Inclusive Semitauonic B-meson decays up to order $\mathcal{O}(\Lambda^3_{QCD}/m^3_b)$

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7th QFET workshop,
Dortmund, 30.11.2016

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Motivation

- 4.0σ tension between SM prediction and experimental data on the combination of the ratios $R(D)$ and $R(D^*)$:

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu})}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu})}$$

	Experiment [HFAG 2016]	SM
$R(D)$	$0.397 \pm 0.040 \pm 0.028$	0.300 ± 0.008
$R(D^*)$	$0.316 \pm 0.016 \pm 0.010$	0.250 ± 0.003

- LEP measurement:

$$\mathcal{B}(b \to X\tau^+\bar{\nu}) = (2.41 \pm 0.23)\%$$

- The $R(D^{(*)})$ data imply:

$$\mathcal{B}(B^+ \to D\tau^+\nu_\tau) + \mathcal{B}(B^+ \to D^*\tau^+\nu_\tau) = (2.70 \pm 0.17)\%$$

$\rightarrow$ We re-evaluate of the SM prediction for $B \to X_c\tau\nu_\tau$ in the kinetic scheme including contributions of order $\mathcal{O}(\Lambda^3_{QCD}/m_b^3)$

Calculation of the decay rate of $B \to X_c \ell \bar{\nu}$

- Electroweak effective Hamiltonian $B \to X_c \ell \nu_\ell$ decay:

$$H_w = \frac{4 G_F V_{cb}}{\sqrt{2}} J_H^\alpha J_{L\alpha}, \quad J_H^\alpha = \bar{c}\gamma^\alpha (1-\gamma_5) b, \quad J_L^\alpha = \bar{\ell}\gamma^\alpha (1-\gamma_5)\nu$$

- Triple differential distribution:

$$\frac{d\Gamma}{dE_\nu dq^2 dE_\ell} = \frac{G_F^2 |V_{cb}|^2}{16\pi^3} W_{\mu\nu} L^{\mu\nu}, \quad q = p_\ell + p_\nu$$

- Hadronic tensor

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_b - q - p_{X_c}) \langle B | J_H^{\mu\dagger} | X_c \rangle \langle X_c | J_H^\nu | B \rangle \sim \mathrm{Im} \int dx\, e^{-iq\cdot x} \langle B | T\{ J_H^{\mu\dagger}(x) J_H^\nu(0)\} | B \rangle$$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 v^\mu v^\nu + i W_3 \epsilon^{\mu\nu\rho\sigma} v_\rho q_\sigma + W_4 q^\mu q^\nu + W_5 (q^\mu v^\nu + q^\nu v^\mu)$$

Triple differential decay rate

$$\frac{d\Gamma}{dE_\nu dq^2 dE_\ell} = \frac{G_F^2 |V_{cb}|^2}{2\pi^3} \Big\{ q^2 W_1 + \Big(2E_\ell E_\nu - \frac{q^2}{2} \Big) W_2 + q^2 (E_\ell - E_\nu) W_3$$
$$+ \frac{1}{2} m_\ell^2 \Big(-2W_1 + W_2 - 2(E_\nu + E_\ell) W_3 + (q^2 - m_\ell^2) W_4 + 4 E_\nu W_5 \Big) \Big\}$$

OPE for hadronic tensor [Dassinger, Mannel, Turczyk (2006)]

Background field c-quark propagator:

$$iS_{BGF} = \frac{1}{\not{Q} + i\not{D} - m_c}, \quad Q = m_b v - q$$

- Expanding of the iS_{BGF} to n^{th} order in $1/m_b$
- Evaluating the forward matrix elements of operators

$$\langle B(p)|\bar{b}_{v,\alpha}(iD_{\mu_1})\ldots(iD_{\mu_n})b_{v,\beta}|B(p)\rangle$$

  in terms of the nonperturbative parameters for a certain order in $1/m_b$
- Basic parameters (up to $1/m_b^3$):

$$2M_B\mu_\pi^2 = -\langle B(p)|\bar{b}_v(iD)^2 b_v|B(p)\rangle$$

$$2M_B\mu_G^2 = \langle B(p)|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v|B(p)\rangle$$

$$2M_B\rho_D^3 = \langle B(p)|\bar{b}_v(iD_\mu)(ivD)(iD^\mu)b_v|B(p)\rangle$$

$$2M_B\rho_{LS}^3 = \langle B(p)|\bar{b}_v(iD_\mu)(ivD)(iD_\nu)(-i\sigma^{\mu\nu})b_v|B(p)\rangle$$

Structure functions

$$W_i = w_i^{(0)} + \frac{\mu_\pi^2}{m_b^2}w_i^{(\pi)} + \frac{\mu_G^2}{m_b^2}w_i^{(G)} + \frac{\rho_D^3}{m_b^3}w_i^{(D)} + \frac{\rho_{LS}^3}{m_b^3}w_i^{(LS)}$$

Phase space limits

Dimensionless variables:

$$y = \frac{2E_\tau}{m_b}, \quad x = \frac{2E_\nu}{m_b}, \quad \eta = \frac{m_\tau^2}{m_b^2}, \quad \rho = \frac{m_c^2}{m_b^2}, \quad \hat{q}^2 = \frac{q^2}{m_b^2}$$

$$y_\pm = \frac{1}{2}\left(y \pm \sqrt{y^2 - 4\eta}\right)$$

- Neutrino energy E_ν in $q^2 - E_\tau$ plan

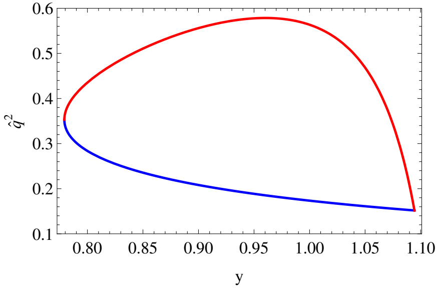
$$\frac{\hat{q}^2 - \eta}{2(y + \sqrt{y^2 - \eta})} \leq x \leq \frac{\hat{q}^2 - \eta}{2(y - \sqrt{y^2 - \eta})}$$

- Momentum transfer q^2

$$y_-\left(1 - \frac{\rho}{1 - y_-}\right) \leq \hat{q}^2 \leq y_+\left(1 - \frac{\rho}{1 - y_+}\right)$$

- τ-lepton energy

$$2\sqrt{\eta} \leq y \leq (1 + \eta - \rho)$$



Decay width of the $B \to X_c \tau \nu_\tau$

Our final result

$$\Gamma(B \to X_c \tau \nu_\tau) = \Gamma_0 \left(1 + A_{ew}\right) \left[C_0^{(0)} + \frac{\alpha_s}{\pi} C_0^{(1)} + C_{\mu_\pi^2} \frac{\mu_\pi^2}{m_b^2} + C_{\mu_G^2} \frac{\mu_G^2}{m_b^2} + C_{\rho_D^3} \frac{\rho_D^3}{m_b^3} + C_{\rho_{LS}^3} \frac{\rho_{LS}^3}{m_b^3} \right]$$

- $\Gamma_0 = \dfrac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3}$
- Coefficients C_i depend on ρ and η
- Calculation reveals: $C_{\rho_{LS}^3} = 0$
- Radiative corrections are taken from [M. Jezabek, L. Motyka (1997)]

Numerical analysis and results

Numerical analysis is done in the kinetic scheme;
We use input of [A. Alberti, P. Gambino, K. J. Healey, S. Nandi (2015)]

$C_0^{(0)}$	$C_0^{(1)}$	$C_{\mu_\pi^2}$	$C_{\mu_G^2}$	$C_{\rho_D^3}$	$C_{\rho_{LS}^3}$
0.152	−0.135	−0.076	−0.586	−7.52	0

- Our prediction

$$\mathrm{Br}(B^\pm \to X_c \tau^\pm \nu_\tau) = (2.37 \pm 0.07)\,\%$$

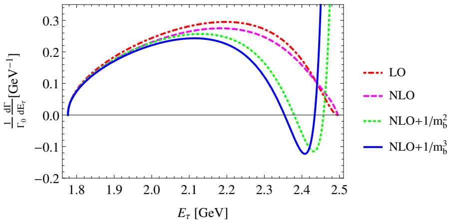
- LEP measurement

$$\mathrm{Br}(B^\pm \to X_c \tau^\pm \nu_\tau) = (2.41 \pm 0.23)\,\%$$

- [Z. Ligeti, J. Tackmann (2014)] (up to $1/m_b^2$ in $1S$ scheme)

$$\mathcal{B}(B^- \to X_c \tau \bar{\nu}) = (2.42 \pm 0.06)\,\%$$

- τ-lepton energy distribution

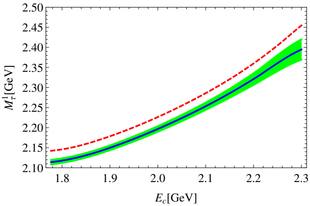


Numerical analysis and results

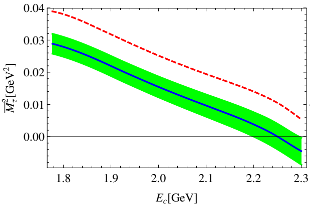
The moments of the τ-lepton energy distribution as functions of the cut energy E_c :

$$M_\ell^n \equiv \langle E_\ell^n \rangle_{E_\ell > E_c} = \frac{\int_{E_c}^{E_m} dE_\ell \, E_\ell^n \, \dfrac{d\Gamma}{dE_\ell}}{\int_{E_c}^{E_m} dE_\ell \, \dfrac{d\Gamma}{dE_\ell}}, \qquad \bar{M}_\ell^n \equiv \langle (E_\ell - \langle E_\ell \rangle)^n \rangle_{E_\ell > E_c}$$

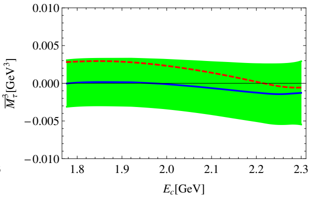
$M_\tau^1(E_c)$



$\bar{M}_\tau^2(E_c)$



$\bar{M}_\tau^3(E_c)$



Turning to the exclusive modes

- The summary concerning the exclusive $B \to D^{(*,**)}\tau\nu_\tau$ decays

	Theory (SM)	Experiment (HFAG + PDG)
$\mathrm{Br}(B^\pm \to D^0\tau^\pm\nu_\tau)$ $\mathrm{Br}(B^\pm \to D^{*0}\tau^\pm\nu_\tau)$	$(0.75 \pm 0.13)\ \%$ $(1.25 \pm 0.09)\ \%$	$(0.90 \pm 0.12)\ \%$ $(1.80 \pm 0.12)\ \%$
$\mathrm{Br}(B^\pm \to (D^0 + D^{*0})\tau^\pm\nu_\tau)$	$(2.00 \pm 0.16)\ \%$	$(2.70 \pm 0.17)\ \%$
$\sum_{D^{**}} \mathrm{Br}(B^\pm \to D^{**0}\tau^\pm\nu_\tau)$	$(0.14 \pm 0.03)\ \%$	—

- SM prediction for the sum of exclusive modes

$$\mathcal{B}(B^\pm \to D^0\tau^\pm\nu_\tau) + \mathcal{B}(B^\pm \to D^{*0}\tau^\pm\nu_\tau) + \sum_{D^{**}} \mathcal{B}(B^\pm \to D^{**0}\tau^\pm\nu_\tau) = (2.14 \pm 0.16)\ \%$$

- vs our prediction for inclusive mode

$$\mathrm{Br}(B^\pm \to X_c\tau^\pm\nu_\tau) = (2.37\ \pm 0.07)\ \%$$

Outlook

- The decay width of the inclusive $B \to X_c \tau \nu_\tau$ including higher $\mathcal{O}(\Lambda^3_{QCD}/m_b^3)$ power correction predicted (in the kinetic scheme)
- The moments of τ-lepton energy distribution calculated
- Results obtained compared with corresponding data on exclusive modes
- Implementation of the possible contribution of New physics is in progress:
  - Searching for the NP operators solving the tension in $R(D)$ and $R(D^*)$
  - Implementation of these operators in the analysis of the inclusive mode