Progress in the Calculation of Collider Processes

Thomas Becher, University of Bern

CHIPP meeting 2009,
Seminarhotel Appenberg, Aug. 24-25, 2009

Progress in the Calculation of Collider Processes

since August 1st

Thomas Becher, University of Bern

CHIPP meeting 2009,
Seminarhotel Appenberg, Aug. 24-25, 2009

Introduction and Apologies

- I *will not* try to review all high-energy theory activities in Switzerland. For the talk to be at least semi-coherent, I will focus on the area of perturbative calculations of collider processes at high energies:
    - strong effort in Switzerland,
    - relevant for LHC,
    - several interesting new developments in the past few years.

PREDICTIONS FOR THE LHC

- Precise predictions are crucial for the physics program at the LHC.
  - QCD effects present the main challenge:
    - rich phenomenology, produces backgrounds to many new physics signals,
    - large coupling constant, leading order perturbation theory is often not sufficient,
    - need to control non-perturbative effects (hadronisation, PDFs, ...).
  - LHC: higher energy and luminosity → higher precision and higher multiplicity final states.

Challenges

- Many loops
  - NNLO to Higgs production
  - NNLO corrections to $e^+e^- \to 3\,\text{jets}$
  - Towards top production at NNLO
- Many legs
  - Unitarity methods, recursion relations
- Many scales
  - Effective theory calculations and resummation of large perturbative logarithms

many legs

Many Legs @ NLO

- A pressing problem in pQCD are NLO calculations of processes with high multiplicity final states
  - pp $\rightarrow \geq 4$ jets, pp $\rightarrow$ (W, Z) + ≥ 3 jets, pp $\rightarrow$ (W W, W Z, WW) + ≥ 2 jets, ...
- Obtaining the NLO description is important because
  - NLO corrections are larger for high-multiplicity final states,
  - they become more abundant at the LHC, and
  - such final states are backgrounds to many New Physics searches.

MANY LEGS

- Standard Feynman diagram approach to perturbative calculations becomes increasingly difficult:

  1. Number of diagrams grows factorially with the number of external legs.
  2. Passarino-Veltman reduction to scalar integrals produces large number of terms and is numerically unstable when external momenta are linearly dependent.

- Two approaches
  - improve traditional method, in particular tensor reduction,
  - abandon diagrammatic technique in favor of an approach based on recursion relations and unitarity.

New Approach: Recycling

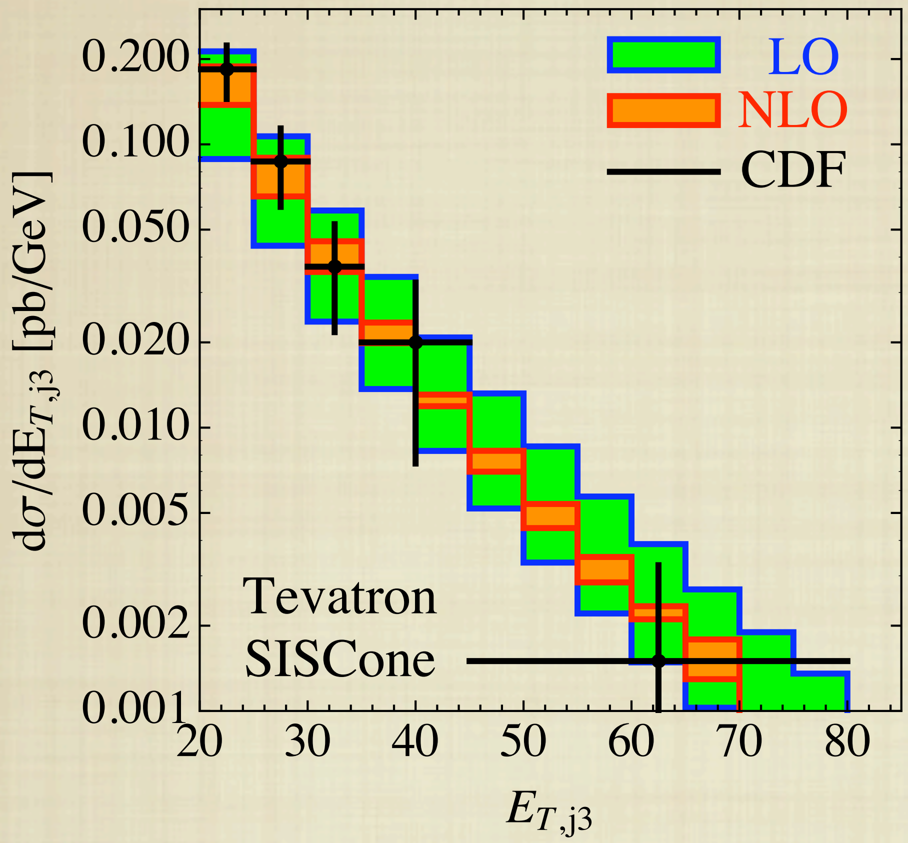
- At tree level, *recursion relations* provide an efficient way to calculate processes with large multiplicites
  - avoids factorial growth in time
- Use recursion relations and generalized unitarity to construct one-loop amplitudes. Bern et al., Britto, Cachazo and Feng. Simple numerical implementation: Ossola, Padadopoulos and Pittau.
  - Recycles trees into loops!
  - Initially, only part of the answer could be obtained. Elegant construction of missing rational part provided by Ellis, Giele, Kunszt, Melnikov '08

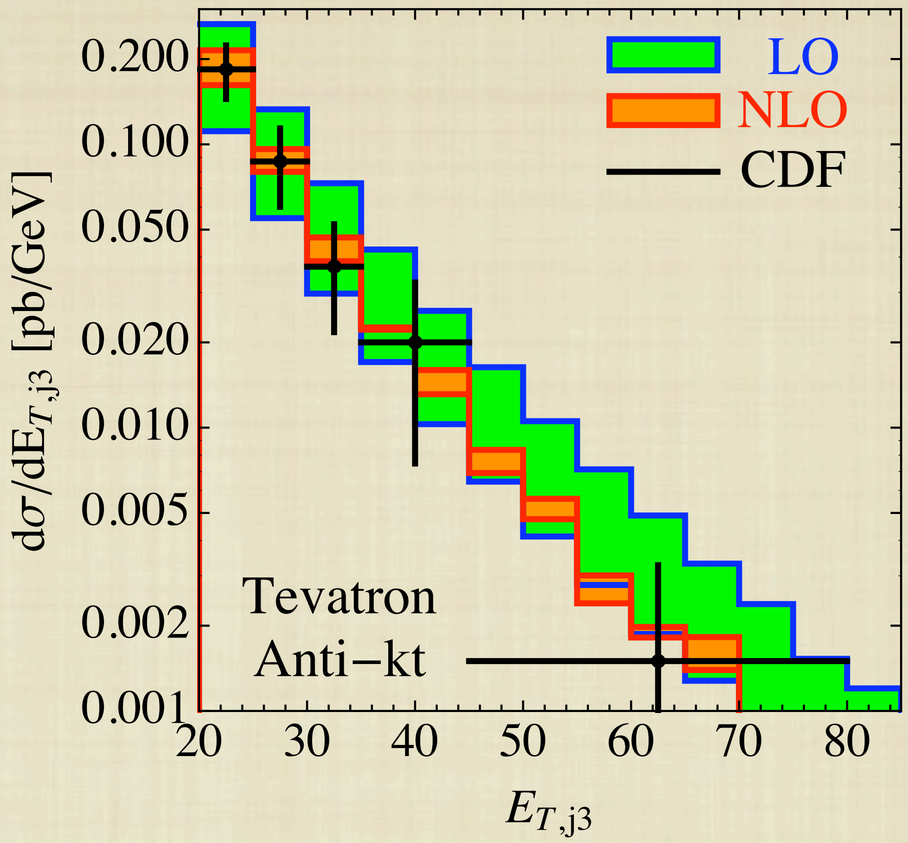
$2 \rightarrow 4$ at Hadron colliders

- This year, the first full results for $2 \rightarrow 4$ processes at hadron colliders were obtained.
  - $pp \longrightarrow t\bar{t}\,b\bar{b}$ Bredenstein, Denner, Dittmaier and Pozzorini. Important background to $pp \rightarrow t\bar{t}\,H$
    - Highly optimized diagrammatic calculation
  - $pp \longrightarrow W + 3\,\text{jets}$ by two groups: Ellis, Melnikov and Zanderighi; *Black Hat Collaboration*: Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre
    - based on unitarity and recursion relations. First calculation is based on method by Ellis, Giele, Kunszt and Melnikov

TEVATRON PREDICTIONS

Ellis, Melnikov and Zanderighi ’09





- Comparison is complicated by the fact that the jet-algorithm used by CDF is not infrared safe.
  - Introduces hard-to-estimate systematical uncertainty into comparison with theory.

many loops

NNLO calculations

- Only a handful of processes are known at NNLO, in particular for *fully differential* cross sections
  - Higgs boson production Anastasiou, Melnikov and Petriello '04,
  - Drell-Yan lepton pair production Melnikov and Petriello '06
  - $e^+e^- \to 2$ jets Anastasiou, Melnikov and Petriello '04, Weinzierl '06
  - $e^+e^- \to 3$ jets Gehrmann, Gehrmann-De Ridder , Glover and Heinrich '07, Weinzierl '09

Higgs production

- Recent theoretical developments
  - Faster codes, suitable for detailed phenomenological studies Anastasiou, Dissertori, Stöckli '08; Grazzini '08
  - Two-loop gg $\to h, H$ amplitudes in the MSSM Anastasiou, Boughezal, Petriello '08
  - Effective theory analysis of the total cross section, all-order resummation of dominant corrections to gg $\to H$ form factor Ahrens, TB, Neubert and Yang '08
  - Calculation of mixed QCD-electroweak corrections Anastasiou, Beerli, Daleo '08
  - Recalculation of finite heavy-quark mass effects Anastasiou, Bucherer, Kunszt '09

Higgs production

- Updated predictions for the NNLO total production cross section by several groups.

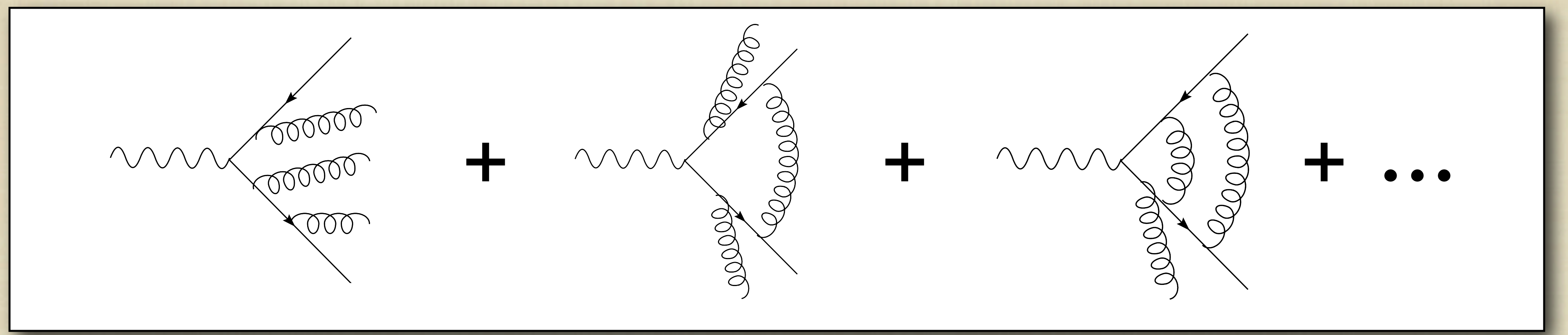
	MRST01	MRST04	MRST06	MSTW08
Tevatron m_H=170 GeV	0.3833	0.3988	0.3943±5%	0.3444±10%
LHC,10 TeV m_H=120 GeV	28.9	29.9	32.6	35.4

Anastasiou, Boughezal, Petriello '08

- Detailed phenomenological studies of NNLO QCD effects in $H \to W^+ W^- \to \ell^+ \nu \ell^- \bar{\nu}$
  - in the Tevatron search Anastasiou, Dissertori, Grazzini, Stöckli, Webber '09
  - at the LHC Anastasiou, Dissertori, Stöckli, Webber '08

$e^+e^- \longrightarrow$ 3 JETS @ NNLO

Gehrmann, Gehrmann De Ridder, Glover, Heinrich'07   Weinzierl '09



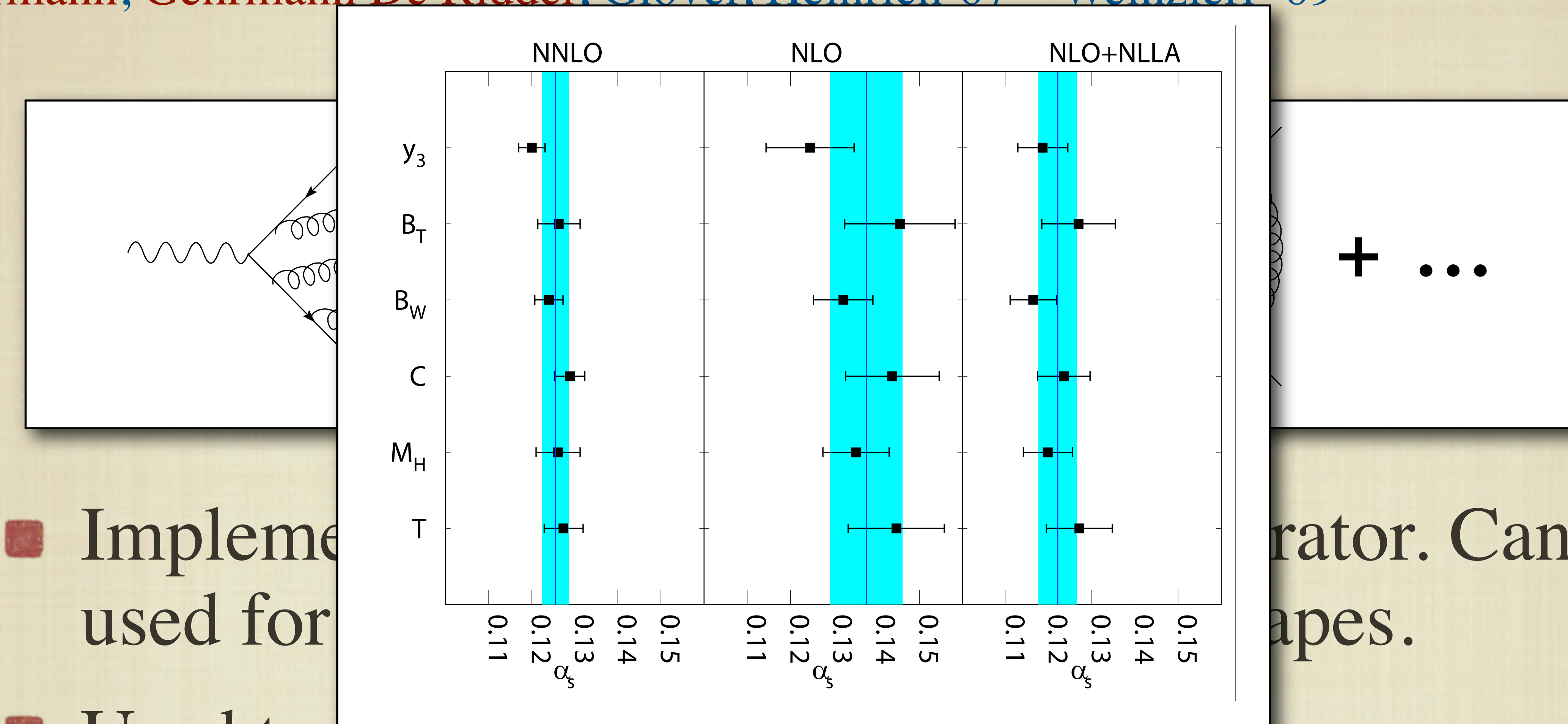
- Implemented in fixed order event generator. Can be used for NNLO evaluation of event shapes.
- Used to determine α_s from LEP data:

$$\alpha_s(M_Z^2) = 0.1240 \pm 0.0008\,(\text{stat}) \pm 0.0010\,(\text{exp}) \pm 0.0011\,(\text{had}) \pm 0.0029\,(\text{theo})$$

- Even at NNLO perturbative uncertainty dominates.

$e^+e^- \longrightarrow$ 3 jets @ NNLO

Gehrmann, Gehrmann De Ridder, Glover, Heinrich’07   Weinzierl ’09



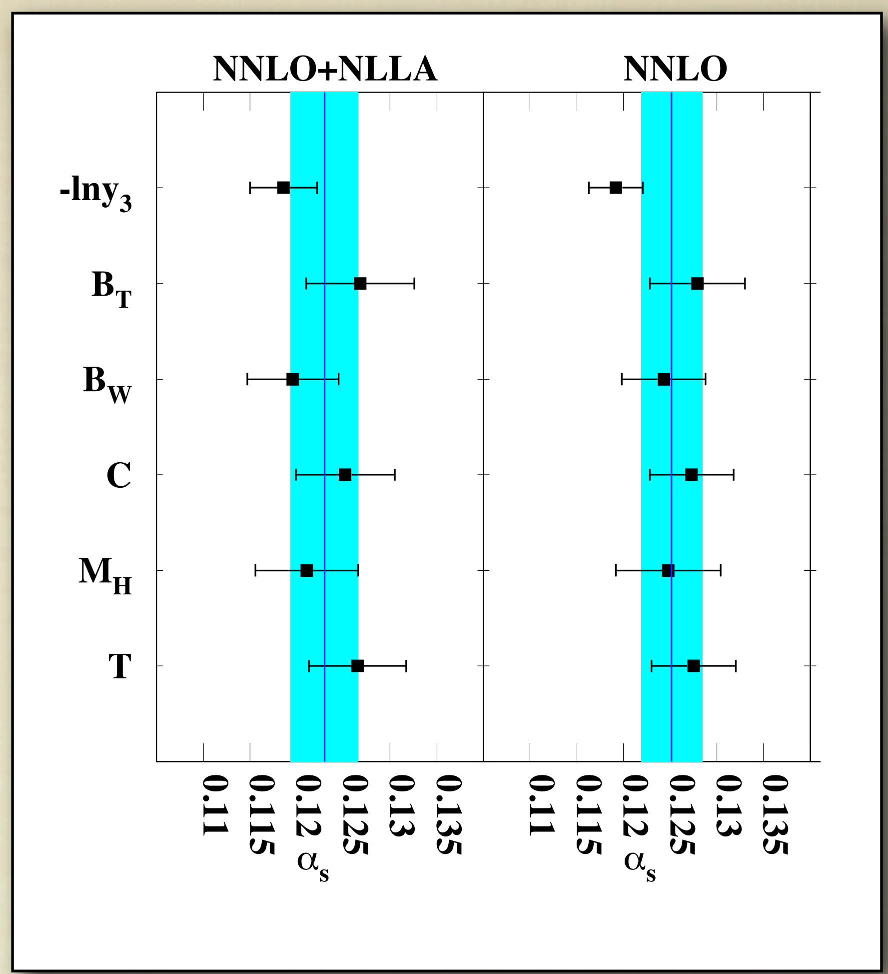
- Impleme… rator. Can be used for … apes.
- Used to determine α_s from LEP data:

$$\alpha_s(M_Z^2) = 0.1240 \;\pm\; 0.0008\,(\text{stat}) \;\pm\; 0.0010\,(\text{exp}) \;\pm\; 0.0011\,(\text{had}) \;\pm\; 0.0029\,(\text{theo})$$

- Even at NNLO perturbative uncertainty dominates.

$e^+e^- \longrightarrow$ 3 JETS

- Recent progress
  - Calculation of NLO electroweak corrections Denner, Dittmaier, Gehrmann, Kurz '09;
  - New fit including NLL resummation Dissertori, Gehrmann, Gehrmann-De Ridder Heinrich, Luisoni, Stenzel '09



$$\alpha_s(M_Z) = 0.1224 \pm 0.0009\,(\mathrm{stat}) \pm 0.0009\,(\mathrm{exp}) \pm 0.0012\,(\mathrm{had}) \pm 0.0035\,(\mathrm{theo})$$

  - without resummation NNLO value is 0.1228
  - NLL result has slightly *larger* uncertainty

Top production @ NNLO

- So far, Tevatron has been the only source, but in the future the LHC will produce millions of $t\bar{t}$ pairs.

$$\sigma_{t\bar{t}}^{\text{NLO}}(\text{LHC}, m_{\text{top}} = 171\text{GeV}) = 875^{+102(11.6\%)}_{-100(11.5\%)}(\text{scales})^{+30(3.4\%)}_{-29(3.3\%)}(\text{PDFs})\,\text{pb}$$

Cacciari et al. '08

- Efforts are underway to increase the accuracy of the theoretical prediction to NNLO:
  - (1-loop)^2 Anastasiou, Mert Aybat '08, [and other papers!]
  - 2-loop $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ numerically Czakon '08
  - Leading-color $q\bar{q} \to t\bar{t}$ amplitudes analytically Bonciani, Ferroglia, Gehrmann and Studerus '09


accel.
Rit.
Fer - ne komm' ich
mf espress.
rit.
her, ü-ber Gip - fel, ü-ber Schlun - - de, ü-ber ein
wogend

many scales

Effective Field theory

- Fixed order calculations become unreliable in situations where several disparate scales are relevant. Higher order corrections enhanced by large logarithms of scale ratios.
- Effective field theories can be used to
  - expand in powers of small scale ratios,
  - resum the enhanced higher-order corrections.
- Many examples of EFTs in particle physics...
  - Euler-Heisenberg action, Nonrelativistic EFTs, Fermi theory, Heavy-Quark Effective Theory, ...
- ... but not (yet!) a common tool in collider physics

Soft-Collinear Effective Theory

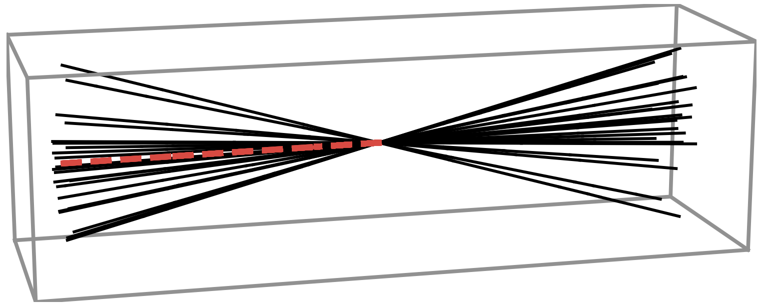
Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke et al. 2002

- EFTs split physics into high- and low-energy part. In collider processes, we have an *interplay of three momentum regions*
  - Hard } high-energy
  - Collinear
  - Soft

  } low-energy part
- Correspondingly, EFT for such processes has two low-energy modes:
  - Collinear fields describing the energetic partons propagating in a given direction, and
  - soft fields which mediate long range interactions among them.

Resummation for Thrust

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



$$1 - T \approx \frac{M_1^2 + M_2^2}{Q^2}$$

- The perturbative result for the thrust distribution contains logarithms $\alpha_s^n \ln^{2n} \tau$, where $\tau = 1\text{-}T$.
  - Near the end-point $\tau \to 0$ these logarithmic terms dominate.
- Using SCET one can derive a factorization theorem

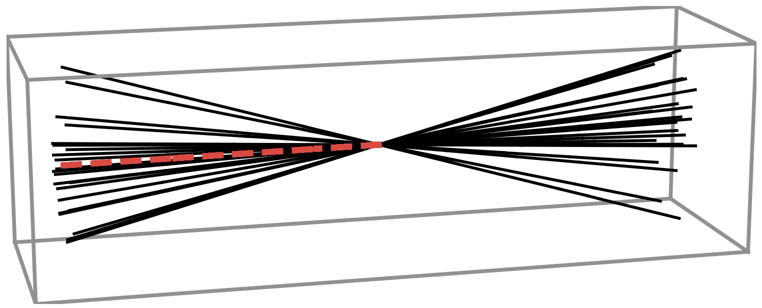
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2\, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T(\tau\, Q - \frac{M_1^2 + M_2^2}{Q}, \mu)$$

Q^2	$\gg$	$M_1^2 \sim M_2^2 \sim \tau\, Q^2$	$\gg$	$\tau^2 Q$
hard		collinear		soft

  - scale dependence governed by RG

Resummation for Thrust

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$



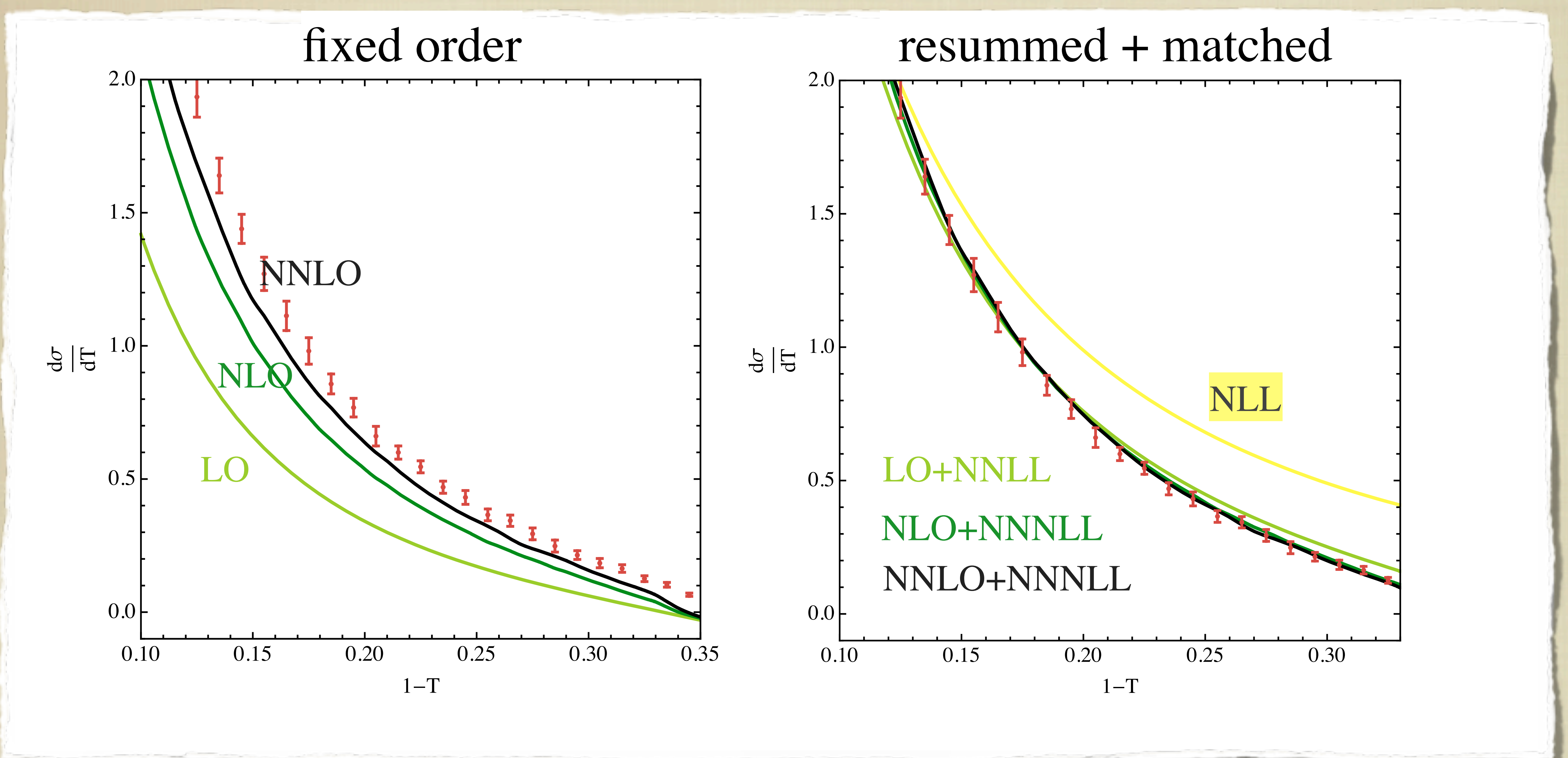
$$1 - T \approx \frac{M_1^2 + M_2^2}{Q^2}$$

- Prediction for event-shape variables dominated by perturbative uncertainty even at NNLO.
  - Traditional methods Catani et al. '93 allowed resummation to NLL but not beyond.
- Using RG evolution in SCET we were able to derive NNNLL resummed distribution matched to NNLO TB and Schwartz '08. Fit to LEP data gives

$$\alpha_s(m_Z) = 0.1172 \pm 0.0010(\text{stat}) \pm 0.0008(\text{sys}) \pm 0.0012(\text{had}) \pm 0.0012(\text{pert})$$
$$= 0.1172 \pm 0.0022\,.$$

most precise α_s at high energy. theory unc. no longer dominant

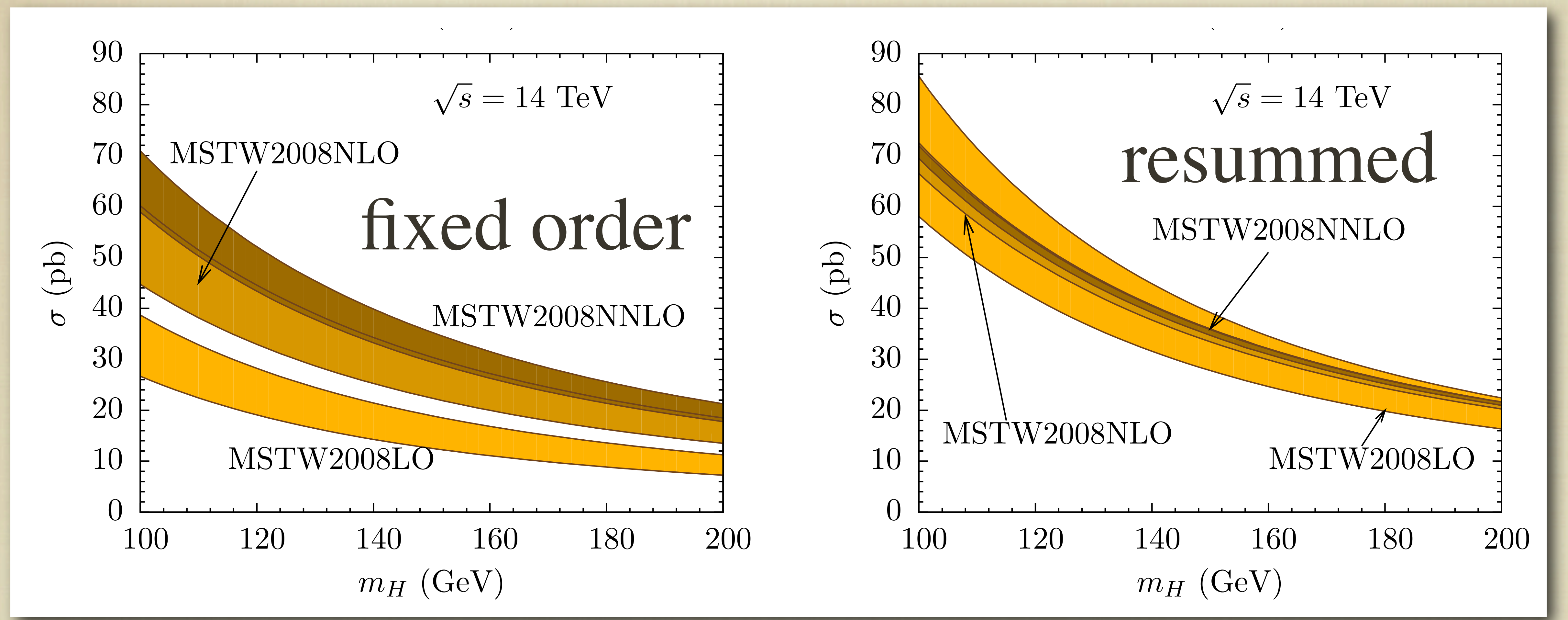
Resummed vs. Fixed Order



- For PDG value $\alpha_s(M_Z)$=0.1176
- This is the region relevant for α_s determination

Higgs production

Ahrens, TB, Neubert, Yang '08



- Includes soft-gluon resummation, but the main effect arises from unconventional scale setting in hard function.
- RG improved NNLO result is 8% larger than fixed order (13% at Tevatron).

Towards n-Jet Processes

- Many collider physics applications of SCET in the past few years. Resummations up to N^3LL, however only for two jet observables, e.g.
  - Drell-Yan rapidity dist. — TB, Neubert, Xu ‘07
  - inclusive Higgs production — Idilbi, Ji, Ma and Yuan ‘06 ; Ahrens, TB, Neubert, Yang ‘08
  - thrust distribution in e^+e^- — TB, Schwartz ’08
- Important to extend method to more complicated processes
  - resummation should be more relevant for more exclusive quantities.
  - parton showers can be used at LL level.

IR singularities of QCD amplitudes

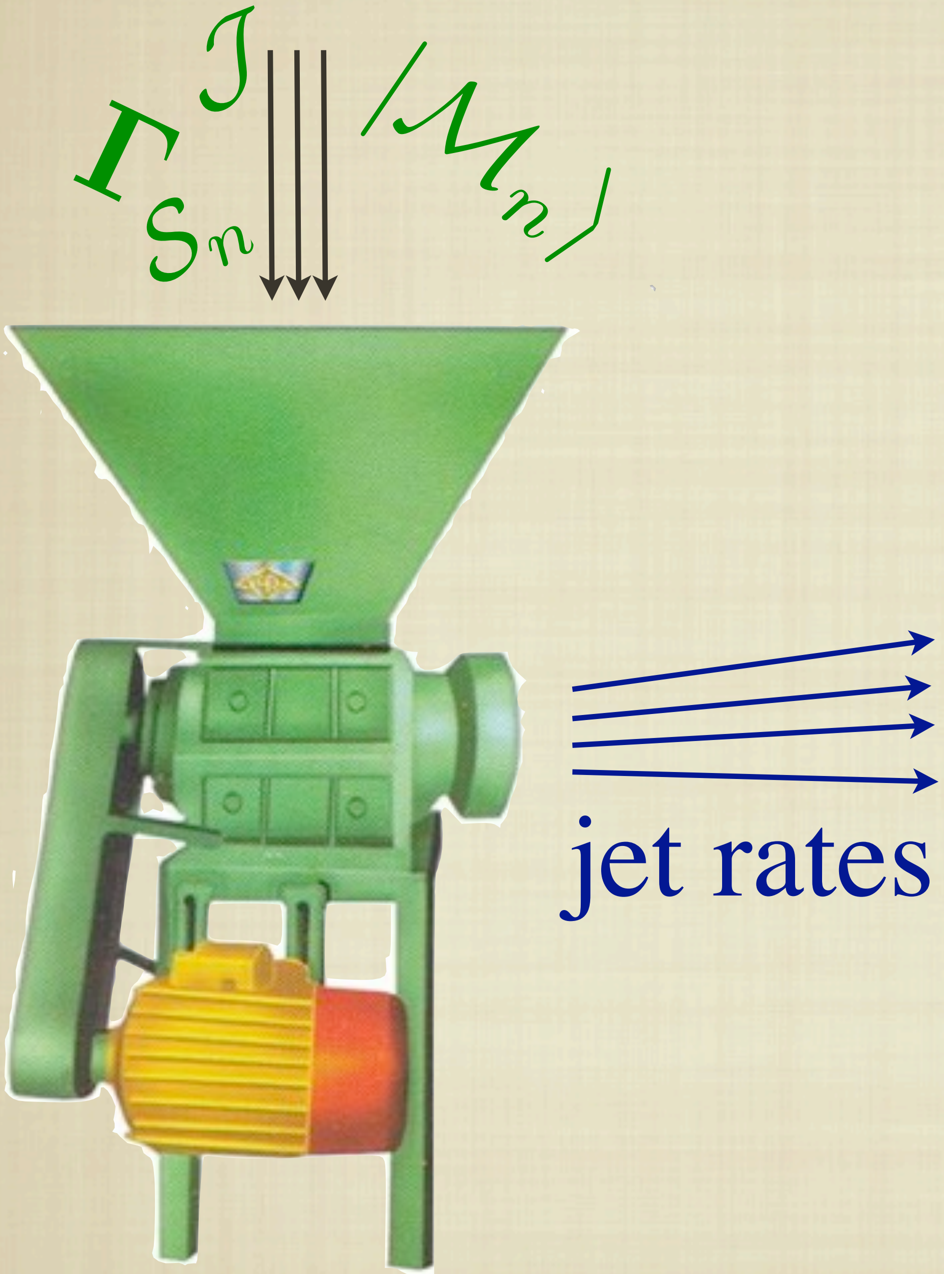
TB, Neubert '09

- Simple conjecture for all-order structure of infrared singularities of massless *n*-point amplitudes:
  - governed by anomalous dimension $\boldsymbol{\Gamma}$ of *n*-jet operators in SCET
  - stringent factorization constraints on form of $\boldsymbol{\Gamma}$.
  - have performed order-by-order analysis of the constraints to three-loop order.
- Knowledge of the anomalous dimension will allow us to perform Sudakov resummation for *n*-jet processes
  - have explicit result for $\boldsymbol{\Gamma}$ for NNLL resummation

N^KLL for N-jet processes

- The necessary ingredients are
  - **hard functions**: from fixed-order results for on-shell amplitudes. New unitarity methods allow calculation of one-loop amplitudes with many legs ($\rightarrow$ NNLL resummation)
  - **jet function:** imaginary part of two-point function, inclusive jet function is known to two loops.
  - **soft function:** matrix element of Wilson lines, one-loop calculation is comparatively simple.
- Then resum log's of different scales using RG evolution.

Automatization



- in the longer term, this will hopefully lead to automated higher-log resummations for jet rates
- goes beyond parton showers, which are only accurate at LL, even after matching
- predicts jets, not individual partons

Summary

- First full results for $2 \rightarrow 4$ processes at NLO
  - $pp \rightarrow t\bar{t}\,b\bar{b}$
  - $pp \rightarrow W + 3\,\text{jets}$
- Updated NNLO predictions for Higgs production, detailed phenomenological studies.
- Event shapes in e^+e^-: NNLO QCD, NLO electroweak effects, N^3LL resummation for thrust, ...
  - ... we are catching up with LEP precision.
- On track to perform higher-log resummation for n-jet processes at LHC using RG evolution SCET.