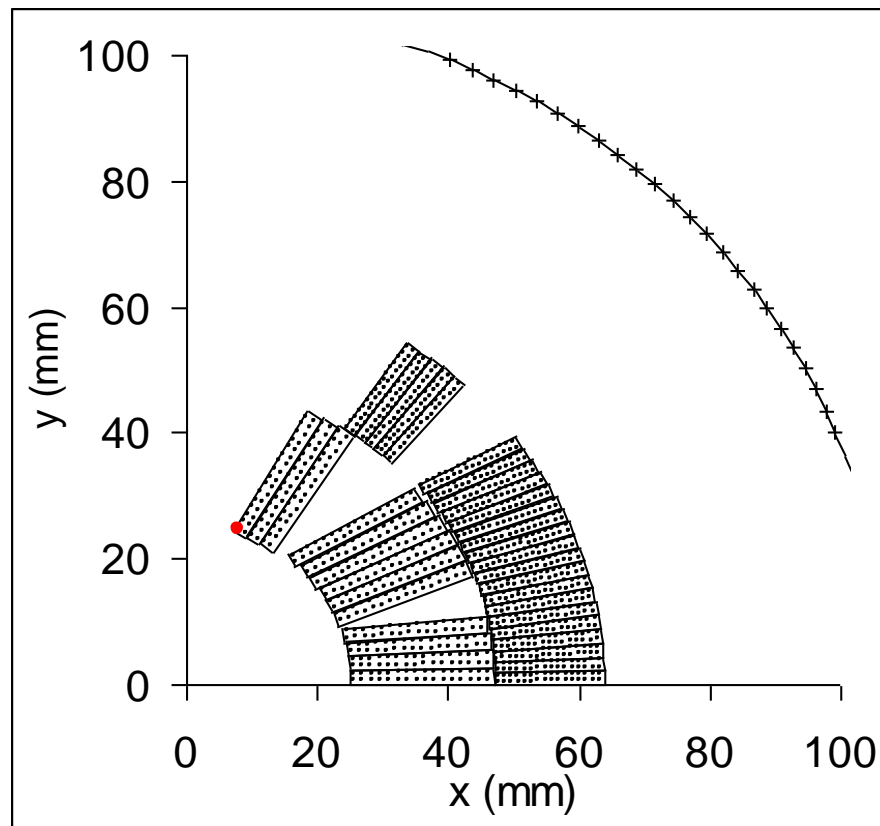
A review of dipole lay-outs

- FRESCA



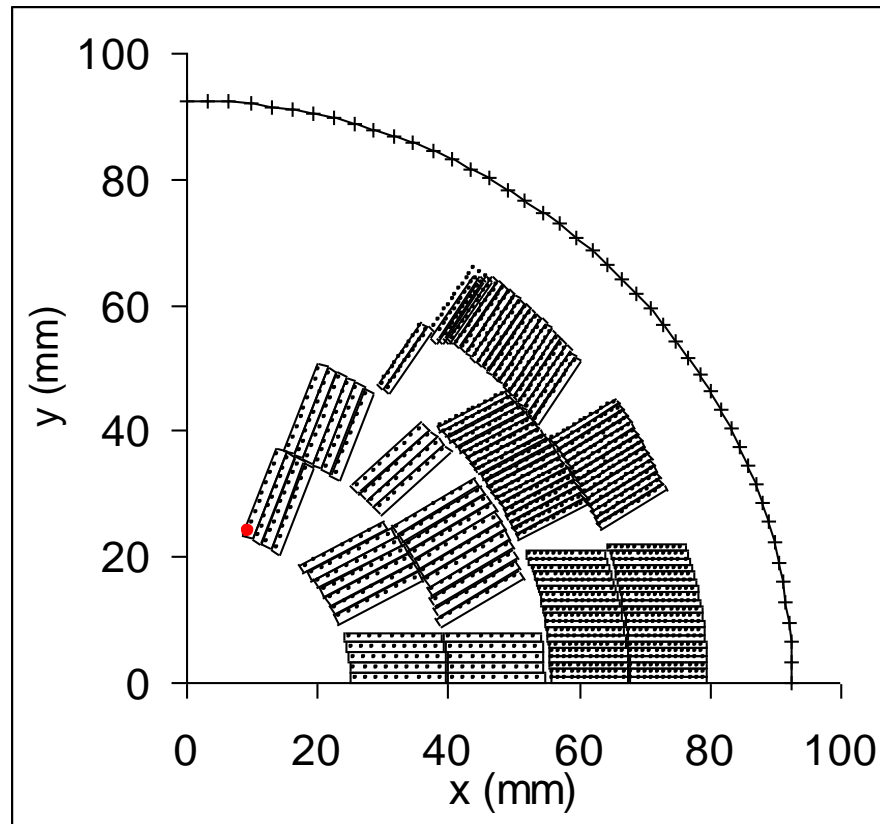
A review of dipole lay-outs

- MSUT



A review of dipole lay-outs

- D20



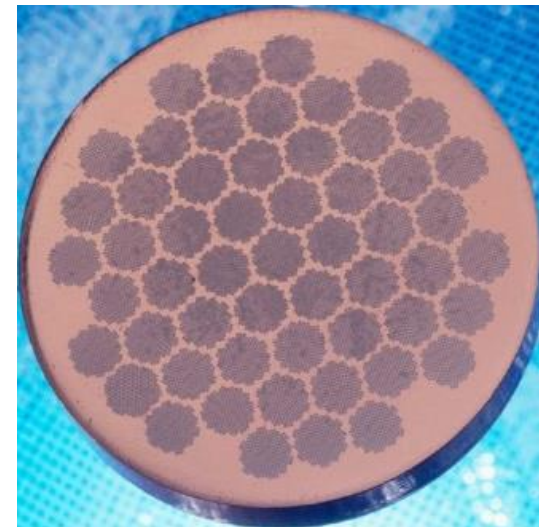
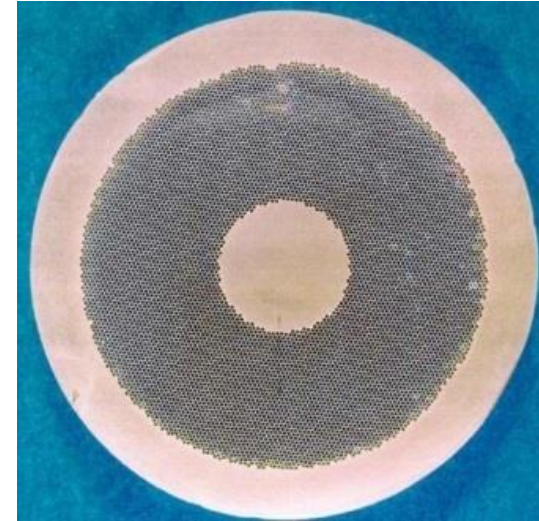


Appendix

Practical superconductors

Fabrication of Nb-Ti multifilament wires

- **Copper to superconductor ratio**
 - ensure quench protection without compromising the overall critical current of wire.
- **Filament diameter**
 - Minimize flux jumps and persistent currents
 - Minimizing the wire processing cost
- **The inter-filament spacing**
 - small so that the filaments, harder than Cu, support each other during drawing operation
 - large enough to prevent filament couplings
- Cu **core** and **sheath** to reduce cable degradation
- Main manufacturing issue: **piece length**
 - It is preferable to wind coils with single-piece wire (to avoid welding)
 - LHC required piece length longer than 1 km



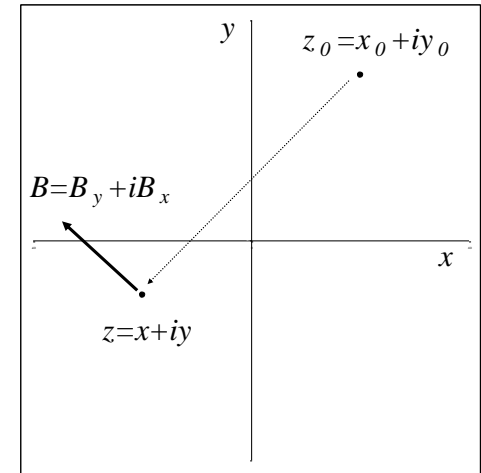
- Important property: starting by the multipolar expansion of a current line (Biot-Savart law)

$$B(z) = B_y(z) + iB_x(z)$$

$$B(z) = \frac{I\mu_0}{2\pi(z - z_0)} = -\frac{I\mu_0}{2\pi z_0} \frac{1}{1 - \frac{z}{z_0}}$$

$$B(z) = -\frac{I\mu_0}{2\pi z_0} \sum_{n=1}^{\infty} \left(\frac{z}{z_0} \right)^{n-1} = -\frac{I\mu_0}{2\pi z_0} \sum_{n=1}^{\infty} \left(\frac{R_{ref}}{z_0} \right)^{n-1} \left(\frac{x + iy}{R_{ref}} \right)^{n-1}$$

$$B_y + iB_x = 10^{-4} B_1 \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x + iy}{R_{ref}} \right)^{n-1} \quad b_n + ia_n = -\frac{I\mu_0 10^4}{2\pi z_0 B_1} \left(\frac{R_{ref}}{z_0} \right)^{n-1}$$



A “good” field quality dipole Sector quadrupole

- Let’s look at the quadrupoles
- First allowed multipole B_6 (dodecapole)

$$B_6 = \frac{\mu_0 j R_{ref}^5}{\pi} \frac{\sin(6\alpha)}{6} \left(\frac{1}{r^4} - \frac{1}{(r+w)^4} \right)$$

for $\alpha = \pi/6$ (i.e. a **30° sector coil**) one has $B_6 = 0$

- Second allowed multipole B_{10}

$$B_{10} = \frac{\mu_0 j R_{ref}^8}{\pi} \frac{\sin(10\alpha)}{10} \left(\frac{1}{r^8} - \frac{1}{(r+w)^8} \right)$$

for $\alpha = \pi/10$ (i.e. a **18° sector coil**) or for $\alpha = \pi/5$ (i.e. a **36° sector coil**) one has $B_{10} = 0$

- The conditions look similar to the dipole case ...

