Enhancing the efficiency of laser and plasma based accelerators using bichromatic driver pulses

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Outline

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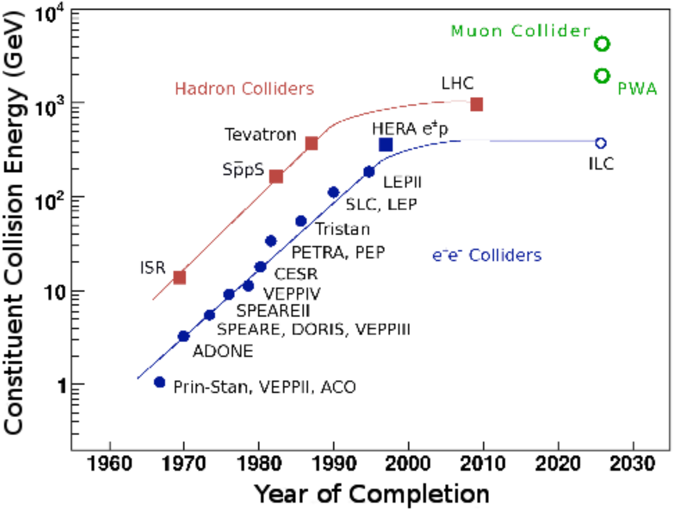
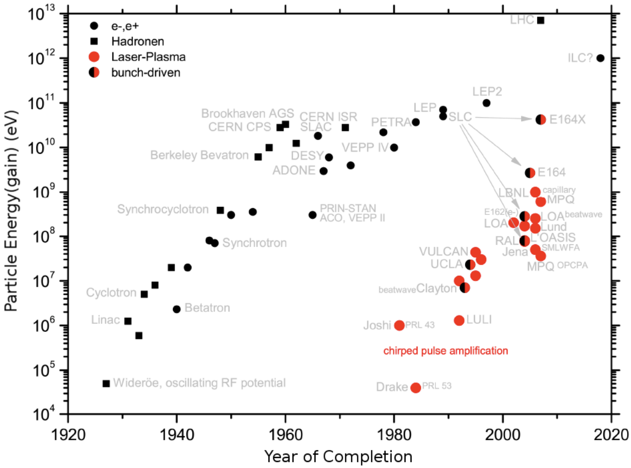
- State-of-the-Art technology: circular accelerator, $8.3\,\mathrm{T}$, $14\,\mathrm{TeV}$
- Limit: accelerating field $< 50\,\mathrm{MV/m}$



How further → VLHC? Extremely Expensive!
(Future Circular Collider Kick Off Meeting, Feb. 2014, Geneva)
New, cheaper technologies are needed → Plasma based acceleration!



New technology: particle acceleration by **plasma waves.** The possible methods are:

- **PWFA:** electron/proton bunch drives the wakes.
- **LWFA:** Short ($\approx 1\,\mathrm{ps}$), ultra intense $I \geq 10^{18}\mathrm{W}\cdot\mathrm{cm}^{-2}$ pulse. $L = c\tau_p \approx \lambda_p = 2\pi c/\omega_p$, $n = 10^{15}\,\mathrm{cm}^{-3}$.
- **PBWA:** Two laser pulses, $\omega_1 - \omega_2 \sim \omega_p$, $n = 10^{16} - 10^{17}\,\mathrm{cm}^{-3}$. An alternative for LWFA.
- **SMLWFA:** LWFA on higher plasma densities. $n = 10^{19}\,\mathrm{cm}^{-3}$, $I \approx 10^{19}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$, $L > \lambda_p$. The plasma "chops up" the long laser pulse. The length of the equidistently spaced train of smaller pulses mathces the plasma wavelength. This train of pulses resonantly excites the plasma.
- **Multiple bunches or pulses:** larger amplitude plasma waves.

Outline

The Lorentz-Force acting on the electron:

$$\mathbf{F} = e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{1}$$

Equations of Motion for a relativistic electron:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{1}{m_e c^2}\mathbf{F} \cdot \mathbf{v} \tag{2a}$$

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = e\left(\mathbf{E} + \frac{\mathbf{p}}{m_e\gamma} \times \mathbf{B}\right) \tag{2b}$$

$\mathbf{E}(t, \mathbf{r}) = \mathbf{E}(\Theta(t, \mathbf{r}))$ and $\mathbf{B}(t, \mathbf{r}) = \mathbf{B}(\Theta(t, \mathbf{r}))$, respectively, with

$$\Theta(t, \mathbf{r}) := t - \mathbf{n} \cdot \frac{\mathbf{r}}{c}. \tag{3}$$

being the retarded time

The presence of an Underdense Plasma can be taken into account via it's n_m index of refraction! [1]

$$n_m = \sqrt{1 - \frac{\omega_p^2}{\omega_L^2}} \quad \text{and} \quad \omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e} \tag{4}$$

The retarded time, including the index of refraction:

$$\Theta(t, \mathbf{r}, n_m) := t - n_m \mathbf{n} \cdot \frac{\mathbf{r}}{c}. \tag{5}$$

Gaussian beams can be derived from the paraxial approximation. For a Gaussian pulse, the electric field has the following form:

$$E_x = E_0 \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W^2(z)}\right] \exp\left(-\frac{\Theta^2}{T^2}\right) \times \cos\left[\frac{kr^2}{2R(z)} - \Phi(z) + \omega\Theta + \sigma\Theta^2 + \varphi\right] \quad (6a)$$

$$E_y = 0 \quad (6b)$$

$$E_z = -\frac{x}{R(z)} E_x + E_0 \frac{2x}{kW^2(z)} \cdot \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W^2(z)}\right] \times \exp\left[-\frac{\Theta^2}{T^2}\right] \sin\left[\frac{kr^2}{2R(z)} - \Phi(z) + \omega\Theta + \sigma\Theta^2 + \varphi\right] \quad (6c)$$

For details, see [2]

Gaussian beams can be derived from the paraxial approximation. For a Gaussian pulse, the magnetic field has the following form:

$$B_x = 0 \tag{7a}$$

$$B_y = \frac{E_x}{c} \tag{7b}$$

$$B_z = \frac{y}{cR(z)} E_x + \frac{1}{c} E_0 \frac{2y}{kW^2(z)} \cdot \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W^2(z)}\right] \times \exp\left[-\frac{\Theta^2}{T^2}\right] \sin\left[\frac{kr^2}{2R(z)} - \Phi(z) + \omega\Theta + \sigma\Theta^2 + \varphi\right] \tag{7c}$$

For details, see [2]

A Gaussian pulse given with eqs. (6) and (7) is an approximate solution of Maxwell's equations.

The parameters of the Gaussian pulse are the following:

$$W(z) = W_0 \left[1 + \left(\frac{z}{z_R}\right)^2\right]^{1/2} \quad \text{the spot size,} \tag{8a}$$

$$R(z) = z \left[1 + \left(\frac{z_R}{z}\right)^2\right] \quad \text{the radius of curvature,} \tag{8b}$$

$$\Phi(z) = \tan^{-1} \frac{z}{z_R} \quad \text{the Gouy phase, and} \tag{8c}$$

$$W_0 = \left(\frac{\lambda z_R}{\pi}\right)^{1/2} \quad \text{the beam waist.} \tag{8d}$$

and z_R being the Rayleigh-length.

A monochromatic pulse looks like:

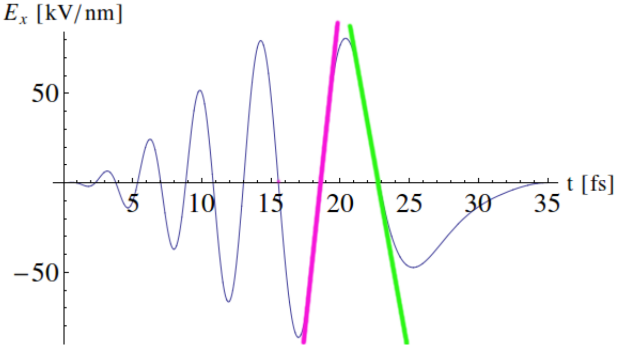


FIGURE : The x-component of the chirped electric field. $\lambda = 800\,\mathrm{nm}$, $T = 35\,\mathrm{fs}$, $I = 10^{17}\,\mathrm{Wcm}^{-2}$, $\sigma = -0.03886\,\mathrm{fs}^{-2}$, $\varphi = 0$.

A bichromatic pulse looks like:

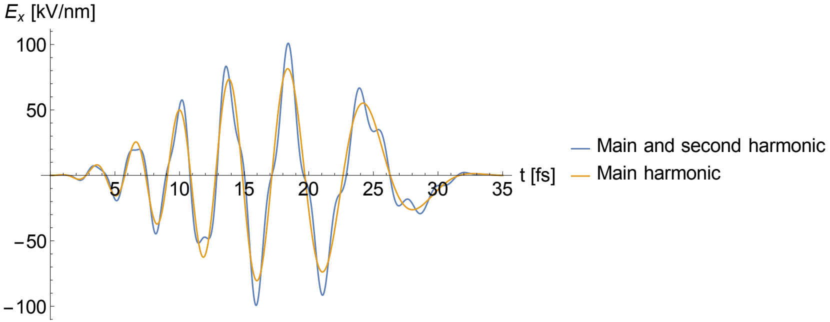


FIGURE : A bichromatic (main and second harmonic) pulse, compared with the corresponding monochromatic (main harmonic) component.

Mathematical form of the Higher Harmonic (HH):

$$\lambda_{\mathrm{HH}} = \frac{\lambda}{q} \quad W_{0,\,\mathrm{HH}} = \left(\frac{\lambda_{\mathrm{HH}} z_R}{\pi} \right)^{1/2} \tag{9}$$

with $q = 2, 3, \cdots$ and $A \in [0, 1]$.

The new beam waist has to be inserted into eqs. (6)–(8c) in order to obtain the HH part. The HH part has to be added to the MH part. Numerically very problematic!

Outline

The relevant plasma densities are far below the critical density. At $\lambda = 800\,\mathrm{nm}$, $n_c = 1.74196 \cdot 10^{21}\,\mathrm{cm}^{-3}$.

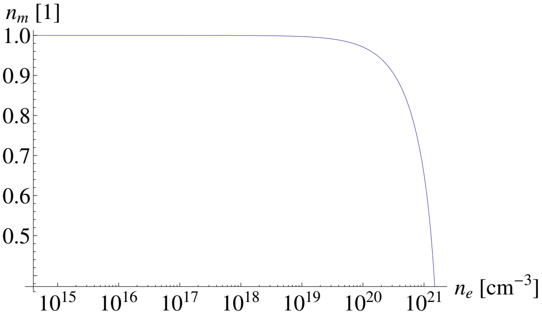


FIGURE : The index of refraction as a function of plasma electron density.

$n_m(10^{15}\,\mathrm{cm}^3) \approx n_m(0) \Rightarrow \Theta(t, \mathbf{r}, n_m) \approx \Theta(t, \mathbf{r})$

ACCELERATION IN UNDERDENSE PLASMAS CAN BE WELL APPROXIMATED BY ACCELERATION IN VACUUM!

For monochromatic laser pulses, the following results can be obtained [3]:

- Negatively chirped pulses provide the best acceleration.
- The electron has to be on-axis and propagate parallel with the pulse.
- Larger beam waists provide more energy gain.
- Shorter pulses provide more energy gain.
- With $\lambda = 800\,\mathrm{nm}$ wavelength, $T = 30\,\mathrm{fs}$ pulse duration, $I = 10^{21}\,\mathrm{W \cdot cm^{-2}}$ intensity and $W_0 = 100\lambda$ beam waist, an energy gain of $270\,\mathrm{MeV}$ pro pulse can be achieved.

	Our Results [3]	Experimental data [4]
Wavelength	$800\,\mathrm{nm}$	$800\,\mathrm{nm}$
Pulse Duration	$30\,\mathrm{fs}$	$55\,\mathrm{fs}$
Intensity	$10^{21}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$	$10^{19}\,\mathrm{W}\cdot\mathrm{cm}^{-2}$
Beam Waist	100λ	$10\,\mathrm{mm}$
Total Pulse Energy	$9.6\,\mathrm{J}$	$10\,\mathrm{J}$
Average Power	$320\,\mathrm{TW}$	$180\,\mathrm{TW}$
Energy gain	$275\,\mathrm{MeV}$ (on $5\,\mathrm{mm}$)	$420\,\mathrm{MeV}$ (on $5\,\mathrm{mm}$) $800\,\mathrm{MeV}$ (on $10\,\mathrm{mm}$)
Accelerating Gradient	$58\,\mathrm{GVm}^{-1}$	$80\,\mathrm{GVm}^{-1}$

OUR RESULTS AGREE WITHIN A FACTOR OF TWO WITH THE EXPERIMENTAL DATA!

Starting with $\lambda = 800\,\mathrm{nm}$ wavelength, $T = 3\,\mathrm{fs}$ pulse duration, $I = 10^{21}\,\mathrm{W} \cdot \mathrm{cm}^{-2}$ intensity, $\varphi = 4.21\,\mathrm{rad}$ CEP and $\sigma_1 = -0.03968\mathrm{fs}^{-2}$, then adding the second harmonic yields [5]:

- The presence of the second harmonic shifted the optimal value of the chirp parameter to a smaller value ($\sigma_1 = -0.00553\mathrm{fs}^{-2}$)
- The energy gain of the electron depends very weakly on the chirp parameter of the second harmonic.
- The CEP has non-trivial optima at $\varphi \approx \pi/3$ and $\varphi \approx 4\pi/3$. Certain values of the CEP yield zero energy gain!
- A bichromatic pulse is capable to transfer about even 30 % more energy than a same intensity monochromatic pulse!

- Conventional radiotherapy devices deliver electron beams with around 2 MeV–20 MeV energy [6,7].
- These electron energies can be achieved even with monochromatic driver pulses.
- Some modern applications use 170–200 MeV energy electron beams which prove they are more efficient.
- These or higher energies can be reached easier with bichromatic driver pulses than with monochromatic ones.
- Plasma electron accelerators with bichromatic driver pulses can make radiotherapy cheaper and be an alternative of the expensive proton (hadron) therapy.

Outline

- A simple but (computationally) efficient model has been presented.
- Negatively chirped monochromatic pulses can transfer up to 270 MeV energy to a single electron.
- The addition of the second harmonic boosts the energy transfer to the electron by even 30 %—it is tempting to use a bichromatic driver pulse for electron acceleration.
- The results obtained with our simple model agree quite well with the experimental data.
- Plasma electron accelerators with bichromatic driver pulse may be used also for radiotherapy or radiography.

Outline

[1] S. Varró, *Nucl. Instr. Meth. Phys. Res. A* ***740*** *(2014),* 280.
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THANK YOU FOR YOUR ATTENTION!