# $B_s \to K \ell \bar{\nu}_\ell$ and $B_{(s)} \to \pi(K)\ell^+\ell^-$ at large recoil and CKM matrix elements

Aleksey Rusov

University of Siegen, Germany

7th QFET Workshop
Dortmund, 30 November 2016

In collaboration with Alexander Khodjamirian

UNIVERSITÄT SIEGEN

DFG FOR 1873

# Motivation

- $\overline{B}_s \to K^+ \ell^- \bar{\nu}_\ell$ is an additional source of the extraction of the CKM matrix element $|V_{ub}|$
- Analysis of the observables in the flavour changing neutral current (FCNC) induced decays $B \to \pi \ell^+ \ell^-$, $B \to K \ell^+ \ell^-$ and $B_s \to K \ell^+ \ell^-$ also allows to additionally constrain the relevant CKM matrix elements
- One needs to calculate accurately the hadronic input: form factors and hadronic amplitudes of nonlocal effects

# Effective Hamiltonian for $B \to P\ell^+\ell^-$ decays

Effective Hamiltonian describing FCNC $B \to P\ell^+\ell^-$ decays

$$H_{\rm eff}^{b\to q} = \frac{4G_F}{\sqrt{2}}\left(\lambda_u^{(q)}\sum_{i=1}^{2} C_i\,\mathcal{O}_i^u + \lambda_c^{(q)}\sum_{i=1}^{2} C_i\,\mathcal{O}_i^c - \lambda_t^{(q)}\sum_{i=3}^{10} C_i\,\mathcal{O}_i\right) + h.c.$$

$$\lambda_p^{(q)} = V_{pb}V_{pq}^*, \quad p = u, c, t, \quad q = d, s$$

- $B \to K\ell^+\ell^-$: $\lambda_t^{(s)} \approx -\lambda_c^{(s)} \sim \lambda^2 \gg \lambda_u^{(s)} \sim \lambda^4$
- $B \to \pi\ell^+\ell^-$: $\lambda_t^{(d)} \sim \lambda_c^{(d)} \sim \lambda_u^{(d)} \sim \lambda^3$
- $B_s \to K\ell^+\ell^-$: $\lambda_t^{(d)} \sim \lambda_c^{(d)} \sim \lambda_u^{(d)} \sim \lambda^3$

# Amplitude

$$A(B \to P\ell^+\ell^-) =$$

$$= \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} \lambda_t^{(q)} f^+_{BP}(q^2) \Bigg[ (\bar{\ell}\gamma^\mu \ell)\, p_\mu \left( C_9 + \frac{2(m_b + m_q)}{m_B + m_P} C_7^{\rm eff} \frac{f^T_{BP}(q^2)}{f^+_{BP}(q^2)} \right)$$

$$+ (\bar{\ell}\gamma^\mu\gamma_5\ell)\, p_\mu C_{10} - (\bar{\ell}\gamma^\mu\ell) p_\mu \frac{16\pi^2}{f^+_{BP}(q^2)} \left( \frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} \mathcal{H}^{(u)}_{BP}(q^2) + \frac{\lambda_c^{(q)}}{\lambda_t^{(q)}} \mathcal{H}^{(c)}_{BP}(q^2) \right) \Bigg]$$

$f^+_{BP}(q^2), f^T_{BP}(q^2)$ — $B \to P$ transitions form factors

$\mathcal{H}^{(u,c)}_{BP}(q^2)$ — nonlocal hadronic amplitudes

- $B \to K\ell^+\ell^-$: only one amplitude $\mathcal{H}^{(c)}_{BK}(q^2)$ ($\lambda_u^{(s)}$ neglected)
- $B \to \pi\ell^+\ell^-$: two amplitudes $\mathcal{H}^{(u)}_{B\pi}(q^2)$ and $\mathcal{H}^{(c)}_{B\pi}(q^2)$
- $B_s \to K\ell^+\ell^-$: two amplitudes $\mathcal{H}^{(u)}_{B_sK}(q^2)$ and $\mathcal{H}^{(c)}_{B_sK}(q^2)$

# Hadronic input

- Form Factors

$$\langle P(p)|\bar{q}\gamma^\mu b|B(p+q)\rangle = f^+_{BP}(q^2)\,(2p^\mu + q^\mu) + \left(f^+_{BP}(q^2) - f^0_{BP}(q^2)\right)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

$$\langle P(p)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = \frac{i f^T_{BP}(q^2)}{m_B + m_P}\left[2q^2 p^\mu + \left(q^2 - \left(m_B^2 - m_P^2\right)\right)q^\mu\right]$$

- Nonlocal effects via correlation functions

$$\mathcal{H}^{(p)}_{BP,\mu} = i\int d^4x e^{iqx}\langle P(p)|\mathrm{T}\Big\{j^{\rm em}_\mu(x), \Big[C_1\mathcal{O}^p_1(0) + C_2\mathcal{O}^p_2(0)$$

$$+\sum_{k=3-6,8g} C_k\mathcal{O}_k(0)\Big]\Big\}|B(p+q)\rangle = \left[(p\cdot q)q_\mu - q^2 p_\mu\right]\mathcal{H}^{(p)}_{BP}(q^2)$$

# Hadronic amplitudes

Use the results for invariant hadronic amplitudes:

- $\mathcal{H}^{(c)}_{BK}(q^2)$ for $B \to K\ell^+\ell^-$:
  [A. Khodjamirian, Th. Mannel, Y.M. Wang (2013)]
- $\mathcal{H}^{(c)}_{B\pi}(q^2)$ and $\mathcal{H}^{(u)}_{B\pi}(q^2)$ for $B \to \pi\ell^+\ell^-$:
  [Ch. Hambrock, A. Khodjamirian, A. V. Rusov (2015)]
- [New] $\mathcal{H}^{(c)}_{B_sK}(q^2)$ and $\mathcal{H}^{(u)}_{B_sK}(q^2)$ for $B_s \to K\ell^+\ell^-$

# Form Factors from Light-Cone Sum Rules

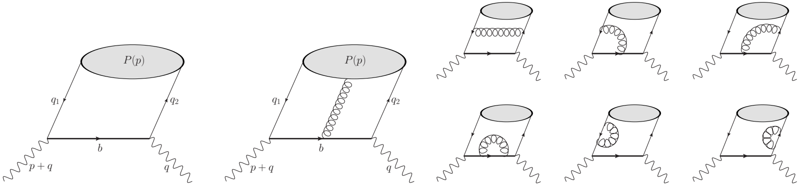
Underlying correlation functions:

$$F^{\mu}_{BP}(p,q) = i\int d^4x e^{iqx}\langle P(p)|T\{\bar{q}_1(x)\Gamma^{\mu}b(x), m_b\bar{b}(0)i\gamma_5 q_2(0)\}|0\rangle$$

$$= \begin{cases} F_{BP}(q^2,(p+q)^2)p^{\mu} + \tilde{F}_{BP}(q^2,(p+q)^2)q^{\mu}, & \Gamma^{\mu} = \gamma^{\mu} \\ F^{T}_{BP}(q^2,(p+q)^2)\left[q^2 p^{\mu} - (q\cdot p)q^{\mu}\right], & \Gamma^{\mu} = -i\sigma^{\mu\nu}q_{\nu} \end{cases}$$

$$B^+ \to K^+: \quad q_1 = u,\ q_2 = s, \qquad B^+ \to \pi^+: \quad q_1 = u,\ q_2 = d$$

$$B^0_s \to K^+: \quad q_1 = s,\ q_2 = u, \qquad B^0_s \to \bar{K}^0: \quad q_1 = s,\ q_2 = d$$



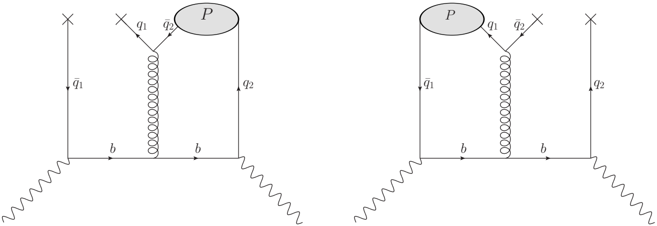
# The OPE result

$$\{F^{(T)}_{B\pi}(q^2), F^{(T)}_{BK}(q^2), F^{(T)}_{B_sK}(q^2)\} \Rightarrow \mathrm{OPE} \Rightarrow \{f^{+,T}_{B\pi}(q^2), f^{+,T}_{BK}(q^2), f^{+,T}_{B_sK}(q^2)\}$$

$$\mathrm{OPE} = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_P^{(2)}$$
$$+\frac{\mu_P}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_P^{(3)} + \frac{\delta_\pi^2}{m_b\chi} T_0^{(4)} \otimes \varphi_P^{(4)}$$
$$+\langle\bar{q}q\rangle\left(T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)}\right)$$

- LO twist 2, 3, 4 $q\bar{q}$ and $\bar{q}qG$ terms
  [V.Belyaev, A.Khodjamirian, R.Rückl (1993); V.Braun, V.Belyaev, A.Khodjamirian, R.Rückl (1996)]
- NLO $O(\alpha_s)$ twist 2 (collinear factorization)
  [A.Khodjamirian, R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997)]
- NLO $O(\alpha_s)$ twist 3 (coll. factorization for asympt. DA)
  [P. Ball, R. Zwicky (2001); G.Duplancic, A.Khodjamirian, B.Melic, Th.Mannel, N.Offen (2007) ]
- Part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 [A. Bharucha (2012)]
- [New] LO twist 5 and twist 6 in factorization approximation
  [A. V. Rusov (paper in preparation)]

# Higher twist effects in $B \to P$ form factors




- In the framework of factorization approximation

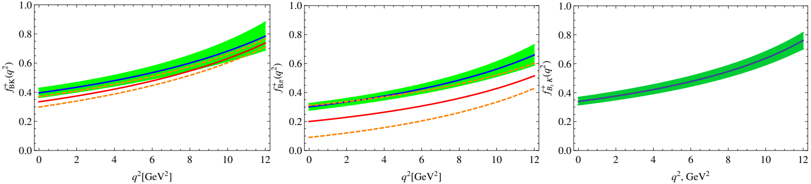
$$f_{BP}^{+(5,6)}(q^2) \sim \langle \bar{q} q \rangle \left( T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)} \right)$$

- Calculation reveals [A. V. Rusov (paper in preparation)]:

$$f_{BP}^{+(5,6)}(0)/f_{BP}^{+}(0) \lesssim 0.02\%,$$

- $\Rightarrow$ Truncation up to twist 4 contributions is reasonable

# $B \to P$ form factors from LCSR: results




Fitting of the LCSR results to the $z$-expansion (BCL parametrization)

$$f^{+,T}_{BP}(q^2) = \frac{f^{+,T}_{BP}(0)}{1 - q^2/m^2_{B^*_{(s)}}} \left\{ 1 + b^{+,T}_{1,BP} \left[ z(q^2) - z(0) - \frac{1}{3}\left( z(q^2)^3 - z(0)^3 \right) \right] + b^{+,T}_{2,BP} \left[ z(q^2)^2 - z(0)^2 + \frac{2}{3}\left( z(q^2)^3 - z(0)^3 \right) \right] \right\}$$

$$z(q^2) = \frac{\sqrt{(m_B + m_P)^2 - q^2} - \sqrt{(m_B + m_P)^2 - t_0}}{\sqrt{(m_B + m_P)^2 - q^2} + \sqrt{(m_B + m_P)^2 - t_0}}, \qquad t_0 = (m_B + m_P)(\sqrt{m_B} - \sqrt{m_P})^2$$

# $B_s \to K\ell\nu_\ell$ at large hadronic recoil

- The integrated over an interval $0 \leq q^2 \leq q_0^2$ differential decay width

$$\Delta\zeta_{B_sK}\,[0, q_0^2] \equiv \frac{1}{|V_{ub}|^2 \tau_{B_s}} \int_0^{q_0^2} dq^2 \frac{d\mathcal{B}(\overline{B}_s \to K^+ \ell \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2}{24\pi^3} \int_0^{q_0^2} dq^2 p_{B_sK}^3 |f_{B_sK}^+(q^2)|^2$$

- [New] Our prediction

$$\Delta\zeta_{B_sK}\,[0, 12\,\mathrm{GeV}^2] = 6.92^{+1.09}_{-0.90}\,\mathrm{ps}^{-1}$$

- For comparison

$$\Delta\zeta_{B\pi}\,[0, 12\,\mathrm{GeV}^2] = 5.30^{+0.87}_{-0.82}\,\mathrm{ps}^{-1}$$

- The ratio $\mathcal{B}(B_s \to K\ell\nu_\ell)/\mathcal{B}(B \to \pi\ell\nu_\ell) = \Delta\zeta_{B\pi}/\Delta\zeta_{B_sK}$ is independent of $V_{ub}$

# Observables in $B \to P\ell^+\ell^-$ decays

- Binned branching fraction

$$\mathcal{B}(\overline{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(q)}|^2}{192\pi^5} \Big\{ \mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2 \mathcal{H}_{BP}[q_1^2, q_2^2]$$

$$+ 2\kappa_q \Big( \cos\xi_q \mathcal{C}_{BP}[q_1^2, q_2^2] - \sin\xi_q \mathcal{S}_{BP}[q_1^2, q_2^2] \Big) \Big\} \tau_B$$

- Binned $CP$-asymmetry

$$\mathcal{A}_{BP}[q_1^2, q_2^2] = \frac{\mathcal{B}(\overline{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) - \mathcal{B}(B \to \overline{P}\ell^+\ell^-[q_1^2, q_2^2])}{\mathcal{B}(\overline{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) + \mathcal{B}(B \to \overline{P}\ell^+\ell^-[q_1^2, q_2^2])}$$

$$= \frac{-2\kappa_q \sin\xi_q \, \mathcal{S}_{BP}[q_1^2, q_2^2]}{\mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2 \mathcal{H}_{BP}[q_1^2, q_2^2] + 2\kappa_q \cos\xi_q \, \mathcal{C}_{BP}[q_1^2, q_2^2]}$$

$$\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} = \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \equiv \kappa_q \, e^{i\xi_q}, \quad q = d, s$$

# CKM factors: way of extraction

- In terms of Wolfenstein parameters:

$$\kappa_d = \left(1 - \frac{\lambda^2}{2}\right) \frac{\sqrt{(\rho(1-\rho)-\eta^2)^2+\eta^2}}{(1-\rho)^2+\eta^2}$$

$$\sin\xi_d = \frac{-\eta}{\sqrt{(\rho(1-\rho)-\eta^2)^2+\eta^2}}, \quad \cos\xi_d = \frac{\rho(1-\rho)-\eta^2}{\sqrt{(\rho(1-\rho)-\eta^2)^2+\eta^2}}$$

$$|\lambda_t^{(d)}| = A\lambda^3\sqrt{(1-\rho)^2+\eta^2}, \qquad |\lambda_t^{(s)}| = A\lambda^2$$

- $\lambda = 0.22537 \pm 0.00061$ is obtained accurately from global CKM fit
- $\mathcal{B}(\overline{B} \to \pi\ell^+\ell^-[q_1^2, q_2^2])/\mathcal{B}(\overline{B} \to K\ell^+\ell^-[q_1^2, q_2^2])$ and $\mathcal{A}_{B\pi}[q_1^2, q_2^2]$ $\Rightarrow$ extraction of $\rho$ and $\eta$
- $\mathcal{B}(\overline{B} \to K\ell^+\ell^-[q_1^2, q_2^2])$ $\Rightarrow$ extraction of $A$

# Numerical results on observables

| Process | $\mathcal{F}_{BP}[\mathrm{GeV}^3]$ | $\mathcal{H}_{BP}[\mathrm{GeV}^3]$ | $\mathcal{C}_{BP}[\mathrm{GeV}^3]$ | $\mathcal{S}_{BP}[\mathrm{GeV}^3]$ |
|---|---|---|---|---|
| $B^- \to K^-\ell^+\ell^-$ | $72.8^{+14.2}_{-11.8}$ | — | — | — |
| $B^- \to \pi^-\ell^+\ell^-$ | $46.6^{+8.6}_{-7.0}$ | $16.1^{+2.8}_{-10.1}$ | $14.1^{+7.6}_{-5.6}$ | $-9.5^{+6.9}_{-7.1}$ |
| $\overline{B}_s \to K^0\ell^+\ell^-$ | $59.4^{+10.8}_{-8.6}$ | $8.2^{+3.4}_{-2.5}$ | $-12.9^{+2.5}_{-2.3}$ | $-3.2^{+1.1}_{-2.6}$ |

Using values of CKM matrix elements from global fit [PDG 2016]

| Process | $\mathcal{B}_{BP}[\mathrm{GeV}^{-2}]$ | $\mathcal{A}_{BP}$ |
|---|---|---|
| $B^- \to K^-\ell^+\ell^-$ | $4.30^{+0.84}_{-0.70} \times 10^{-8}$ | 0 |
| $B^- \to \pi^-\ell^+\ell^-$ | $1.11^{+0.27}_{-0.21} \times 10^{-9}$ | $-0.15^{+0.11}_{-0.11}$ |
| $\overline{B}_s \to K^0\ell^+\ell^-$ | $1.48^{+0.27}_{-0.22} \times 10^{-9}$ | $-0.04^{+0.01}_{-0.03}$ |

$$[q_1^2, q_2^2] = [1.0, 6.0]\,\mathrm{GeV}^2$$

# Conclusion

- LCSR for $B \to \pi$, $B \to K$ and $B_s \to K$ form factors revisited including higher twist effects
- Binned decay width of $B_s \to K\ell\nu_\ell$ process predicted
- Hadronic and CKM structures of observables in $B \to P\ell^+\ell^-$ analysed (taking into account the nonlocal effects)
- Numerical values of the binned values of observables in $B \to P\ell^+\ell^-$ updated (including new prediction for $B_s \to K\ell^+\ell^-$)

# Backup

# Binned parts of the decay width

$$\mathcal{F}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 p_{BP}^3 |f_{BP}^+(q^2)|^2 \Big( \big|c_{BP}(q^2)\big|^2 + |C_{10}|^2 \Big)$$

$$\mathcal{H}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 p_{BP}^3 \, \big|h_{BP}(q^2)\big|^2$$

$$\begin{pmatrix} \mathcal{C}_{BP}[q_1^2, q_2^2] \\ \mathcal{S}_{BP}[q_1^2, q_2^2] \end{pmatrix} = \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 p_{BP}^3 \, \big|f_{BP}^+(q^2) c_{BP}(q^2) h_{BP}(q^2)\big| \begin{pmatrix} \cos\delta_{BP}(q^2) \\ \sin\delta_{BP}(q^2) \end{pmatrix}$$

$$c_{BP}(q^2) = C_9 + \frac{2(m_b + m_q)}{m_B + m_P} C_7^{\rm eff} \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} + 16\pi^2 \frac{\mathcal{H}_{BP}^{(c)}(q^2)}{f_{BP}^+(q^2)}$$

$$h_{BP}(q^2) = 16\pi^2 \Big( \mathcal{H}_{BP}^{(c)}(q^2) - \mathcal{H}_{BP}^{(u)}(q^2) \Big) \quad \delta_{BP}(q^2) = {\rm Arg}(h_{BP}(q^2)) - {\rm Arg}(c_{BP}(q^2))$$