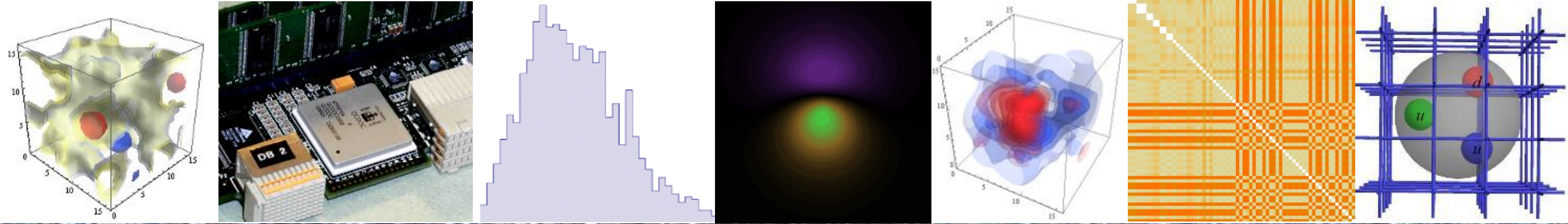


# Lattice QCD Overview


## Huey-Wen Lin

# Outline

## § Lattice QCD for Nucleon Structure

- A brief introduction and
  selected results near physical pion mass

## § Spotlight on New Calculations

- Work in progress with great future prospects:
  strangeness and PDF Bjorken-$x$ dependence

## § Appendix

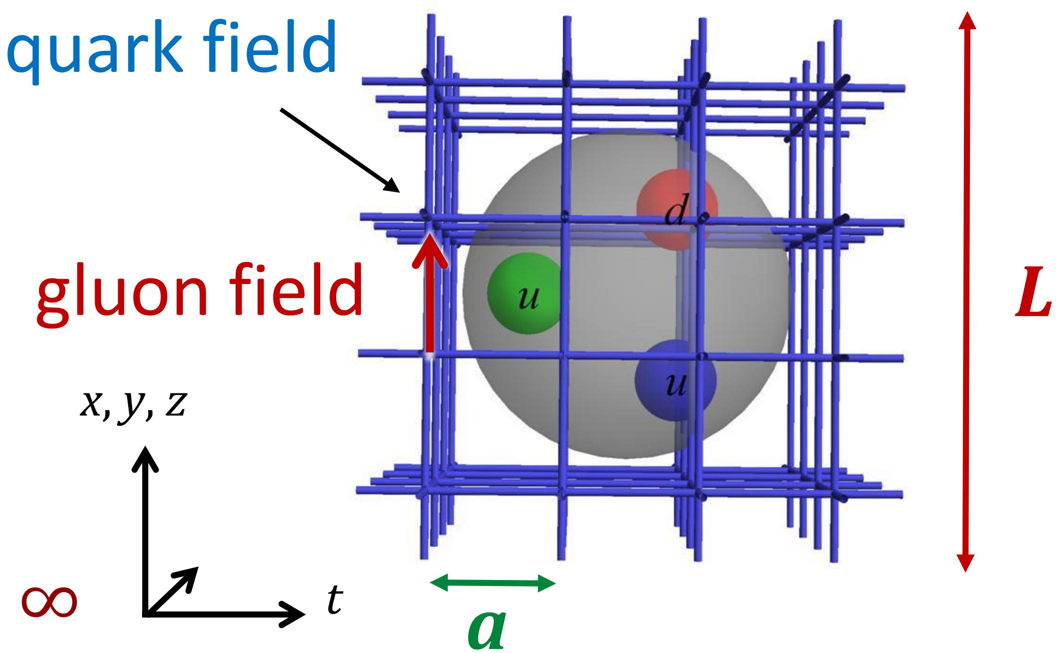
- Lattice systematics study

# What is Lattice QCD?

- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}\,O\left(\bar{\psi},\psi,A\right)$$

  in **Euclidean** space
  - Quark mass parameter (described by $m_\pi$)
  - Impose a UV cutoff
    discretize spacetime
  - Impose an infrared cutoff
    finite volume



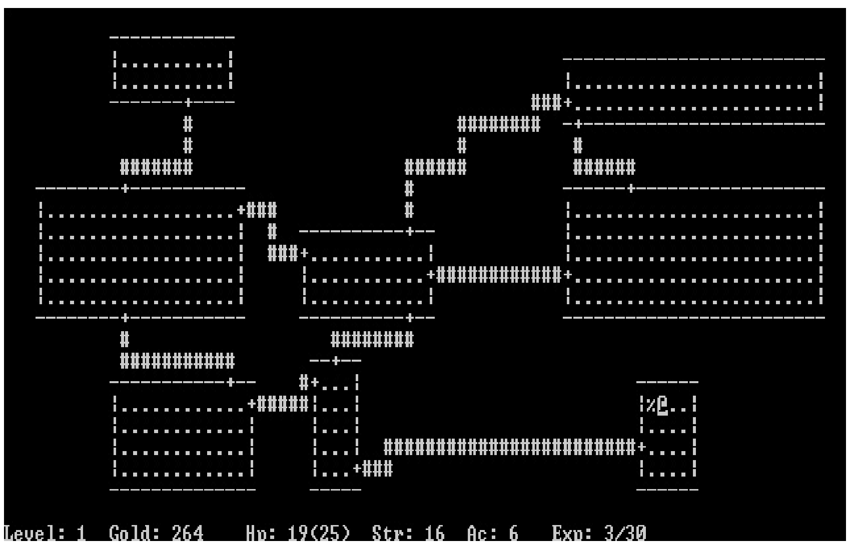

- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

# Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

1980s




Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

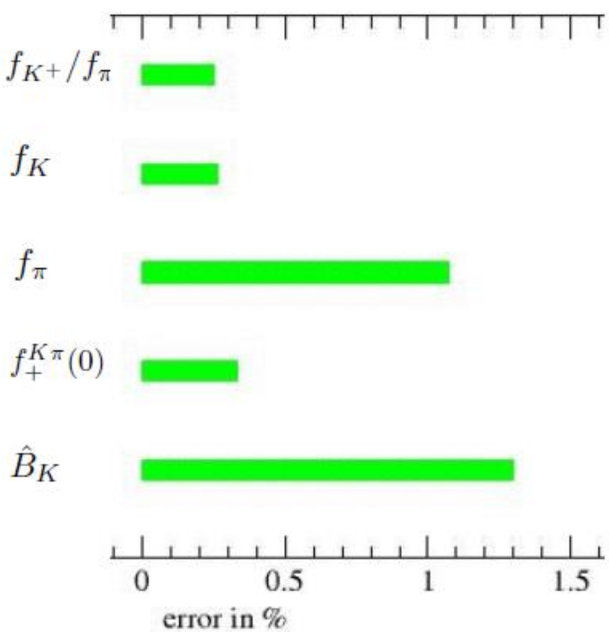
# Successful Examples

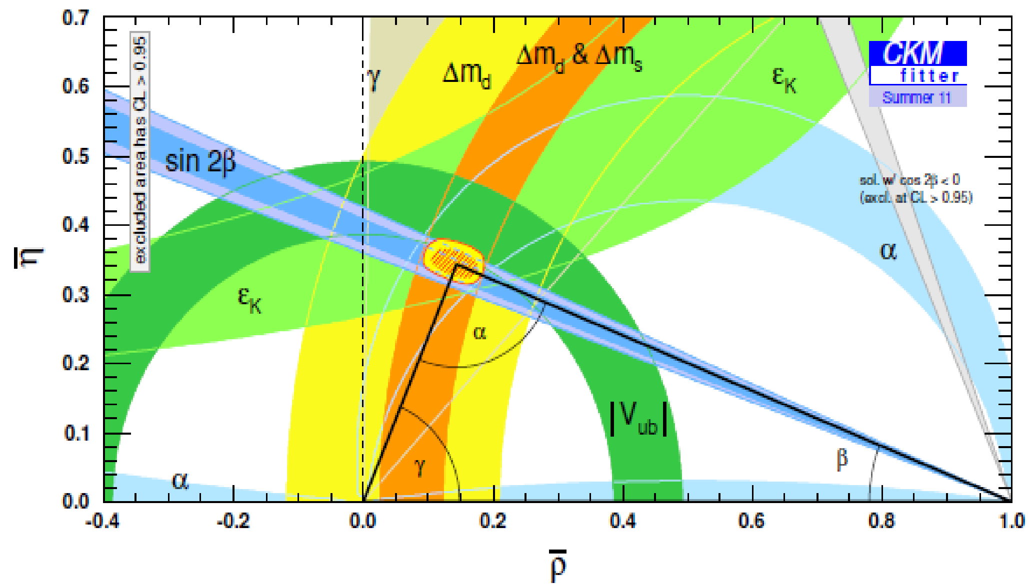
§ Lattice flavor physics provides precise inputs from the SM

A. El-Khadra, Sep. 2015, INT workshop "QCD for New Physics at the Precision Frontier"

- Very precise results in many meson systems

errors (in %) (preliminary) FLAG-3 averages




§ We are beginning to do precision calculations in nucleons

# The Trouble with Nucleons

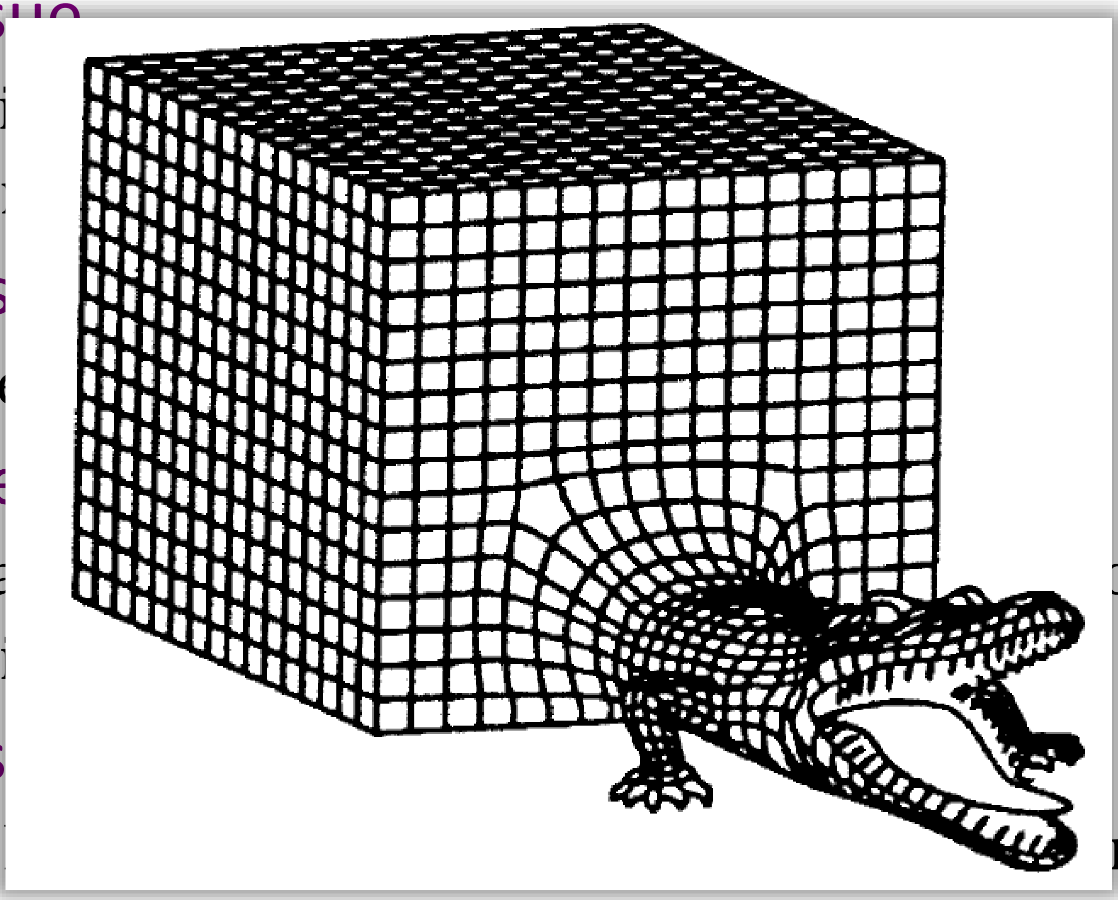
Nucleons are more complicated than mesons because…

- Noise issue
  - Signal diminishes at large $t_E$ relative to noise
  - Gets worse when quark mass decreases
- Excited-state contamination
  - Nearby excited state: Roper(1440)
- Hard to extrapolate in pion mass
  - $\Delta$ resonance nearby; multiple expansions, poor convergence…
  - Less an issue in the physical pion-mass era
- Requires larger volume and higher statistics
  - Ensembles are not always generated with nucleons in mind
  - **High-statistics**: large measurement and long trajectory

# The Trouble with Nucleons

Nucleons are more complicated than mesons because…

- Noise issue
  - Signal di
  - Gets wo
- Excited-s
  - Nearby
- Hard to e
  - Δ resona… onvergence…
  - Less an
- Requires
  - Ensembl… ns in mind
  - **High-s**tatistics: large measurement and long trajectory



PROCEED WITH CAUTION

# The Trouble with Nucleons

Nucleons are more complicated than mesons because…

- § Noise issue
  - Signal di
  - Gets wo
- § Excited-s
  - Nearby e
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  - Δ resona … onvergence…
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  - Ensembl … ns in mind
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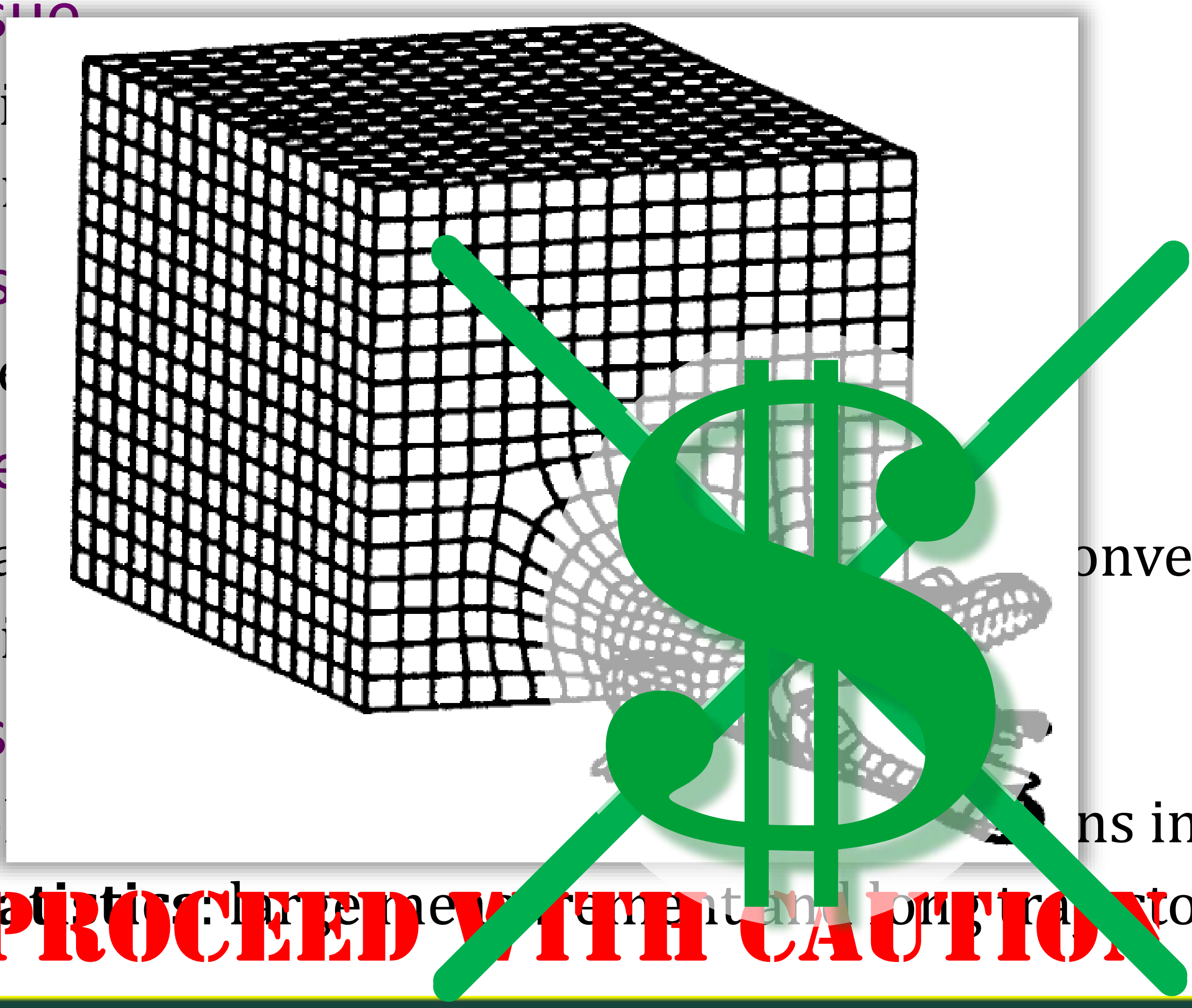


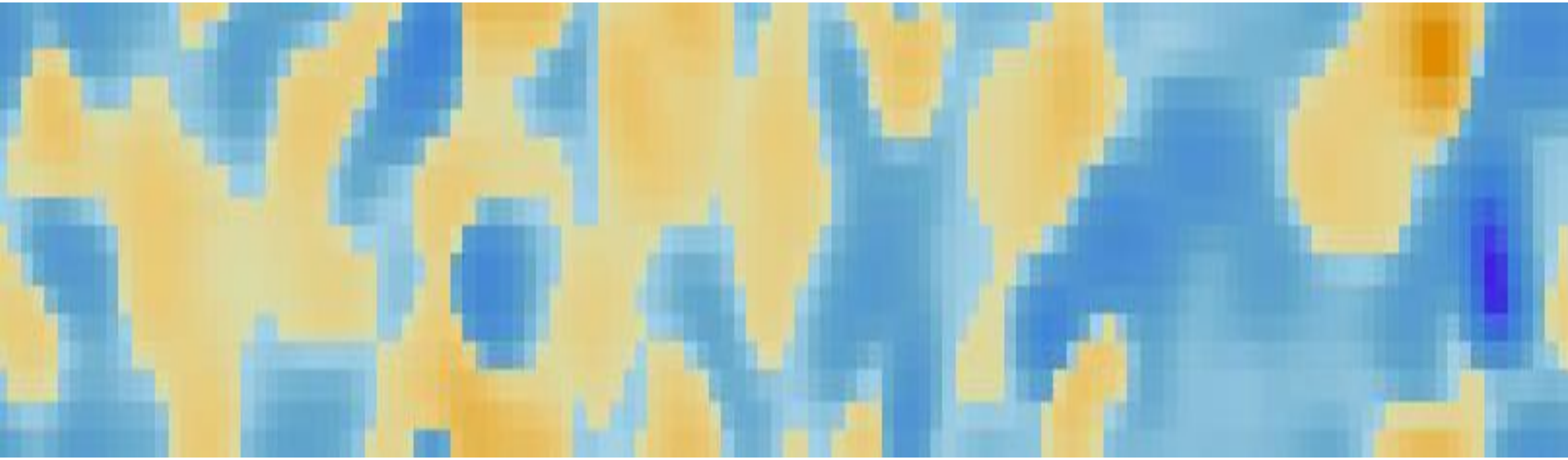
PROCEED WITH CAUTION

# Thank you!


Slide from M. Savage
**Exascale Requirements Review for Nuclear Physics**
Gaithersburg, MD 20877
June 15–17, 2016

§ More computational resources ⟹ better chance your favorite quantities are calculated

# Nucleon Matrix Elements

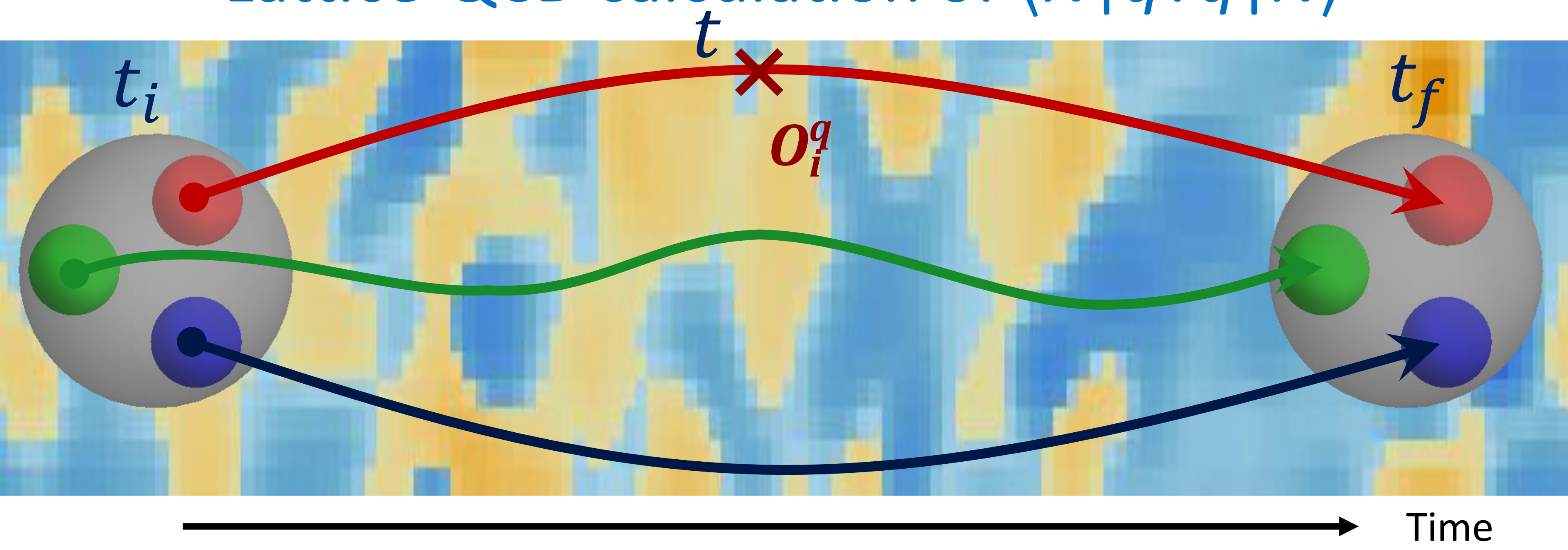


§ Pick a QCD vacuum

- Gauge/fermion actions, flavour (2, 2+1, 2+1+1), $m_\pi$, $a$, $L$, …

# Nucleon Matrix Elements

## Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

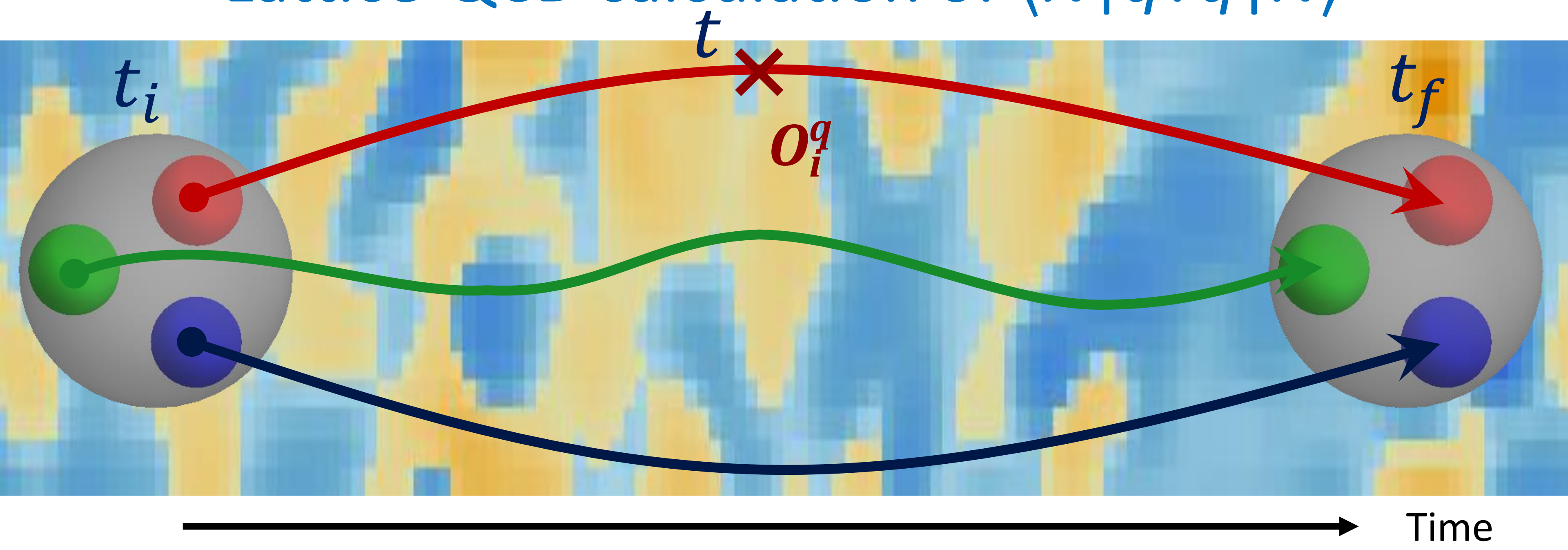



§ Construct correlators (hadronic observables)

- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

# Nucleon Matrix Elements

## Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



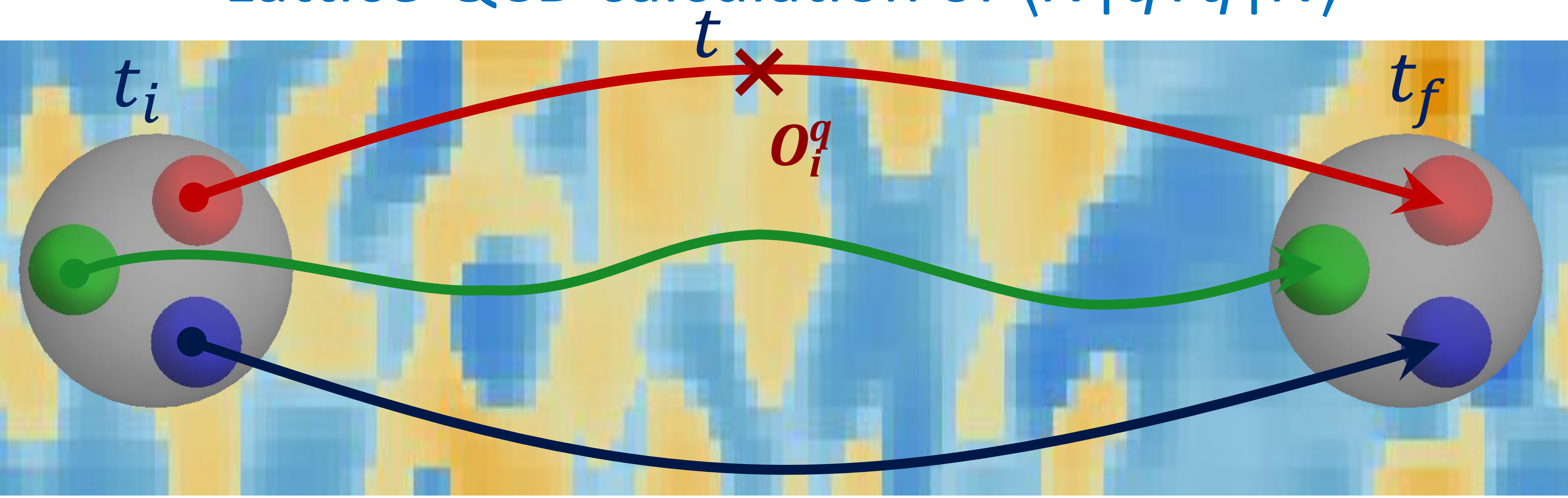

§ Analysis (extract couplings)

$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

$$C^{2\mathrm{pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)}+\ldots$$

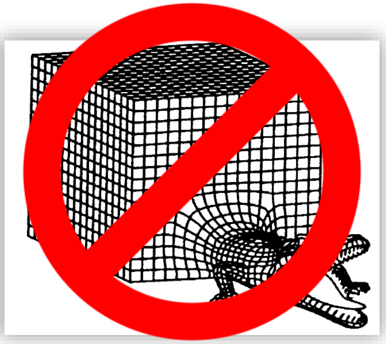
# Nucleon Matrix Elements

## Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$




§ Systematic Uncertainty (nonzero *a*, finite *L*, etc.)

- Contamination from excited states (see Appendix)
- Nonperturbative renormalization
  e.g. RI/SMOM scheme $\overline{\text{MS}}$ 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



# Precision Nucleon Couplings

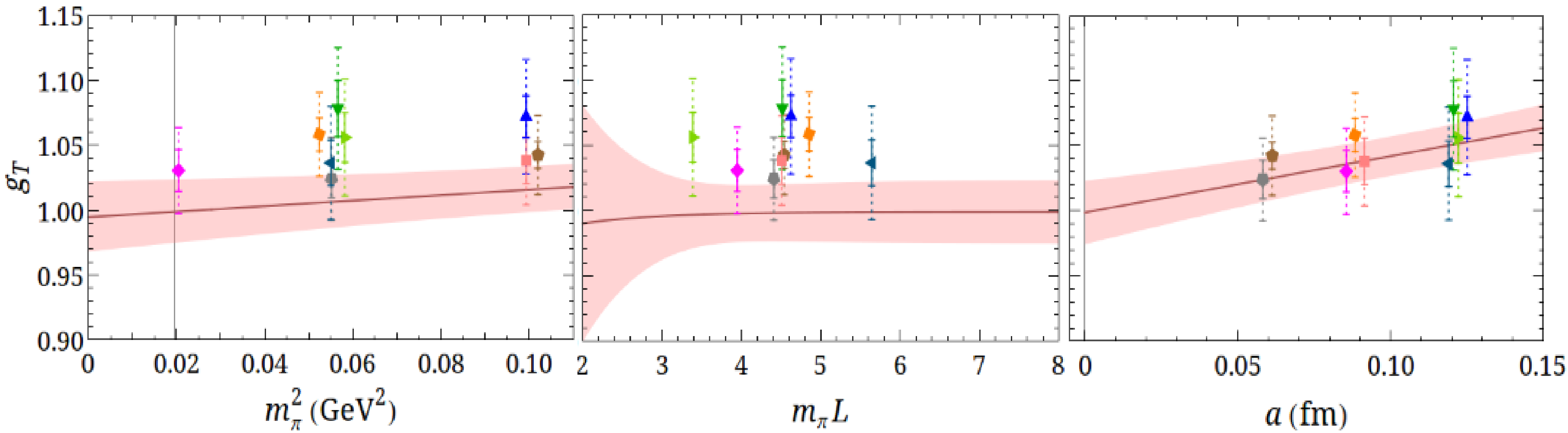
§ $g_T$: zeroth moment of transversity $\Gamma = \sigma_{\mu\nu}$

$$g_T = \int_{-1}^{1} dx\, \delta q(x)$$

§ A state-of-the art calculation (PNDME)

- Extrapolate to the physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

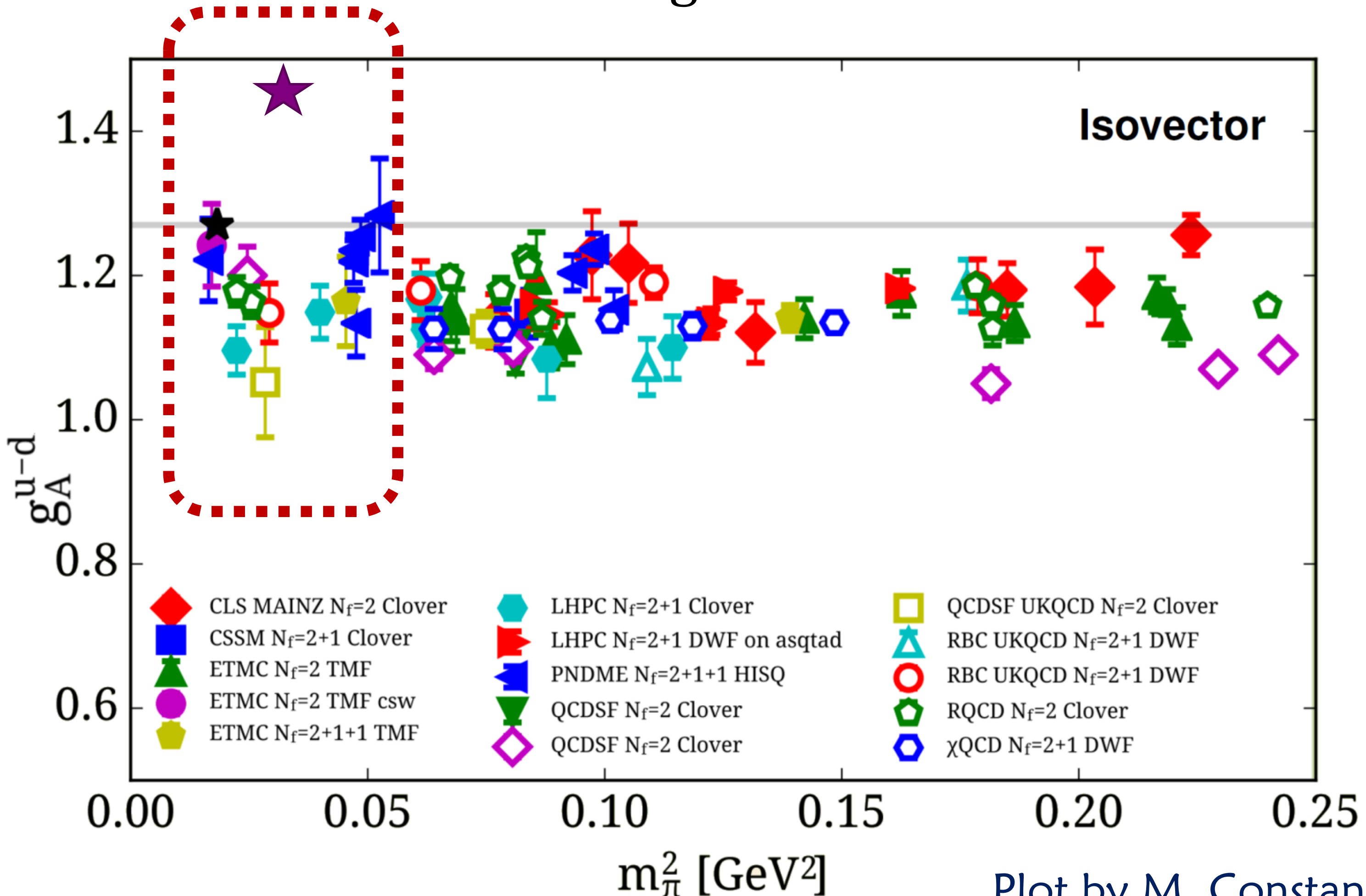



First extrapolation to the physical limit
of a nucleon matrix element!

# Precision Nucleon Couplings

§ Usually more than one LQCD calculation

- Lattice results should agree in the continuum limit




$\Gamma = \gamma_\mu \gamma_5$

Plot by M. Constantinou

# Precision Nucleon Couplings

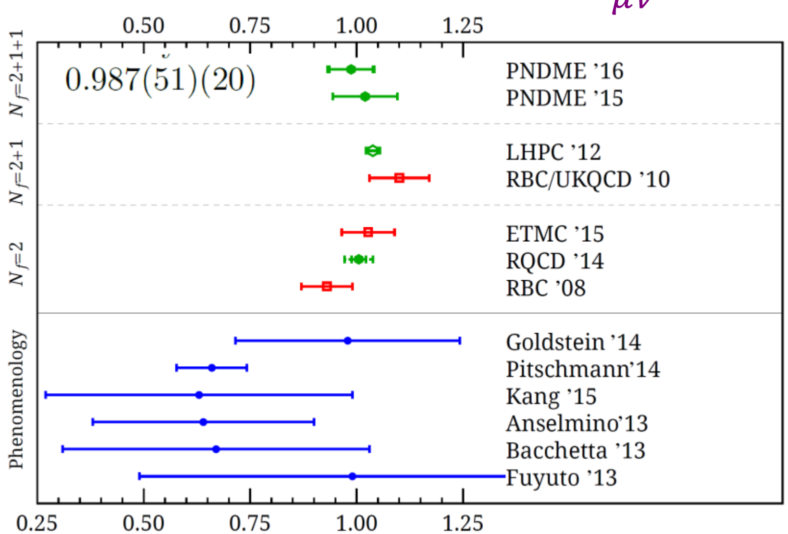
## FLAG rating system

PNDME, 1506.06411; 1606.07049

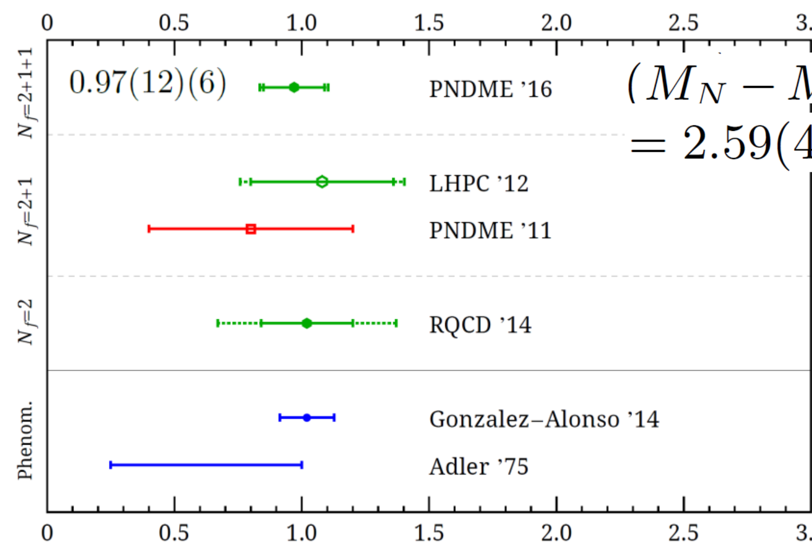
## New: excited-state rating

| Collaboration | Ref. | publication status | $N_f$ | chiral extrapolation | continuum extrapolation | finite volume | excited state | renormalization | $g_T$ |
|---|---|---|---|---|---|---|---|---|---|
| PNDME'15 | This work | P | 2+1+1 | ★ | ★ | ★ | ★ | ★ | 1.020(76)[a] |
| ETMC'13 | [30] | C | 2+1+1 | ■ | ○ | ○ | ■ | ★ | 1.11(3)[b] |
| LHPC'12 | [28] | A | 2+1 | ★ | ○ | ★ | ○ | ★ | 1.037(20)[c] |
| RBC/UKQCD'10 | [29] | A | 2+1 | ○ | ■ | ★ | ★ | ★ | 1.10(7)[d] |
| RQCD'14 | [31] | P | 2 | ★ | ★ | ★ | ○ | ★ | 1.005(17)(29)[e] |
| ETMC'13 | [30] | C | 2 | ★ | ■ | ○ | ■ | ○ | 1.114(46)[f] |
| RBC'08 | [32] | P | 2 | ■ | ■ | ★ | ■ | ★ | 0.93(6)[g] |

$g_T$ $\Gamma = \sigma_{\mu\nu}$



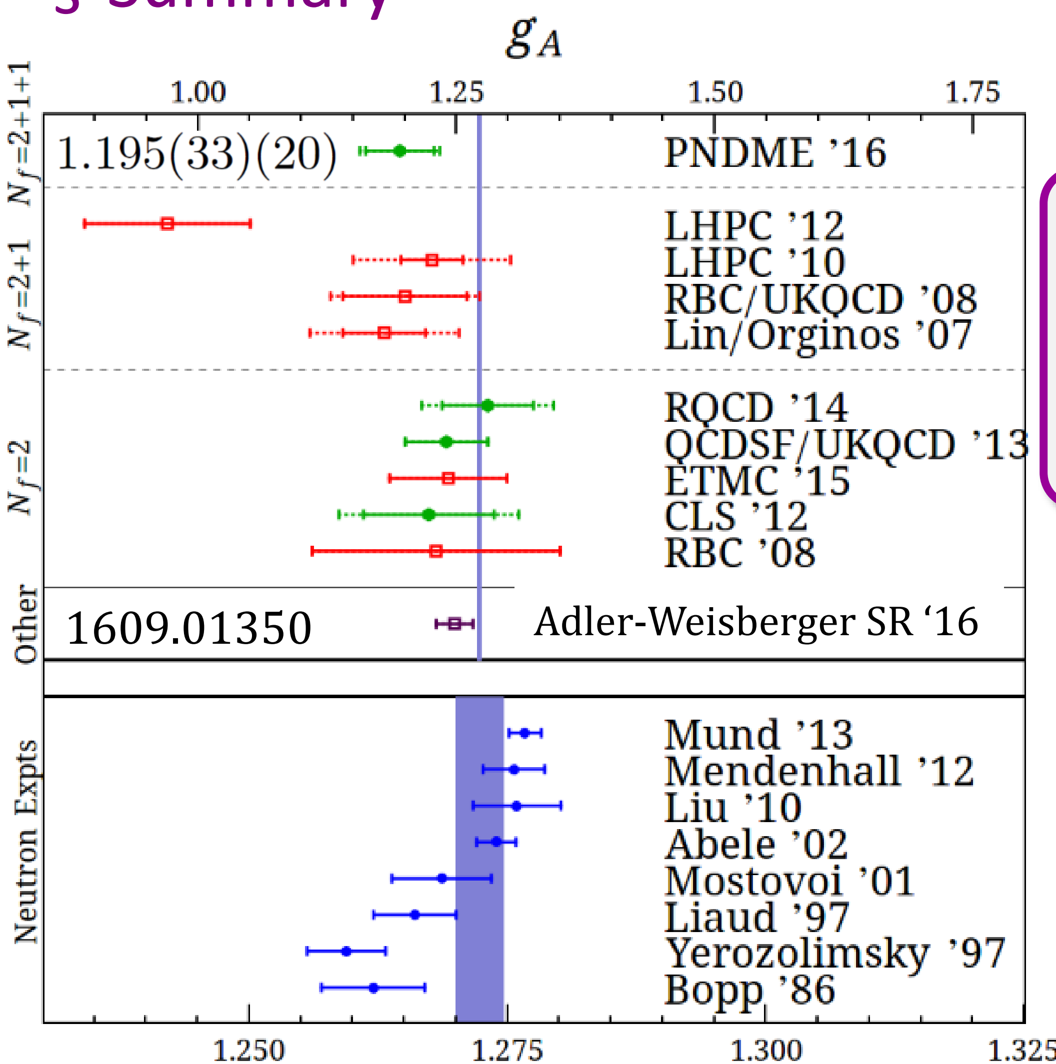

$g_S$ $\Gamma = 1$




$$(M_N - M_P)^{\rm QCD} = 2.59(49)\ \text{MeV}$$

# Nucleon Axial Charge

## § Summary




## § Implications?

- $2\sigma$ might go away with greater statistics

**Lattice 2016 Prelim.**

- RBC* 2+1f 1.15(4)
- PACS* 2+1f 1.8(4)

## § New physics?

- $\lambda = g_A/\, g_V\, f_{NP}$

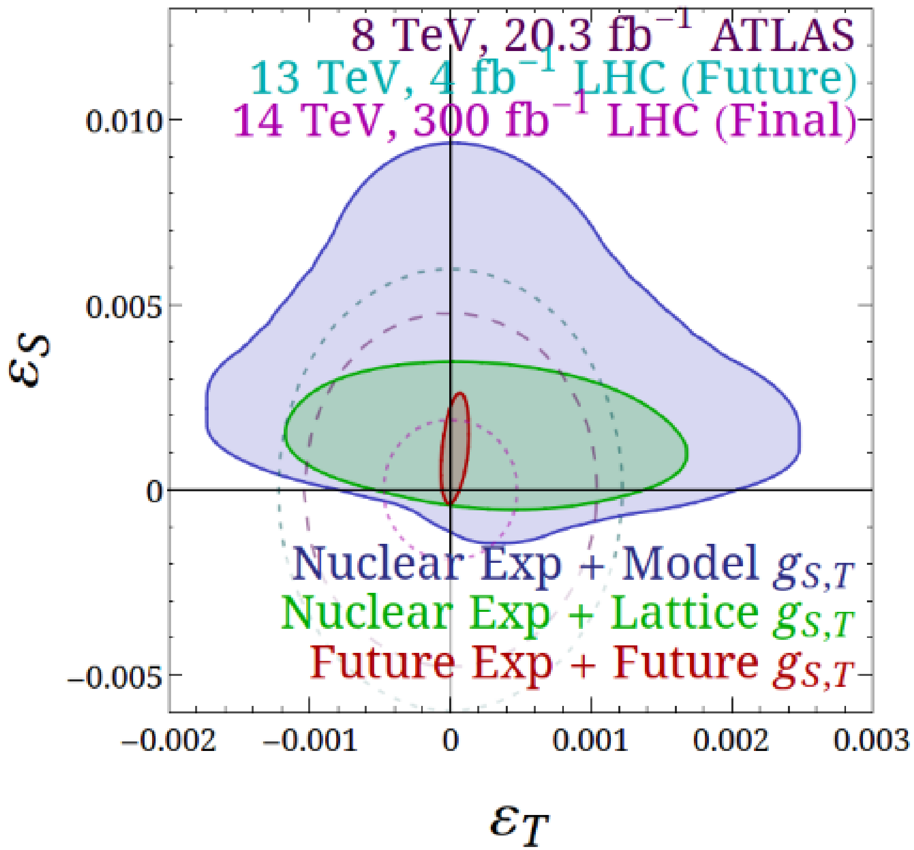
$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$$

## § Stay tuned…

# Beta Decays & TeV Physics

§ Given precision $g_{S,T}$ and $O_{\mathrm{BSM}}$, predict new-physics scales

Low-Energy Expt → $O_{\mathrm{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)




$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$

Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$|B_1 - b|_{\mathrm{BSM}} < 10^{-3}$

$|b|_{\mathrm{BSM}} < 10^{-3}$

CENPA: $^6$He($b_{\mathrm{GT}}$) at $10^{-3}$

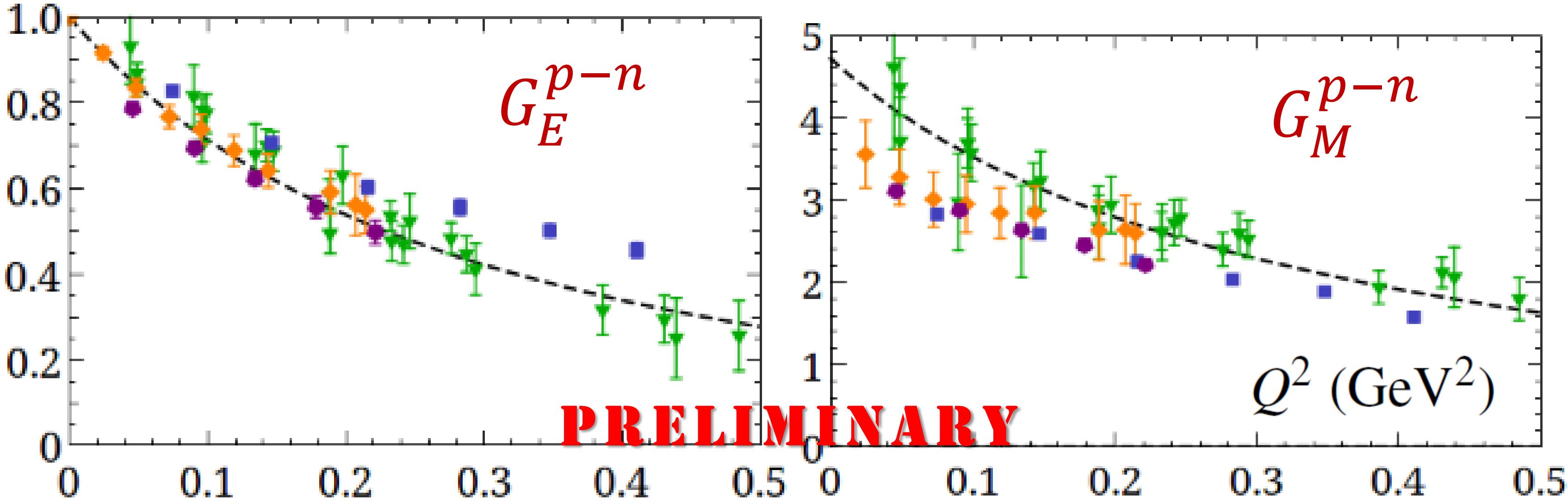
PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049

$\Lambda_S > 7$ TeV

$\Lambda_T > 13$ TeV

# Form Factors

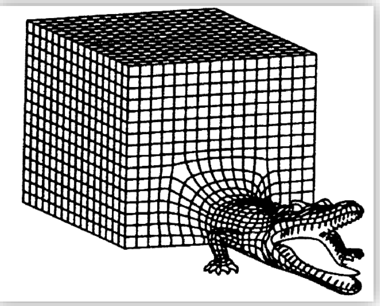
- EM form factors very close to physical pion mass
  - PNDME (2+1+1f): $M_\pi = 130$ MeV, $a = 0.09$fm, $M_\pi L = 3.9$, 2-state, 56K
  - ETMC (2f): $M_\pi = 131$ MeV, $a = 0.09$fm, $M_\pi L = 3$, $t_{sep} = 1.3$fm, O(10K)?
  - LHPC (2+1f): $M_\pi = 149$ MeV, a = 0.12 fm, $M_\pi L = 4.2$, sum., O(7K)
  - PACS (2+1f): $M_\pi = 145$ MeV, $a = 0.085$ fm, $M_\pi L = 6$, $t_{sep} = 1.3$fm, O(9K)




- Expecting more precise results in the next couple years
- New way of calculating form factors

C. Chang, Tue. Lattice, "Form factors from moments of correlation functions"

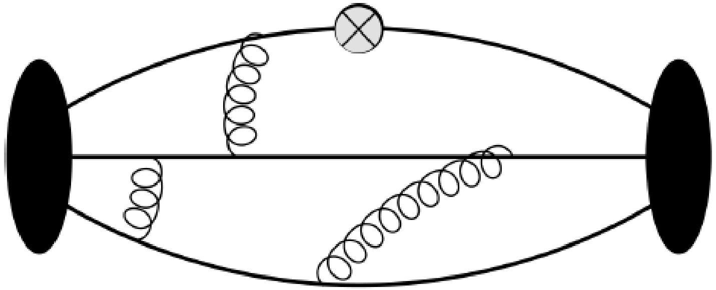
# *Spotlight on New Calculations*

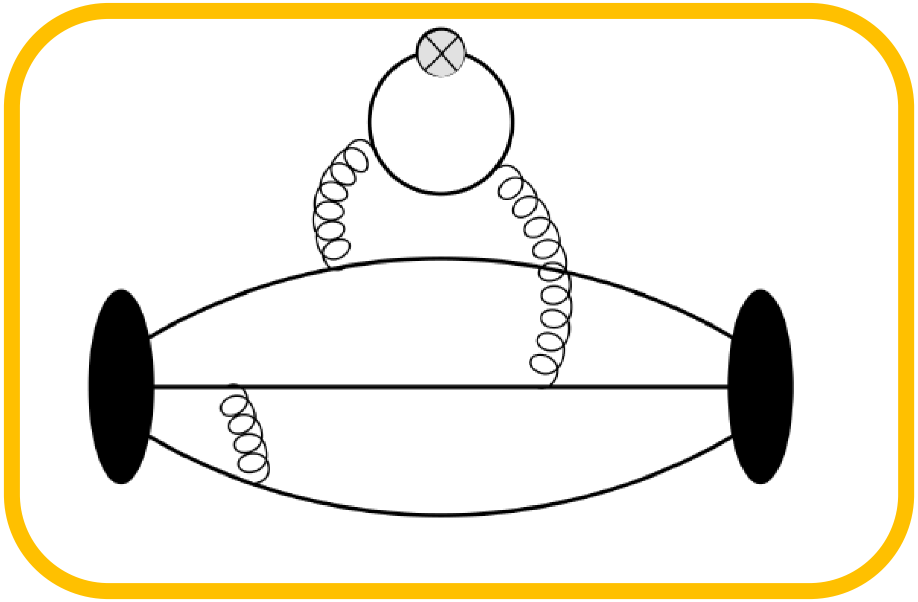


# Disconnected Diagrams

## § Disconnected diagram

- Multiple ways to calculate this notorious contribution…
- Truncated solver, hopping-parameter expansion, hierarchical probing, …





# Strangeness

## § Importance of $g_A^s$

- Strange-quark intrinsic-spin contribution to proton
- Astrophysics application: the CCSN "problem"

  3D explosions require $g_A^s \approx -0.2$   Janka, Melson, & Summa (2016)

## § Global fit: $g_A^s \approx -0.1$

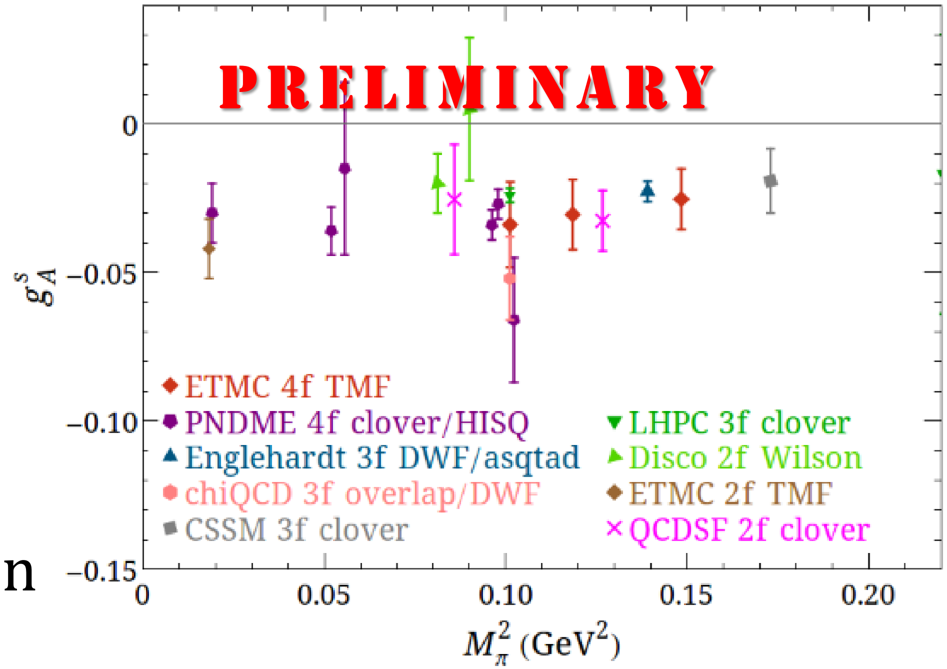
assumptions often used:

$$\Delta\bar{s}(x,Q^2) = \Delta\bar{u}(x,Q^2) = \Delta\bar{d}(x,Q^2) = \frac{1}{2}\Delta s^+(x,Q^2)$$

## § Lattice status

More players since the last Spin

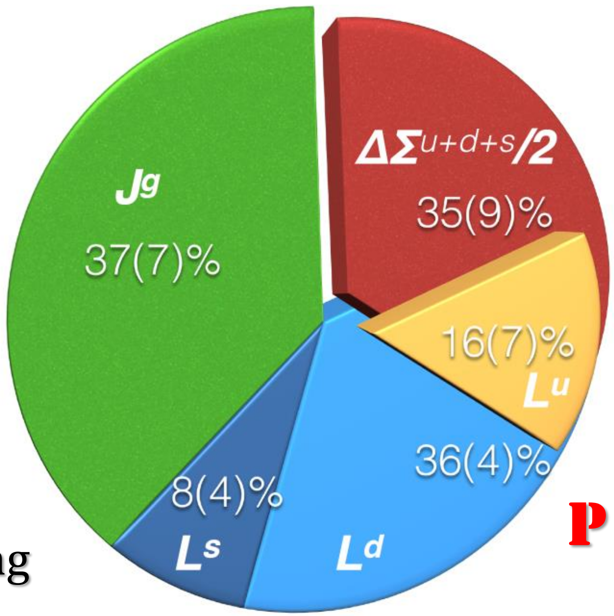
Lighter pion masses



# Origin of Proton Spin

## § What is the makeup of the nucleon?

- Decomposition using Ji's GPD moment connection
- Preliminary result from χQCD (2+1f Ov/DWF, 400 MeV)




$$J^q = \frac{1}{2}(A_{20}^q + B_{20}^q)$$

$\Delta\Sigma$: quark spin

$L = J - \Delta\Sigma$: orbital angular momentum

Plots by Yi-Bo Yang

**PRELIMINARY**

J. Liang, Monday Lattce+Helicity

- ETMC (2f TMF 130 MeV) $M_\pi L = 3$ Preliminary

$$\Delta\Sigma^{u+d+s} = 0.214(61), L^{u+d+s} = 0.168(60), J^g = 0.118(57)$$

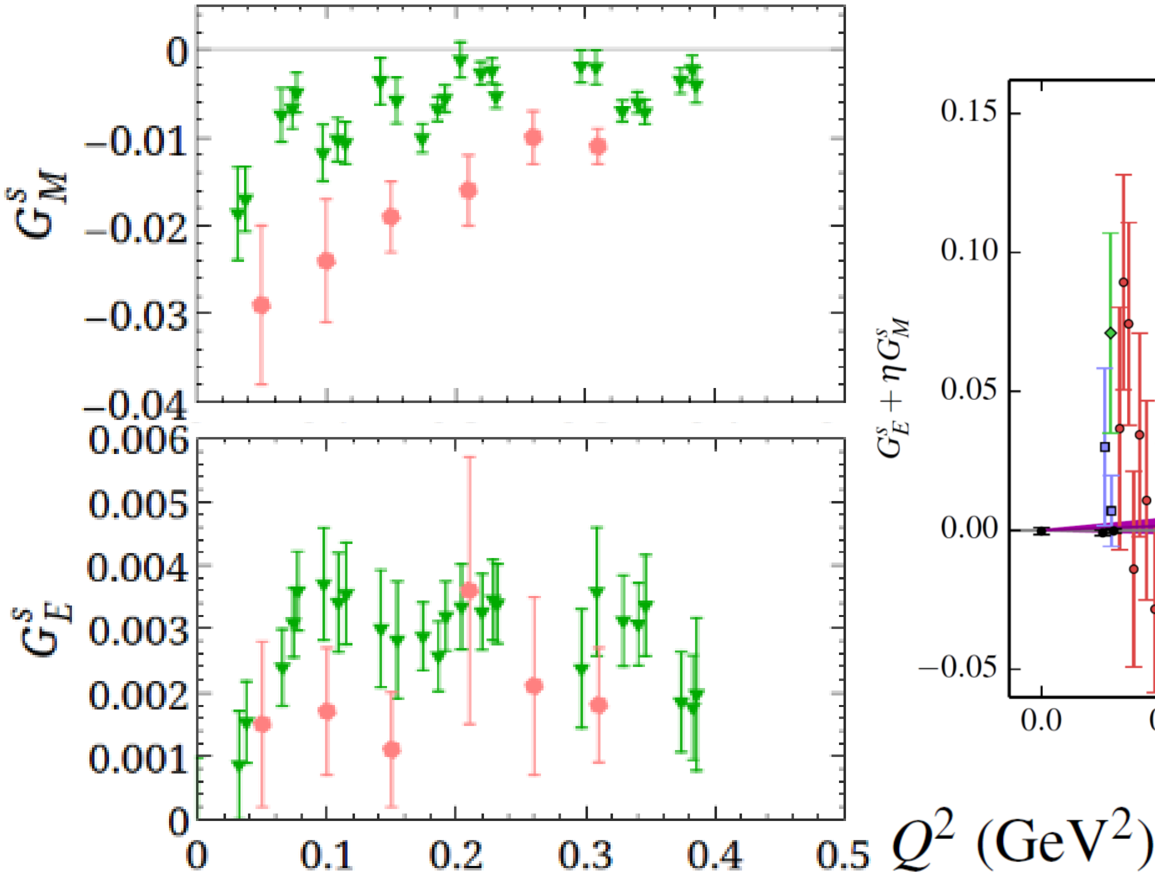
M. Constantinou, Mon., Lattice

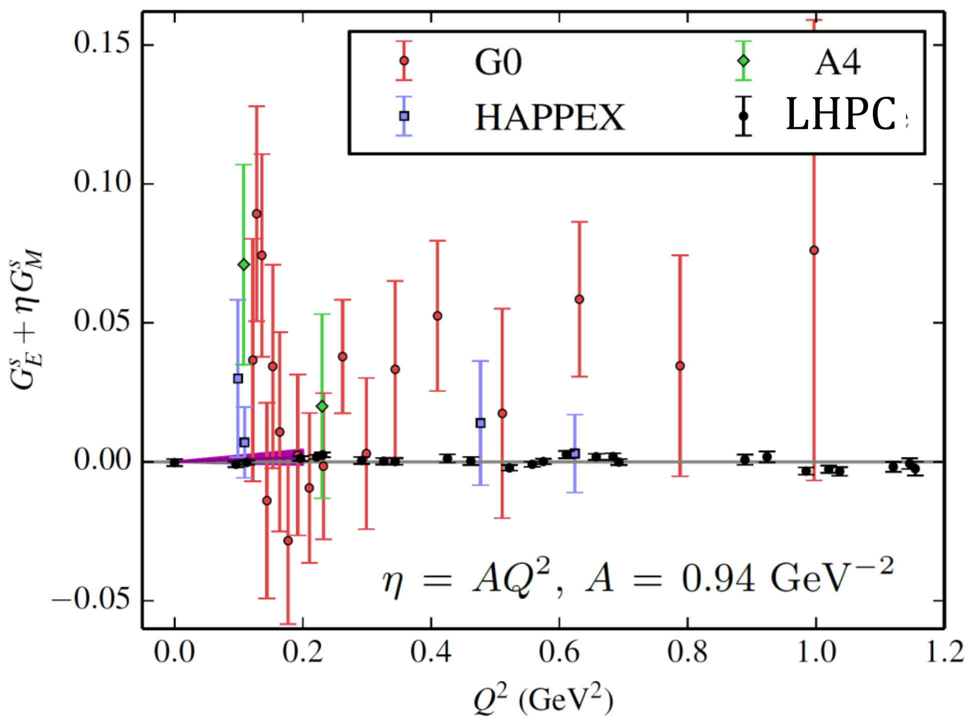
# Strange Form Factors

K. Orginos/R. Sufian, Tue. Lattice

## § Better determined strange form factors

- LHPC (2+1f): clover $M_\pi = 317$ MeV, $a = 0.11$ fm
- χQCD (2+1f): ov/DWF $M_\pi = 207{,}140$ MeV, $a = 0.11$ fm

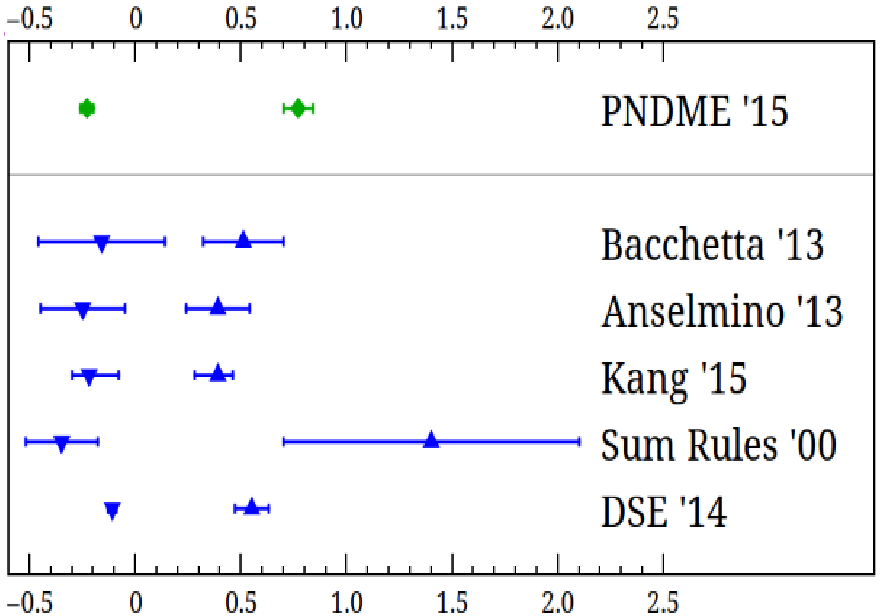


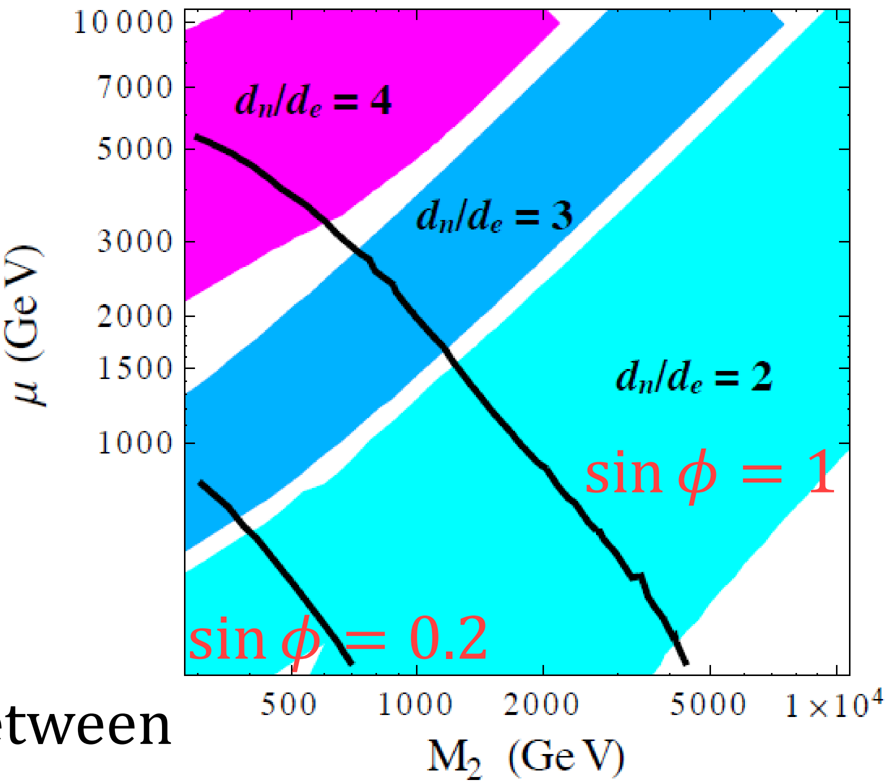



# Quark EDM

§ Extrapolate to the physical limit PNDME, 1506.04196; 1506.06411

$$g_T^d = -0.233(28), g_T^u = 0.774(66), g_T^s = 0.008(9)$$




Observation of a neutron EDM between the current limit and $4\times10^{-28} e\cdot\text{cm}$ would falsify the split-SUSY scenario with gaugino mass unification

# Ultimate QCD Machine

**Electron Ion Collider: The Next QCD Frontier**

Imaging of the proton

**How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?**

How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction?

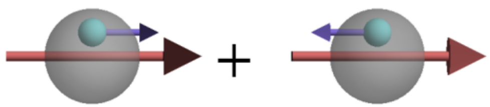
What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin?

EIC White Paper, 1212.1701

# PDFs on the Lattice

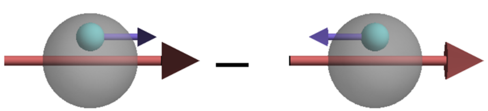
§ Lattice calculations rely on operator product expansion, only provide moments

most well known



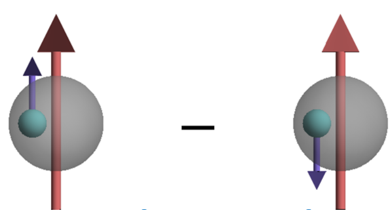
spin-averaged/unpolarized

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

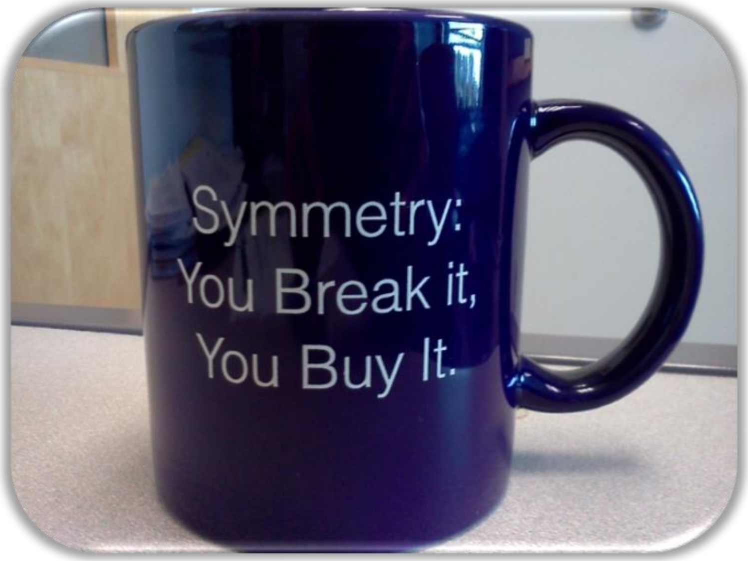
very poorly known

§ True distribution can only be recovered with all moments

# PDFs on the Lattice

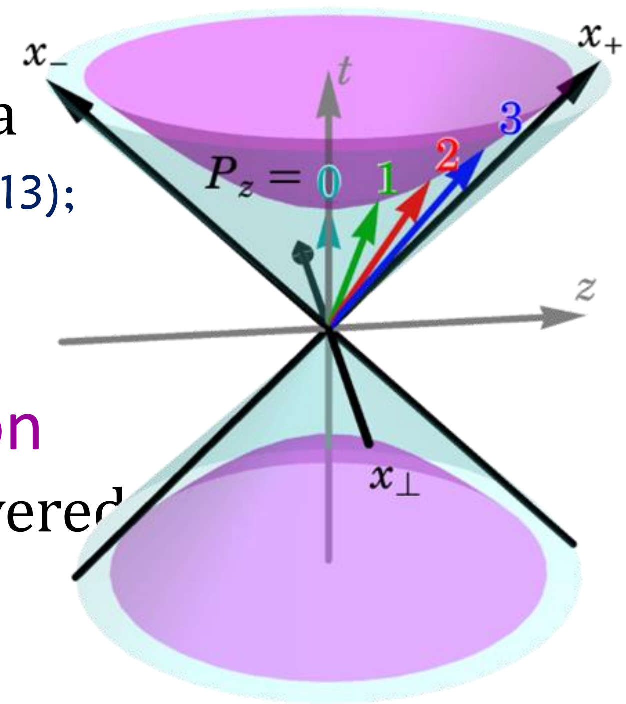
- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea

Z. Davoudi, Tuesday Lattice



# PDFs on the Lattice

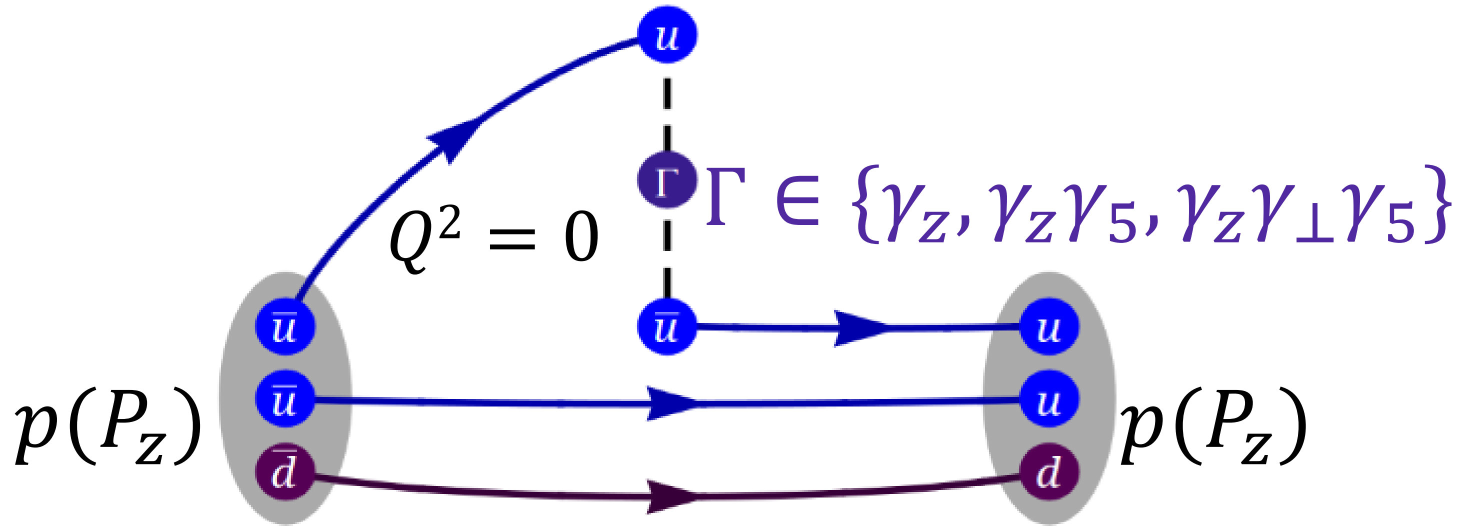
- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea
- New Strategy: Xiangdong Ji, PRL 111, 039103 (2013); J. Chen, Monday Lattce+Helicity
- Adopt lightcone description for PDFs
- Calculate finite-boost quark distribution
  - In $P_z \to \infty$ limit, parton distribution recovered
  - For finite $P_z$, corrections are applied through effective theory
- Demonstration: Feasible with today's resources!



# Parton Distribution Functions

## Large-Momentum Effective Theory for PDFs

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\, \Gamma \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$




$P_z \to \infty$ limit

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

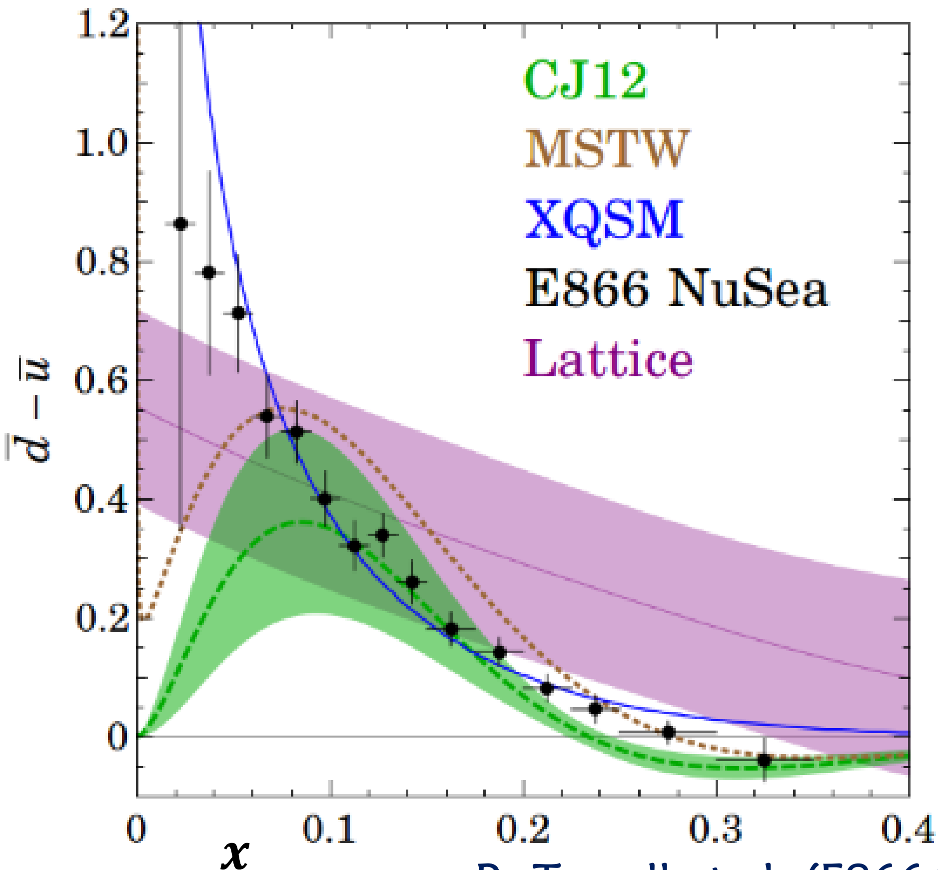
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

# Sea Flavor Asymmetry

## § Lattice exploratory study

- $M_\pi \approx 310$ MeV

HWL et al 1402.1462




**Compared with E866**

Too good to be true?

**Lost resolution in small-*x* region**

Future improvement to have larger lattice volume

$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

| Experiment | $x$ range | $\int_0^1 [\bar{d}(x) - \bar{u}(x)]dx$ |
|---|---|---|
| E866 | $0.015 < x < 0.35$ | $0.118 \pm 0.012$ |
| NMC | $0.004 < x < 0.80$ | $0.148 \pm 0.039$ |
| HERMES | $0.020 < x < 0.30$ | $0.16 \pm 0.03$ |

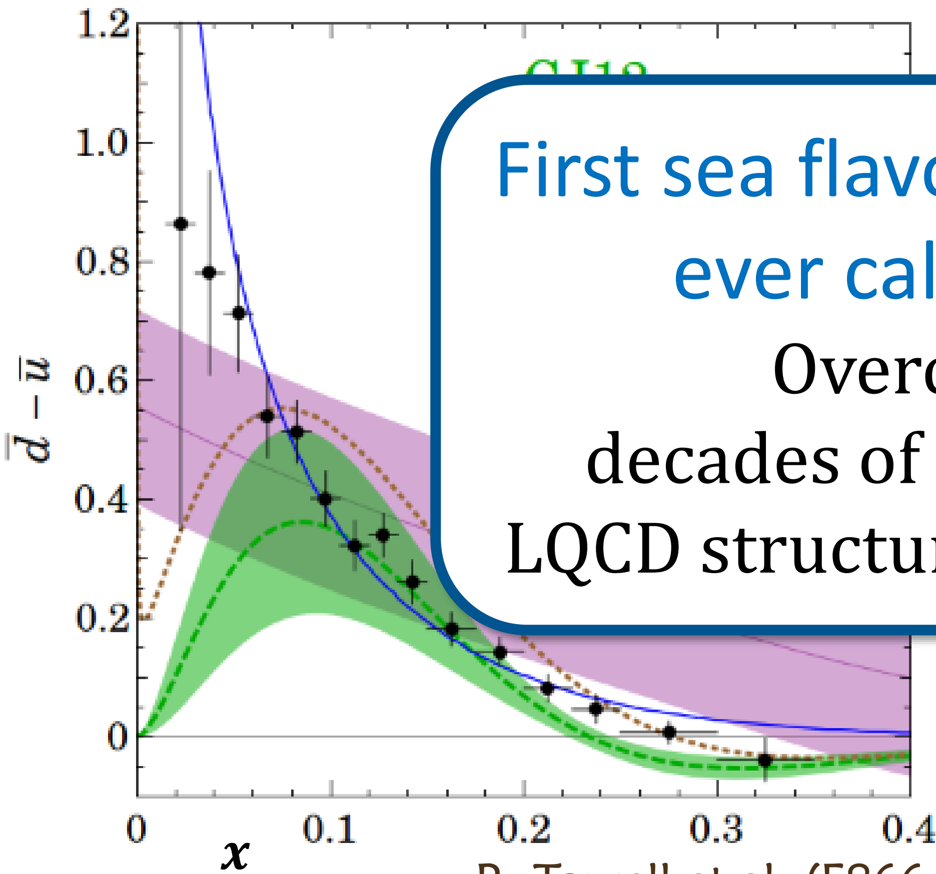
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

# Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx 310$ MeV

HWL et al 1402.1462




Compared with E866

Too good to be true?

Lost resolution in

ment to

ce volume

$) \approx -0.16(7)$

**First sea flavor asymmetry ever calculated!**

Overcomes decades of obstacles in LQCD structure calculations

| | | $\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$ |
|---|---|---|
| E866 | $0.015<x<0.35$ | $0.118\pm0.012$ |
| NMC | $0.004<x<0.80$ | $0.148\pm0.039$ |
| HERMES | $0.020<x<0.30$ | $0.16\pm0.03$ |

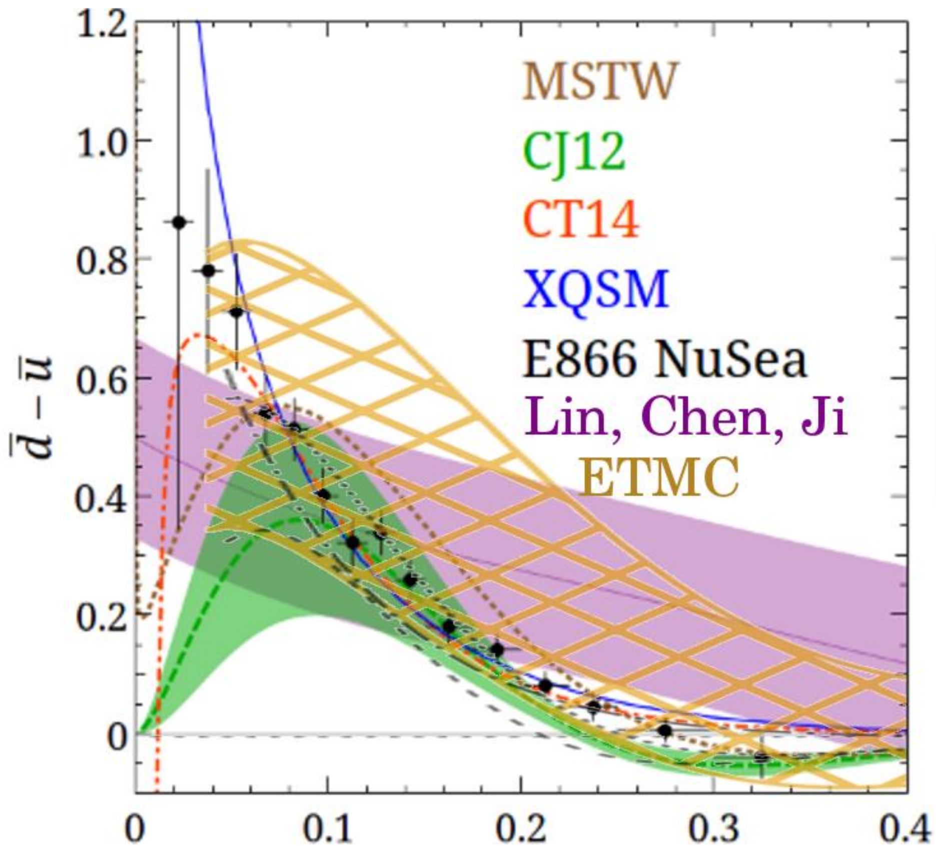
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

# Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx 310$ MeV

HWL et al 1402.1462




Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Similar results repeated by ETMC, at $M_\pi \approx 373$ MeV
ETMC, 1504.07455

(7)

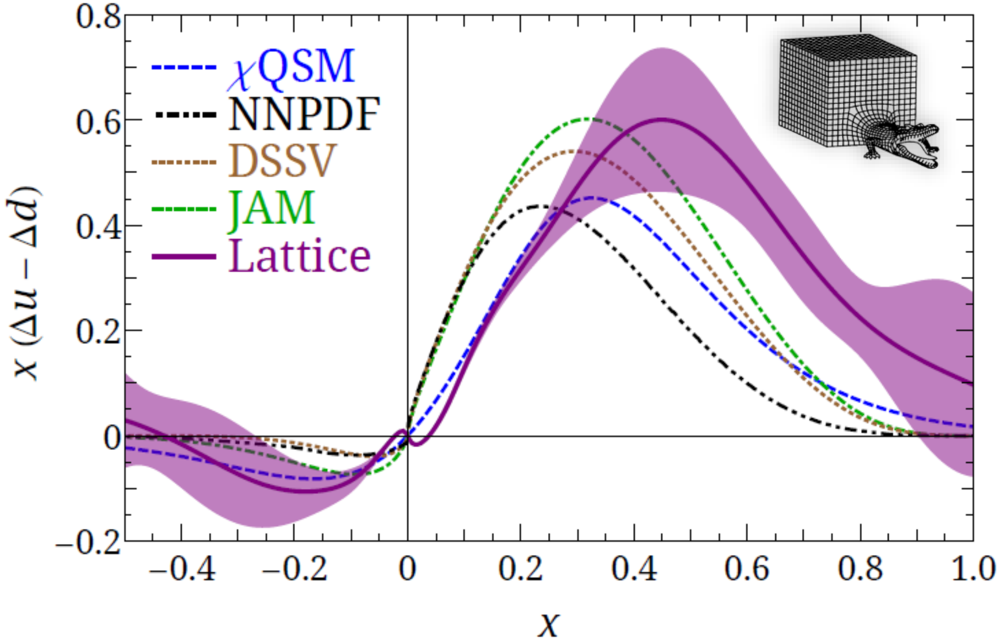
| Experiment | $x$ range | $\int_0^1[\overline{d}(x)-\overline{u}(x)]dx$ |
|---|---|---|
| E866 | $0.015<x<0.35$ | $0.118\pm0.012$ |
| NMC | $0.004<x<0.80$ | $0.148\pm0.039$ |
| HERMES | $0.020<x<0.30$ | $0.16\pm0.03$ |

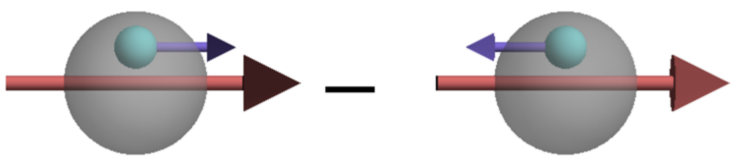
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

# Helicity Distribution

§ Exploratory study   $M_\pi \approx 310$ MeV

1603.06664,
Frontier Article in NPB911, 246



Removing
$O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$
+ $O(\Lambda_{QCD}^2/P_z^2)$

- We see polarized "sea asymmetry" $\int dx\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.14(9)$
- Both STAR and PHENIX at RHIC see $\Delta\bar{u} > \Delta\bar{d}$

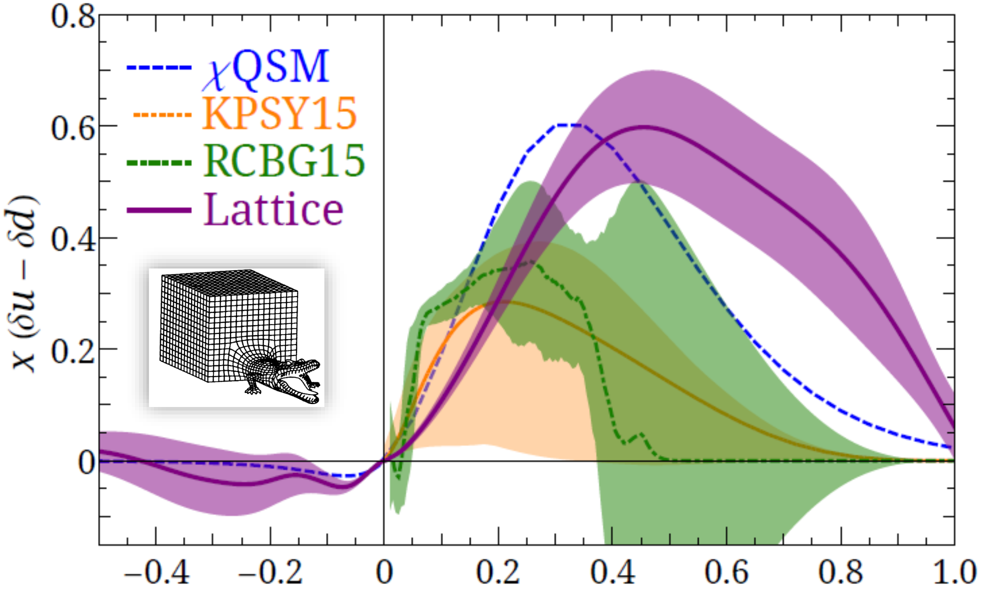
1404.6880 and 1504.07451

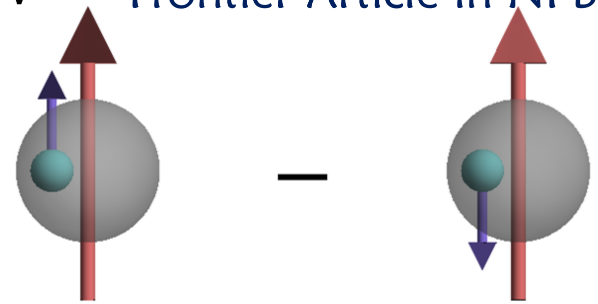
- Other experiments, Fermilab DY exp'ts (E1027/E1039), future EIC

# *Transversity Distribution*

§ Exploratory study ☙ $M_\pi \approx 310$ MeV

1603.06664,
Frontier Article in NPB911, 246




Removing
$O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$
+ $O(\Lambda_{QCD}^2/P_z^2)$

$\delta\bar{q}(x) = -\delta q(-x)$ 1505.05589; 1503.03495

☙ We found sea asymmetry of $\int dx\, (\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.10(8)$

☙ Chiral quark-soliton model $\int dx\, (\delta\bar{u}(x) - \delta\bar{d}(x)) \approx -0.082$

P. Schweitzer et al., PRD 64, 034013 (2001)

☙ SoLID at JLab, Drell-Yan exp't at FNAL (E1027+E1039), EIC, ...

# Transversity Distribution

§ E

## A few notes

- The method can be applied to GPDs, TMDs, …
- More work to be done on the LQCD side
  high statistics, lighter pion mass, …
- Improving large-momentum signal
  RQCD, Phys. Rev. D 93, 094515 (2016)
- Renormalization and matching issues
  T. Ishikawa, Tuesday Lattice
  "Improved quasi parton distribution through Wilson line renormalization", Chen et al., 1609.08102
  "Practical quasi parton distribution functions", T. Ishikawa et al., 1609.02018

01)

…

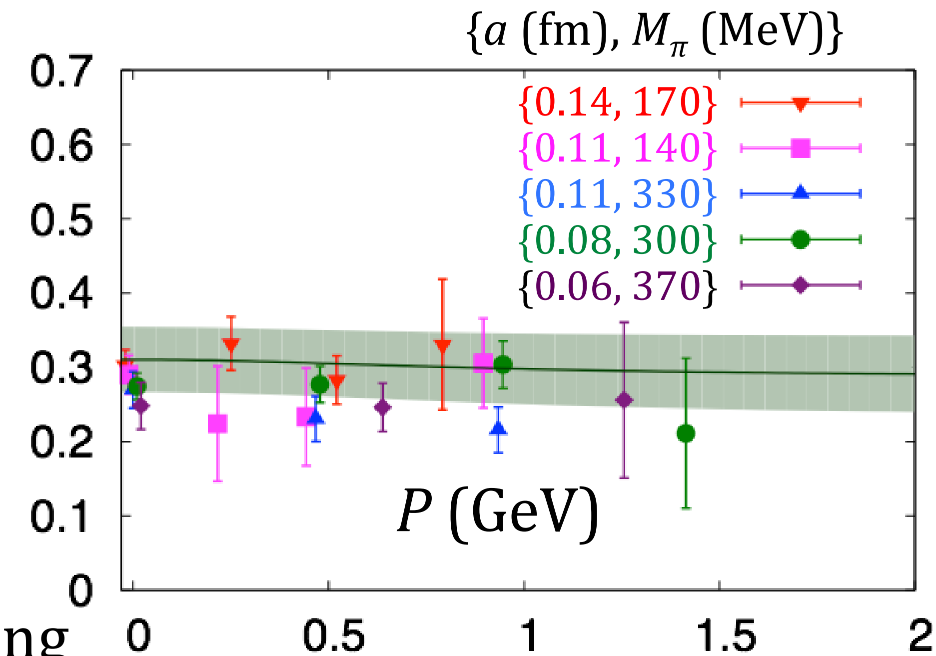
# Gluon Helicity

§ Jaffe & Manohar, 1990 $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q^z + \mathcal{L}_g^z$

§ Can be calculated through large-momentum frame

X. Ji et al., PRL. 111 (2013) 112002; 110 (2013) 262002; PRD 89, 085030 (2014)

$$S_G(P)\,S_z = \frac{\left\langle PS \left| \int d^3x \left(\vec{E} \times \vec{A}_{\rm phys}\right)_z \right| PS \right\rangle}{2E_P}$$




§ First results by χQCD

$$\Delta G\left(\mu^2 = 10\ \text{GeV}^2\right)$$
$$\approx S_G\left(\infty, \mu^2 = 10\ \text{GeV}^2\right)$$
$$= 0.287(55)(16)$$

Y. Yang, Monday Lattce+Helicity

- Future improvement on matching

§ Current limit

- DSSV14 $\int_{0.05}^{1} dx\, \Delta G\left(10^2\ \text{GeV}, x\right) \approx [0.14, 0.24]$

# A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

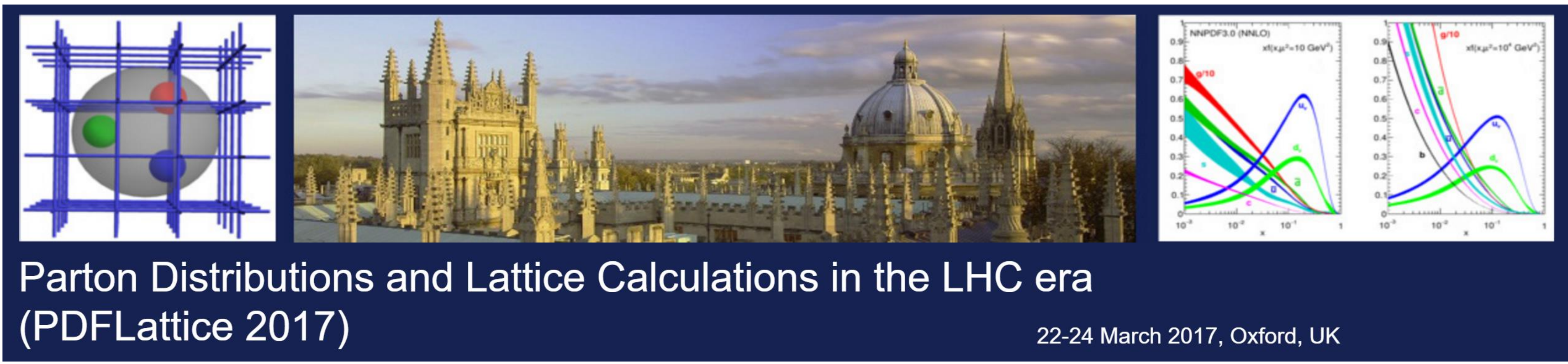
The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

# *Future Prospects*

§ A first joint workshop with global-fitting community to address key LQCD inputs

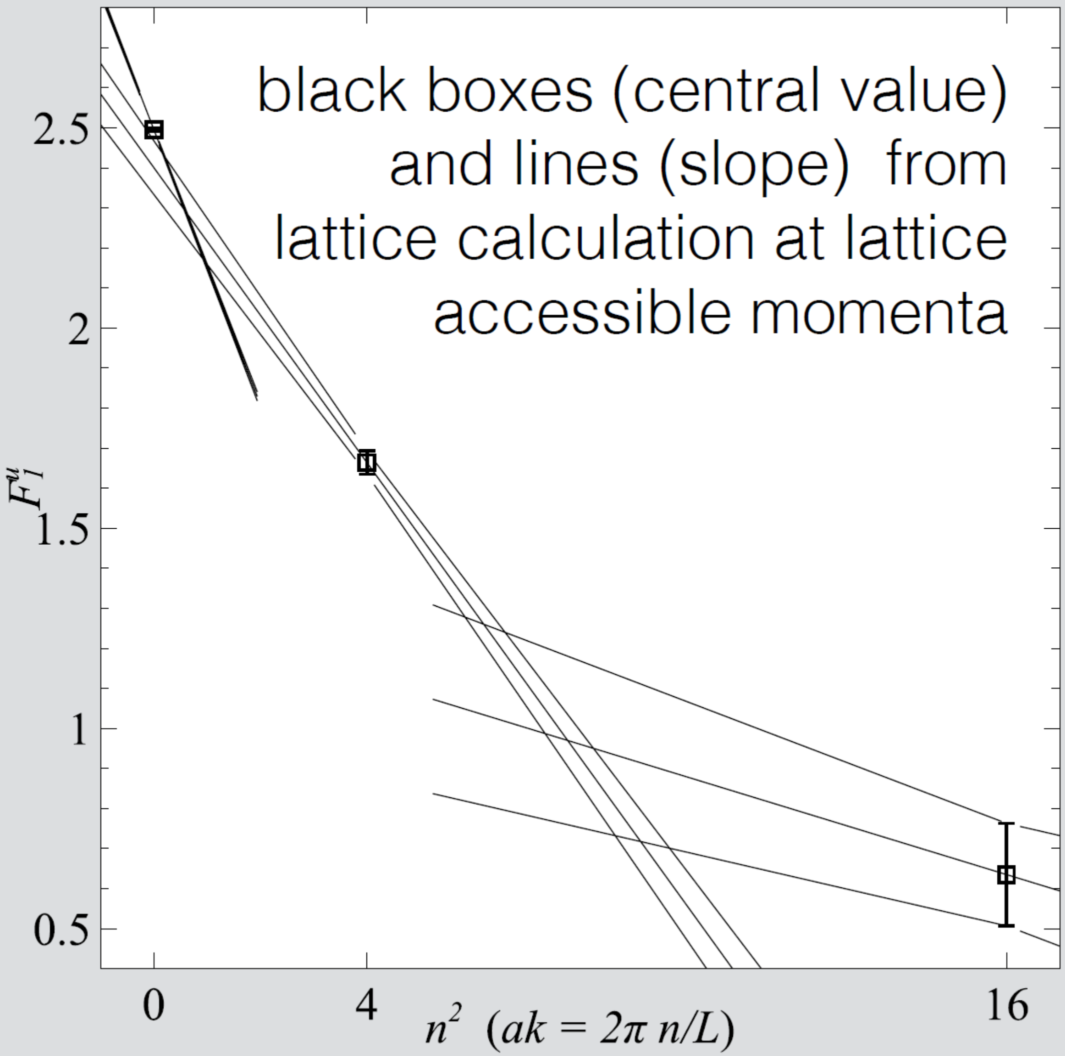
- http://www.physics.ox.ac.uk/confs/PDFlattice2017




*"The goal of this workshop is to* ***bring together the global PDF analysis and lattice-QCD communities*** *to explore ways to improve current PDF determinations. In particular, we plan to* ***set precision goals for lattice-QCD*** *calculations so that these calculations, together with experimental input, can achieve more reliable determinations of PDFs. In addition we will discuss what impact such improved determinations of PDFs will have on future new-physics searches."*

# *Backup Slide*

# Form factors for moments of correlation functions




Model independent lattice calculation for slopes of form factors

Some applications:
axial mass ($g_A$)
charge radius ($g_V$)
non-zero momentum transfer slopes ($F_A$,$F_1$)

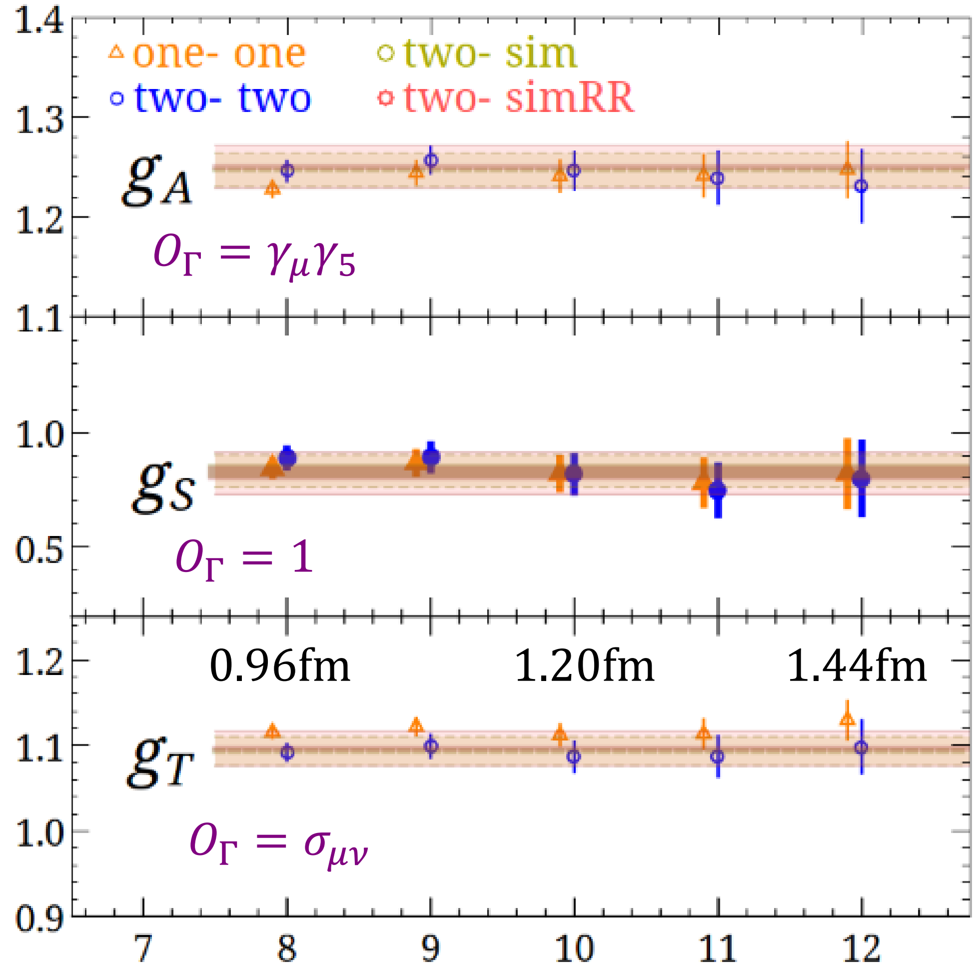
Chris Bouchard
Chia Cheng Chang*
Kostas Orginos
David Richards

*speaker

# Appendix A
# Lattice Systematics Study

# Systematic Control

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME) $a = 0.12$ fm, 310-MeV pion




- Move the **excited-state systematic** into the statistical error

$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$

$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0| \ldots 1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$

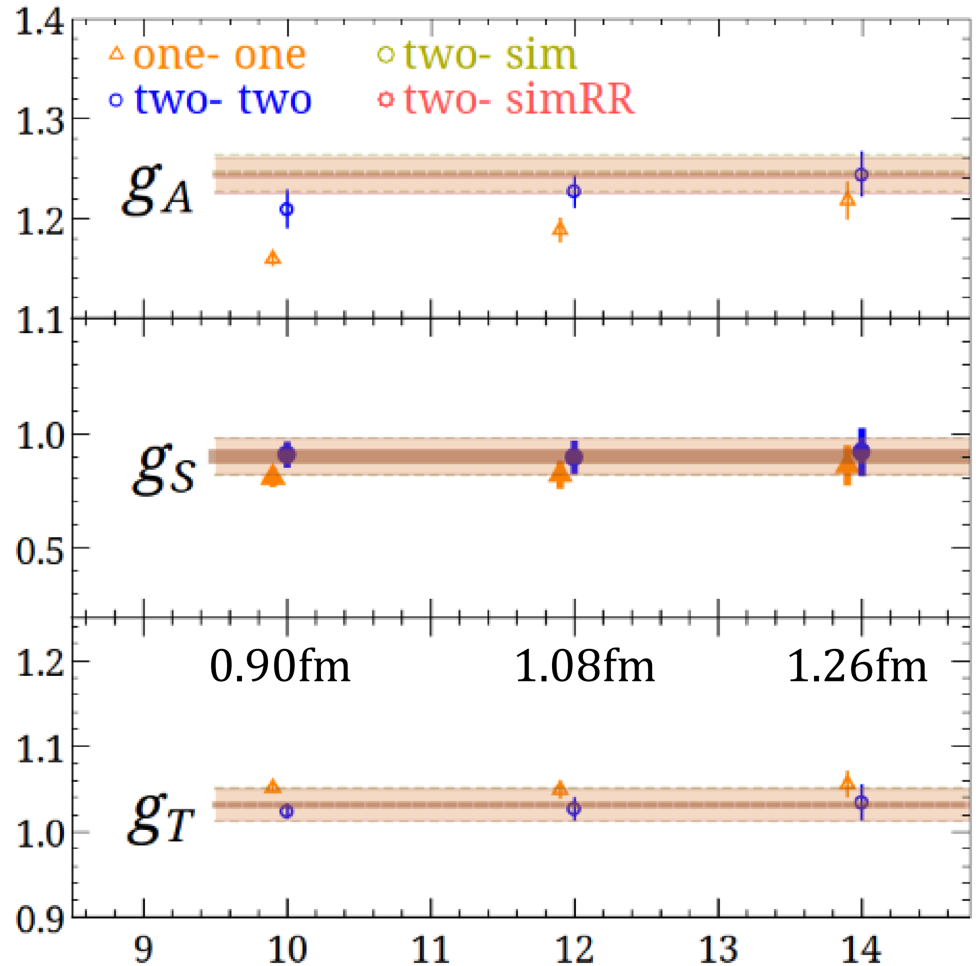
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e \ldots {}^{(t-t_i)}e^{-M_0(t_f-t)}$$

$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e \ldots {}^{t_i)}$$

- No obvious contamination between 0.96 and 1.44 fm separation

# Systematic Control

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME) $a = 0.09$ fm, 310-MeV pion




- Move the **excited-state systematic** into the statistical error

$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0| \ldots 1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e \ldots (t-t_i)\, e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e \ldots t_i)$$

- Much stronger effect at finer lattice spacing!
- Needs to be studied case by case

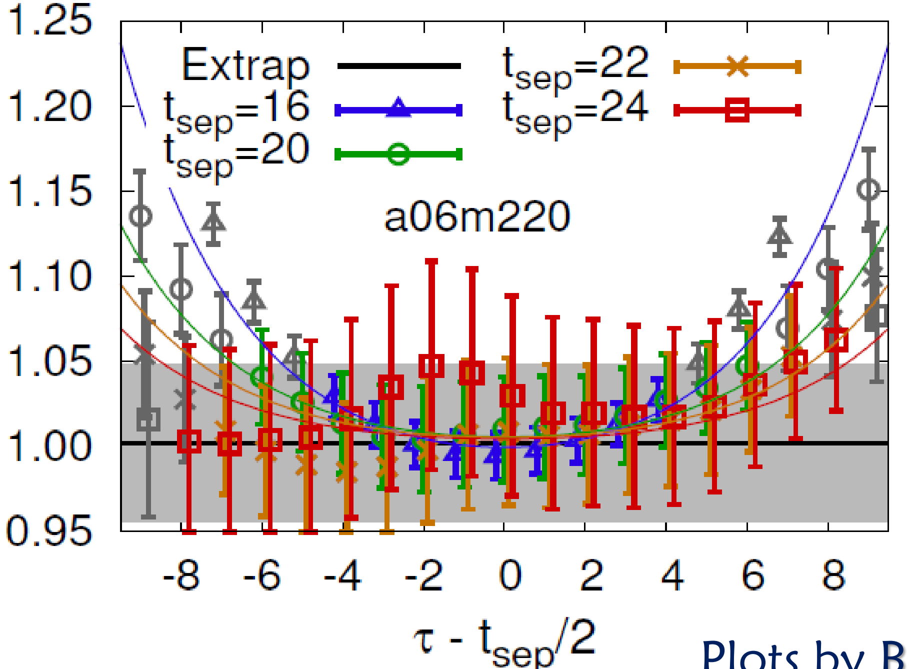
# Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)
  - Statistical effect

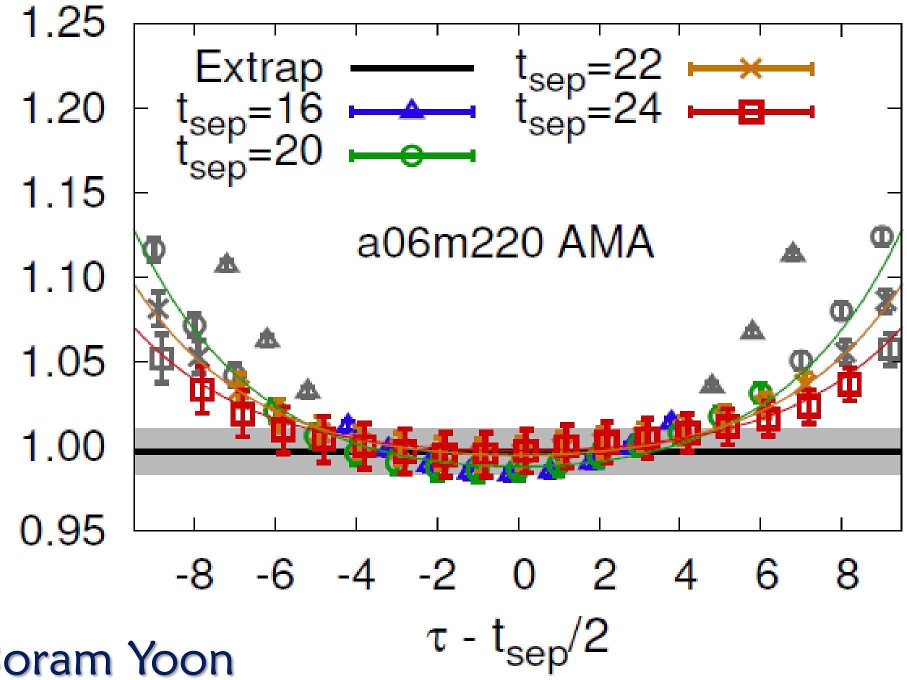
**$a$ = 0.06 fm**, 220-MeV pion

$g_T^{\text{bare}}$

2.6k




41.6k




Plots by Boram Yoon

# Systematic Control

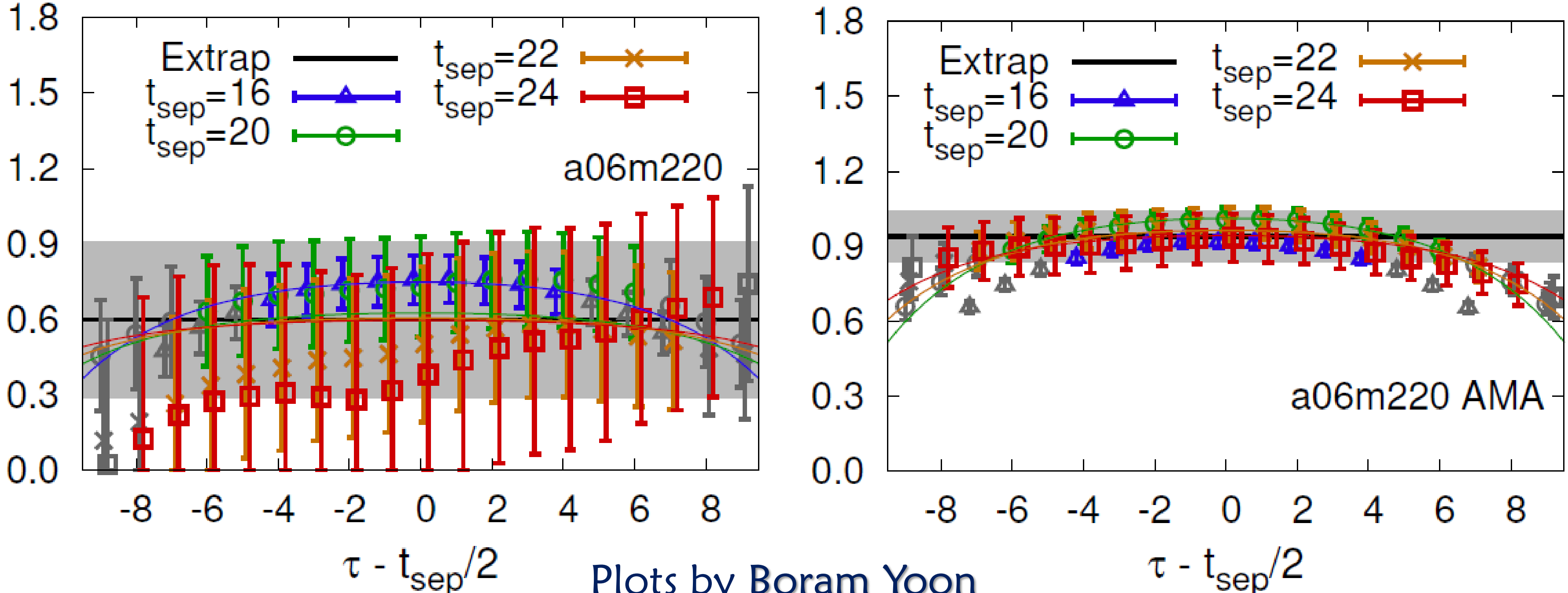
- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME)

❧ Statistical effect

**$a$ = 0.06 fm**, 220-MeV pion

$g_S^{\mathrm{bare}}$

2.6k 41.6k




Plots by Boram Yoon

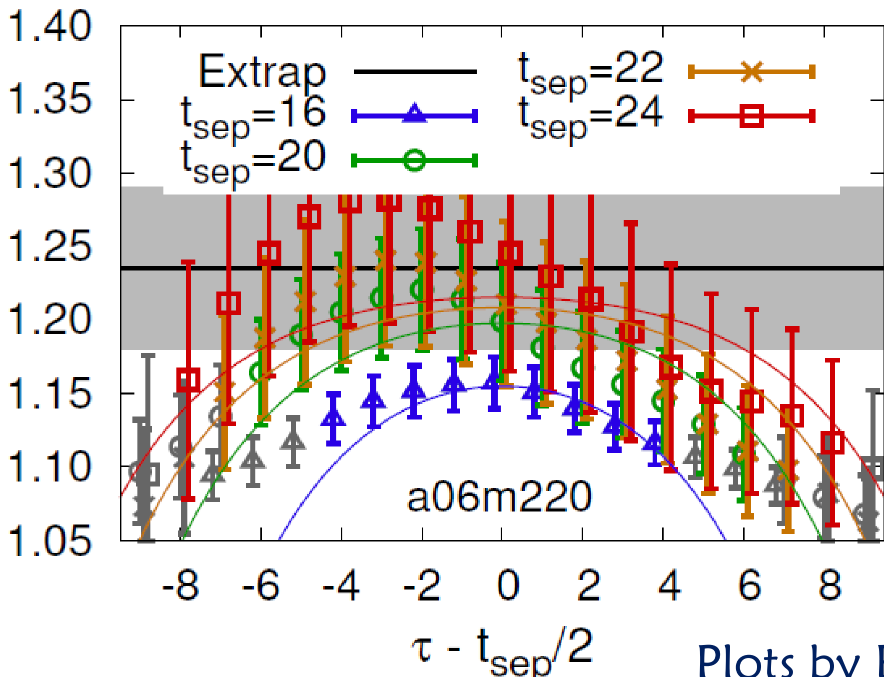
# Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

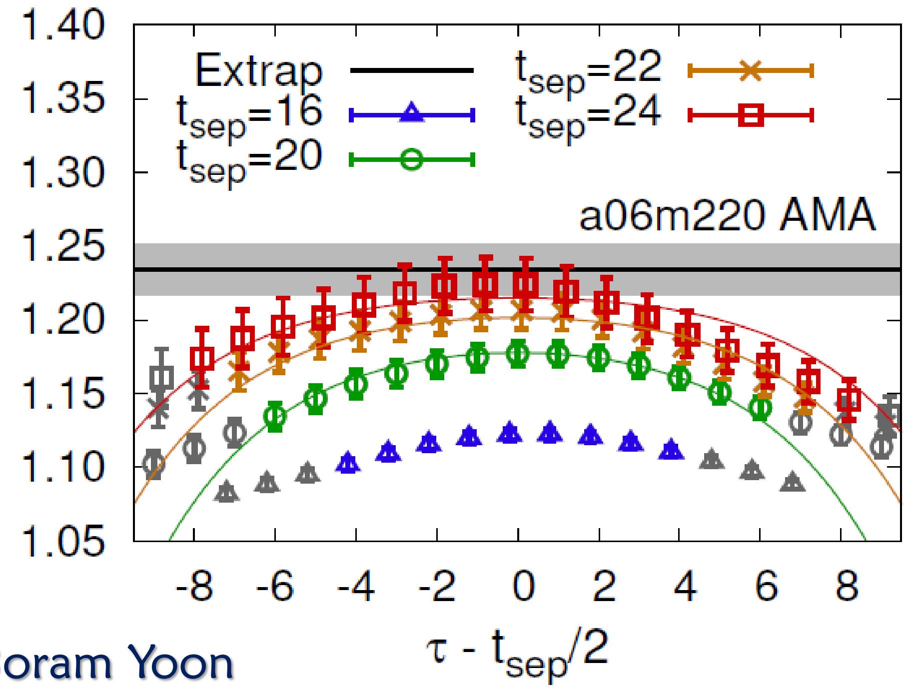
  - Statistical effect (worst case) $a = 0.06$ fm, 220-MeV pion

$g_A^{\text{bare}}$

2.6k




41.6k




Plots by Boram Yoon

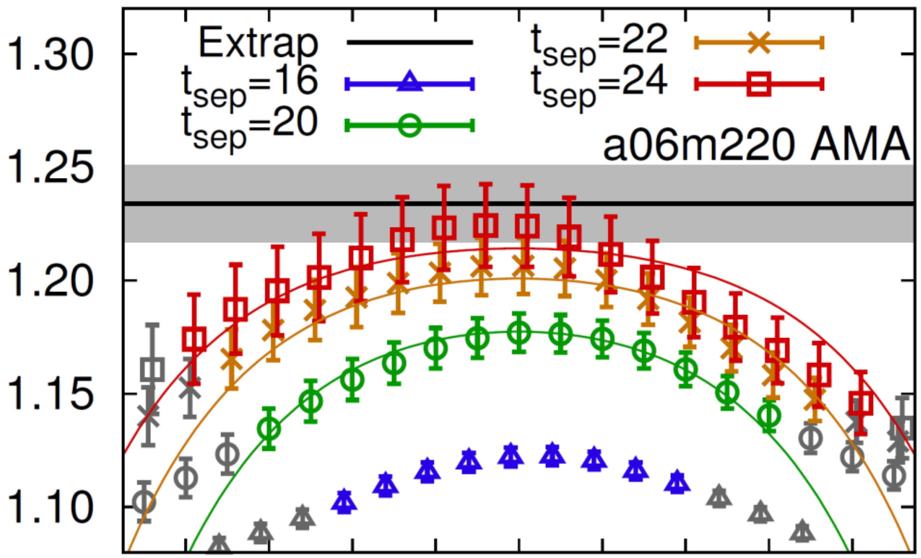
# Systematic Control

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)
  - Robustness of the 2-state fit

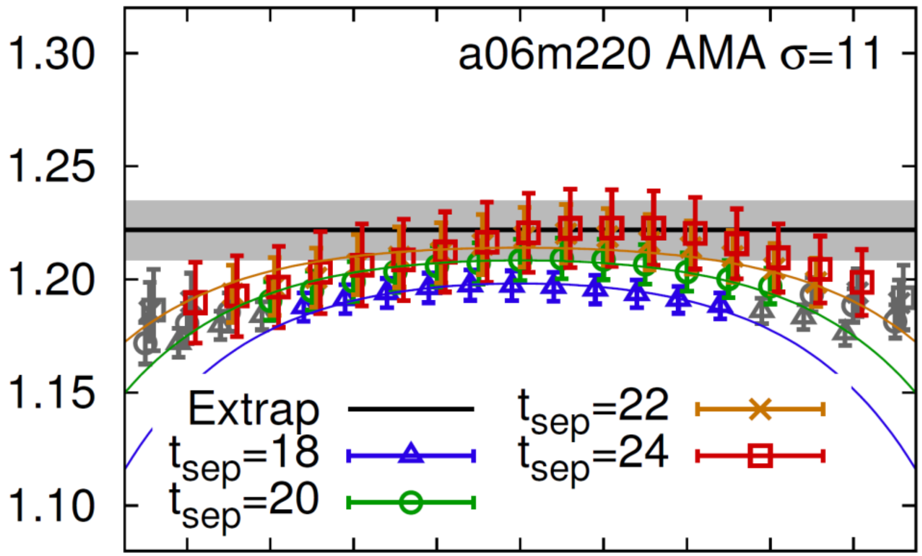
$a = 0.06$ fm, 220-MeV pion

$g_A^{\text{bare}}$

2.6k




41.6k



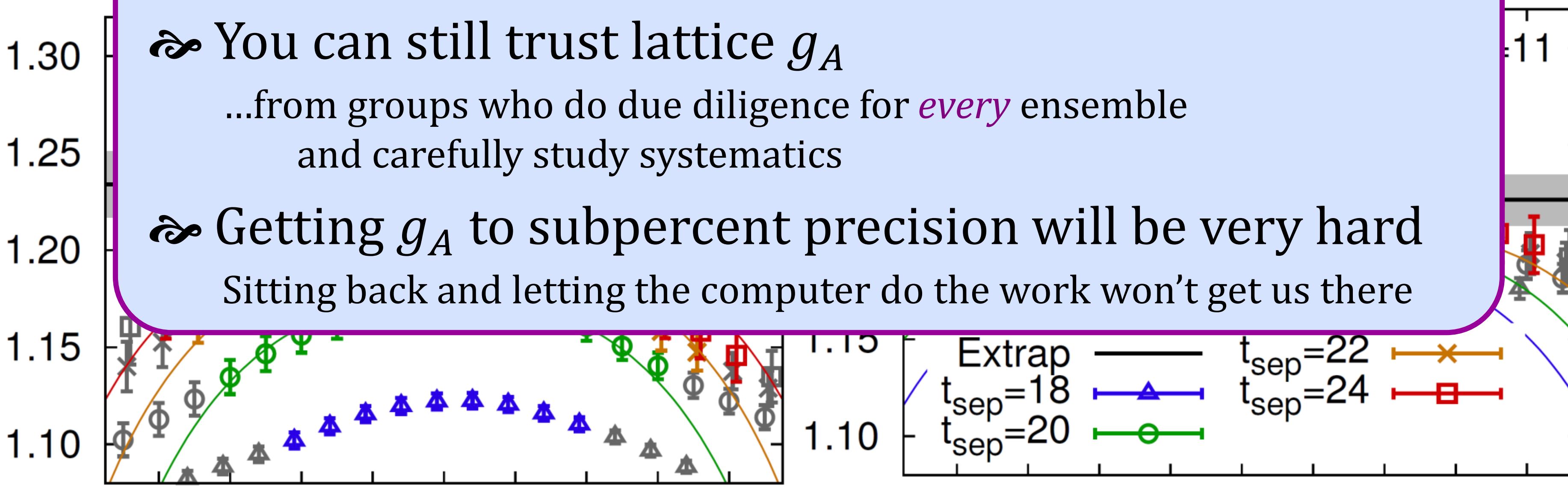

Plots by Boram Yoon

# Systematic Control

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME)

## My Two Cents

- $g_A$ is *not* a gold-plated quantity
  - Early impressions that $g_A$ would be easy underestimated systematics
- You can still trust lattice $g_A$
  - …from groups who do due diligence for *every* ensemble and carefully study systematics
- Getting $g_A$ to subpercent precision will be very hard
  - Sitting back and letting the computer do the work won't get us there

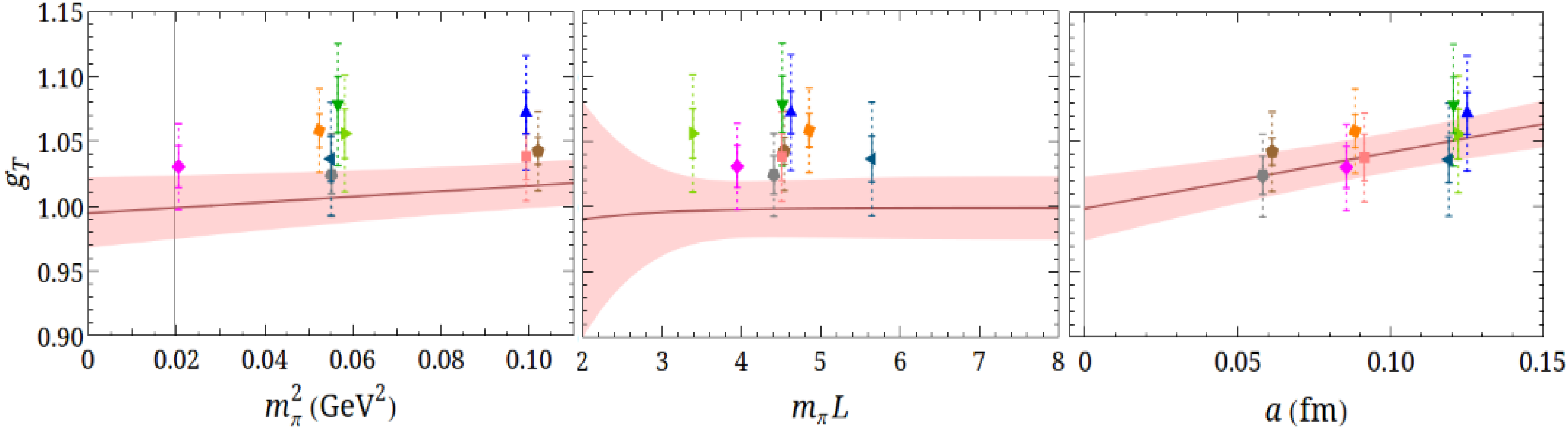



Plots by Boram Yoon

# Precision Nucleon Couplings

- § Much effort has been devoted to controlling **systematics**
- § A state-of-the art calculation (PNDME)
  - Extrapolate to the physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

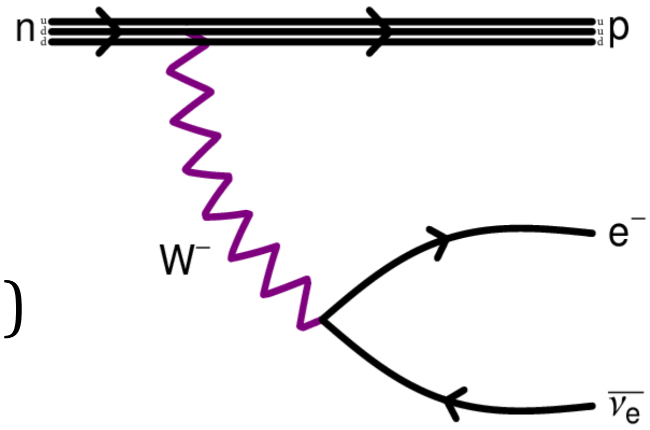



First extrapolation to the physical limit
of a nucleon matrix element!

# Nucleon Axial Charge

- A fundamental measure of nucleon structure
- Axial-vector–current matrix element

$$g_A = G_A^{u-d}(Q_2 = 0)$$

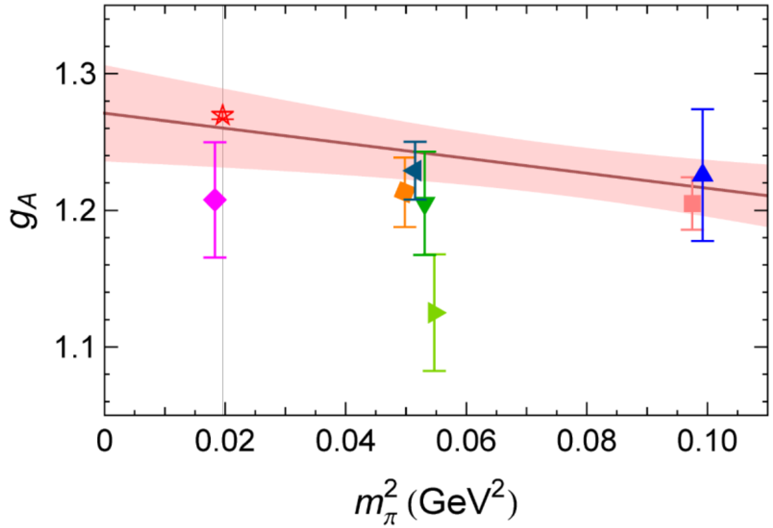


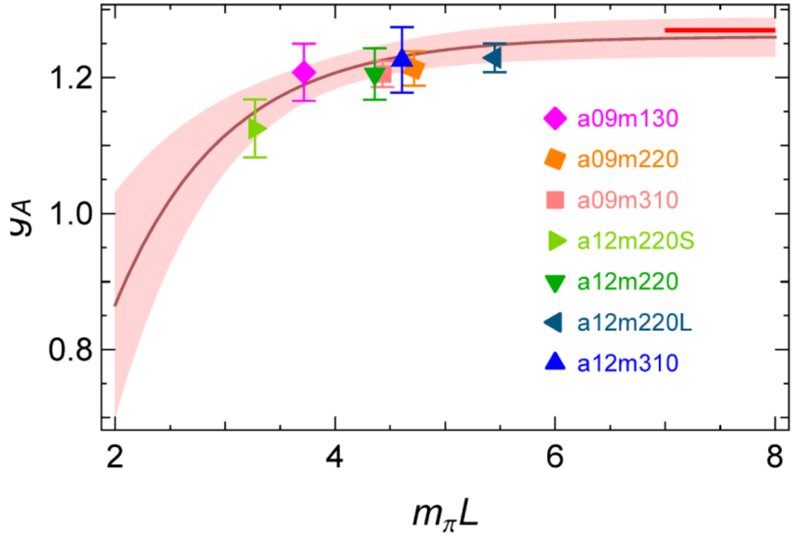

- Important to many nuclear processes
  - The rate of *pp* fusion (as in Sun-like stars)
  - $n$-lifetime when combining with $V_{ud}$
  - New-physics searches such as right-handed neutrinos
  - $0\nu\beta\beta$ searches, "quenching" $g_A^4$

- In lattice QCD: A benchmark for nucleon structure

# Nucleon Axial Charge

## § Finite-volume/statistical effects

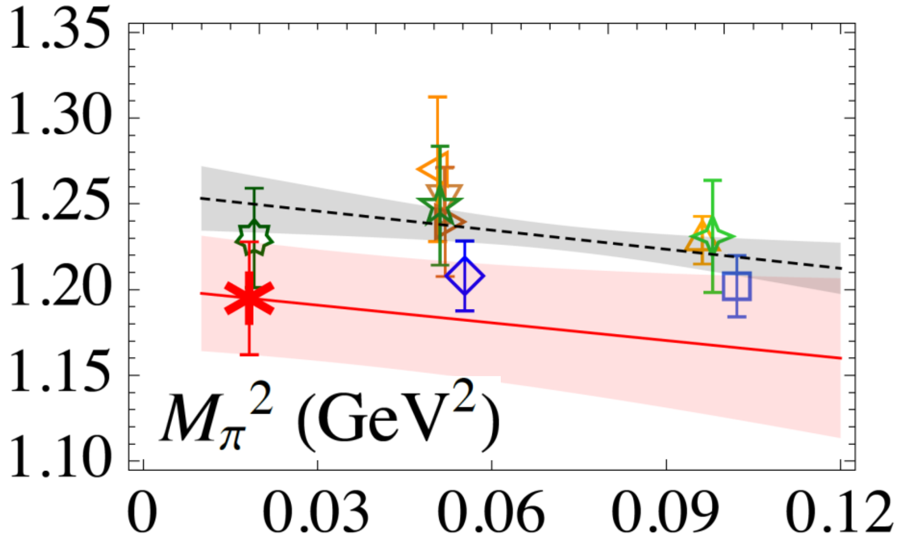


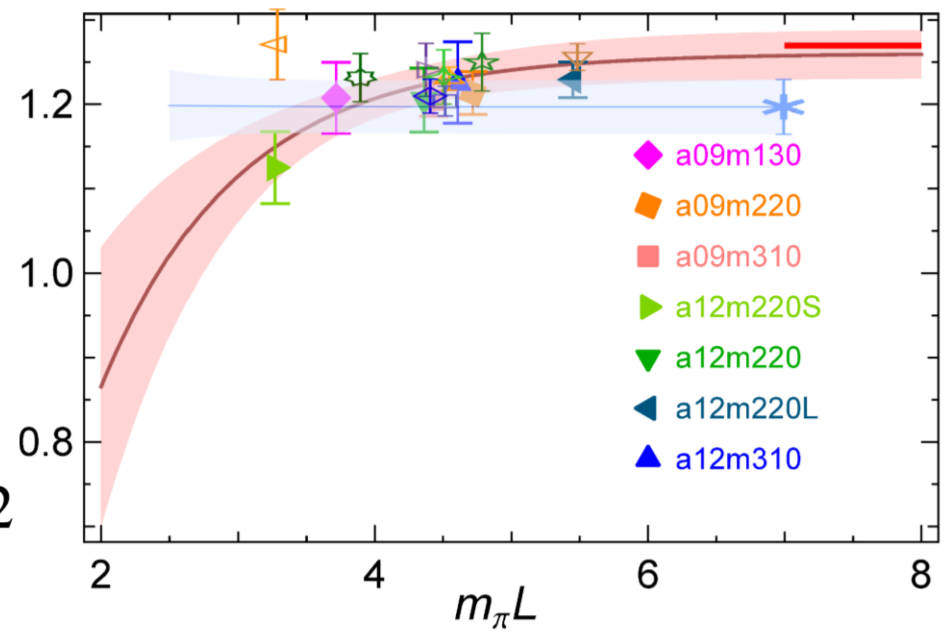



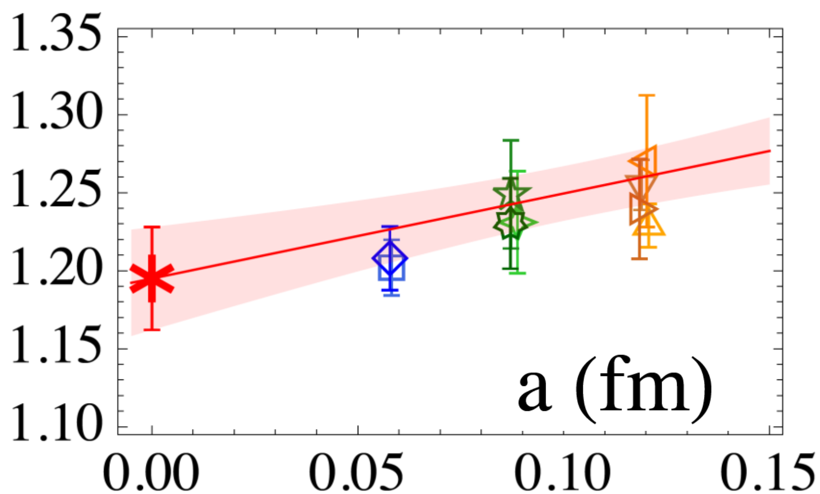

2013 Results

# Nucleon Axial Charge

## § Finite-volume/statistical effects




2016 Results