Joint University Accelerator School

Examination of Transverse Beam Dynamics

JUAS 2017

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1 Exercise: FODO cell [5 points]

We want to design a storage ring based on a periodic FODO lattice, for electrons with nominal momentum 1 GeV/c. The ring is composed by 32 cells, and its circumference is 160 m long. Each cell features 2 dipole magnets, whose total filling factor is 60%.

1. Characterise the dipole magnet. What are its: length? bending angle? bending radius? magnetic field?
2. What values of the phase advance are permitted in order to have $\beta_{\text{max}} \leq 10$ m?
3. The quadrupoles are 0.5 m long. What is the quadrupole gradient required to achieve the maximum μ of point 2?
4. For the gradient computed in the previous point, what are the values of the maximum and of the minimum dispersion?
5. Estimate the momentum compaction, α, and the length variation of the closed orbit for a particle with $\frac{\Delta P}{P} = -1\%$.

2 Exercise: Dispersion in a transfer line [3 points]

A misaligned quadrupole introduces dispersion. Consider a quadrupole, with gradient $G = 10$ T/m and length 1 m, installed in a region where the nominal momentum is 7 GeV/c. If the quadrupole is installed with a horizontal offset of 1 mm, what are the values of dispersion and dispersion prime 10 m downstream?

3 Exercise: CLIC lattice [3 points]

The main linac of CLIC consists of 12 sectors filled with FODO cells with increasing length and fixed phase advance, $\mu = 72°$ per cell, in both axes. In each sector the cell length scales approximatively with the square root of the energy: $L_{\text{cell}} \propto \sqrt{E}$. If one wants to keep fixed the quadrupole gradient G in all sectors, how must the length of the quadrupole magnets scale? Justify your answer.

4 Exercise: the Achromatic Telescopic Squeeze (ATS) [9 points]

The ATS is a proposed alternative optics for the LHC and the baseline optics for its upgrade: the HL-LHC. The ATS allows to further squeeze the beam at the IP overcoming the limitation imposed by the sextupole strengths for the chromatic correction.

In the traditional squeeze the reduction of beta at the IP is entirely produced within the straight section and the dynamics in the arcs is not changed.

1. Write the expression for $\beta(s)$ starting from a waist ($\beta_w = \beta^*$, $\alpha_W = 0$) and show that as β^* is reduced, the chromaticity of the ring generated by the squeezing magnets (placed at a distance L^* from the IP) increases.

   The ATS scheme adopts a FODO cell matched to 90° to transport the beam in the arcs, connected by means of dispersion suppressors to the straight section, where the Interaction Point (IP) and the detector are placed. The sextupoles, which perform the chromatic correction, are placed next to each quadrupole in the arc.

2. Briefly explain why it is useless to place sextupoles for the chromatic correction in the straight section.

   In order to get additional sextupolar strength we can mismatch the beta function in the arcs. We shall see how to do this in a controlled way. Use the thin lens approximation and neglect the dipole magnets.

3. Express the focal length of the quadrupoles in term of the distance L between them, so that we achieve a 90° FODO.

4. Compute the transfer matrix of a cell, starting from the centre of a focussing quadrupole (use thin-lens approximation).

5. What are M^2 and M^4? Briefly comment the result.

6. Write down T, the 3x3 matrix that transport the Twiss and show that $T^2 = I$, the identity matrix.

7. If we enter in a cell with: $\beta = \beta_0$, $\alpha = 0$ what are their values β_1, α_1 after one cell? And after two cells?

8. Which value of β_0 is such that $\beta_0 = \beta_1 \equiv \beta_{\text{Match}}$? Is this result in agreement with the formula for the matched FODO?

9. Now consider a system made of two cells starting at the centre of the focussing quadrupole. Remember: this system is periodic for any initial Twiss parameters as $T^2 = I$. We enter into this lattice with $\alpha_0 = 0$, $\beta_0 = r\beta_{\text{Match}}$, where $r \in \mathbb{R}^+$. Write the expressions of β at the focussing quadrupoles as a function of the enhancing parameter: r, assuming $L = 29.3$ m.

10. In the arc, sextupole magnets are installed next to each quadrupole and tuned to compensate for the natural chromaticity. Show that if the chromaticity of the arc is corrected to zero, it remains null for each value of r (you can consider only the focussing quadrupoles).

11. If we want to correct for the natural chromaticity produced in the straight section, we need to generate some additional positive chromaticity in the arc so that: $\xi^{\text{ARC}} = -\xi_N^{\text{IP}}$. Suppose that for $r = 1$ the total chromaticity of the arc is some positive value: ξ_1^{ARC}. Write an expression and sketch the plot for $\frac{\xi_r^{\text{ARC}}}{\xi_1^{\text{ARC}}}$. Briefly comment on how this can help in squeezing β_{IP} down to lower values.