Parton fragmentation within spin-dependent TMD and collinear observables

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Outline

- Motivation

- FFs in transverse momentum dependent (TMD) observables
  - Definitions
  - Electron-positron annihilation: $e^+e^- \to h_a h_b X$
  - Semi-inclusive deep-inelastic scattering (SIDIS): $e\,N \to e'\,h\,X$
  - Proton-proton collisions (hadron in a jet): $p\,p \to (h\,jet)\,X$

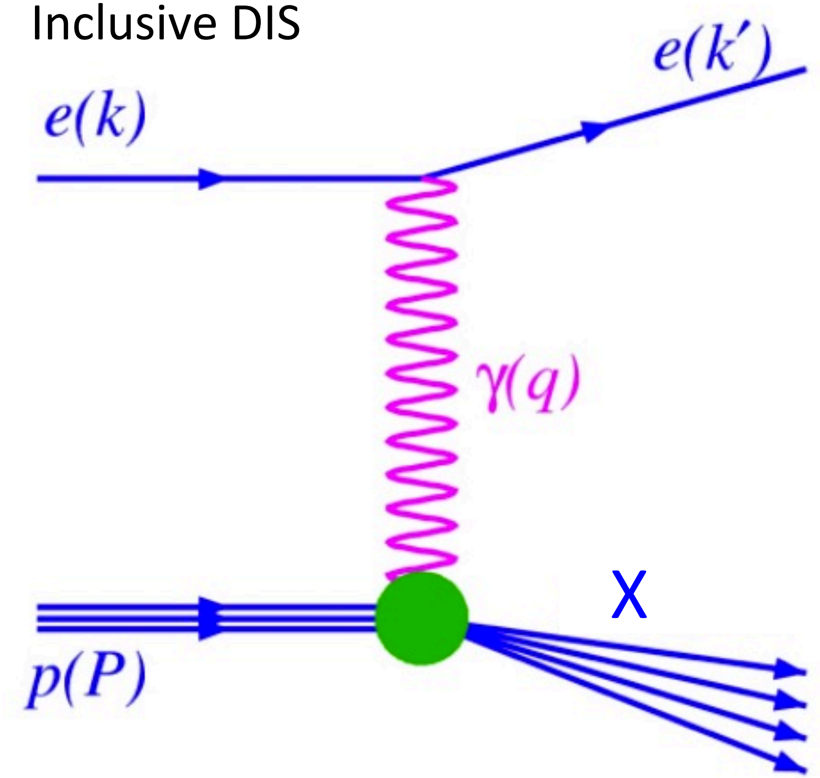
- FFs in collinear observables
  - Definitions (twist-3)
  - Proton-proton collisions (A_N): $p\,p \to h\,X$
  - Definitions (di-hadron)
  - Electron-positron/SIDIS/proton-proton:

    $e^+e^- \to (h_{a1}\,h_{a2})\,(h_{b1}\,h_{b2})\,X \,/\, e\,N \to e'\,(h_a\,h_b)\,X \,/\, p\,p \to (h_a\,h_b)\,X$

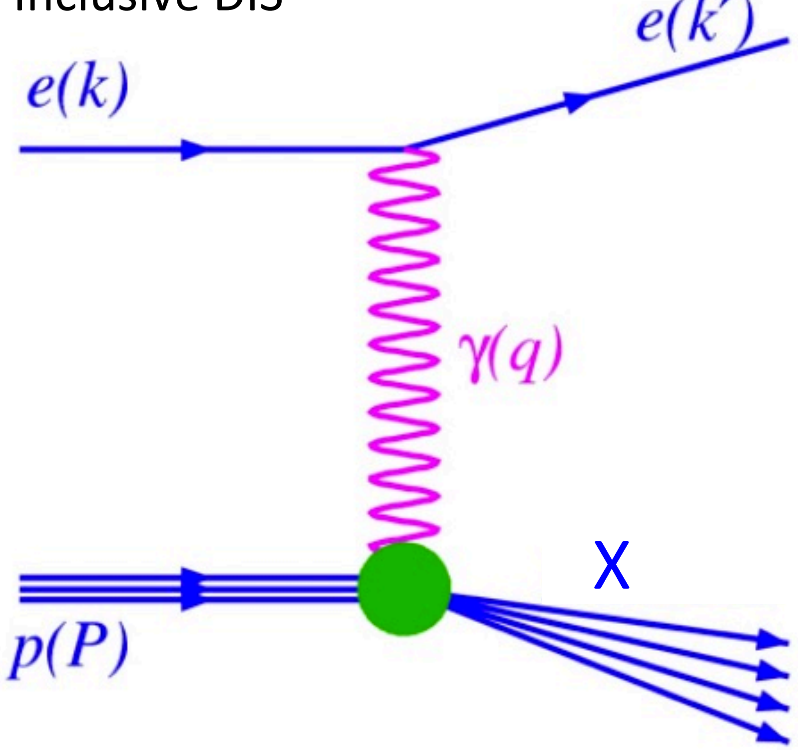
- Summary and outlook

Motivation

Inclusive DIS



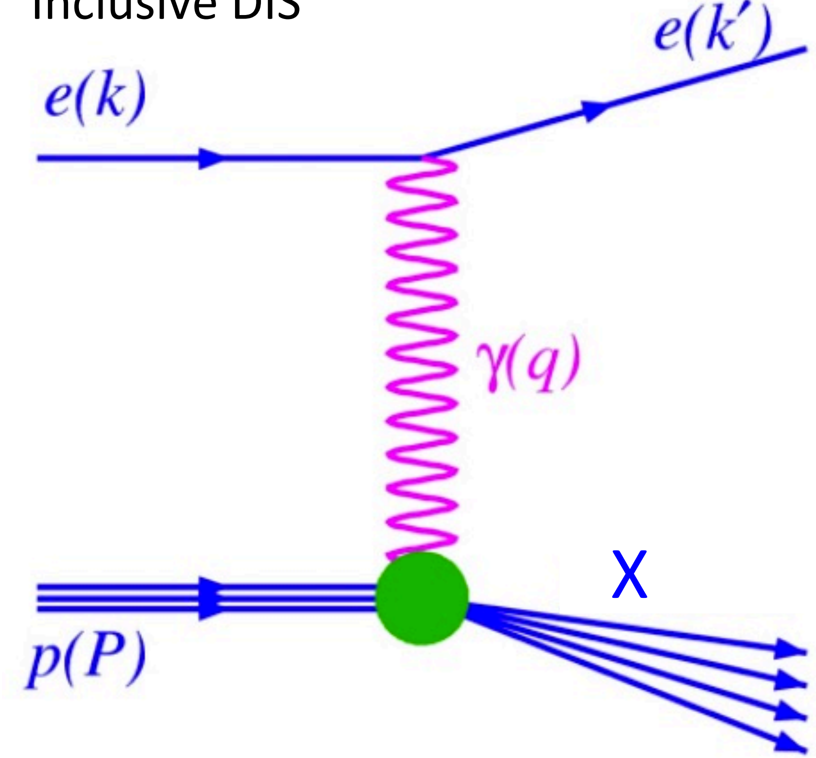
Inclusive DIS



Twist-2 collinear PDFs (x)

H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			

Inclusive DIS

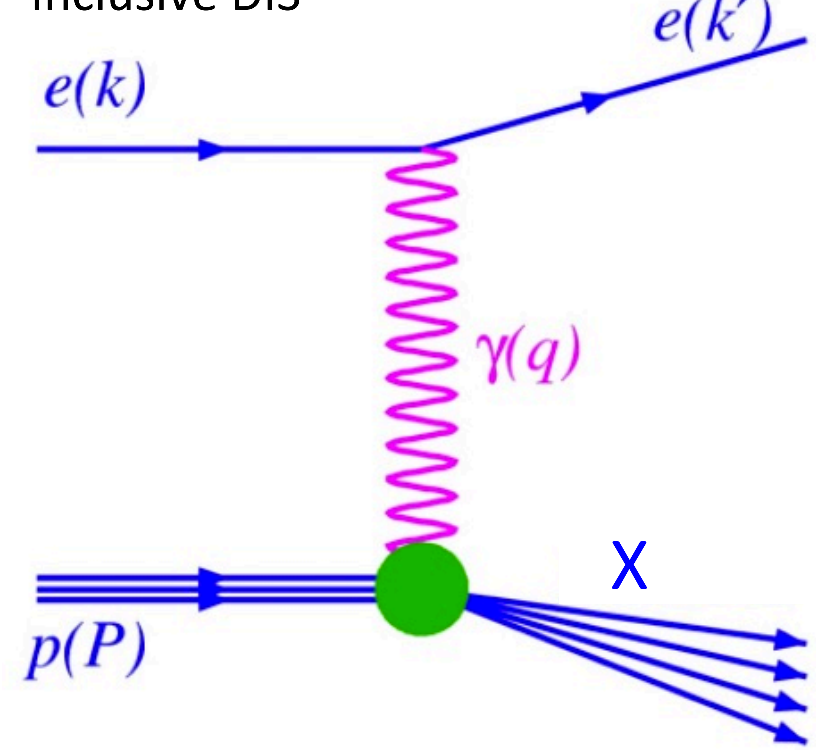


Twist-2 collinear PDFs (x)

q pol. / H pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			h_1 transversity

allows us to calculate the **tensor charge** of the nucleon

Inclusive DIS



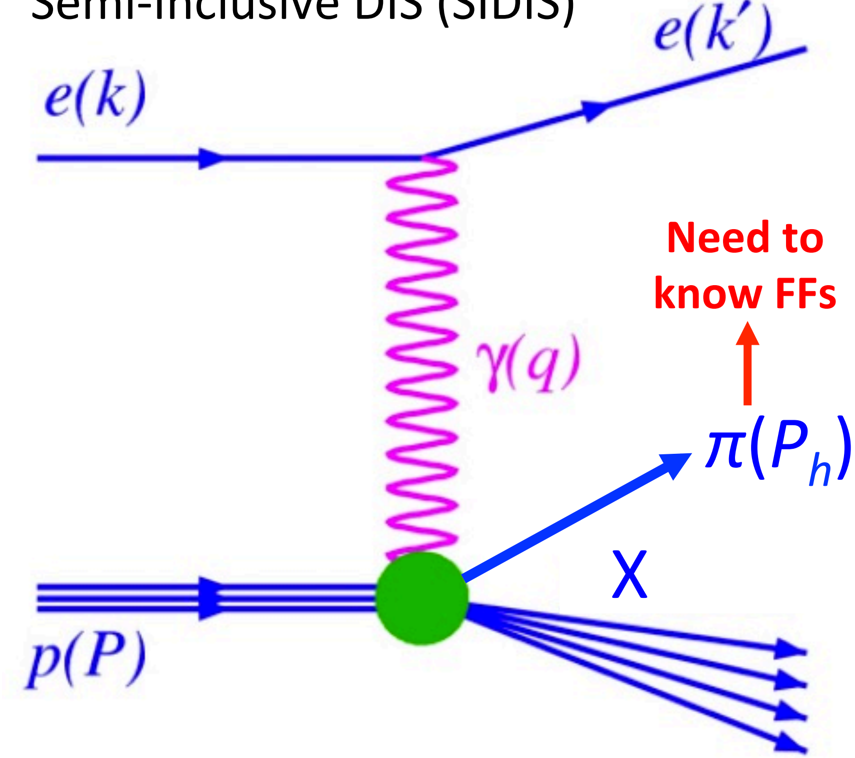
Twist-2 collinear PDFs (x)

H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			h_1 transversity

chiral-odd

CANNOT be accessed in inclusive DIS!

Semi-Inclusive DIS (SIDIS)



Twist-2 collinear PDFs (x)

H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
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chiral-odd

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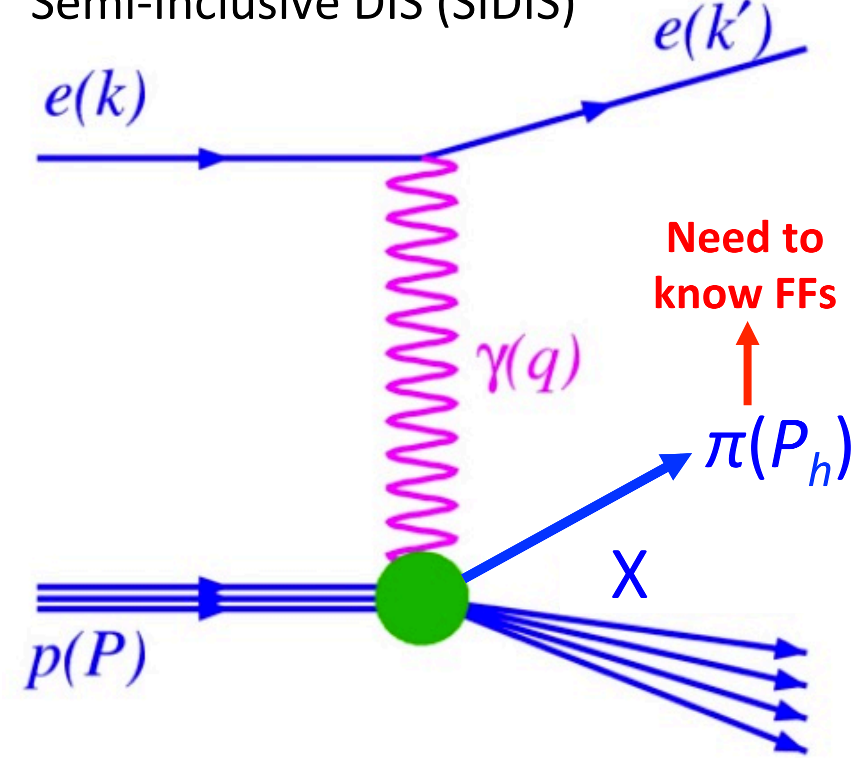
Semi-Inclusive DIS (SIDIS)



Twist-2 collinear PDFs (x)

H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			h_1 transversity

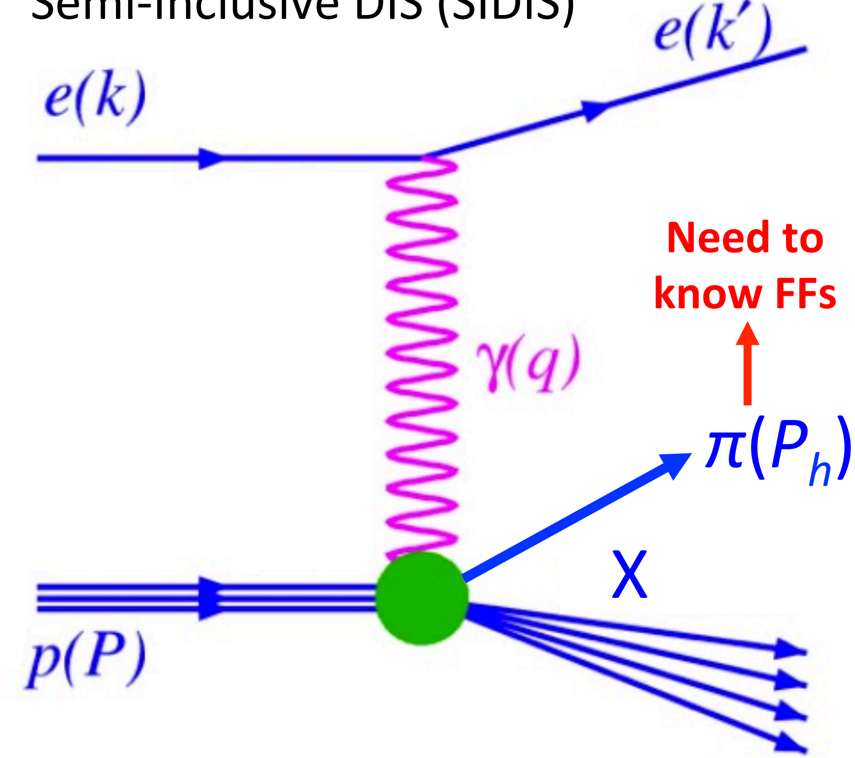
$\Delta\Sigma$ strange quark helicity -> kaon FFs

See talk by Leader

chiral-odd

CANNOT be accessed in inclusive DIS!

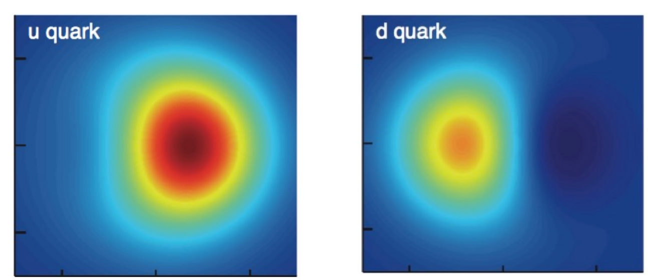
Semi-Inclusive DIS (SIDIS)



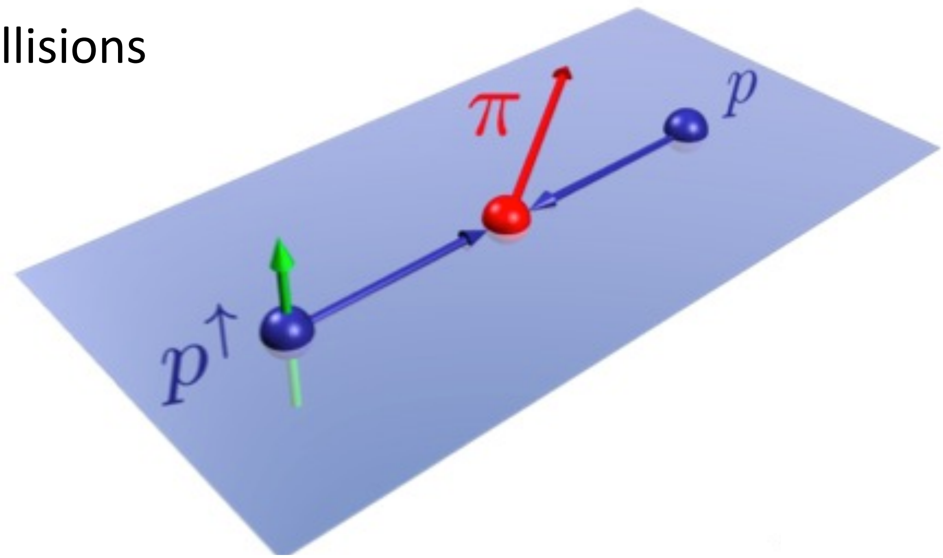
Twist-2 TMD PDFs (x, k_T)

H pol. \ q pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

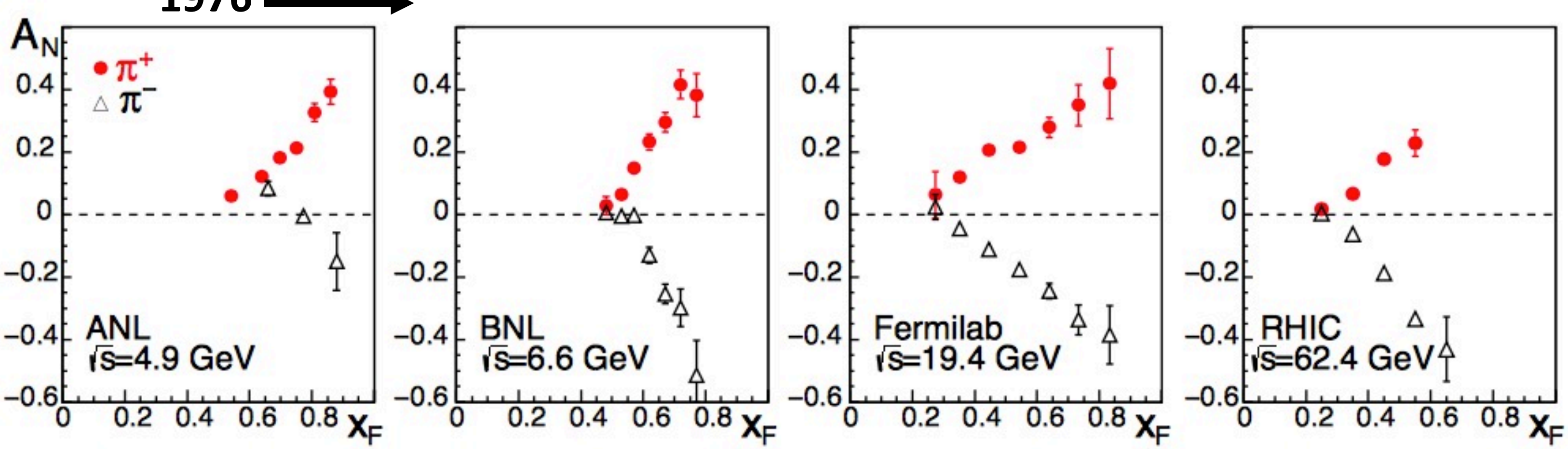
3-DIMENSIONAL structure of the nucleon



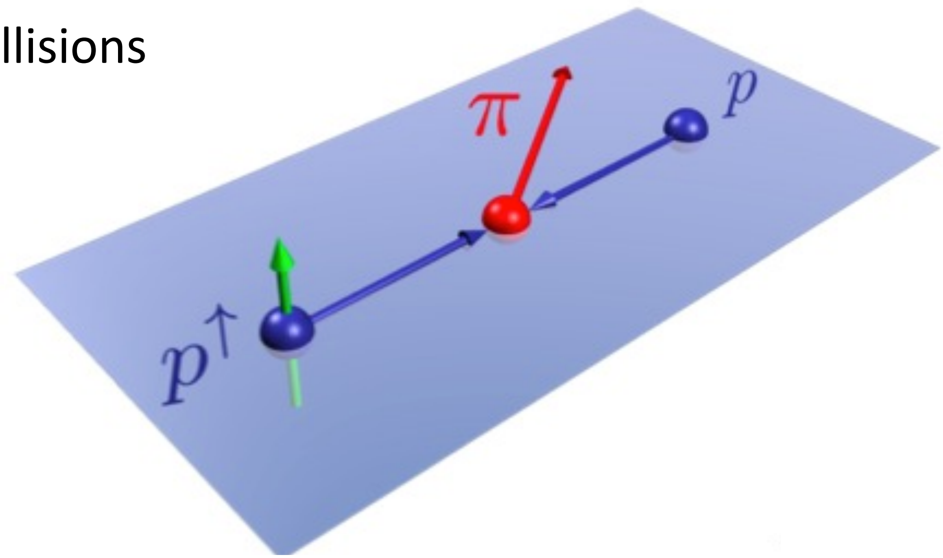
A_N in proton-proton collisions



1976 →

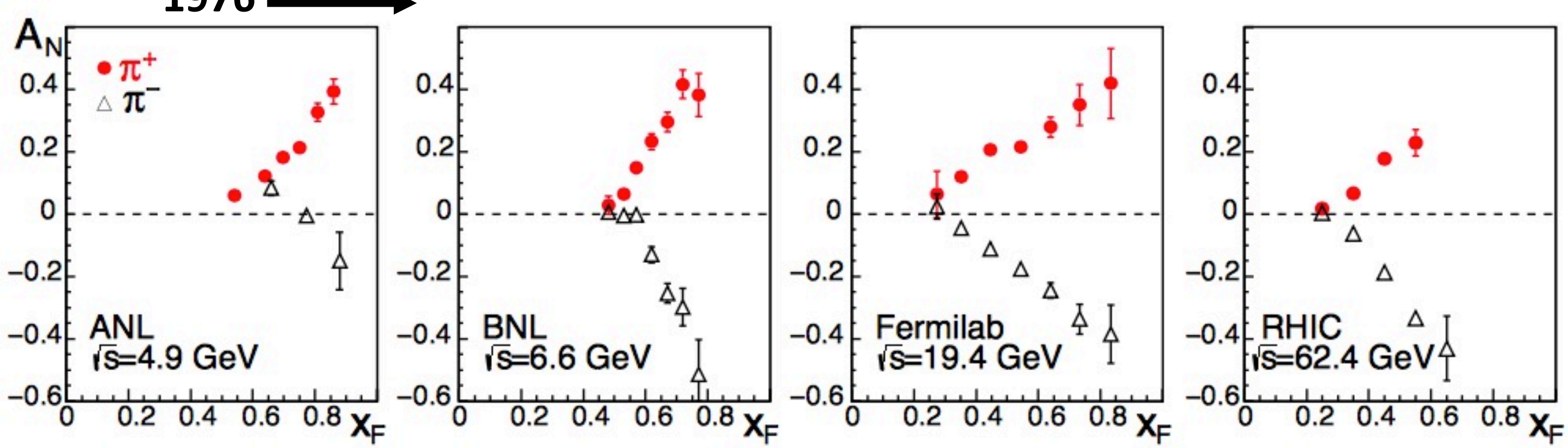


A_N in proton-proton collisions



quark-gluon-quark FFs could resolve 40 year-old puzzle of what causes A_N

1976 →



Twist-2 TMD FFs ($z, p_\perp$)

q pol. / H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

Twist-3 collinear FFs ((z) or (z, z_1))

H pol.	intrinsic	kinematical	dynamical
U	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Unpolarized) Di-hadron FFs (DiFF)
$(z, \zeta, R_\perp^2, p_\perp \cdot R_\perp, p_\perp^2)$

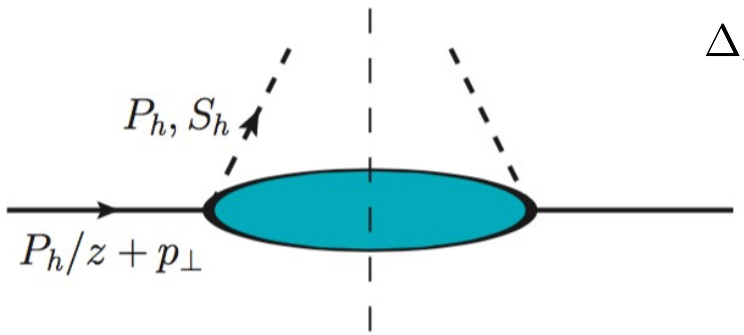
$$D_1, G_1^\perp, H_1^\sphericalangle, H_1^\perp$$

fragmentation sector is rich in its own right – functions & particles

$\pi^0, \pi^\pm, K^0, K^\pm, \Lambda, \eta, D^0, D^\pm$ ...

FFs in TMD Observables

➢ Definitions



$$\Delta^q_{ij}(z, p_\perp) = \frac{1}{N_c}\sum_X \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\int\frac{d^2 z_\perp}{(2\pi)^2}\, \mathrm{e}^{-i\frac{\lambda}{z} - i p_\perp\cdot z_\perp}\langle 0|q_i(0)|P_h S_h;\, X\rangle \times \langle P_h S_h;\, X|\,\bar{q}_j(\lambda m + z_\perp)\,|0\rangle$$

$$\Delta^{h/q[\gamma^-]} = D_1^{h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{\epsilon_\perp^{ij} p_\perp^i S_{h\perp}^j}{M_h}\, D_{1T}^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2})\,,$$

$$\Delta^{h/q[\gamma^-\gamma_5]} = \Lambda_h\, G_{1L}^{h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{\vec{p}_\perp\cdot\vec{S}_{h\perp}}{M_h}\, G_{1T}^{h/q}(z, z^2\vec{p}_\perp^{\,2})\,,$$

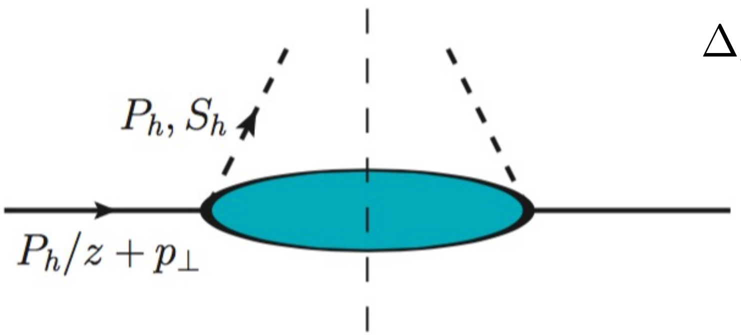
$$\Delta^{h/q[i\sigma^{i-}\gamma_5]} = S_{h\perp}^i\, H_{1T}^{h/q}(z, z^2\vec{p}_\perp^{\,2}) - \frac{\epsilon_\perp^{ij} p_\perp^j}{M_h}\, H_1^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{p_\perp^i}{M_h}\left[\Lambda_h\, H_{1L}^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{\vec{p}_\perp\cdot\vec{S}_{h\perp}}{M_h}\, H_{1T}^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2})\right]$$

(Boer, Jakob, Mulders (1997))

Twist-2 TMD FFs ($z, p_\perp$)

H pol. \ q pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

➢ Definitions



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Twist-2 TMD FFs (z, $p_\perp$)

q pol. / H pol.	U	L	T
U	D_1	Collins function	$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T}, $H_{1T}^\perp$

$$\Delta^{h/q[\gamma^-]} = D_1^{h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{\epsilon_\perp^{ij} p_\perp^i S_{h\perp}^j}{M_h} D_{1T}^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2}),$$

$$\Delta^{h/q[\gamma^-\gamma_5]} = \Lambda_h\, G_{1L}^{h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{\vec{p}_\perp \cdot \vec{S}_{h\perp}}{M_h} G_{1T}^{h/q}(z, z^2\vec{p}_\perp^{\,2}),$$

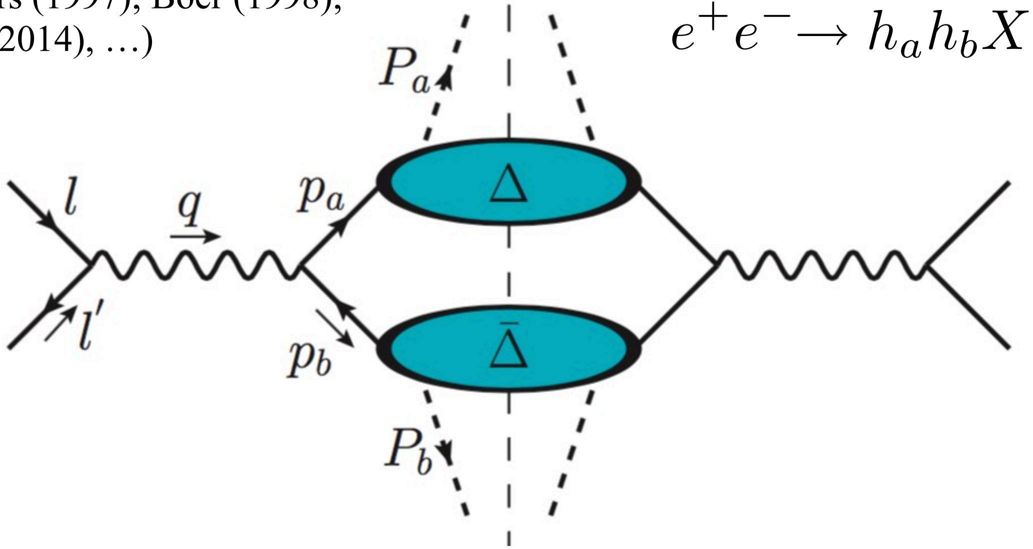
$$\Delta^{h/q[i\sigma^{i-}\gamma_5]} = S_{h\perp}^i\, H_{1T}^{h/q}(z, z^2\vec{p}_\perp^{\,2}) - \frac{\epsilon_\perp^{ij} p_\perp^j}{M_h} H_1^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{p_\perp^i}{M_h}\left[\Lambda_h\, H_{1L}^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2}) + \frac{\vec{p}_\perp \cdot \vec{S}_{h\perp}}{M_h} H_{1T}^{\perp\, h/q}(z, z^2\vec{p}_\perp^{\,2})\right]$$

(Boer, Jakob, Mulders (1997))

- Electron-positron annihilation

(Boer, Jakob, Mulders (1997); Boer (1998); DP, Metz, Schlegel (2014), …)

$$e^+e^- \to h_a h_b X$$



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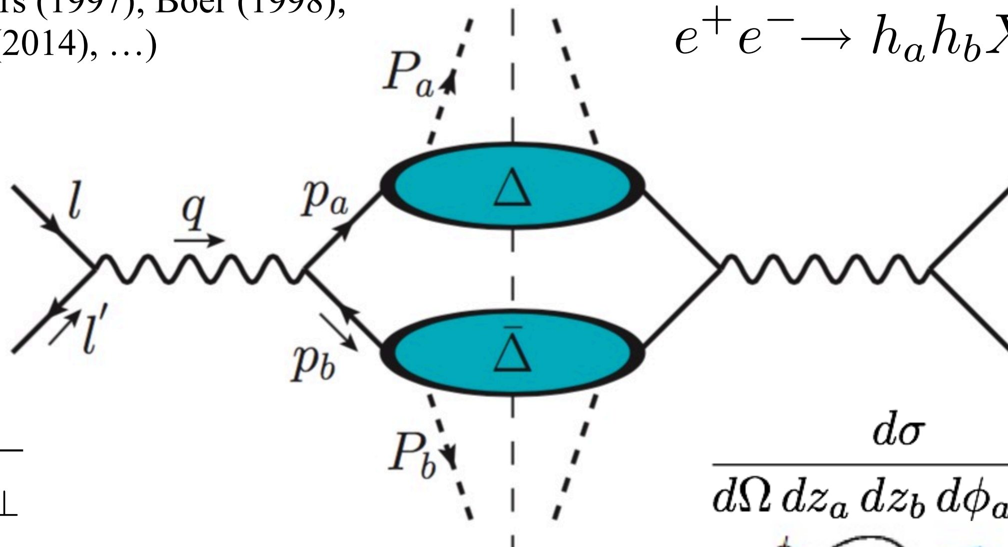
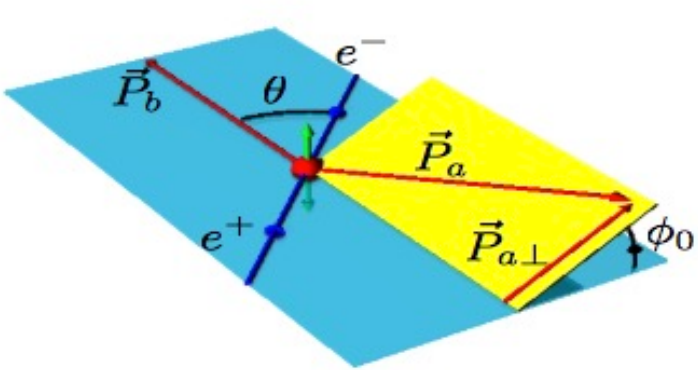
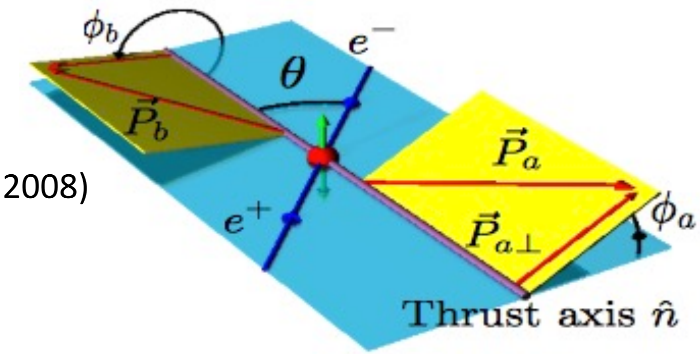


$$\frac{d\sigma}{d\Omega\, dz_a\, dz_b\, d^2\vec{P}_{a\perp}}$$

$$\frac{d\sigma}{d\Omega\, dz_a\, dz_b\, d\phi_a d\phi_b}$$



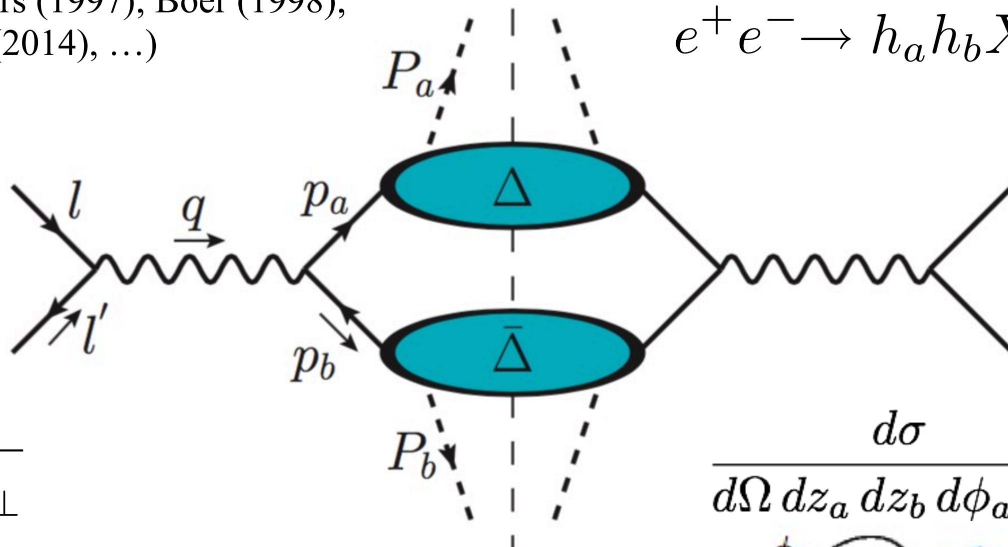
Figures from Seidl, et al. (2008)



➢ Electron-positron annihilation

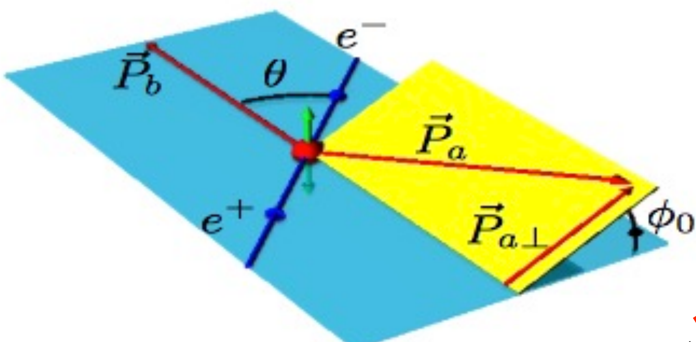
(Boer, Jakob, Mulders (1997); Boer (1998); DP, Metz, Schlegel (2014), …)

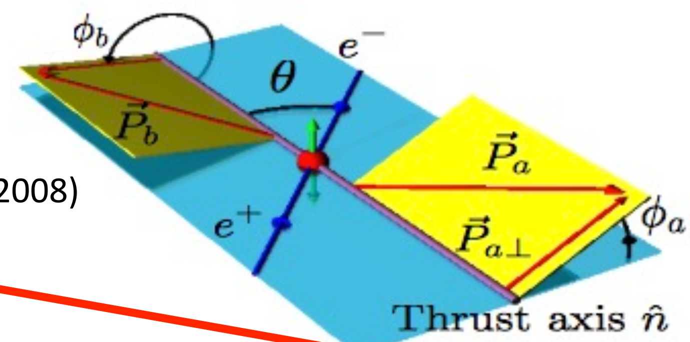
$$e^+e^- \longrightarrow h_a h_b X$$



$$\frac{d\sigma}{d\Omega\, dz_a\, dz_b\, d^2\vec{P}_{a\perp}}$$

$$\frac{d\sigma}{d\Omega\, dz_a\, dz_b\, d\phi_a d\phi_b}$$





Figures from Seidl, et al. (2008)

Collins effect

$$\propto \cdots + B(y)\ \cos(2\phi_0)\ F_{UU}^{\cos(2\phi_0)}$$

$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\ \hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\ H_1^\perp\, \bar{H}_1^\perp\right]$$

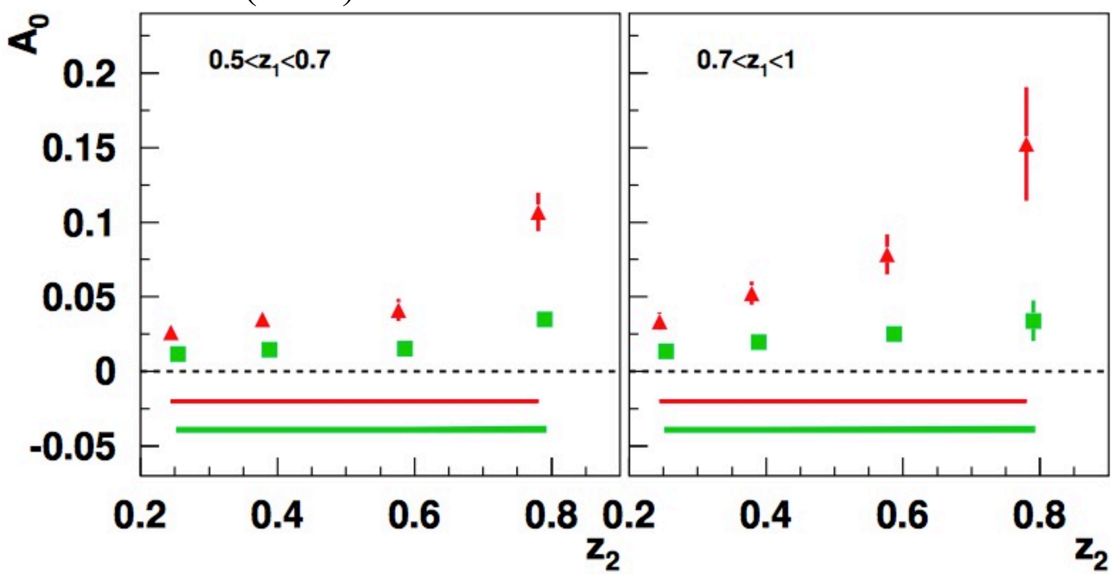
$$\propto \cdots + B(y)\ \cos(\phi_a+\phi_b)\ F_{UU}^{\cos(\phi_a+\phi_b)}$$

$$F_{UU}^{\cos(\phi_a+\phi_b)} = \sum_q e_q^2\, B(y)\, H_1^{\perp(1)}(z_a)\, \bar{H}_1^{\perp(1)}(z_b)$$

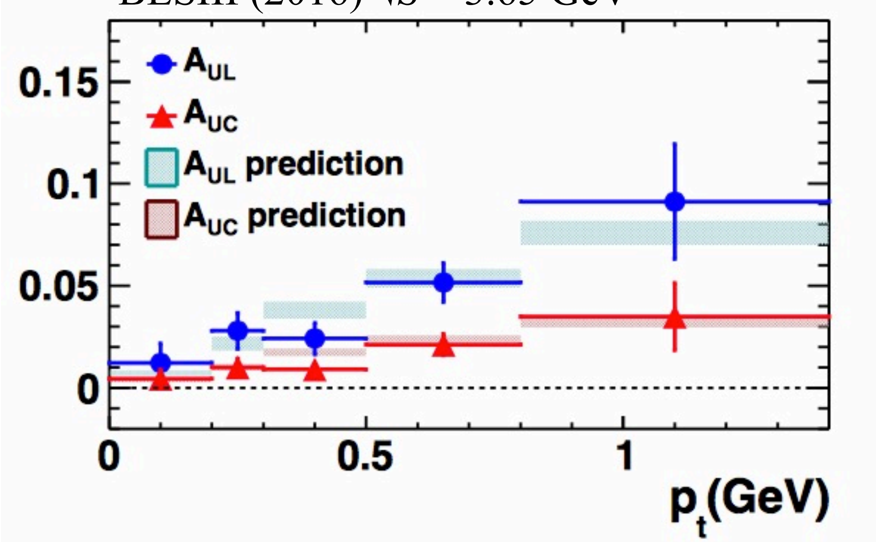
See talks by Anulli, Seidl

- Clear nonzero Collins asymmetry for $\pi\pi$ pairs
- BaBar (2015) also measured KK and πK ➔ access to kaon Collins FF
- Measurements at different $\sqrt{S}$ gives information on TMD evolution

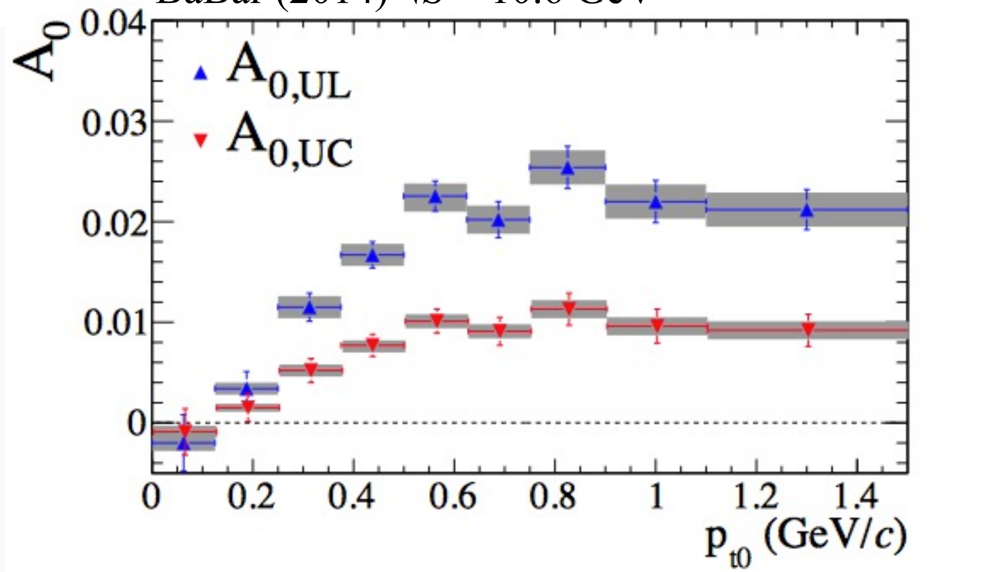
Belle (2008) $\sqrt{S}$ = 10.52 GeV



BESIII (2016) $\sqrt{S}$ = 3.65 GeV

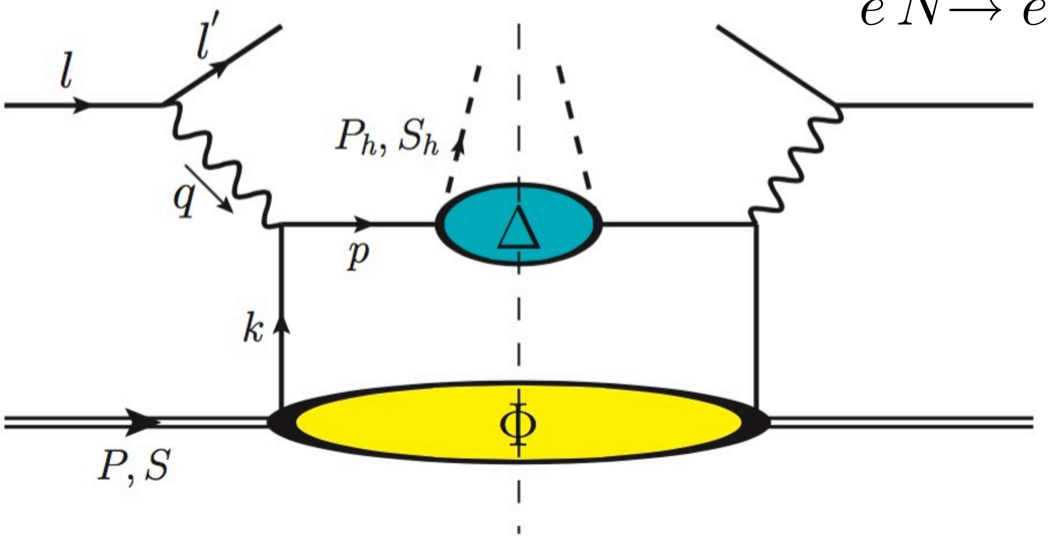


BaBar (2014) $\sqrt{S}$ = 10.6 GeV



- Semi-inclusive DIS (SIDIS)
  (Mulders, Tangerman (1996); Boer, Jakob, Mulders (2000); Bacchetta, et al. (2007), …)

$$e\,N \to e'\,h\,X$$



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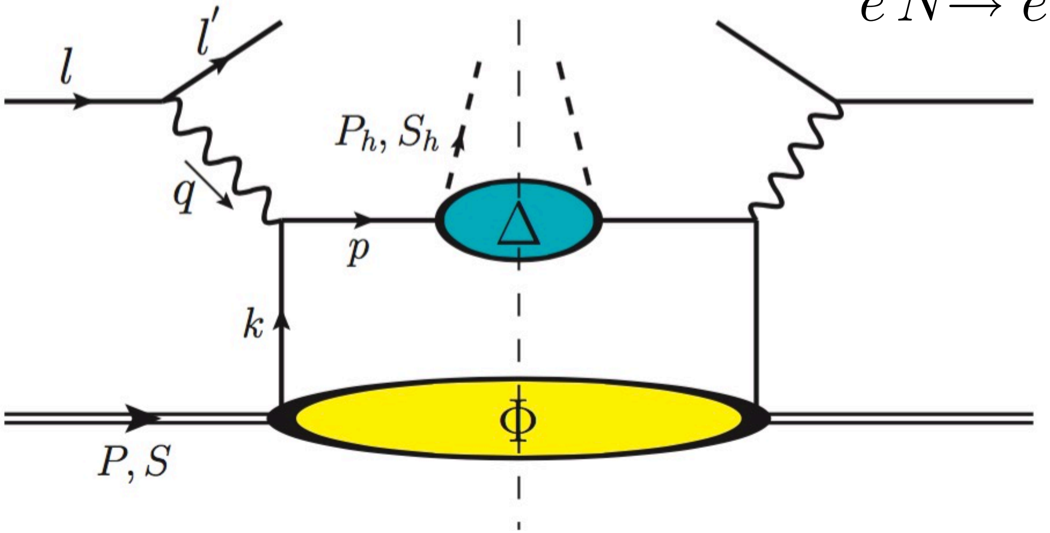
$$e\,N \rightarrow e'\,h\,X$$



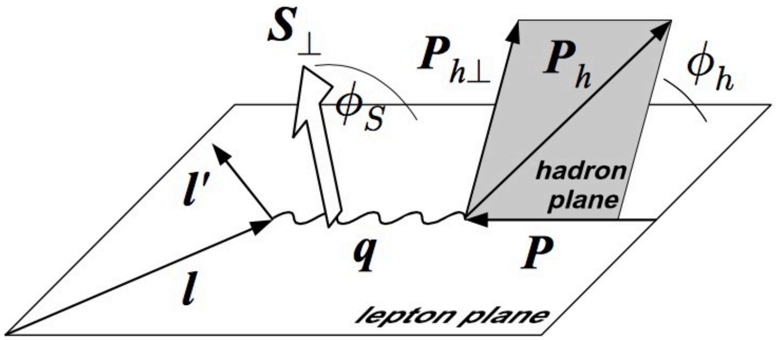
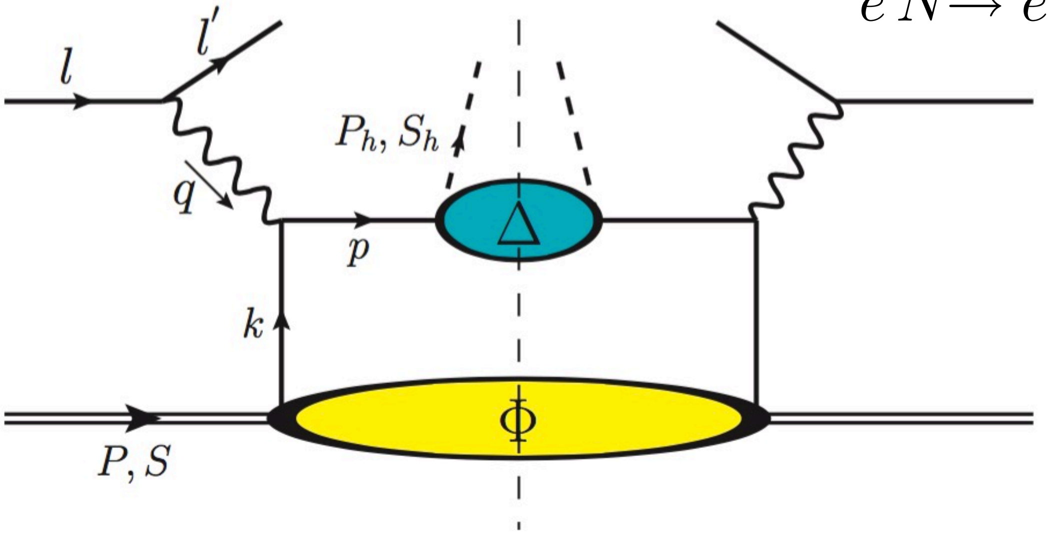


Figure from
Boer, et al. (2011)

➢ Semi-inclusive DIS (SIDIS)

(Mulders, Tangerman (1996); Boer, Jakob, Mulders (2000); Bacchetta, et al. (2007), …)

$$e\,N \rightarrow e'\,h\,X$$



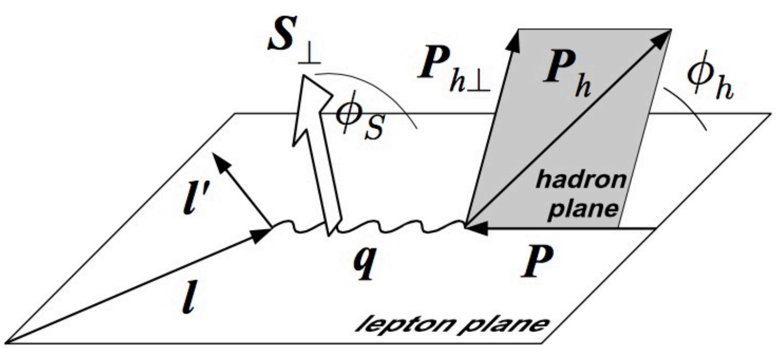


Figure from Boer, et al. (2011)

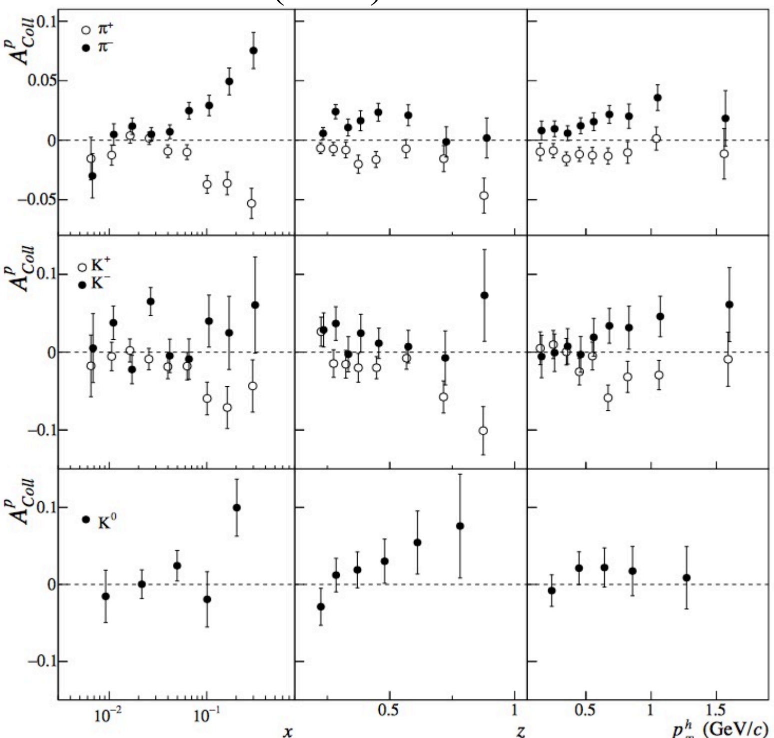
$$\frac{d\sigma}{dx\,dy\,d\phi_s\,dz\,d\phi_h\,dP_{h\perp}^2}$$

Collins effect

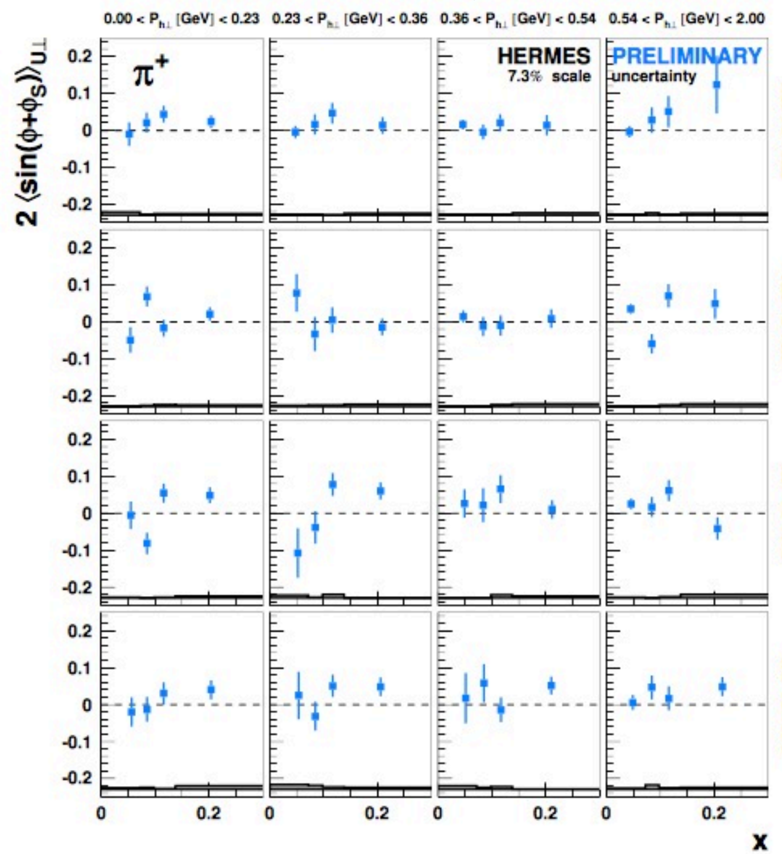
$$\propto \left\{\cdots + |\vec{S}_\perp| \left[\sin(\phi_h+\phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \ldots\right] + \ldots\right\}$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h} h_1 H_1^\perp\right]$$

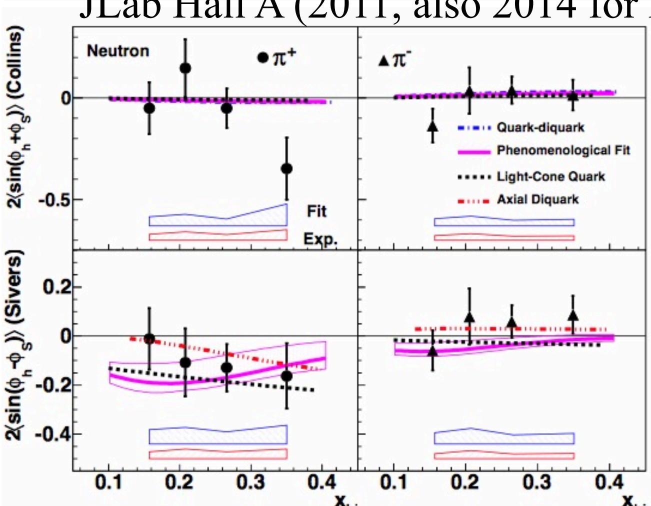
COMPASS (2015)



HERMES (preliminary)

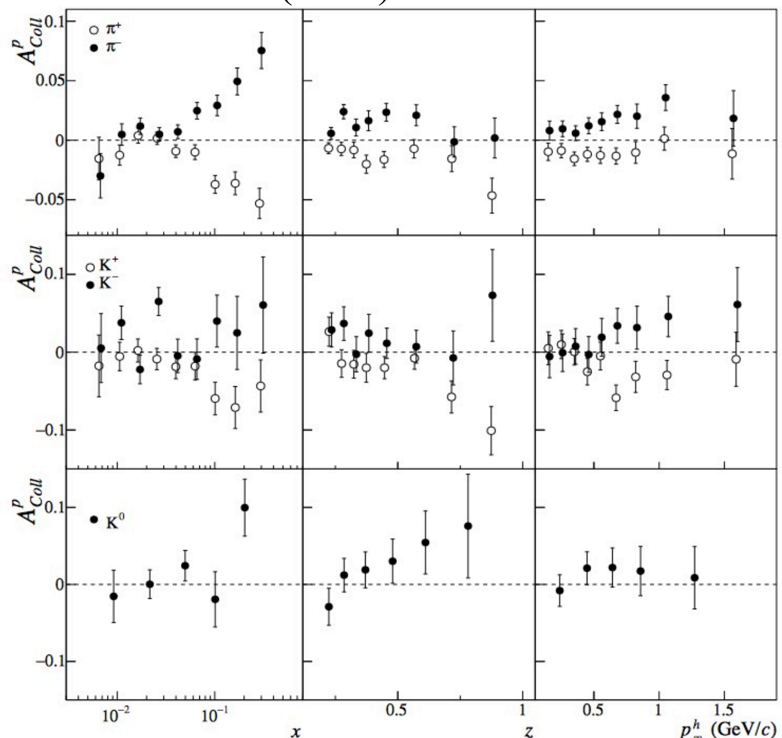


JLab Hall A (2011, also 2014 for kaons)



See talks by Puckett, Schnell

COMPASS (2015)



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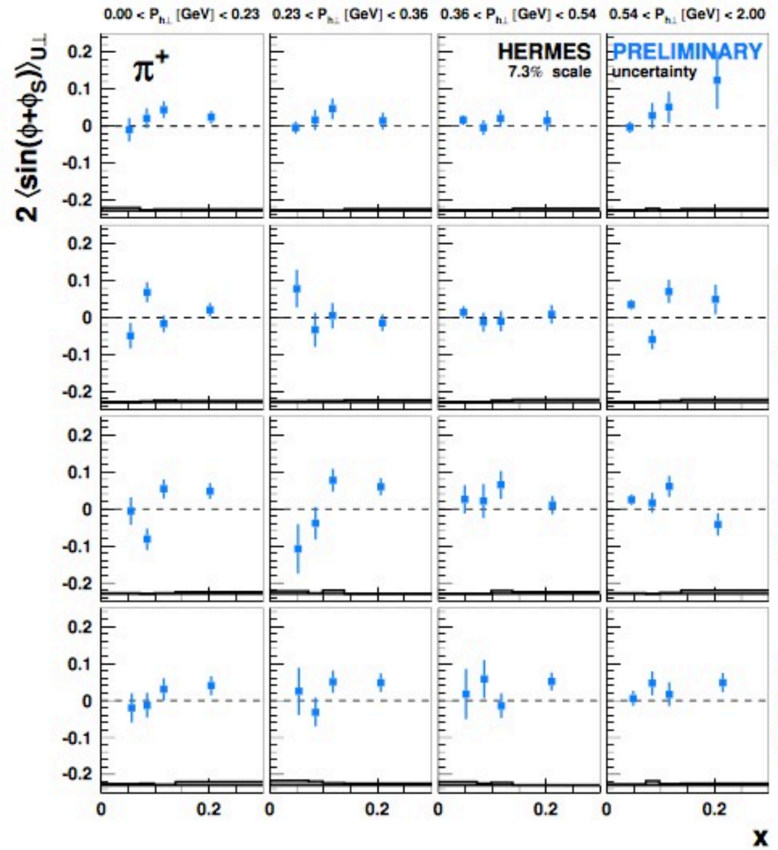

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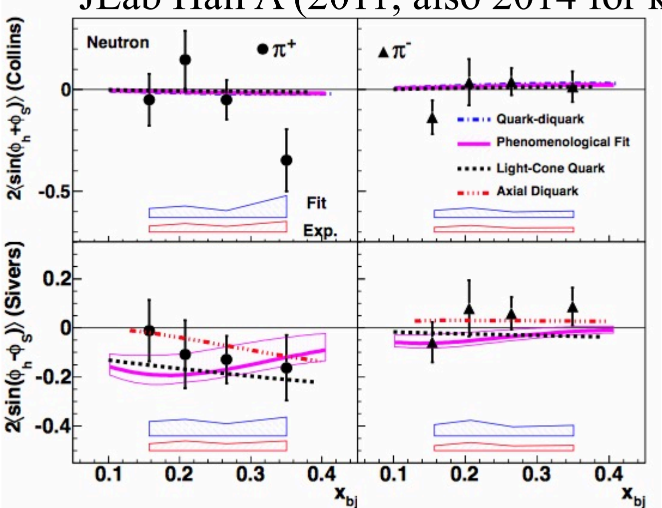
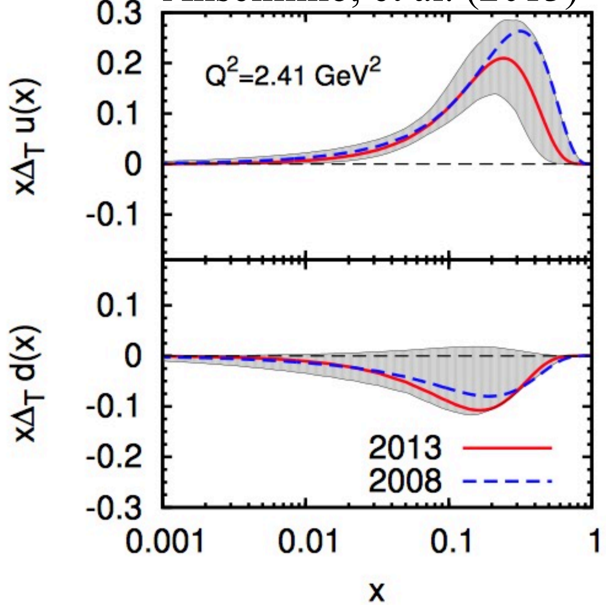


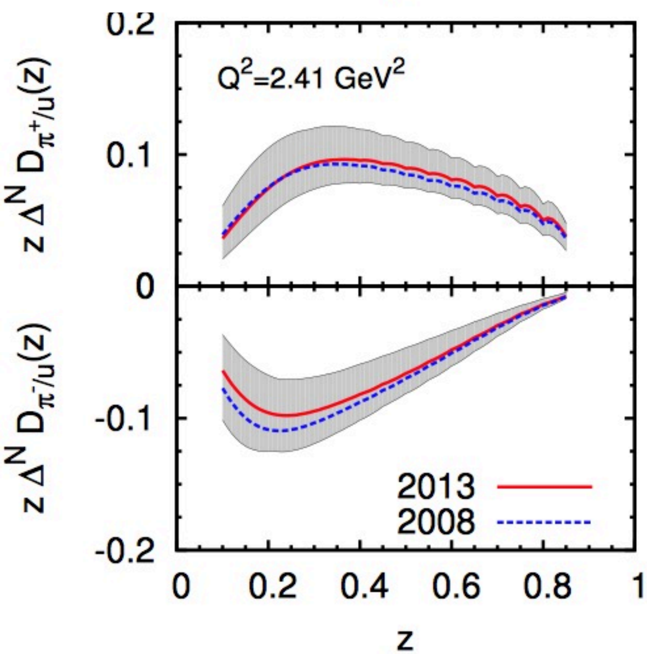
Simultaneously extract Collins & transversity from SIDIS and e^+e^-

IMPORTANT: Collins function is universal

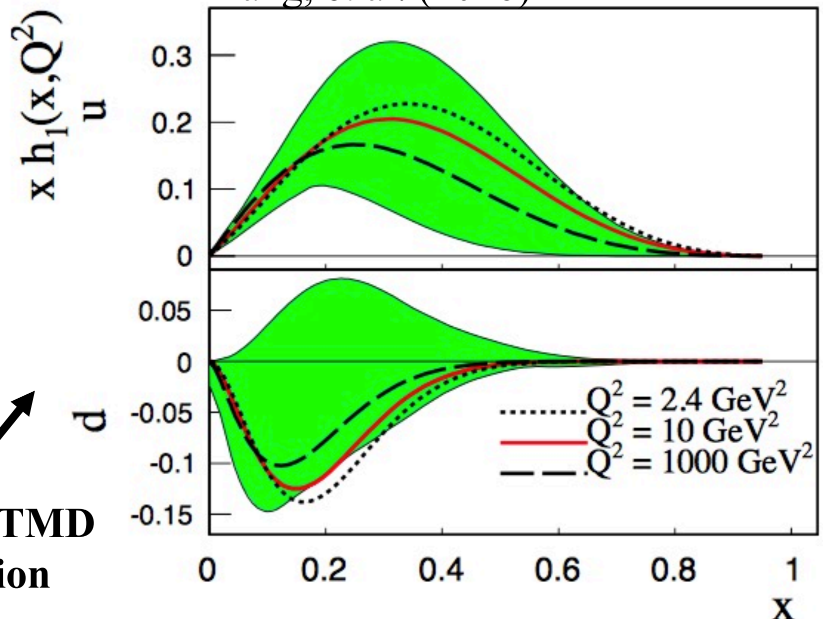
(Metz (2002); Collins, Metz (2004), …)

Anselmino, et al. (2013)

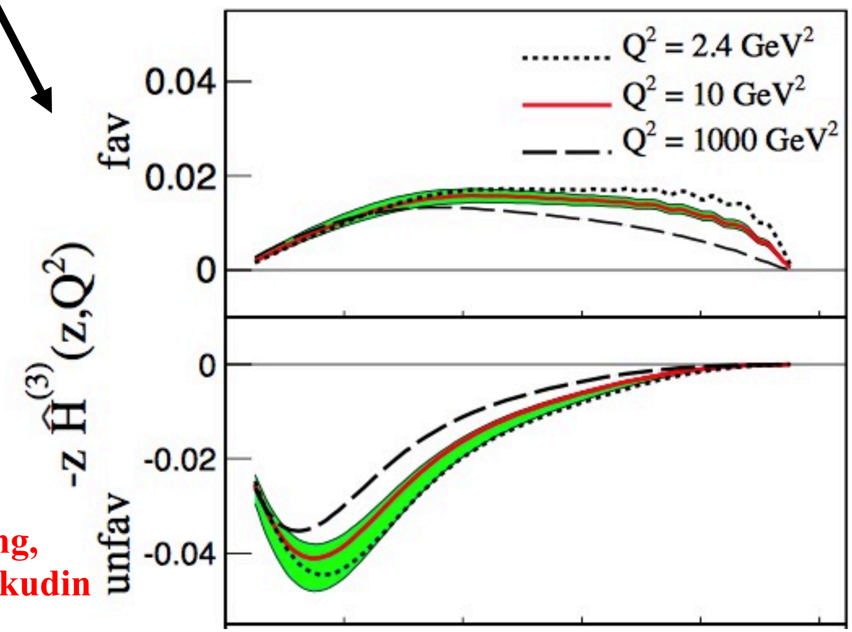




Kang, et al. (2016)



uses full TMD evolution



See talks by Kang, Echevarria, Prokudin

- **Proton-proton collisions (hadron in a jet)**
  (Yuan (2008); D'Alesio, Murgia, Pisano (2011, 2014))

$$p\,p \rightarrow (h\,jet)\,X$$

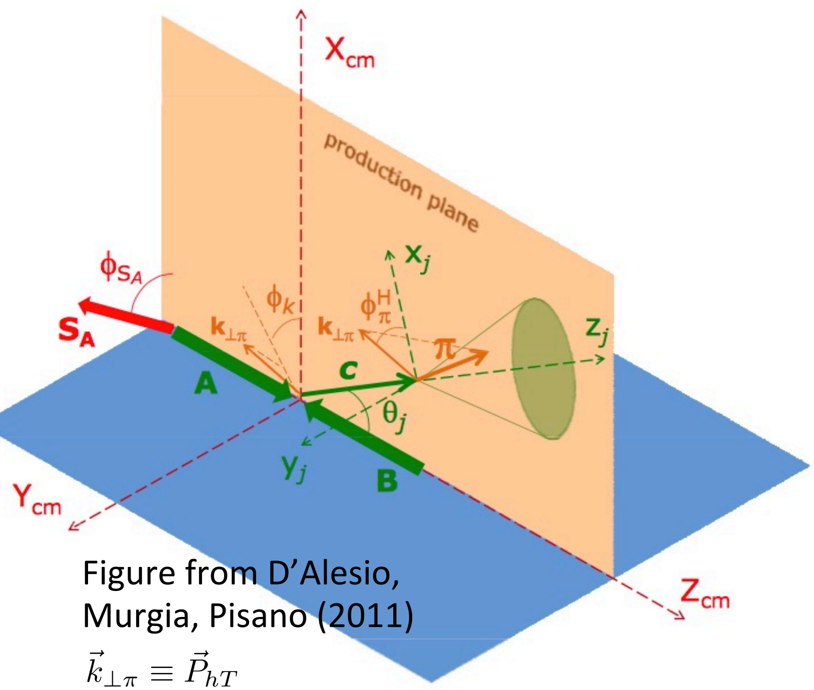


Figure from D'Alesio, Murgia, Pisano (2011)

$\vec{k}_{\perp\pi} \equiv \vec{P}_{hT}$

$$\frac{d\sigma}{d^3\vec{P}_J\,dz\,d^2\vec{P}_{hT}} \propto \sin(\phi_s - \phi_\pi^H)\,h_1(x_a,\vec{k}^2_{\perp a}) \otimes f_1(x_b,\vec{k}^2_{\perp b}) \otimes H_1^\perp(z,\vec{k}^2_{\perp\pi}) \otimes \hat{\sigma}_{pol}$$

➢ Proton-proton collisions (hadron in a jet)

(Yuan (2008); D'Alesio, Murgia, Pisano (2011, 2014))

$$p\,p \rightarrow (h\,jet)\,X$$

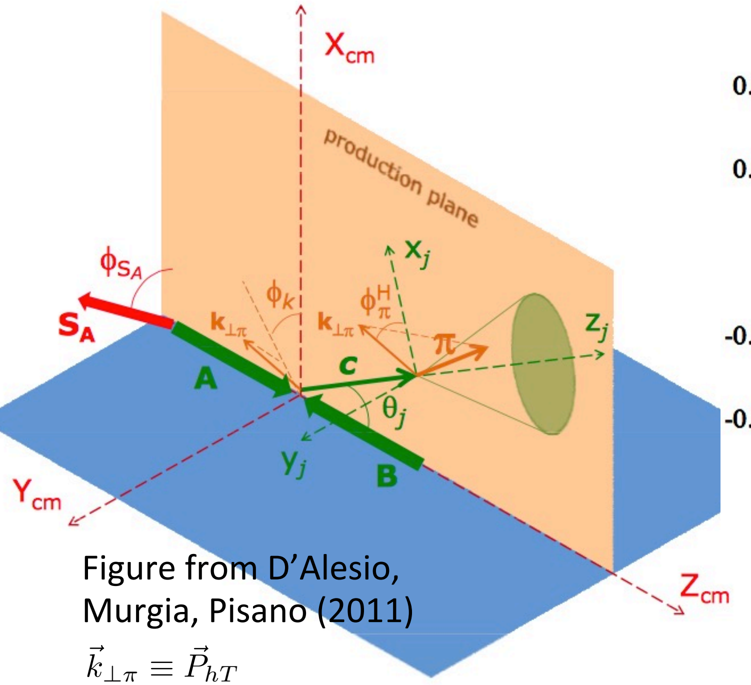
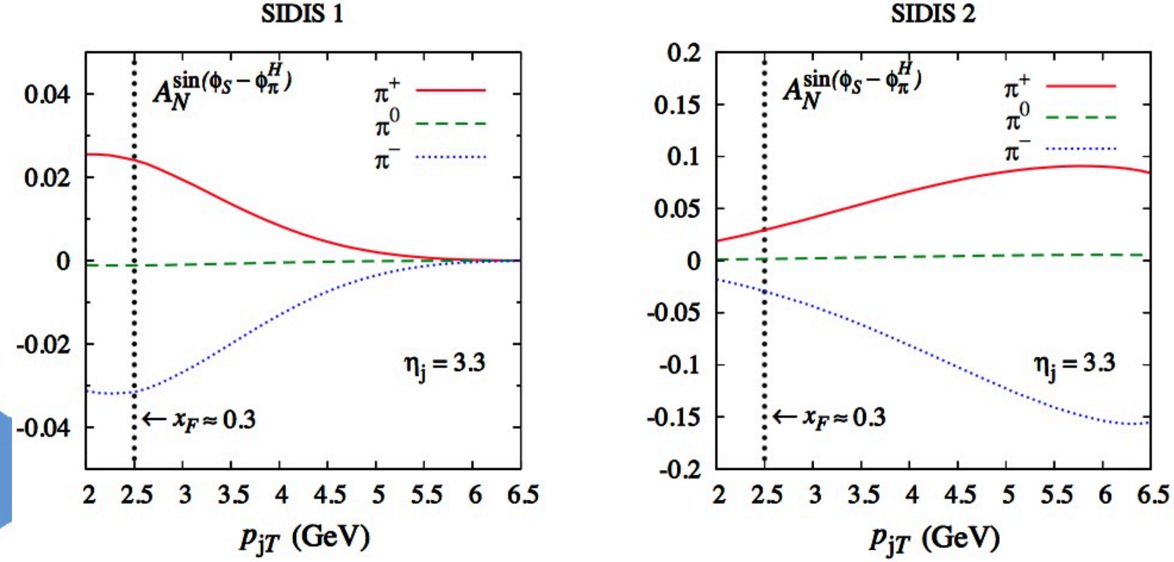


Figure from D'Alesio, Murgia, Pisano (2011)

$\vec{k}_{\perp\pi} \equiv \vec{P}_{hT}$

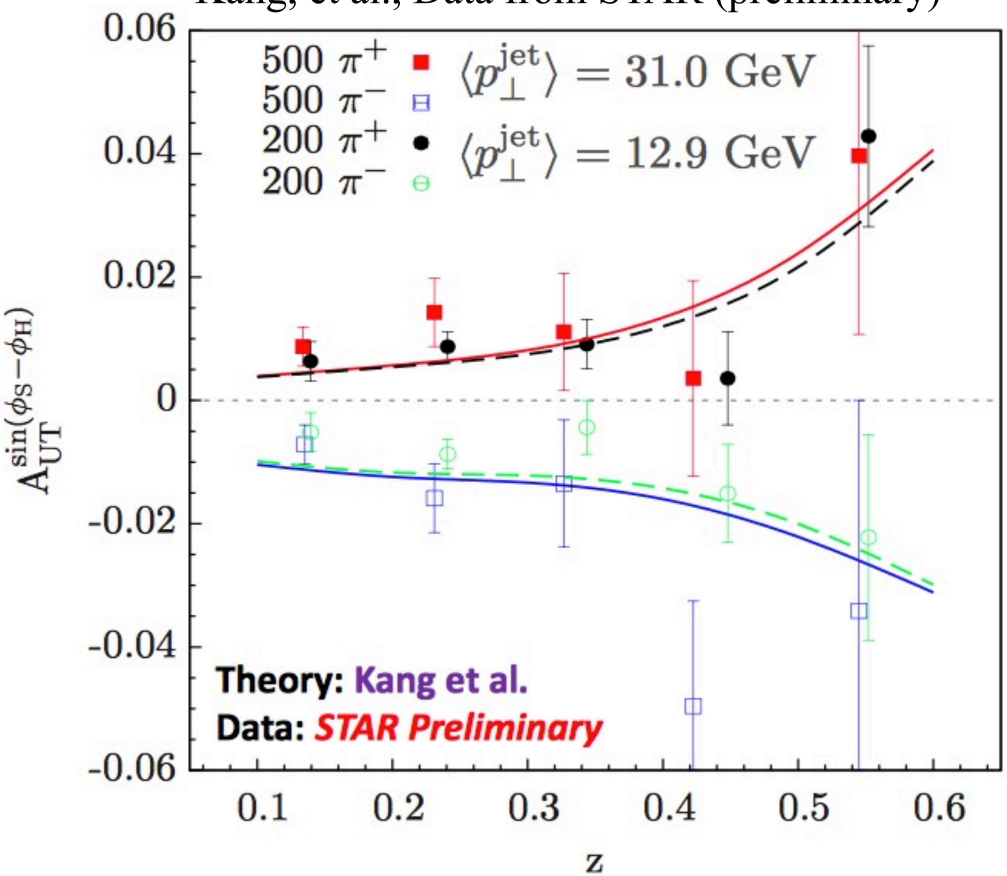
D'Alesio, Murgia, Pisano (2014)



$$\frac{d\sigma}{d^3\vec{P}_J\,dz\,d^2\vec{P}_{hT}} \propto \sin(\phi_s - \phi_\pi^H)\,h_1(x_a, \vec{k}_{\perp a}^2) \otimes f_1(x_b, \vec{k}_{\perp b}^2) \otimes H_1^\perp(z, \vec{k}_{\perp\pi}^2) \otimes \hat{\sigma}_{pol}$$

See talk by Drachenberg

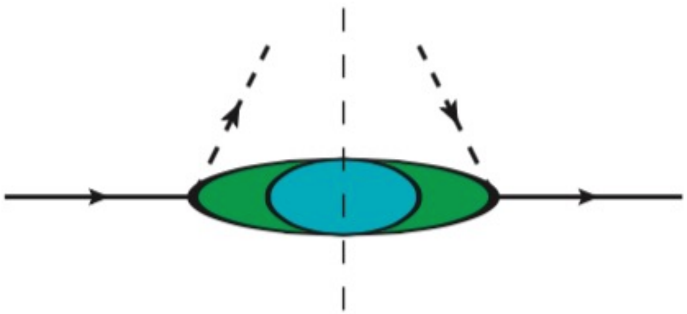
Kang, et al., Data from STAR (preliminary)



- Clear nonzero Collins asymmetry for charged pions
- Similar magnitude for $\sqrt{S}$ = 200 GeV and $\sqrt{S}$ = 500 GeV (cf. Belle and BaBar much smaller asymmetry than BESIII)
- No evolution? or Cancellation of evolution effects in the asymmetry? or Simply a kinematical effect?
- Data not yet included in a global fit (test the universality of the Collins function)

FFs in Collinear Observables

➢ Definitions (twist-3)



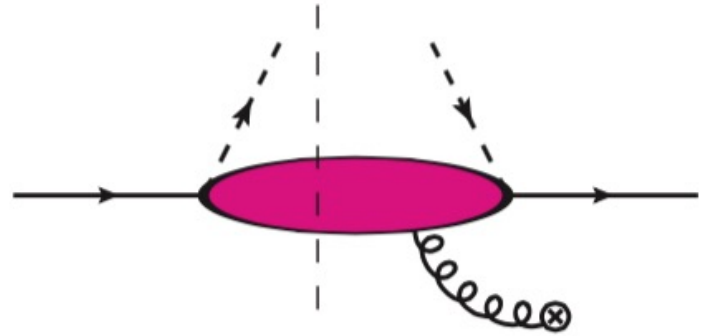
intrinsic

$$\Delta^q_{ij}(z) = \frac{1}{N_c}\sum_X \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi}\, e^{-i\frac{\lambda}{z}} \langle 0|\; q_i(0)\, |P_h S_h;\, X\rangle \times \langle P_h S_h;\, X|\, \bar{q}_j(\lambda m)\, |0\rangle$$

and

kinematical

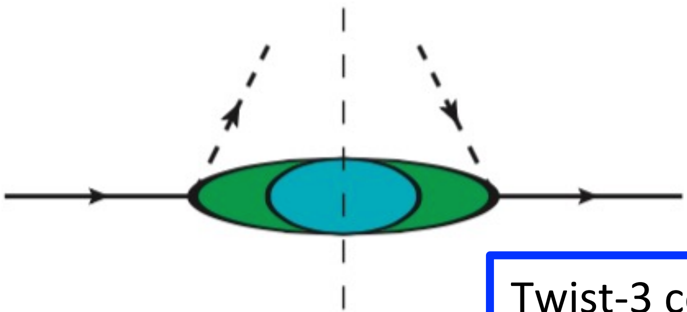
$$\Delta^{q,\rho}_{\partial,ij}(z) = \int d^2p_\perp\, p^\rho_\perp\, \Delta^q_{ij}(z,p_\perp)$$

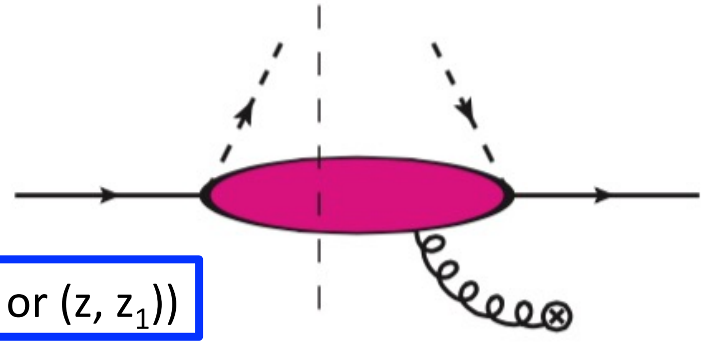


dynamical

$$\Delta^{q,\rho}_{F,ij}(z,z_1) = \frac{1}{N_c}\sum_X \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_{-\infty}^{\infty} \frac{d\mu}{2\pi}\, e^{i\frac{\lambda}{z_1}+i(\frac{1}{z}-\frac{1}{z_1})\mu} \langle 0|\, ig m_\eta F^{\eta\rho}(\mu m) q_i(\lambda m)\, |P_h S_h\,;\, X\rangle \times \langle P_h S_h\,;\, X|\, \bar{q}_j(0)\, |0\rangle$$

- Definitions (twist-3)



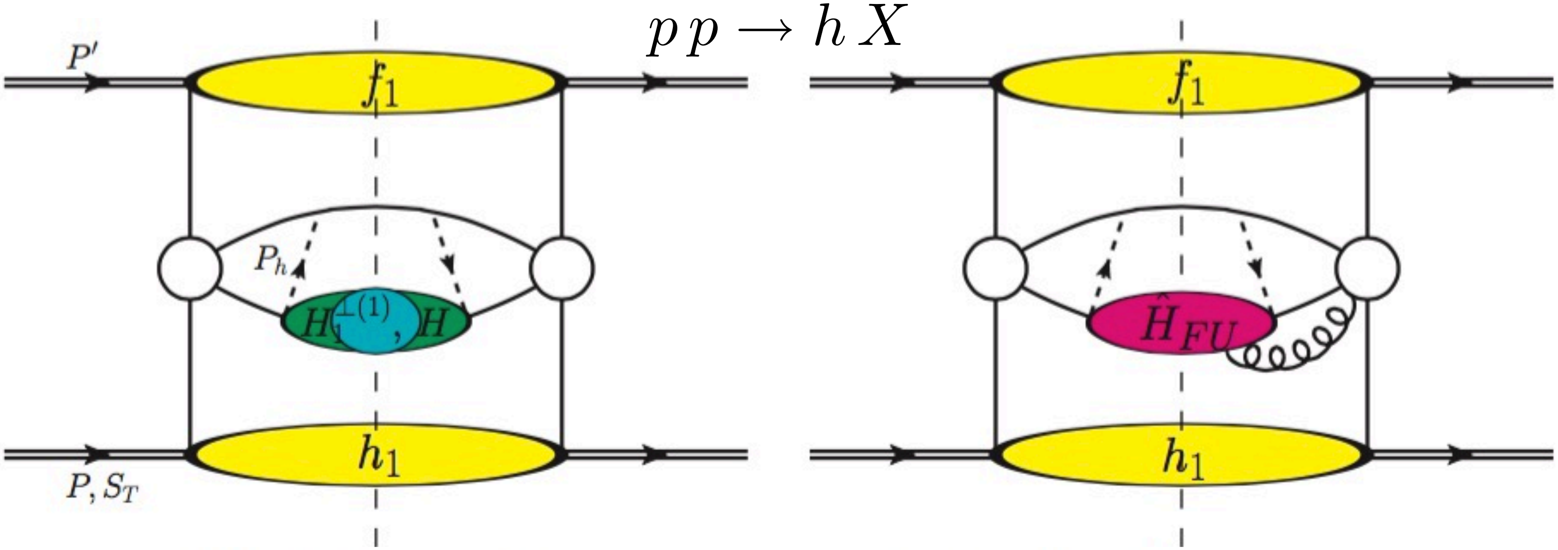


Twist-3 collinear FFs ((z) or (z, z_1))

H pol.	intrinsic	kinematical	dynamical
U	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

- Proton-proton collisions (A_N)

(Kang, Yuan, Zhou (2010); Metz, DP (2013); Kanazawa, Koike, Metz, DP (2014); Gamberg, Kang, Metz, DP, Prokudin (2014); Koike, DP, Takagi, Yoshida (2016), Kanazawa, Koike, Metz, DP, Schlegel (2016))



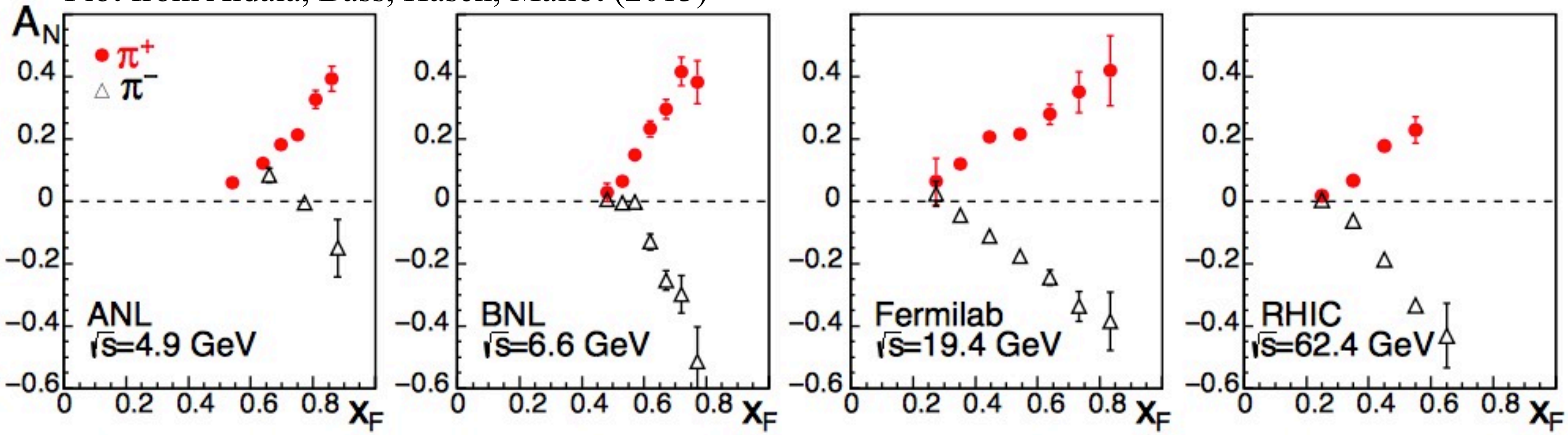
$$\frac{E_h\, d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_P}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\,\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}\,h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

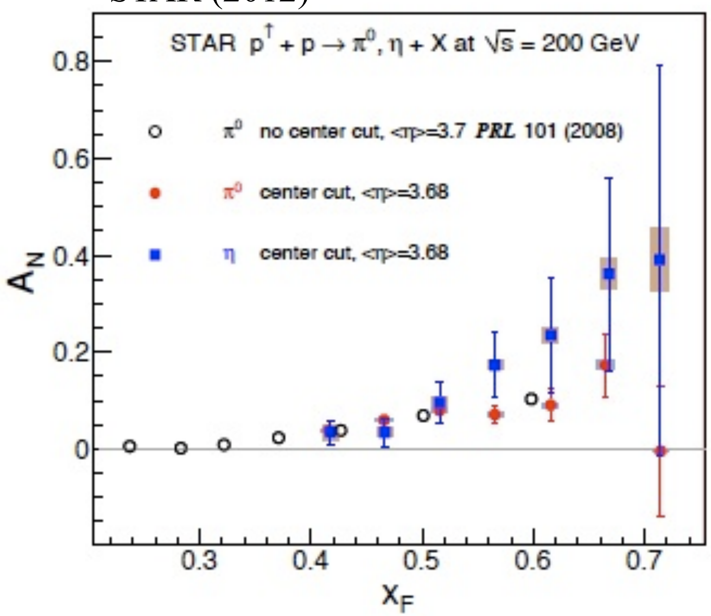
$$\left. + \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

Metz, DP (2013)

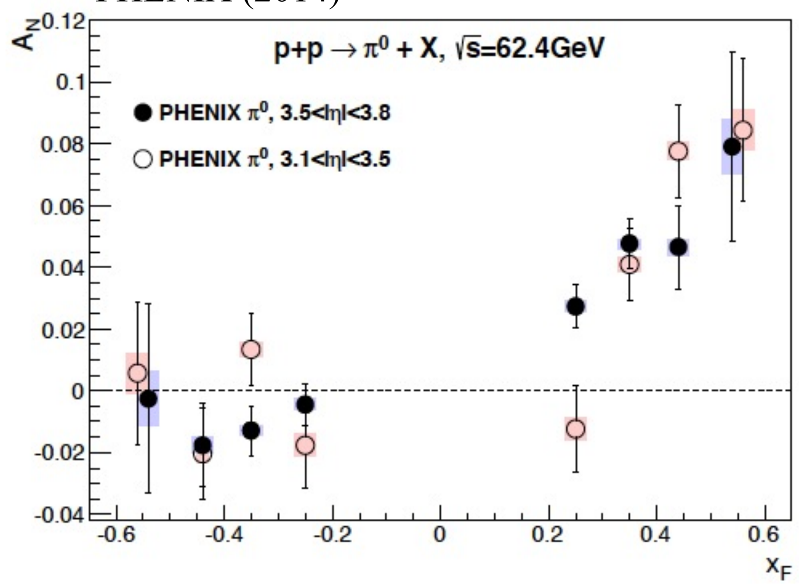
Plot from Aidala, Bass, Hasch, Mallot (2013)



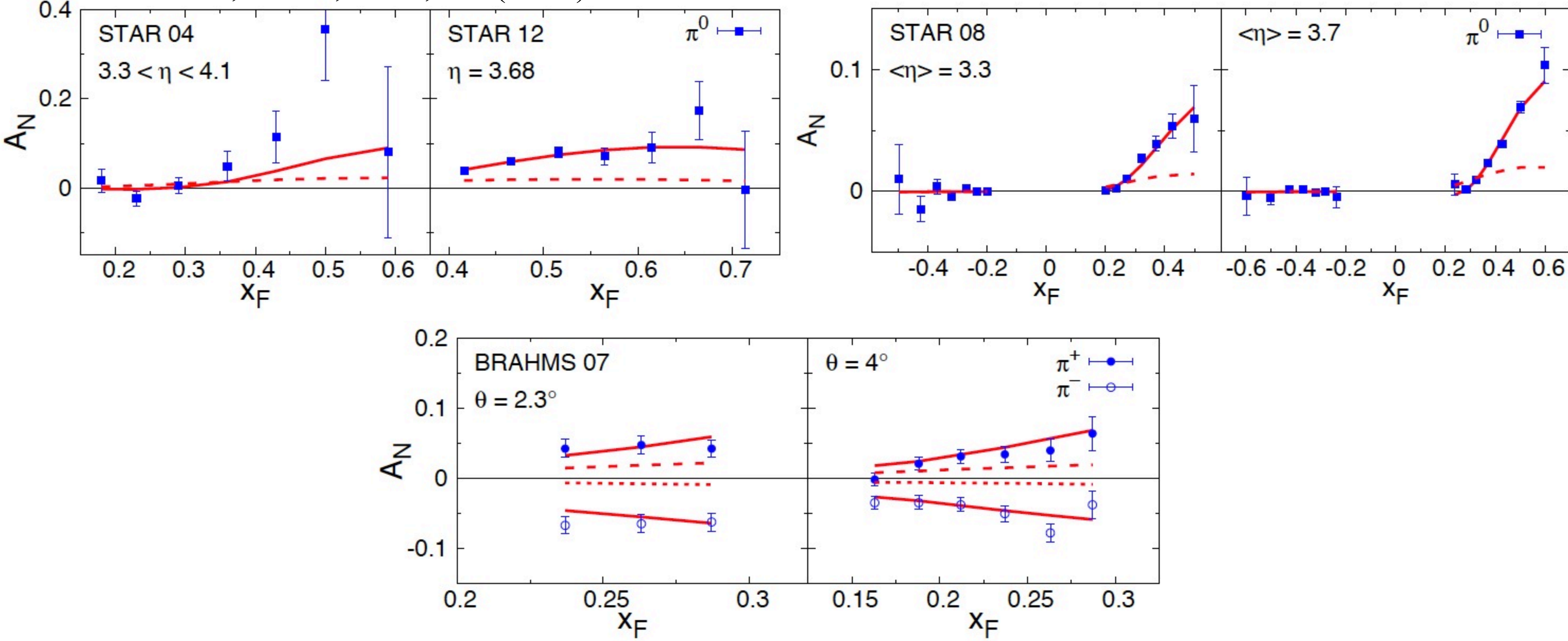
STAR (2012)



PHENIX (2014)



Kanazawa, Koike, Metz, DP (2014)



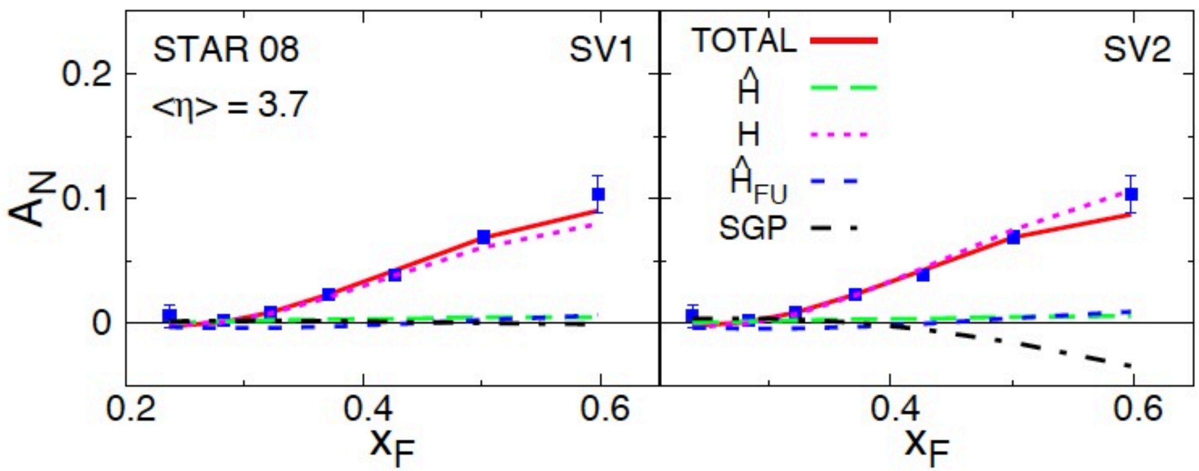
-Used Sivers function from SIDIS as input for Qiu-Sterman function $f_{1T}^{\perp(1)}(x) \propto T_F(x,x)$

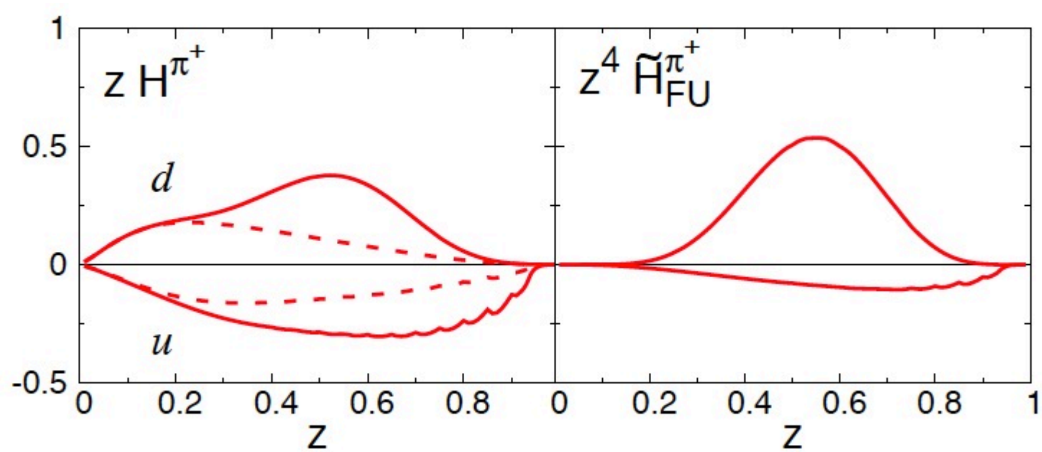
-Used Collins and transversity extracted from SIDIS/e^+e^-

-Used EOM relation for H

-Extracted $\hat{H}_{FU}^{\Im}(z,z_1)$

See talks by Koike, Gamberg


STAR 08
<η> = 3.7
SV1
SV2
TOTAL
Ĥ
H
Ĥ_FU
SGP
A_N
0.2
0.1
0
0.2
0.4
0.6
x_F


z H^π+
z^4 H̃^π+_FU
d
u
1
0.5
0
-0.5
0
0.2
0.4
0.6
0.8
1
z

EOM relation + Lorentz invariance relation (LIR) ➔

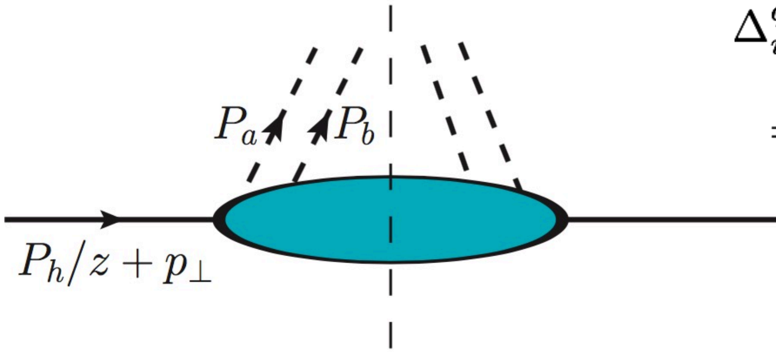
$$\boldsymbol{H(z)} = \int_z^1 dz_1 \int_{z_1}^{\infty} \frac{dz_2}{z_2^2}\, 2\left[\frac{\left(2(\frac{2}{z_1}-\frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)\delta\left(\frac{1}{z_1}-\frac{1}{z}\right)\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\boldsymbol{\hat{H}_{FU}^{\Im}(z_1,z_2)}\right]$$

$$\boldsymbol{H_1^{\perp(1)}(z)} = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^{\infty} \frac{dz_2}{z_2^2}\, \frac{\left(\frac{2}{z_1}-\frac{1}{z_2}\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\boldsymbol{\hat{H}_{FU}^{\Im}(z_1,z_2)}$$

Kanazawa, Koike, Metz, DP, Schlegel (2016)

quark-gluon-quark FF $\hat{H}_{FU}^{\Im}(z,z_1)$ could be the main cause of A_N

- Definitions (di-hadron)



$$\Delta^q_{ij}(z, p_\perp; P_a, P_b) = \frac{1}{N_c} \sum_X \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int \frac{d^2 z_\perp}{(2\pi)^2} \, \mathrm{e}^{-i\frac{\lambda}{z} - ip_\perp \cdot z_\perp} \langle 0 | q_i(0) | P_a, P_b;\, X\rangle \times \langle P_a, P_b;\, X | \, \bar{q}_j(\lambda m + z_\perp) \, |0\rangle$$

(Unpolarized) DiFFs
$(z, \zeta, R_\perp^2, p_\perp \cdot R_\perp, p_\perp^2)$

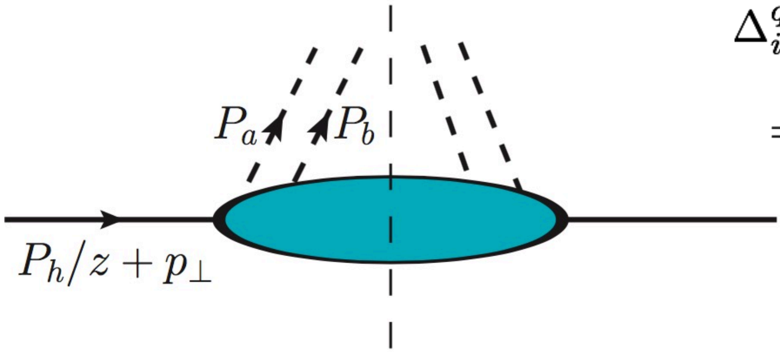
$$D_1, G_1^\perp, H_1^\sphericalangle, H_1^\perp$$

$$\Delta^{q\,[\gamma^-]}(z, \vec{p}_\perp; P_a, P_b) = D_1^{h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2)\,,$$

$$\Delta^{q\,[\gamma^-\gamma_5]}(z, \vec{p}_\perp; P_a, P_b) = \frac{\epsilon_\perp^{ij} R_\perp^i \, p_\perp^j}{M_a \, M_b} \, G_1^{\perp\, h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2)\,,$$

$$\Delta^{q\,[i\sigma^{i-}\gamma_5]}(z, \vec{p}_\perp; P_a, P_b) = -\frac{\epsilon_\perp^{ij} \, R_\perp^j}{M_a + M_b} \, H_1^{\sphericalangle\, h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2) - \frac{\epsilon_\perp^{ij} \, p_\perp^j}{M_a + M_b} \, H_1^{\perp\, h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2)$$

➢ Definitions (di-hadron)



$$\Delta^q_{ij}(z, p_\perp; P_a, P_b) = \frac{1}{N_c} \sum_X \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int \frac{d^2 z_\perp}{(2\pi)^2}\, \mathrm{e}^{-i\frac{\lambda}{z} - i p_\perp \cdot z_\perp} \langle 0 | q_i(0) | P_a, P_b;\, X\rangle \times \langle P_a, P_b;\, X |\, \bar{q}_j(\lambda m + z_\perp)\, |0\rangle$$

Integrated (Unpolarized) DiFFs (z, ζ, M_h^2)

$$D_1,\ H_1^{\sphericalangle}$$

$$\Delta^{q\,[\gamma^-]}(z, \vec{p}_\perp; P_a, P_b) = D_1^{h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2)\,,$$

$$\Delta^{q\,[\gamma^-\gamma_5]}(z, \vec{p}_\perp; P_a, P_b) = \frac{\epsilon_\perp^{ij}\, R_\perp^i\, p_\perp^j}{M_a\, M_b}\, G_1^{\perp\, h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2)\,,$$

$$\Delta^{q\,[i\sigma^{i-}\gamma_5]}(z, \vec{p}_\perp; P_a, P_b) = -\frac{\epsilon_\perp^{ij}\, R_\perp^j}{M_a + M_b}\, H_1^{\sphericalangle\, h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2) - \frac{\epsilon_\perp^{ij}\, p_\perp^j}{M_a + M_b}\, H_1^{\perp\, h_a h_b/q}(z, \zeta, \vec{R}_\perp^2, \vec{p}_\perp \cdot \vec{R}_\perp, \vec{p}_\perp^2)$$

➢ Electron-positron/SIDIS

(Artru, Collins (1996); Bianconi, Boffi, Jakob, Radici (1999); Boer, Jakob, Radici (2003); de Florian, Vanni (2004); Bacchetta, Courtoy, Radici (2013); Radici, Courtoy, Bacchetta, Guagnelli (2015); Pisano, Radici (2016))

$$e^+e^- \to (h_{a1}\, h_{a2})\,(h_{b1}\, h_{b2})\, X$$

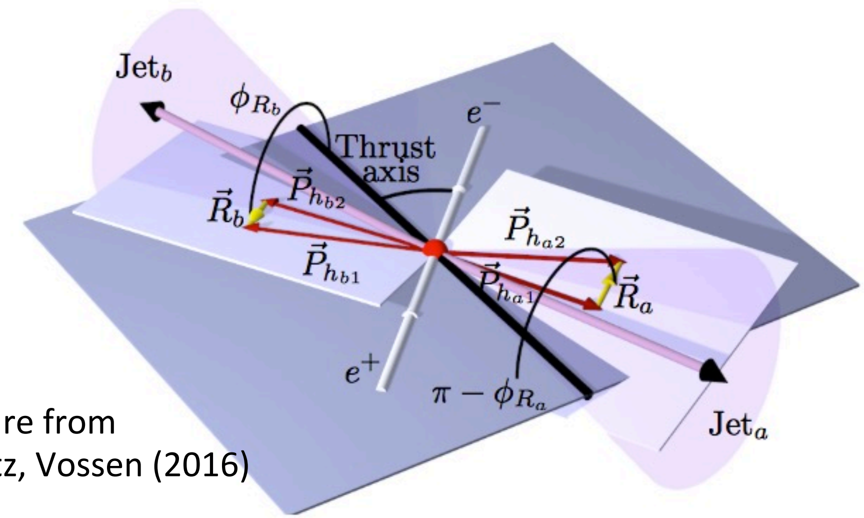


Figure from
Metz, Vossen (2016)

$$e\, N \to e'\,(h_a\, h_b)\, X$$

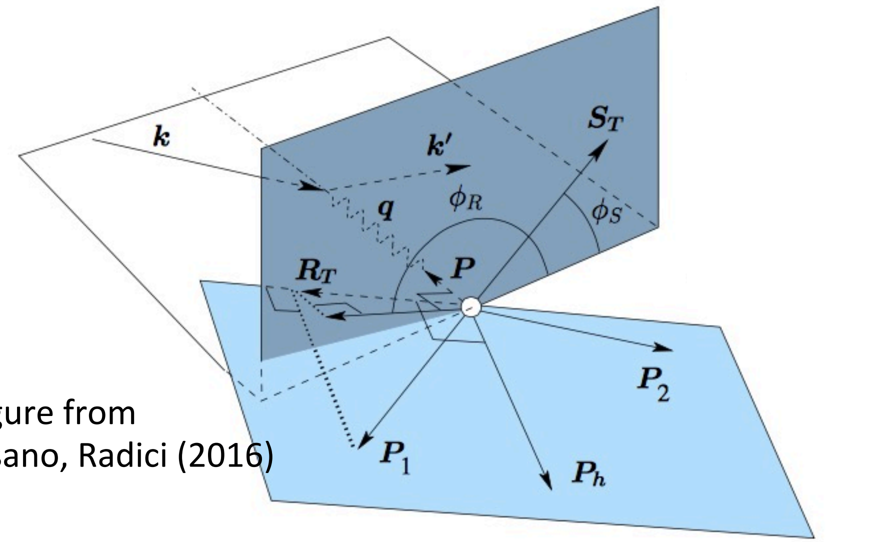


Figure from
Pisano, Radici (2016)

$$\frac{d\sigma}{d\Omega\, dz_a\, dz_b\, d\phi_{Ra}\, d\phi_{Rb}} \propto \cdots + B(y)\cos(\phi_{Ra} + \phi_{Rb}) \times H_1^{\sphericalangle}(z_a, M_{ha})\, \bar{H}_1^{\sphericalangle}(z_b, M_{hb})$$

$$\frac{d\sigma}{dx\, dy\, dz\, d\phi_R\, dM_h^2} \propto \cdots - |\vec{S}_\perp||\vec{R}_\perp| B(y)\sin(\phi_R + \phi_S) \times h_1(x)\, H_1^{\sphericalangle}(z, M_h)$$

➢ Electron-positron/SIDIS

(Artru, Collins (1996); Bianconi, Boffi, Jakob, Radici (1999); Boer, Jakob, Radici (2003); de Florian, Vanni (2004); Bacchetta, Courtoy, Radici (2013); Radici, Courtoy, Bacchetta, Guagnelli (2015); Pisano, Radici (2016))

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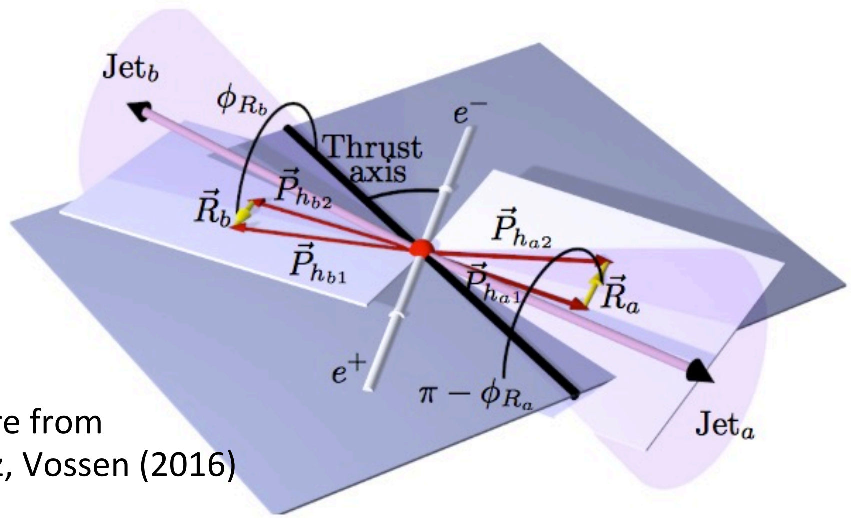


Figure from Metz, Vossen (2016)

$$e\, N \to e'\,(h_a\, h_b)\, X$$

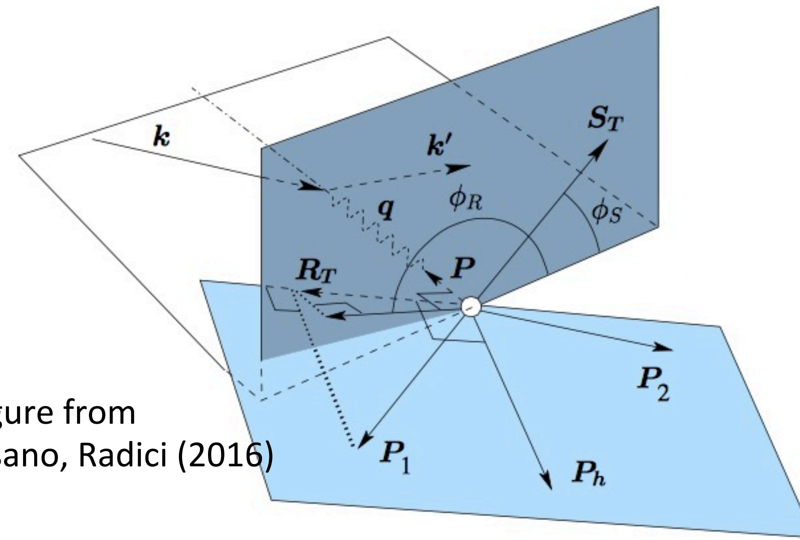


Figure from Pisano, Radici (2016)

$$\frac{d\sigma}{d\Omega\, dz_a\, dz_b\, d\phi_{Ra}\, d\phi_{Rb}} \propto \cdots + B(y)\cos(\phi_{Ra}+\phi_{Rb}) \times H_1^{\sphericalangle}(z_a, M_{ha})\, \bar{H}_1^{\sphericalangle}(z_b, M_{hb})$$

$$\frac{d\sigma}{dx\, dy\, dz\, d\phi_R\, dM_h^2} \propto \cdots - |\vec{S}_\perp||\vec{R}_\perp| B(y)\sin(\phi_R+\phi_S) \times h_1(x)\, H_1^{\sphericalangle}(z, M_h)$$

Extract transversity in a collinear framework*

*evolution of DiFFs different than single-hadron FFs

(Konishi, Ukawa, Veneziano (1979); Sukhatme, Lassila (1980); de Florian, Vanni (2004))

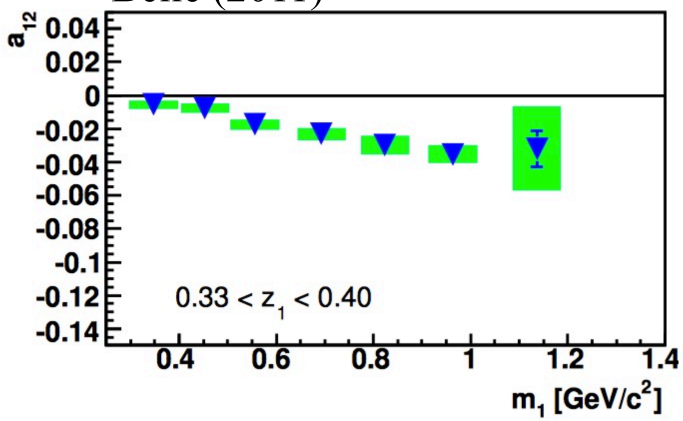
➢ Electron-positron/SIDIS

(Artru, Collins (1996); Bianconi, Boffi, Jakob, Radici (1999); Boer, Jakob, Radici (2003); de Florian, Vanni (2004); Bacchetta, Courtoy, Radici (2013); Radici, Courtoy, Bacchetta, Guagnelli (2015); Pisano, Radici (2016))

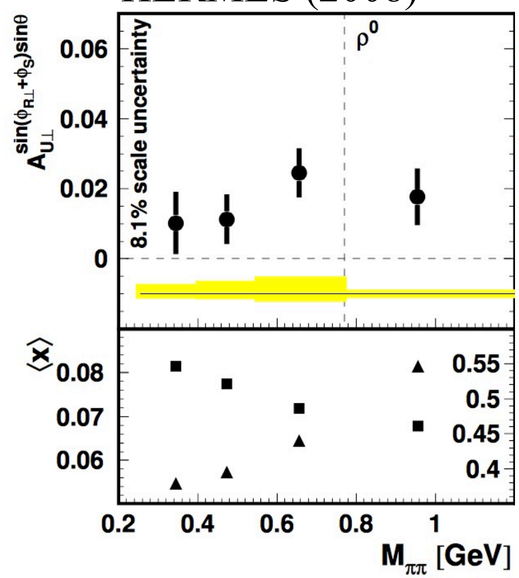
$$e^+e^- \to (h_{a1}\, h_{a2})\,(h_{b1}\, h_{b2})\, X$$

$$e\, N \to e'\,(h_a\, h_b)\, X$$

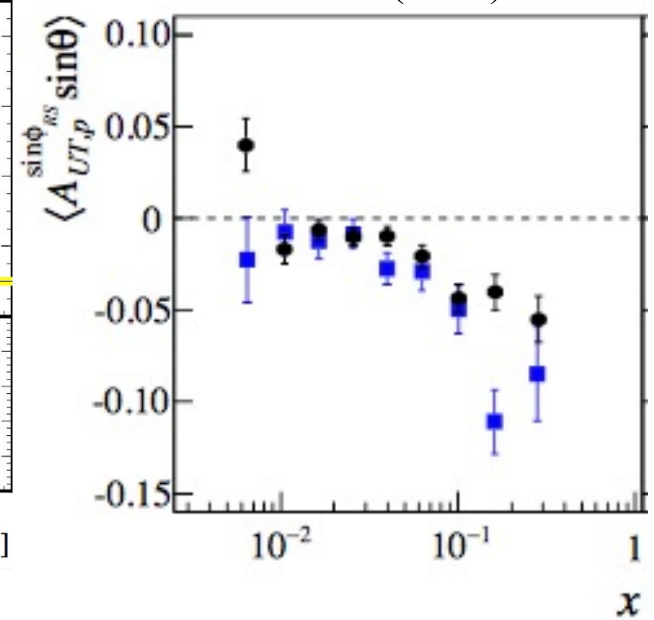
Belle (2011)



HERMES (2008)

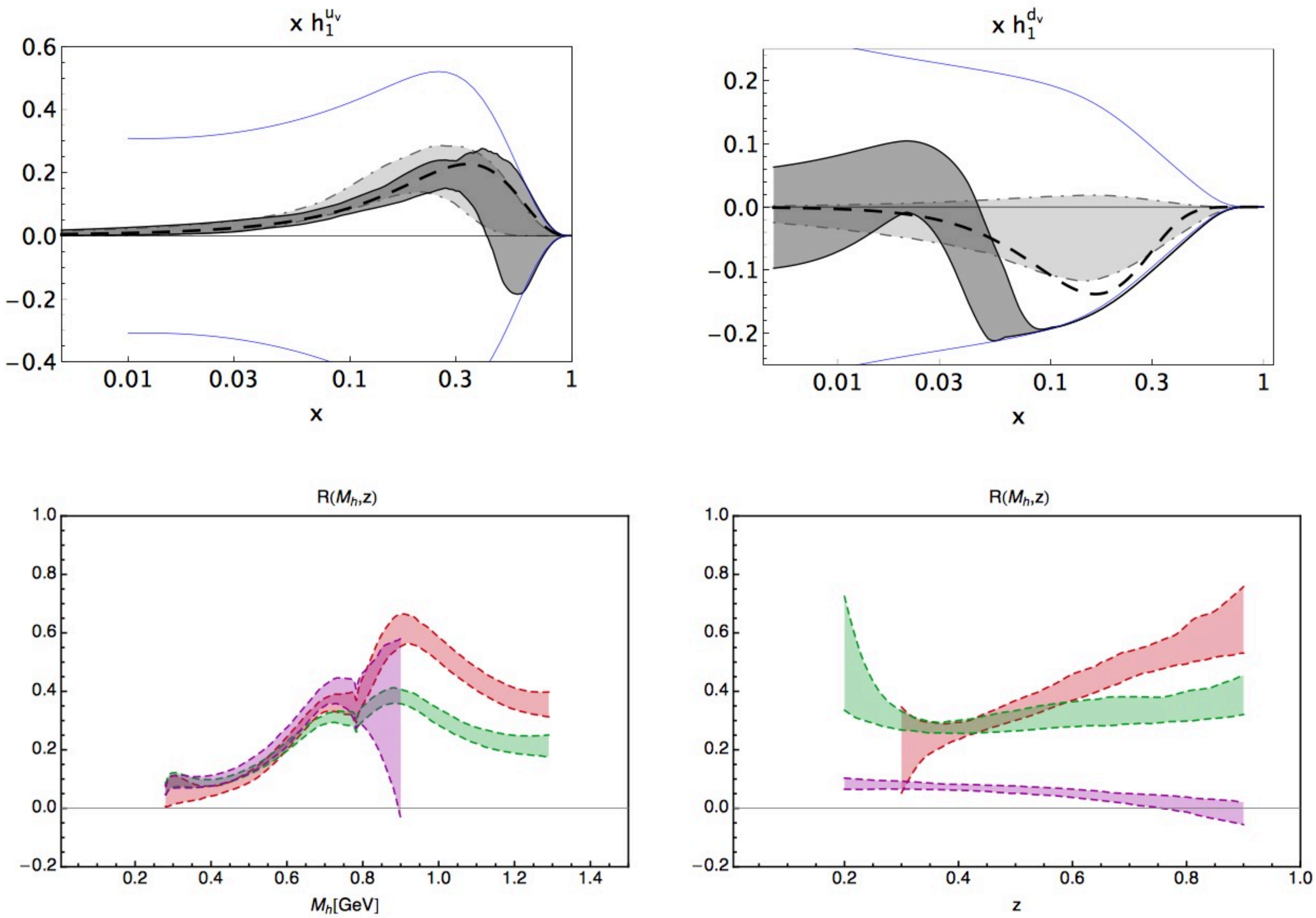


COMPASS (2014)



Radici, et al. (2015)

See talk by Radici



- Proton-proton collisions

(Bacchetta, Radici (2004); Radici, Ricci, Bacchetta, Mukherjee (2016))

$$p\,p \longrightarrow (h_a\,h_b)\,X$$

$$d\sigma_{UT} \propto \sin(\phi_R - \phi_{S_a})\,h_1(x_a) \otimes f_1(x_b) \otimes H_1^{\sphericalangle}(z, M_h^2) \otimes \hat{\sigma}_{pol}$$

➢ Proton-proton collisions

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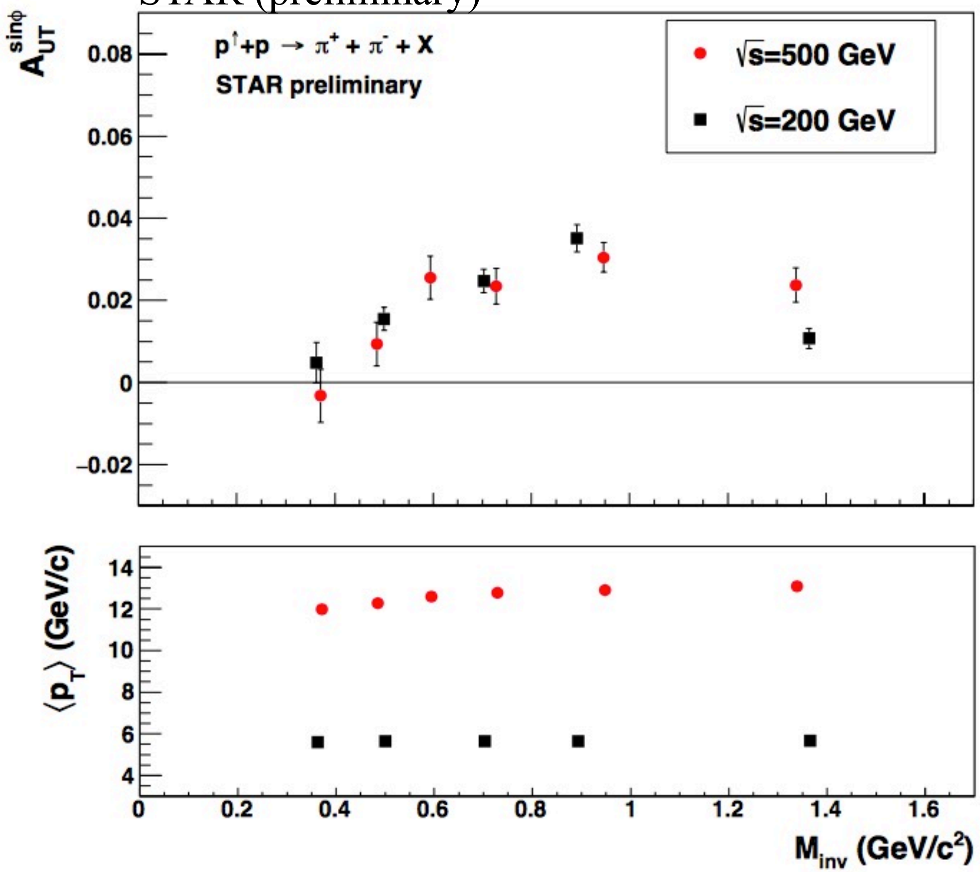
STAR (preliminary)



See talks by Drachenberg, Skoby

➢ Proton-proton collisions

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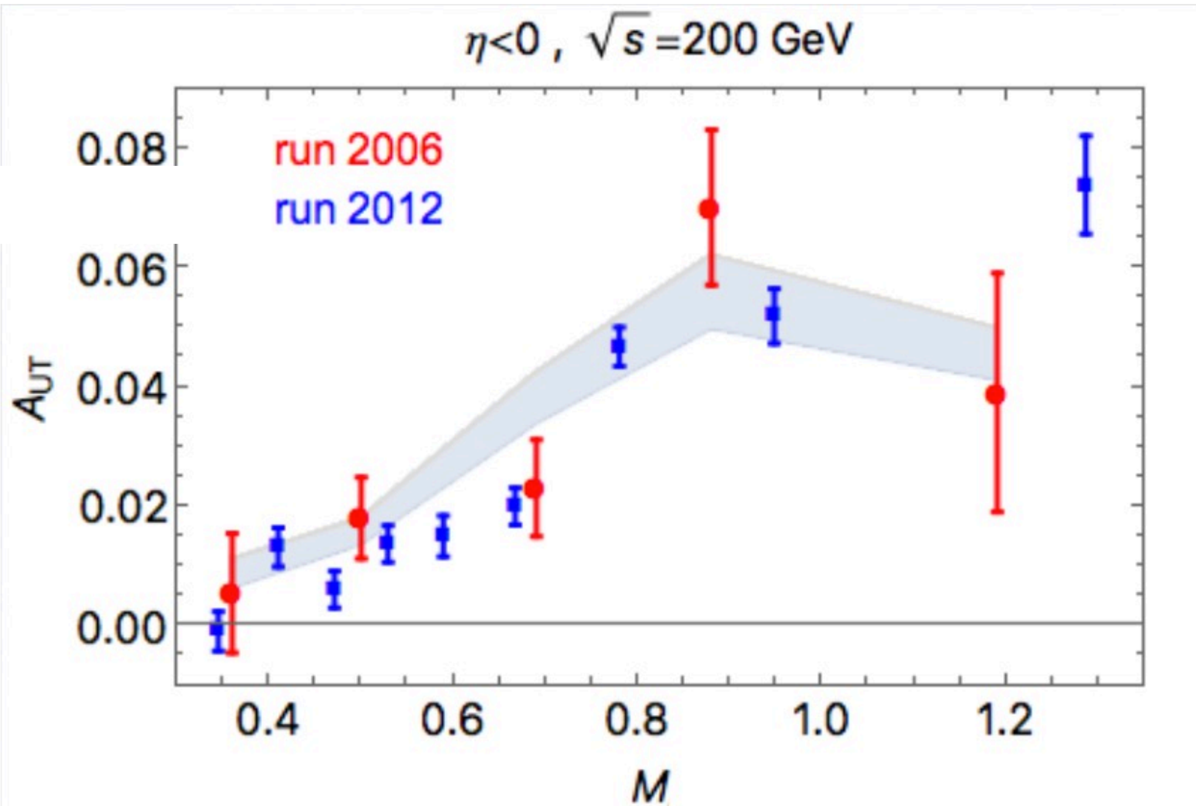
$$p\,p \longrightarrow (h_a\,h_b)\,X$$

$$d\sigma_{UT} \propto \sin(\phi_R - \phi_{S_a})\,h_1(x_a) \otimes f_1(x_b) \otimes H_1^{\sphericalangle}(z, M_h^2) \otimes \hat{\sigma}_{pol}$$

Radici, et al. (2016), Data from STAR (2015, blue is preliminary)



- Another probe of transversity
- Possible issues in describing A_{UT} vs. η and A_{UT} vs. P_T in the forward region
- In general, no knowledge of D_1 for DiFFs
- Need global fit with SIDIS/e^+e^-

See talks by Drachenberg, Radici, Skoby

➢ Other topics of importance

- Extraction of unpolarized FF $D_1(z)$, $D_1(z, z^2\vec{p}_\perp^{\,2})$ **See talks by Gonzalez, Leader, Nocera, Seidl**
- Other SIDIS azimuthal modulations involve Collins - access to Boer-Mulders, pretzelocity
- $e^+e^- \to h_a h_b X$ with lepton and/or hadron (Lambda) polarization and EW effects
  See talk by Kaibao for $V\pi X$ final state **See talk by Guan**
- Model calculations of FFs (CANNOT compute FFs on lattice) **See talks by Kerbizi, Schweitzer**
- Sum rules (or lack there-of) providing constraints on FFs
- Twist-3 TMD FFs
- A_N for Lambda production **See talk by Yabe**
- Measurement of TMD DiFFs

⋮

See recent review by Metz and Vossen - arXiv:1607.02521

➢ Other topics of importance

- Extraction of unpolarized FF $D_1(z)\,,\ D_1(z, z^2\vec{p}_\perp^{\,2})$ **See talks by Gonzalez, Leader, Nocera, Seidl**
- Other SIDIS azimuthal modulations involve Collins - access to Boer-Mulders, pretzelocity
- $e^+e^- \to h_a\, h_b\, X$ with lepton and/or hadron **(Lambda) polarization** and EW effects
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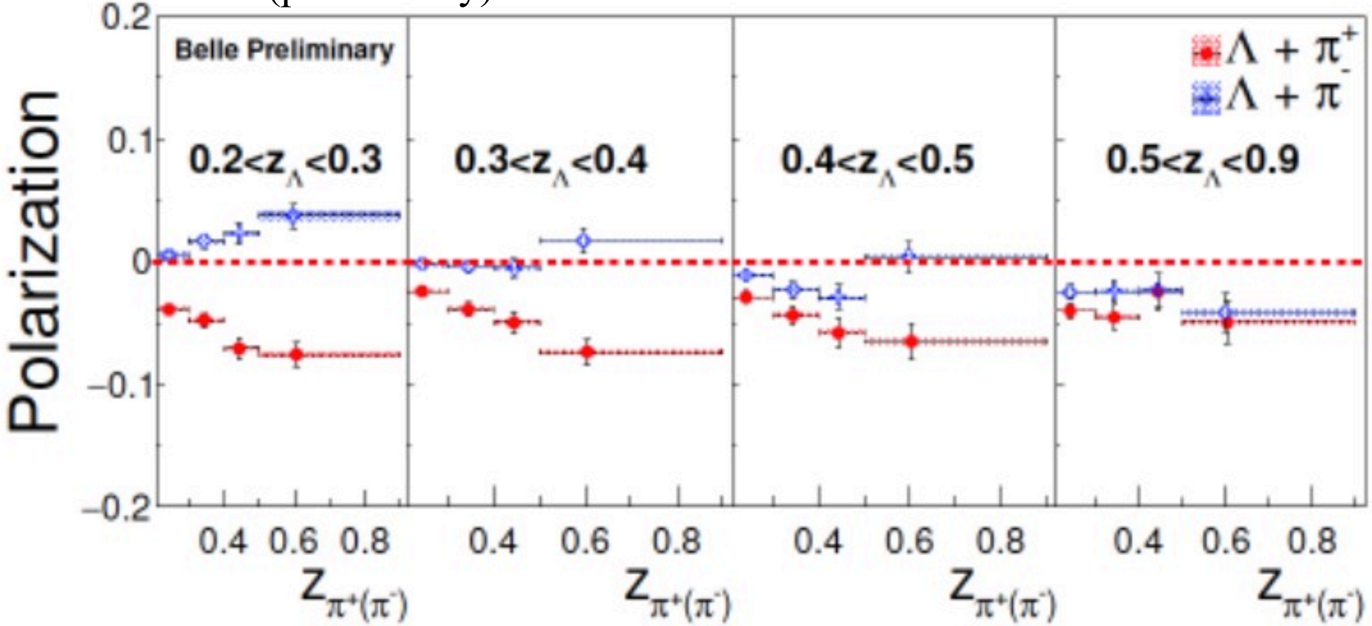
⋮

See recent review by Metz and Vossen - arXiv:1607.02521

$e^+e^- \to \Lambda^{\uparrow}\,\pi^{\pm}\,X$

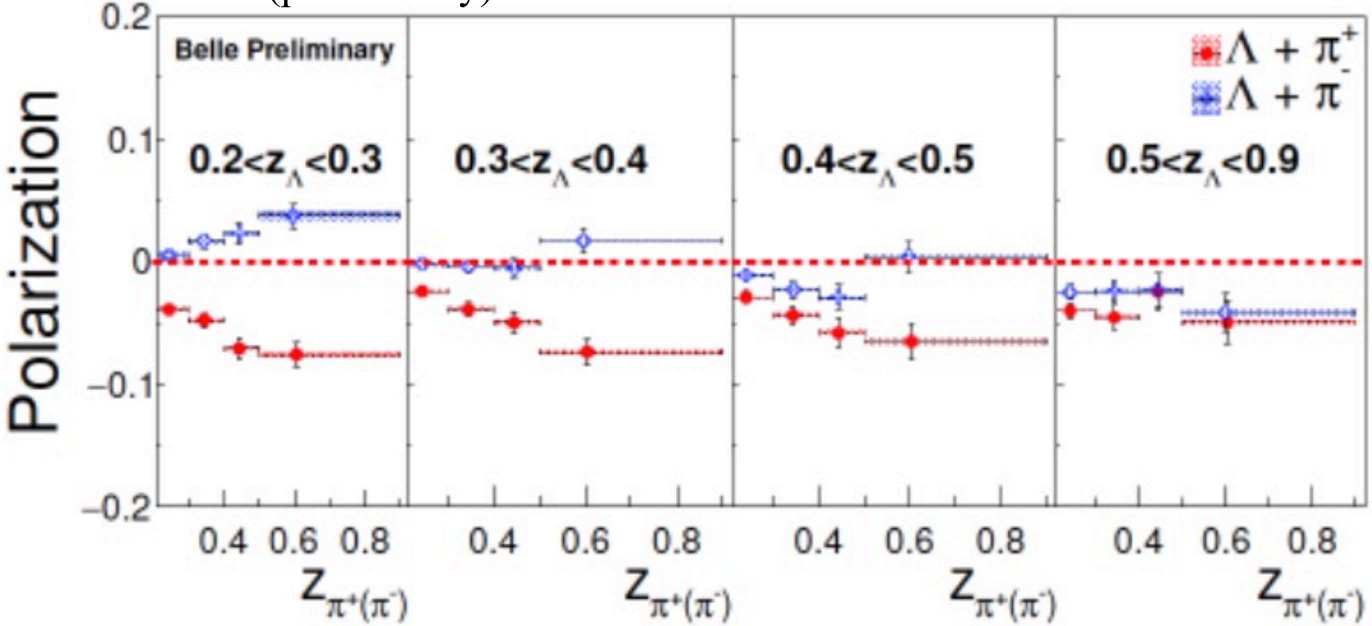
See talk by Guan

Belle (preliminary)



$$e^+e^- \to \Lambda^\uparrow \pi^\pm X$$

Belle (preliminary)



$$e^+e^- \to \Lambda^\uparrow \Lambda^\uparrow X \quad + \quad e\,p^\uparrow \to e'\,\Lambda^\uparrow X \,/\, p^\uparrow p \to \Lambda^\uparrow X$$

See talk by Mei

$H_1(z_1)$ x $H_1(z_2)$

$h_1(x)$ x $H_1(z)$

extract transversity in "true" collinear factorization

➢ Summary and outlook

- Knowledge of fragmentation functions are crucial to understand nucleon structure, and, moreover, provide their own rich source of measurements and phenomenology

- Much progress has been made in understanding FFs in spin-dependent observables (Collins effect, A_N in *pp*, A_{UT} di-hadron, …), yet many open questions remain

- More precise measurements (Belle II, COMPASS, EIC, JLab12, RHIC, SuperKEKB, …) and phenomenological extractions (NLO, NNLO, proper TMD evolution, …) will be needed in order to fully grasp the 3D structure of hadrons