$B^0 - \bar{B}^0$ mixing: three loop QCD SR analysis

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Motivation

SM is complete: Era of precision tests

B-physics is beautiful, interesting and suitable

Plenty of data: LHC, BaBar, Belle I,II,...

Stumbling block – QCD: twofold difficulties

i) PT part: α_s is not small, SM has differ scales,
$m_t = 170\ \mathrm{GeV}$, $m_b = 5\ \mathrm{GeV}$, $\Lambda_{\mathrm{QCD}} \sim 0.5\ \mathrm{GeV}$,
expansions are in $\alpha_s \ln(m_t/m_b)$ and
eventually $\alpha_s \ln(m_t/\mu)$ with $\mu \sim \Lambda_{\mathrm{QCD}}$
ii) NonPT: quark-gluons vs hadrons

The point (i) is technical, while (ii) still (un/partly)solved

How does it work for $B^0 - \bar{B}^0$?

$B^0 - \bar{B}^0$ mixing phenomenology: what we see

Produced is $(B^0, \bar{B}^0)$ that evolves

$$i\frac{d}{dt}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = H_{eff}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

with H_{eff} being a 2×2 (nondiagonal !) matrix

$$H_{eff} = (M - i\Gamma/2)_{ij}\,, \quad i,j = 1,2$$

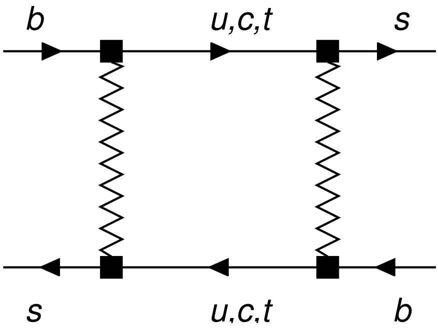
Eigenstates are (B_L, B_H) with fuzzy beauty

Observables of $B^0 - \bar{B}^0$ system:

mass difference: $\Delta m = M_{heavy} - M_{light} \approx 2\,|M_{12}|$
decay rates difference:
$\Delta\Gamma = \Gamma_L - \Gamma_H \approx -2\,|\Gamma_{12}|\cos\Phi$, $\Phi = \arg(-M_{12}/\Gamma_{12})$

$B^0 - \bar{B}^0$ mixing: SM (EW) picture

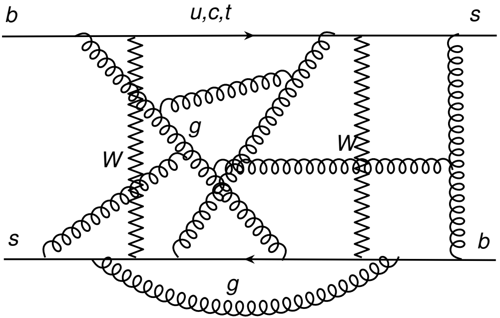
EW skeleton diagram



Famous box diagram that describes flavor changing

$B^0 - \bar{B}^0$ mixing: SM (EW+QCD) picture

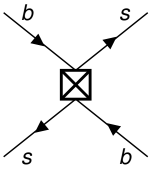
Full SM diagrams with QCD corrections



$m_t = 170$ GeV, $m_b = 5$ GeV, $\Lambda_{\rm QCD} \sim 0.5$ GeV
Expansion parameter enhanced $\sim \alpha_s \ln(m_t/m_b)$

Eff Theory simplification

Heavy fields (t, W)



integrated away

$$H_{eff} = \frac{G_F^2 M_W^2}{4\pi^2} \left(V_{tb}{}^* V_{td}\right)^2 C(m_t, m_W, m_b, \alpha_s)\{\bar{b}_L \gamma_\sigma d_L \bar{b}_L \gamma^\sigma d_L\}$$

$C(m_t, m_W, m_b, \alpha_s(\mu))$ is known at NLO (two loop graphs)

Thus one needs hadronic ME $\langle \bar{B}^0 | \bar{b}_L \gamma_\sigma d_L \bar{b}_L \gamma^\sigma d_L (m_b) | B^0 \rangle$

Still $m_b \gg \Lambda$ and one can use PT

Eff Theory simplification: HQET

PT part of ME is extracted by matching to HQET:

$$\bar{b}_L\gamma_\sigma d_L \bar{b}_L\gamma^\sigma d_L(m_b) = \left(1 - \frac{7}{2}\frac{\alpha_s}{\pi}\right)\{\bar{h}_L^+\gamma_\sigma d_L \bar{h}_L^-\gamma^\sigma d_L\}$$
$$-\frac{3}{2}\frac{\alpha_s}{\pi}\{\bar{h}_L^+ d_R \bar{h}_L^- d_R\}$$

h^+, h^- (remnant) fields for quark b,anti-quark b

HQET operators are at the scale of order $\Lambda_{\rm QCD}$
$\to$ genuine nonPT method is required

ME is not computable in model independent way

One computes Green function and extract hadronic ME

Two ways: SR and the lattice

ME

Three-point correlator

$$K = \int d^d x_1\, d^d x_2\, e^{ip_1 x_1 - ip_2 x_2} \langle 0 | T \tilde{\jmath}_2(x_2) \tilde{Q}_1(0) \tilde{\jmath}_1(x_1) | 0 \rangle$$

of a four-quark operator $\tilde{Q}_1 = \bar{h}_+ \gamma_\beta d_L \bar{h}_- \gamma_\beta d_L$ and interpolating current $\tilde{\jmath}$ with

$$\tilde{\jmath}_1(\mu) = \bar{d}\gamma_5 h_+, \qquad \tilde{\jmath}_2(\mu) = \bar{d}\gamma_5 h_-$$

and overlap

$$\langle 0 | \tilde{\jmath}_1(\mu) | \bar{B}(p) \rangle = F(\mu)$$

The dispersion relation for Euclidean times $\tau_{1,2}$ $(\tau = it)$

$$K(\tau_1, \tau_2) = \int_0^\infty d\omega_1\, d\omega_2\, e^{-\omega_1 \tau_1 - \omega_2 \tau_2}\, \rho(\omega_1, \omega_2) + (\text{p. c.})$$

determines the spectral density $\rho(\omega_1, \omega_2)$

Hadronic picture: B-meson pole plus continuum

$$\rho_H(\omega_1, \omega_2) = F^2 \langle \bar{B}^0 | \tilde{Q}_1 | B^0 \rangle \delta(\omega_1 - \bar{\Lambda}) \delta(\omega_2 - \bar{\Lambda}) + \rho_{\rm cont}(\omega_1, \omega_2)$$

Lattice computes $K(\tau_1, \tau_2)$ and fits B-contribution

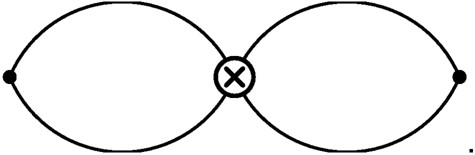
SR method explicitly computes $K(\omega_1, \omega_2)$ and analytically continues it to find

$$F^2 \langle B | \tilde{Q}_1 | B \rangle = \int d\omega_1 d\omega_2 \, \rho^{\rm OPE}(\omega_1, \omega_2).$$

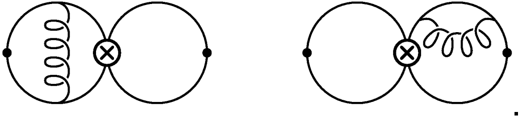
Why are SR still competitive quantitatively?

OPE diagrams

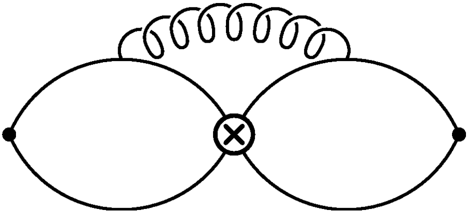
LO:



NLO fact:



NLO nonfact:



Structure of OPE diagrams

OPE diagrams fall into two categories

$$K(\omega_1, \omega_2) = K_{\rm fac}(\omega_1, \omega_2) + \Delta K(\omega_1, \omega_2)$$

The factorized part has the explicit form

$$K_{fac}(\omega_1, \omega_2) = \left(1 + \frac{1}{N_c}\right) \times \Pi(\omega_1)\Pi(\omega_2)$$

with $\Pi(\omega_i)$ - a 2-point correlator

$$p^\alpha \Pi(\omega) = i \int dx e^{ipx} \langle T \tilde{\jmath}(x) \bar{h} \gamma^\alpha (1 - \gamma_5) d(0) \rangle$$

.

SR for the factorized piece $K_{\rm fac}(\omega_1, \omega_2)$ yields $B = 1$.

Results: analytical expressions

We have computed these three loop diagrams for three point correlator (NLO result)

$$\rho(\omega_1,\omega_2) = \left(1+\frac{1}{N_c}\right)\rho(\omega_1)\rho(\omega_2) + \Delta\rho(\omega_1,\omega_2)$$

$$= \left(1+\frac{1}{N_c}\right)\rho(\omega_1)\rho(\omega_2)\left(1-\frac{\alpha_s}{4\pi}\frac{N_c-1}{2N_c}\left(\frac{4}{3}\pi^2-5\right)\right)$$

.

The result is rather simple and ω independent
(only for LL operator)

Numerically $\left(1-2.72\frac{\alpha_s}{4\pi}\right) = 1-0.68\frac{\alpha_s(\mu\sim 1\ \mathrm{GeV})}{\pi}$

Results: numerical values

PT contribution (3-loop)

$$\Delta B_{PT} = -0.10 \pm 0.02 \pm 0.03$$

Quark condensate (2-loop)

$$\Delta B_q = -0.002 \pm 0.001$$

Other condensates (tree-level+2-loop gluon cond)

$$\Delta B_{nonPT} = -0.006 \pm 0.005$$

Total

$$\Delta B = -0.11 \pm 0.04 \pm 0.03$$

Comparison to lattice

Bag parameter SR: $B = 1 - (0.11 \pm 0.04)$

Invariant parameter: $\hat{B} = ZB = 1.34 \pm 0.06$

$$Z = \alpha_s(m_b)^{-\frac{\gamma_0}{2\beta_0}} \left(1 + \frac{\alpha_s(m_b)}{4\pi}\left(\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2}\right)\right)$$

$Z = 1.51$ at $\alpha_s(m_b) = 0.2$.

Latest lattice (A.Bazavov et al. (2016))

$$\hat{B}_{latt} = 1.38(12)(6)$$

Other lattice results

$$\hat{B}_{latt} = 1.26(9) \quad (S.Aoki\ et\ al.,\ Review,\ 2016)$$

$$\hat{B}_{latt} = 1.30(6) \quad (2009(P.Lepage),\ 2015(Y.Aoki))$$

Discussion

SR for $\bar{B}^0 - B^0$ mixing are numerically competitive because of special structure of the OPE: one can take the factorized part out.

The nonfact part ΔB is small and gives useful estimate even if its uncertainty is large
Lattice cannot split correlators

The accuracy for ME ($\hat{B}$ parameter) is better than 10%
$\hat{B}_{SR} = 1.34 \pm 0.06, \qquad \hat{B}_{latt} = 1.26(9), 1.38(12)(6)$

New feature: NLO for PT coefficients is not sufficient
$C_{\mathrm{QCD}\to\mathrm{HQET}} = 1 - \frac{7}{2}\frac{\alpha_s}{\pi} \approx 1 - 0.35 + ? \to 1 - 0.35 \pm 0.1?$

Status and outlook

SR technology works and numerically competitive independent check/confirmation of lattice results

One can compute other operators (not LL):
i) for width differences; ii) for new physics

Prospects/plans

- ✓ Computation of 3-loop correlators for other operators (competition with lattice)

.
Definitely a must for few % precision
(pure PT results)

- ✓ 2-loop matching of QCD operators to HQET
- ✓ 3-loop anomalous dimensions in HQET