QCD Factorization for $B \to \pi\pi$ Form Factors at Large Dipion Masses

Philipp Böer

based on: arXiv:1608.07127 [hep-ph]

in collaboration with Thorsten Feldmann and Danny van Dyk (Zürich, U.)

30. November 2016
7th QFET Workshop, TU Dortmund

UNIVERSITÄT SIEGEN

DFG FOR 1873

Motivation

Determination of V_{ub}

- V_{ub} is one of the least precisely known CKM matrix elements
- use $B \to \pi\pi\ell\nu$ decays for exclusive $|V_{ub}|$ determination
  - exploit full angular distribution of the 4-body final state [Faller et al. '14]
  - rich set of observables

Input

- important input for e.g. $B \to \pi\pi\pi$ in the QCDF approach

→ see talk by Keri Vos

Form Factors

Hadronic Matrix Elements in the SM

$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu b|\bar{B}(p)\rangle \quad \to \quad F_\perp$$

$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle \quad \to \quad F_0, F_\parallel, F_t$$

- four form factors $F_i \equiv F_i(q^2, k^2, \cos\theta_\pi)$
- kinematics:

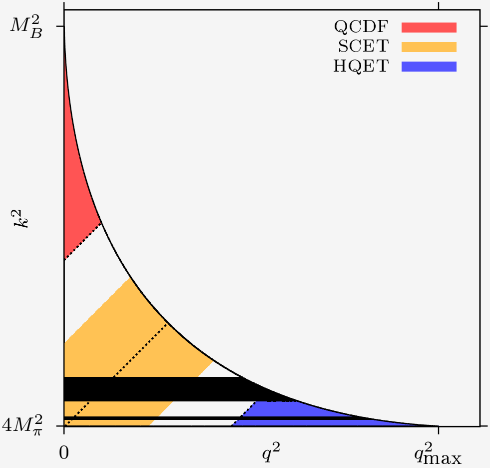
$$k = k_1 + k_2\,, \qquad \bar{k} = k_1 - k_2\,, \qquad q = p - k\,,$$

$$q\cdot\bar{k} \propto \cos\theta_\pi \quad \text{(dipion helicity angle)}$$

- partial wave expansion in Legendre polynomials of $\cos\theta_\pi$

  $\to$ S-, P-, D-, ... waves

Phase Space in $B \to \pi\pi\ell\nu$

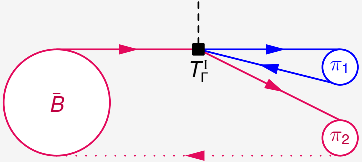


- LCSR at small k^2 and small q^2
  [Hambrock, Khodjamirian '15] $\to$ see talk by AK
- HHχPT at small k^2 and large q^2
  [Kang et al. '13]
- QCDF at large k^2 and small q^2
  $\to$ this talk

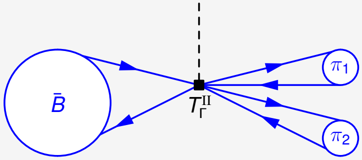
QCD Factorization Formula

- form factors factorize at leading power in HQE

$$
\begin{aligned}
&\langle \pi^+(k_1)\pi^-(k_2)|\,\bar{u}\,\Gamma\,b\,|B^-(p)\rangle \\
&\sim \xi_\pi(E_2)\int_0^1 \mathrm{d}u\,\phi_\pi(u)\,T_\Gamma^{\mathrm{I}}(u,\ldots) \\
&+\int_0^1 \mathrm{d}u\int_0^1 \mathrm{d}v\int_0^\infty \frac{\mathrm{d}\omega}{\omega}\,\phi_\pi(u)\phi_\pi(v)\phi_B(\omega)\,T_\Gamma^{\mathrm{II}}(u,v,\omega,\ldots) \\
&+\text{power corrections}
\end{aligned}
$$



$+$



QCD Factorization Formula

- form factors factorize at leading power in HQE

$$
\begin{aligned}
&\langle \pi^+(k_1)\pi^-(k_2)|\, \bar{u}\,\Gamma\, b\,|B^-(p)\rangle \\
&\sim \xi_\pi(E_2) \int_0^1 \mathrm{d}u\, \phi_\pi(u)\, T_\Gamma^{\mathrm{I}}(u,\ldots) \\
&+ \int_0^1 \mathrm{d}u \int_0^1 \mathrm{d}v \int_0^\infty \frac{\mathrm{d}\omega}{\omega}\, \phi_\pi(u)\phi_\pi(v)\phi_B(\omega)\, T_\Gamma^{\mathrm{II}}(u,v,\omega,\ldots) \\
&+ \text{power corrections}
\end{aligned}
$$

- universal “soft“ $B \to \pi$ form factor $\xi_\pi(E_2)$ for $E_2 \sim m_b/2$ [Charles et al. ’98] [Beneke, Feldmann ’01]
- universal LCDAs $\phi_\pi(u)$, $\phi_B(\omega)$ for light and heavy mesons
- perturbative short-distance kernels $T_\Gamma^{\mathrm{I,II}}$ from *hard* and *hard-collinear* gluon exchange

Diagrammatic Analysis

- our aim: verify factorization formula to leading non-trivial order
  $\to$ calculate short-distance kernels

$$T_\Gamma^{\mathrm{I}} = \mathcal{O}(\alpha_s)\,, \qquad T_\Gamma^{\mathrm{II}} = \mathcal{O}(\alpha_s^2)$$

Tasks

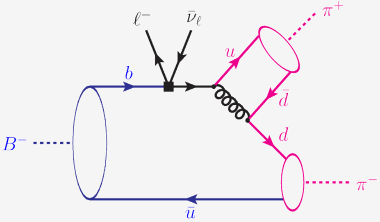
Show that:

1. leading power contributions only involve Twist-2 LCDA of π^+
2. endpoint-divergencies that arise from spectator interactions in T_Γ^{II} are universal and can be absorbed into ξ_π

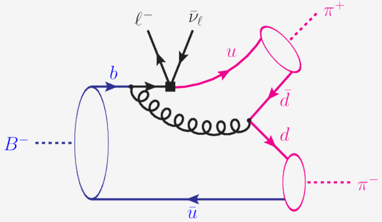
- not (yet) an all order proof within SCET

The Kernel T_Γ^{I}

(A)



(B)



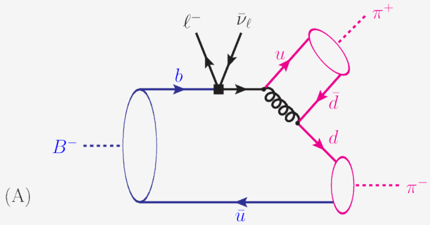
- contains short-distance QCD effects that do not involve the spectator
- hard (perturbative) gluon exchange (large virtuality $\sim \sqrt{k^2} \gg \Lambda$)
- we find, e.g. for $F_\perp$:

$$T_\perp^{\mathrm{I}} = -i\frac{\alpha_s C_F}{N_C} f_2(u)\, s_5\,, \qquad s_5 = \frac{1}{M_B}\mathrm{Tr}\left[\not{k}_2 \not{k}_1 \gamma_5 \Gamma_\perp \frac{1+\not{v}}{2}\right] = -\frac{\sqrt{\lambda}\sqrt{k^2}}{2M_B^3}\,,$$

$$f_2(u) = \frac{1}{\bar{u}}\,\frac{-M_B^2}{\bar{u}(k^2 - 2E_1 M_B) - 2E_2 M_B}$$

- finite convolution integrals with the π^+ LCDA $\phi_\pi(u)$ $\qquad \rightarrow$ task #1 ✓

The Kernel T_Γ^{I}

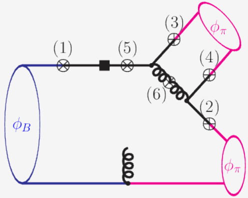


- most general case:
  - basis of „kinematical traces“: $\{s_1, \ldots, s_8\}$
  - only two independent moments at leading order: $f_1(u), f_2(u)$
    $\rightarrow$ 2 independent Form Factors in this approx.
    $\rightarrow$ relations among partial waves

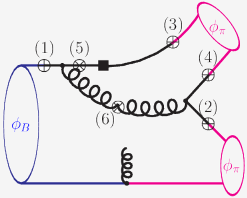
- Twist-3 projections of π^+ numerically important due to large prefactor $\mu_\pi = m_\pi^2/(m_u + m_d) \sim 2.5\,\mathrm{GeV} \sim M_B/2$

NLO Spectator Scattering - The Kernel T_Γ^{II}

(A)



(B)



hard-collinear gluon exchange between spectator quark and any vertex

Endpoint Divergencies:

- endpoint divergencies emerge in the limit $\bar{u},\bar{v} \to 0$ and/or $\omega \to 0$

$$\text{e.g. :} \qquad \int_0^1 \mathrm{d}u \, \frac{\phi_\pi(u)}{\bar{u}^2} \to \infty \,, \qquad \int_0^\infty \mathrm{d}\omega \, \frac{\phi_B^+(\omega)}{\omega^2} \to \infty \qquad (\bar{u} = 1 - u)$$

- cancellation of $\bar{u} \to 0$ divergencies as expected from color-transparency
- what about the other divergent contributions?

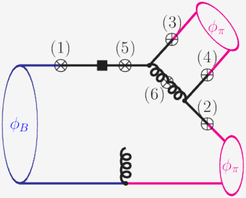
Cancellation of Endpoint-Divergencies (Feynman Gauge)

structure	A1	A2	A3 + A4	A5	A6	A1-A6
$\frac{2E_2 M_B}{\bar{u}^2 k^2}\, s_5\, \frac{\phi_B^+(\omega)}{\omega}\, \frac{\phi_\pi(v)}{2v\bar{v}}$	0	0	$-C_{FA}\, 2v$	0	$C_A\, \frac{v-\bar{v}}{2}$	$2vC_F - \frac{C_A}{2}$
$\frac{S_A}{\bar{u}}\, \frac{\phi_B^-(\omega)}{\omega}\, \frac{\phi_\pi(v)}{\bar{v}^2}$	$C_F\, \frac{1}{v}$	$C_F\, \bar{v}$	$C_{FA}\, \frac{\bar{v}}{v}$	0	$-\frac{C_A}{2}\, \frac{\bar{v}}{v}$	$C_F\, (1+\bar{v})$
$\frac{S_A}{\bar{u}}\, \frac{\phi_B^+(\omega)}{\omega}\, \frac{\mu_\pi \phi_\sigma(v)}{6\bar{v}^3 E_2}$	C_F	0	0	0	0	C_F
$2\mu_\pi\, \frac{S_A}{\bar{u}}\, \frac{\phi_B^+(\omega)}{\omega^2}\, \frac{\phi_P(v)}{\bar{v}}$	0	C_F	0	0	0	C_F

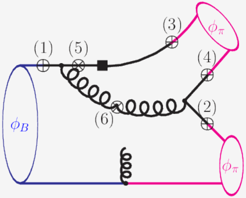
structure	B1	B2	B3+B5	B4	B6	B1-B6
$\frac{2E_2 M_B}{\bar{u}^2 k^2}\, s_5\, \frac{\phi_B^+(\omega)}{\omega}\, \frac{\phi_\pi(v)}{2v\bar{v}}$	0	0	0	$C_{FA}\, 2v$	$C_A\, \frac{\bar{v}-v}{2}$	$\frac{C_A}{2} - 2vC_F$
$\frac{S_B^{(\mathrm{i})}}{\bar{u}}\, \frac{\phi_B^+(\omega)}{\omega}\, \frac{\phi_\pi(v)}{\bar{v}^2}$	0	0	$-C_{FA}\, v_\perp^2$	$C_{FA}\, v_\perp^2$	0	0
$\frac{S_B^{(\mathrm{i})}+S_B^{(\mathrm{ii})}}{\bar{u}}\, \frac{\phi_B^-(\omega)}{\omega}\, \frac{\phi_\pi(v)}{\bar{v}^2}$	$C_F\, \frac{1}{v}$	$C_F\, \bar{v}$	$C_{FA}\, \frac{1}{v}$	$-C_{FA}$	$-\frac{C_A}{2}\, \frac{\bar{v}}{v}$	$C_F\, (1+\bar{v})$
$\frac{S_B^{(\mathrm{i})}+S_B^{(\mathrm{ii})}}{\bar{u}}\, \frac{\phi_B^+(\omega)}{\omega}\, \frac{\mu_\pi \phi_\sigma(v)}{6\bar{v}^3 E_2}$	C_F	0	$-C_{FA}\, v_\perp^2$	$C_{FA}\, v_\perp^2$	0	C_F
$\frac{S_B^{(\mathrm{i})}}{\bar{u}}\, \frac{\phi_B^-(\omega)}{\omega}\, \frac{\mu_\pi \phi_\sigma(v)}{6\bar{v}^3 E_2}$	0	0	$C_{FA}\, v_\perp^2$	$-C_{FA}\, v_\perp^2$	0	0
$2\mu_\pi\, \frac{S_B^{(\mathrm{i})}+S_B^{(\mathrm{ii})}}{\bar{u}}\, \frac{\phi_B^+(\omega)}{\omega^2}\, \frac{\phi_P(v)}{\bar{v}}$	0	C_F	0	0	0	C_F

NLO Spectator Scattering - The Kernel T_Γ^{II}

(A)



(B)



Cancellation of endpoint divergencies:

- in the sum of all diagrams we recover the very same structure of endpoint divergencies as in the $B \to \pi$ case: [Beneke, Feldmann '01]
  - endpoint divergencies are universal and can be absorbed in ξ_π

$$\langle \pi\pi|\bar{u}\Gamma b|B\rangle = \frac{2\pi f_\pi \xi_\pi(E_2)}{k^2} \int_0^1 du\, \phi_\pi(u) T_\Gamma^{\mathrm{I}}(u,\ldots) \quad + \quad \text{finite terms}$$

- factorizable (finite) contributions determine T_Γ^{II} $\rightarrow$ task #2 ✓
- our renormalization prescription: $\xi_\pi(E_2) \equiv f_+((p-k_2)^2)$

Phenomenology

Applicability of the QCDF Approach: $(\Lambda \sim 1\,\mathrm{GeV})$

- factorization of coll. and soft modes: $E_{1,2} \gg \Lambda$
- factorization of coll. and anticoll. modes: $\sqrt{k^2} \gg \Lambda$

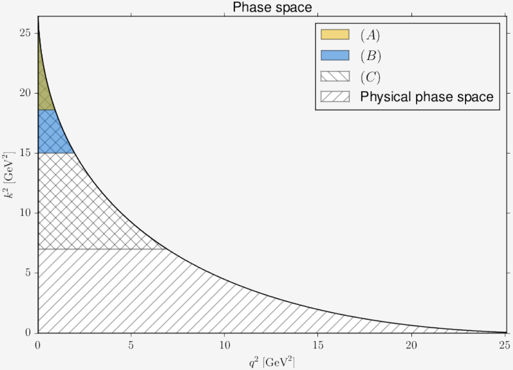
$\rightarrow$ either cut on $E_{1,2} = E_{1,2}(k^2, q^2, \cos\theta_\pi)$ and k^2

$\rightarrow$ or (easier for numerical implementation): cut on k^2 and $|\cos(\theta_\pi)|$

Three scenarios:

(A)	(B)	(C)
$\lvert\cos\theta_\pi\rvert \leq 1$	$\lvert\cos\theta_\pi\rvert \leq 1$	$\lvert\cos\theta_\pi\rvert \leq 1/3$
$k^2 \geq 2/3 M_B^2 \approx 18.6\,\mathrm{GeV}^2$	$k^2 \geq M_B^2/2 \approx 13.9\,\mathrm{GeV}^2$	$k^2 \geq M_B^2/4 \approx 7\,\mathrm{GeV}^2$
$E_{1,2} \geq M_B/3 \approx 1.76\,\mathrm{GeV}$	$E_{1,2} \geq M_B/4 \approx 1.32\,\mathrm{GeV}$	

Phenomenology



Three scenarios:

(A)	(B)	(C)
$\|\cos\theta_\pi\| \leq 1$	$\|\cos\theta_\pi\| \leq 1$	$\|\cos\theta_\pi\| \leq 1/3$
$k^2 \geq 2/3 M_B^2 \approx 18.6\,\mathrm{GeV}^2$	$k^2 \geq M_B^2/2 \approx 13.9\,\mathrm{GeV}^2$	$k^2 \geq M_B^2/4 \approx 7\,\mathrm{GeV}^2$
$E_{1,2} \geq M_B/3 \approx 1.76\,\mathrm{GeV}$	$E_{1,2} \geq M_B/4 \approx 1.32\,\mathrm{GeV}$	

Numerical Results

numerical results to LO in α_s for

- (partially integrated) branching ratio $\mathcal{B}$
- and pionic forward-backward asymmetry A_{FB}^{π}

scenario	$\mathcal{B}(B^- \to \pi^+\pi^-\mu^-\bar{\nu}_\mu)\,/\,\lvert V_{ub}\rvert^2$	$A_{\mathrm{FB}}^{\pi}(B^- \to \pi^+\pi^-\mu^-\bar{\nu}_\mu)$
(A)	$(2.93^{+0.87}_{-0.40}) \cdot 10^{-8}$	$-0.196^{+0.015}_{-0.019}$
(B)	$(9.60^{+2.80}_{-1.30}) \cdot 10^{-7}$	$-0.032^{+0.019}_{-0.021}$
(C)	$(3.18^{+0.63}_{-0.63}) \cdot 10^{-5}$	$+0.125^{+0.007}_{-0.007}$

$\rightarrow$ too small to be measured with reasonable precision!*

(*unless power-suppressed contributions are numerically dominant)

reasons:

- strong phase-space suppression in (A) and (B)
- perturbative suppression: $\mathrm{d}\Gamma \sim \alpha_s^2$

Conclusion and Outlook

- factorization formula for $B \to \pi\pi$ form factors works as expected
  - leading power contributions emerge from twist-2 LCDAs
  - non-trivial cancellation of endpoint-divergencies
- reduction of independent form factors at leading order
- unfortunately the rate is very small

Outlook

- interpolation between diff. phase-space regions
  (using other approaches: [Meißner, Wang '14], [Kang et al. '14], [Hambrock, Khodjamirian '15])
- generalization to off-resonant $B \to K\pi\ell\ell$ decays

Backup-Slides

$B \to \pi\pi\ell\nu$ Kinematics in QCDF Region

$$B^-(p) \to \pi^+(k_1)\,\pi^-(k_2)\;\bar{\nu}_\ell(q_1)\,\ell^-(q_2)$$

- large $k^2 = (k_1 + k_2)^2 \gg \Lambda^2$ $\quad(\Lambda \sim 1\mathrm{GeV})$
- perform calculation in dipion-restframe
  - pions are light-like: $k_1^2 = k_2^2 = 0$
  - use collinear & anticollinear directions: $k_1^\mu = E_\pi n^\mu, \quad k_2^\mu = E_\pi \bar{n}^\mu$ with $n^2 = \bar{n}^2 = 0$ back to back, $E_\pi \sim \mathcal{O}(m_b/2)$

Partonic Momenta in Light-like Directions — up to $\mathcal{O}(\Lambda_{\mathrm{had}}^2/m_b^2)$

IS: $p_1^\mu = m_b v_b^\mu - p_\perp^\mu \qquad p_2^\mu = \frac{\omega}{2} n^\mu + \frac{\tilde{\omega}}{2} \bar{n}^\mu + p_\perp^\mu$

FS: $k_{11}^\mu = u E_\pi n^\mu + k_{1\perp}^\mu \qquad k_{12}^\mu = \bar{u} E_\pi n^\mu - k_{1\perp}^\mu$

$k_{21}^\mu = v E_\pi \bar{n}^\mu + k_{2\perp}^\mu \qquad k_{22}^\mu = \bar{v} E_\pi \bar{n}^\mu - k_{2\perp}^\mu$

- ω, $\tilde{\omega}$ and “perp”-projections are of $\mathcal{O}(\Lambda_{\mathrm{had}}/m_b)$

LO - Kernel T_Γ^{I}

- hard propagators (large virtualities)
  - u-quark: $k^2 = \mathcal{O}(m_b^2)$
  - b-quark: $\bar{u}k^2 = \mathcal{O}(m_b^2)$
  - gluon : $\bar{u}k^2 = \mathcal{O}(m_b^2)$

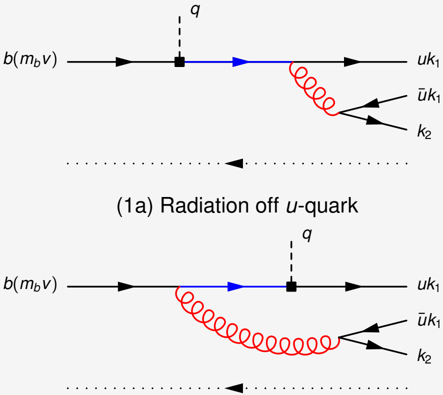
$\Rightarrow$ integrate out gluon and quark propagators!

- (Dirac-) Fierz transformation
- twist-2 projection: (x^2=0)

$$\langle \pi^+ |\, \bar{u}(x)\, X\, d(0)\, |0\rangle \to \mathrm{Tr}\,[X\, M_{\pi_1}]$$

$$M_{\pi_1} = \frac{i f_\pi E_\pi \phi(u)}{4} \not{n} \gamma_5 + \text{twist-3}$$

$b \to ug(\to \bar{d}d)$

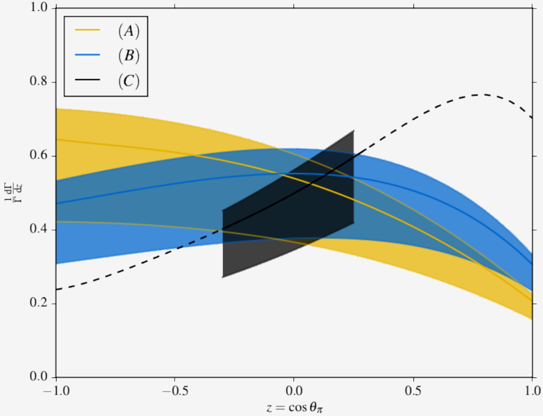


(1a) Radiation off u-quark

(1b) Radiation off b-quark

Differential Distribution

- differential distribution in $z \equiv \cos\theta_\pi$ (integrated over k^2 and q^2)



Input Parameters

parameter	value/interval	unit	prior	source/comments
		QCD input parameter		
$\alpha_s(m_Z)$	0.1184 ± 0.0007	—	gaussian @ 68%	[PDG '12]
μ	$M_B/2 \pm M_B/4$	GeV	gaussian† @ 68%	
$\overline{m}_{u+d}(2\,\mathrm{GeV})$	7.8 ± 0.9	MeV	uniform @ 100%	see [JHEP 02 (2015) 126]
		hadron masses		
m_B	5279.58	MeV	—	[PDG '12]
m_π	139.57	MeV	—	[PDG '12]
		parameters of the pion DAs		
f_π	130.4	MeV	—	[PDG '12]
$a_2^\pi(1\,\mathrm{GeV})$	$[0.09, 0.25]$	—	uniform @ 100%	[Phys. Rev. D83 (2011) 094031]
$\mu_\pi(2\,\mathrm{GeV})$	2.5 ± 0.3	GeV	—	$m_\pi^2/(\overline{m}_{u+d})$

We express the prior distribution as a product of individual priors that are either uniform or gaussian. The prior for the parameters describing the $B \to \pi$ form factor f_+ are not listed here, and taken from [Imsong et al. '14]. †: We artificially restrict the support of the renormalization scale μ to the interval $[M_B/4, M_B]$.