JUAS 2017 exam

The following constant will be used in the exercises

elementary electric charge	$e = 1.6 \cdot 10^{-19}$ C
velocity of light	$c = 2.998 \cdot 10^{8}$ m/s
reduced Planck constant	$\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Kg m^2/ s
vacuum dielectric constant	$\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m
electron rest energy	0.511 MeV
proton rest energy	938 MeV
classical electron radius	$r_e = 1/(4\pi\varepsilon_0)\, e^2/(mc^2) = 2.81 \cdot 10^{-15}$ m

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} = 3.84 \cdot 10^{-13} \text{m}$$

Ex. 1:
Consider an electron storage ring operating at 3 GeV, with a lattice made of 24 DBA cells.

a) give an estimate for the minimum emittance achievable in such ring (take J_x = 1).
b) If the magnetic field of the dipole is 1.4T calculate the critical energy of the bending magnets.

Assume we want to upgrade this lattice to a 48 DBA cells.

c) give an estimate for the minimum emittance achievable in such ring.
d) Assuming that the length of each dipole has not changed in the upgrade, calculate the new critical energy for the dipole radiation.
e) Describe what is the change in the characteristics of the bending magnet radiation (i.e. power, spectrum and angular distribution).

Ex. 2:
Consider an electron storage ring operating at 6 GeV. The injection process requires that the oscillations of the injected beam at 6 GeV are damped within 10 ms, otherwise the injected beam hits the back of the septum magnet and no injection is possible.

a) What is the energy loss per turn necessary to guarantee the damping time required for injection (take J_x = 1 and a revolution time T_0 = 2μs).
b) If the energy of the ring reduced to 3 GeV, what is the energy loss per turn necessary to guarantee the injection with the existing injection system parameters?
c) If we want to use damping wiggler to guarantee the injection at 3GeV and L_W = 50 m are available, what is the magnetic field B_W required from the damping?
d) What are the main effects of the introduction of damping wigglers on the equilibrium beam parameters?

Ex. 3:
In a 3 GeV electron synchrotron, a beamline is made by an undulator of length L = 2 m, operating at a wavelength λ = 1nm. The emittance of the ring is 1 nm in both planes and the optics function at the middle of the straight section are $\beta_x = \beta_y = 5$m and $\alpha_x = \alpha_y = 0$ and the dispersion function is zero.

The beamline has a round slit aperture at 20 m downstream with radius r = 1 mm.

a) is the undulator fan at λ = 1nm passing through the slit (assume a Gaussian beam for the undulator radiation).
b) What happens if the optics function in the middle of the straight section are reduced to $\beta_x = \beta_y = 1$m? Give an estimate of the change in flux through the slit.
c) Describe the criteria to achieve the optimal matching of the electron phase space and the photon phase space at the centre of the undulator.

Ex. 4:
Describe the strategies for the design of the linear optics functions β, α and dispersion D in as low emittance lattice. What are the most common solutions? Sketch the optics function required to achieve the theoretical minimum emittance in a TME and DBA lattice.

Ex 1: solution

a)

A 24 cell DBA lattice has 48 dipoles therefore the bending angle is

$$\theta = \frac{2\pi}{48} = 131 \text{ mrad}$$

The theoretical minimum emittance of a DBA lattice is (including the breaking of the achromatic condition)

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x}$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} = 3.84 \cdot 10^{-13} \text{m}$$

For a 3 GeV beam (γ = 5871) and J_x = 1 we have

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x} \sim \frac{3.84 \cdot 10^{-13}}{12\sqrt{15}} (5870)^2 (0.131)^3 = 640 \text{ pm}$$

b)

The critical energy is

$$\varepsilon_c = \frac{3\hbar c \gamma^3}{2\rho}$$

In order to compute the bending radius we use

$$\frac{1}{B\rho} = \frac{e}{p} \qquad \rightarrow \qquad \rho = \frac{p}{eB} = \frac{\beta E}{ecB} \sim \frac{3\text{GeV}}{ec1.4\text{T}} = \ \sim 7.14 \text{ m}$$

The critical energy reads

$$\varepsilon_c = \frac{3\hbar c \gamma^3}{2\rho} = \frac{3\hbar c (5870)^3}{2 \cdot 7.14} = 8.37 \text{ keV}$$

c)

In a 48 DBA cells lattice there are 96 dipoles hence

$$\theta = \frac{2\pi}{96} = 65 \text{ mrad}$$

The theoretical minimum emittance achievable is

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x} \sim \frac{3.84 \cdot 10^{-13}}{12\sqrt{15}} (5870)^2 (0.65)^3 = 80 \text{ pm}$$

d)

The length if the dipole does not change therefore the bending radius is doubled and the magnetic field is halved. The new critical energy is also halved

$$\varepsilon_c = \frac{3\hbar c\gamma^3}{2\rho} = \frac{3\hbar c(5870)^3}{2\cdot 3.57} = 4.18 \text{ keV}$$

e)
The total instantaneous power is reduced by a factor 4 (since ρ is doubled)

$$P = \frac{e^2}{6\pi\varepsilon_0}\frac{\gamma^4}{\rho^2}$$

The high energy end of the spectrum is reduced as the critical energy decreases from 8 keV to 4 keV.

The critical angle

$$\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$$

increases as the critical frequency decreased

Ex. 2 solution

a)
The damping time in the horizontal plane is

$$\frac{1}{\tau_x} = \frac{J_x U_0}{2T_0 E_0}$$

For a 6GeV beam (T_0 = 2 μs, J_x = 1) and τ_x = 10 ms we must have

$$U_0 = \frac{2T_0E_0}{\tau_x J_x} = \frac{2\cdot 2\mu s\cdot 6\text{GeV}}{0.01} = 2.4 \text{ MeV/turn}$$

b)
The energy loss per turn is

$$U_0(\text{keV}) = 88.46\frac{E(\text{GeV})^4}{\rho(\text{m})}$$

If the ring energy is reduced from 6 GeV to 3 GeV, the energy loss per turn is reduced by a factor 2^4 = 16 therefore

U_0 = 0.15 MeV/turn

and the damping time becomes

$$\frac{1}{\tau_x} = \frac{J_x U_0}{2T_0 E_0} = \frac{0.15}{2\cdot 2\mu s\cdot 3000} = 12.5 \qquad \rightarrow \qquad \tau_x = 80 \text{ ms}$$

In order to guarantee injection, the energy loss per turn has to be increased by a factor of 8, going from 0.15 MeV/turn to 1.2 MeV/turn.

c)
The damping wiggler must provide the additional

ΔU_0 = 1.2 – 0.15 = 1.05 MeV/turm

The energy loss per turn do to a wiggler of field $B_W = B_0/\sqrt{2}$ and length L_W is

$$E_w = \frac{2}{3}\frac{r_e e^2}{m_e^3 c^4} E^2 B_W^2 L_W$$

i.e.

$$E_w(eV) = \frac{2}{3}\frac{r_e c^2}{(m_e c^2)^3} E^2 B_W^2 L_W = \frac{2}{3}\frac{2.81\cdot 10^{-15} 9\cdot 10^{16}}{(0.511\cdot 10^6)^3} E(eV)^2 B_W^2 L_W(m) =$$
$$= 1263\cdot 10^{-18} E(eV)^2 B_W^2 L_W(m) = 1263\cdot 10^{-18}(3\cdot 10^9)^2 B_W^2 L_W(m) = 11367\cdot B_W^2 L_W(m)$$

In order to achieve 1.05 MeV we have/turn

$$E_w(eV) = 11367\cdot B_W^2 L_W(m) = 1.05\cdot 10^6$$

hence

$$B_W^2 L_W(m) = \frac{1.05\cdot 10^6}{11367} = 92.4$$

with a length of 50 m available B_W = 1.34 T.

d)
Beyond the energy loss per turn and the damping time, the damping wiggler also change the emittance and the energy spread (see slides)

Ex. 3 solution

a)
The source size and divergence of the undulator are given by the quadratic sum of the electron and photon beam size

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} \qquad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2}$$

Given the emittance $\varepsilon_x = \varepsilon_y$ = 1 nm and the optics function ($\beta_x = \beta_y$ = 5m) quoted

$$\sigma_x = \sqrt{\varepsilon_x \beta_x} \quad = 71 \text{ um}$$

$$\sigma_{x'} = \sqrt{\frac{\varepsilon_x}{\beta_x}} \quad = 14 \text{ urad}$$

The values in the vertical plane are the same.

An undulator of L = 2 m at λ = 1 nm will have

Photon beam size $\sigma_{ph} = \sqrt{\frac{\lambda}{2L}}$ 15.8 um

Photon beam divergence $\sigma_{ph'} = \frac{\sqrt{2L\lambda}}{2\pi}$ 10.1 urad

The total beam size and divergence read

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} = 72.6 \text{ um}$$

$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = 17.3 \text{ urad}$$

At 20 m downstram the spot size will be

dx = 17.3 urad * 20m = 0.346 mm

Summed with 72.6 um yields 0.418 mm rms. The aperture has a radius of 1 mm therefore about ±2 σ of the Gaussian beam pass through.

b)
If the optics function are reduced to $\beta_x = \beta_y = 1$m

$$\sigma_x = \sqrt{\varepsilon_x \beta_x} \qquad = 31.6 \text{ um}$$

$$\sigma_{x'} = \sqrt{\frac{\varepsilon_x}{\beta_x}} \qquad = 31.6 \text{ urad}$$

The values in the vertical plane are the same.

The total beam size and divergence read

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} = 35.3 \text{ um}$$

$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = 33.2 \text{ urad}$$

At 20 m downstream the spot size will be

dx = 33.2urad * 20m = 0.66 mm

summed with the beam size gives 0.705 mm. In an aperture of 1 mm radius this is about ±0.5 σ. So the radiation is clipped and the flux reduced (±1 σ corresponds to 66% of the flux, ±2 σ corresponds to 95% of the flux).

c)
the ellipses in phase space have to be aligned and

$$\beta_x \sim \frac{L}{\pi} = 0.63 \text{ m}$$

Ex. 4:

See slides (minimise βx and put a waist in the dipole, control D_x, MBA lattices).