

# The muon $g-2$ from lattice QCD

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*22nd International Spin Symposium, University of Illinois*

September 29, 2016

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Luchang Jin (Columbia), Christoph Lehner (BNL),  
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## HVP with staggered fermions

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# Outline I

- 1 Introduction
- 2 The hadronic vacuum polarization contribution
- 3 Hadronic light-by-light (HLbL) scattering contribution
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# Muon g-2 experimental measurement [Bennett et al., 2006]

E821 at BNL measured relative precession of muon spin to it's momentum  $\omega_a = \frac{g-2}{2} \frac{eB}{m} = a_\mu \frac{eB}{m}$ , the muon anomaly

The rate of detected electrons oscillates with  $\omega_a$ , fit to  $N(t) = Be^{-\lambda t}(1 + A \cos \omega_a t + \phi)$

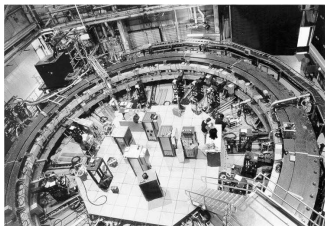


Figure 12. The storage-ring magnet. The cryostats for the inner-radius coils are clearly visible. The kickers have not yet been installed. The racks in the center are the quadrupole pulsed, and a few of the detector stations are installed, especially the quadrant of the ring closest to the person. The magnet power supply is in the upper left, above the plane of the ring. (Courtesy of Brookhaven National Laboratory)

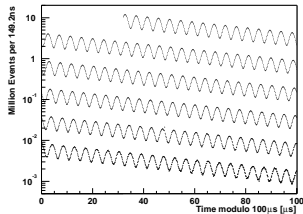


Figure 26. Histogram of the total number of electrons above 1.8 GeV versus time (modulo 100  $\mu$  s) from the 2001  $\mu^-$  data set. The bin size is the cyclotron period,  $\approx 149.2$  ns, and the total number of electrons is 3.6 billion.

$$a_\mu(\text{Expt}) = 11\,659\,208.0(5.4)(3.3) \times 10^{-10} \quad 0.54 \text{ ppm!}$$

# New muon g-2 experiments

Storage ring moved to FNAL for E989, beginning in 2017

(Peter Winter's talk next)



which is aiming for 0.14 ppm,  $4\times$  improvement!

In Japan at J-PARC, the E34 experiment will measure  $a_\mu$  using ultra-cold muons in a “table-top” experiment ( $\sim 2020$ )

# Experiment - Theory

SM Contribution	Value $\pm$ Error ( $\times 10^{11}$ )	Ref
QED (5 loops)	116584718.951 $\pm$ 0.080	[Aoyama et al., 2012]
HVP LO	6923 $\pm$ 42	[Davier et al., 2011]
	6949 $\pm$ 43	[Hagiwara et al., 2011]
HVP NLO	-98.4 $\pm$ 0.7	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	12.4 $\pm$ 0.1	[Kurz et al., 2014]
HLbL	105 $\pm$ 26	[Prades et al., 2009]
HLbL (NLO)	3 $\pm$ 2	[Colangelo et al., 2014]
Weak (2 loops)	153.6 $\pm$ 1.0	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	116591802 $\pm$ 49	[Davier et al., 2011]
(0.43 ppm)	116591828 $\pm$ 50	[Hagiwara et al., 2011]
(0.51 ppm)	116591840 $\pm$ 59	[Aoyama et al., 2012]
Exp (0.54 ppm)	116592080 $\pm$ 63	[Bennett et al., 2006]
Diff (Exp - SM)	287 $\pm$ 80	[Davier et al., 2011]
	261 $\pm$ 78	[Hagiwara et al., 2011]
	249 $\pm$ 87	[Aoyama et al., 2012]

QCD errors largest, discrepancy large

# New experiments+new theory=new physics?

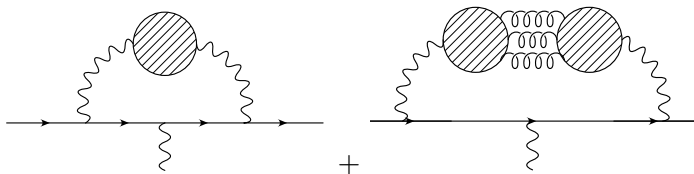
- Fermilab E989 begins 2017, aims for 0.14 ppm  
J-PARC E34  $\sim 2020$ , aims for 0.3-0.4 ppm  
Today  $a_\mu(\text{Expt}) - a_\mu(\text{SM}) \approx 2.9 - 3.6\sigma$
- If both central values stay the same,
  - E989 ( $\sim 4\times$  smaller error)  $\rightarrow \sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%)  $\rightarrow \sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction)  $\rightarrow \sim 8\sigma$
- Good for discriminating models if discovery at LHC [Stckinger, 2013]
- Lattice calculations important to trust theory errors  
(see talks at Lattice 2016 (Southampton) for latest results by many groups)

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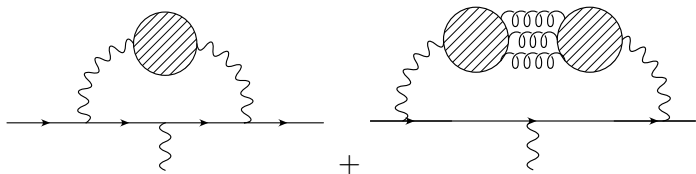
# Hadronic vacuum polarization (HVP) contribution



The blobs (quark loops), which represent all possible intermediate hadronic states ( $\rho$ ,  $\pi\pi$ , ...) are not calculable in perturbation theory, but can be calculated from

- dispersion relation + experimental cross-section for  $e^+e^- \rightarrow \text{hadrons}$
- first principles using lattice QCD

# Lattice QCD method [Blum, 2003, Lautrup et al., 1971]



Using lattice QCD and continuum,  $\infty$ -volume pQED

$$a_\mu(\text{HVP}) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 f(q^2) \hat{\Pi}(q^2)$$

$f(q^2)$  is known,  $\hat{\Pi}(q^2)$  is subtracted HVP,  $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$ ,  
computed directly on Euclidean space-time lattice

$$\begin{aligned} \Pi^{\mu\nu}(q) &= \int e^{iqx} \langle j^\mu(x) j^\nu(0) \rangle & j^\mu(x) &= \sum_i Q_i \bar{\psi}(x) \gamma^\mu \psi(x) \\ &= \Pi(q^2) (q^\mu q^\nu - q^2 \delta^{\mu\nu}) \end{aligned}$$

To improve signal to noise use direct-double-subtraction method

[Bernecker and Meyer, 2011, Lehner and Izubuchi, 2015]

and study Euclidean time dependence

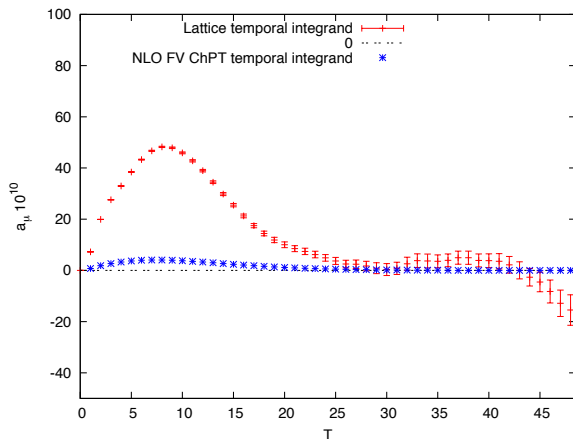
$$\Pi(q^2) - \Pi(0) = \sum_t \left( \frac{\cos qt - 1}{q^2} + \frac{1}{2}t^2 \right) C(t)$$

$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$

$$a_\mu^{\text{HVP}} = \sum_t w(t) C(t)$$

$w(t)$  includes the continuum QED part of the diagram

# Integrand $w(t)C(t)$ , light quark contribution (C. Lehner)

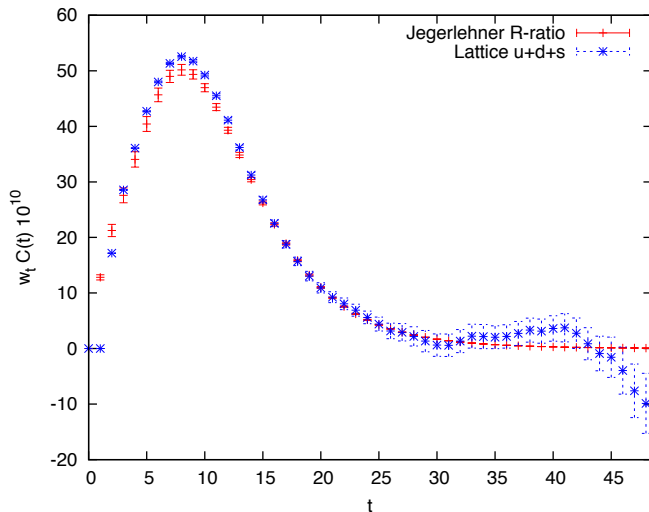


$m_\pi = 140$  MeV,  $a = 0.114$  fm,  $L = 5.5$  fm  
(RBC/UKQCD  $48^3$  ensemble)

Statistical noise comes from long-distance region

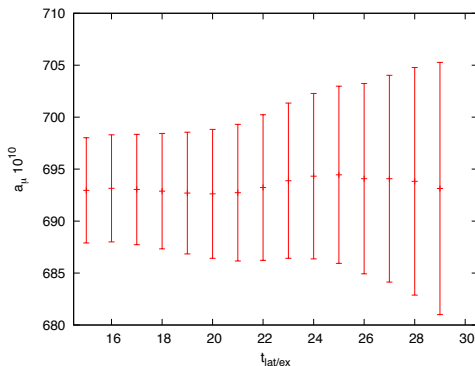
# Comparison to dispersion relation + $\sigma_{e^+e^-}$ method

[Bernecker and Meyer, 2011]



(Using data from Jegerlehner, *et al.*)

# Combined lattice + dispersive result



$t_{\text{lat/ex}} = 15a \approx 1.7 \text{ fm}$  gives 0.7% statistical error!

New numerical techniques (AMA, LMA) crucial

$$a_\mu^{\text{HVP},u,d,s} = 693(5) \times 10^{-10}$$

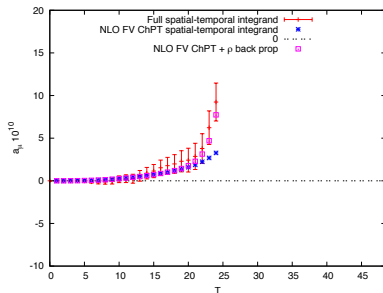
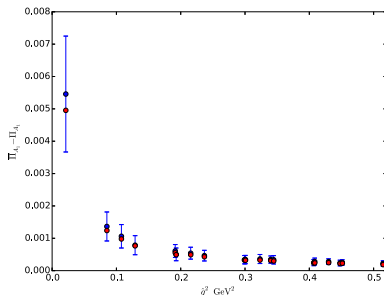
( including strange quark contribution (M. Spraggs) [Blum et al., 2016] )

# Systematic errors

Lattice spacing errors:  $a = 0.086$  fm calculation in progress

Leading finite volume effects from  $\pi\pi$ , use FV  $\chi$ PT

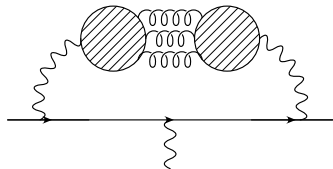
[Aubin et al., 2015]



LO  $\chi$ PT poor for total HVP ( $\rho$  res), look at differences instead

$\pi\pi$  FV effect is about 3% in LO  $\chi$ PT

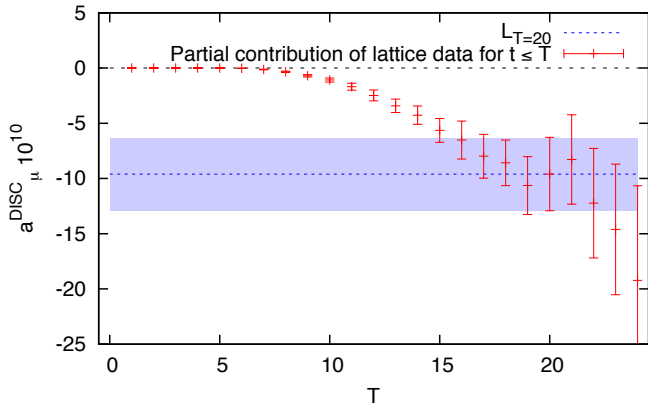
# Disconnected contribution to HVP (C. Lehner)



- quark-disconnected diagrams notoriously difficult, expected to be small (vanishes in  $SU(3)$  limit)
- Still important to reach (sub-) percent precision
- First results at physical masses with statistically resolved signal [Blum et al., 2015a].
- New stochastic estimator allowed us to obtain  
 $-(9.6 \pm 3.3_{\text{stat}} \pm 2.3_{\text{sys}}) \times 10^{-10}$ , or 1.5% of total at  $3\sigma$  level



Partial sum  $\sum_t w(t)C(t)$



We choose  $t = 20$  in plateau region as central value and use conservative resonance model including the  $\rho$  and  $\phi$  + FV  $\chi$ PT for the  $2\pi$  state to estimate the systematic error. With disc. errors,

$$a_\mu^{\text{HVP(LO)DISC}} = -(9.6 \pm 3.3_{\text{stat}} \pm 2.3_{\text{sys}}) \times 10^{-10}$$

# Beyond leading order

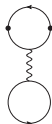
Compute  $O(\alpha)$  corrections to HVP (gluons connect quark loops!):



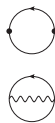
(a) V



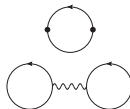
(b) S



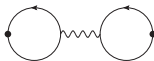
(c) T



(d) D1



(e) D2



(f) F

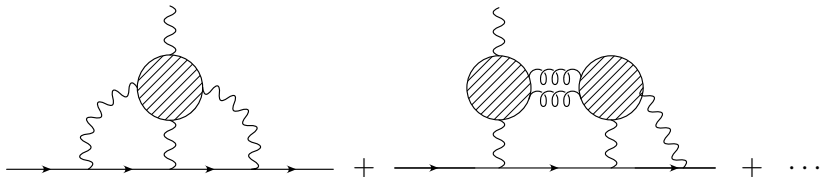


(g) D3

Computation underway on  $48^3$  physical mass ensemble

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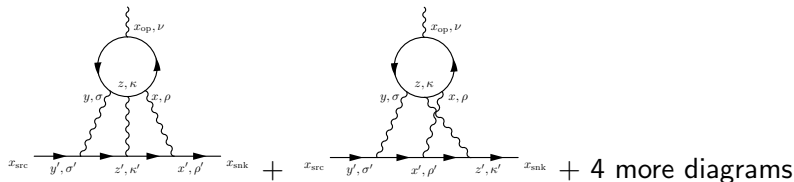
Models:  $(105 \pm 26) \times 10^{-11}$  [Prades et al., 2009, Benayoun et al., 2014]  
 $(116 \pm 40) \times 10^{-11}$  [Jegerlehner and Nyffeler, 2009]

systematic errors difficult to quantify

First lattice results [Blum et al., 2015b, Blum et al., 2015c] promise reliable errors. Using new methods we have found, for physical masses,  $a = 0.114$  fm,  $L = 5.5$  fm

$$\begin{aligned} a_{\mu}^{\text{cHLbL}} &= 11.60 \pm 0.96 \times 10^{-10} \\ a_{\mu}^{\text{dHLbL}} &= -6.25 \pm 0.80 \times 10^{-10} \quad (\text{leading diagram}) \\ a_{\mu}^{\text{HLbL}} &= 5.35 \pm 1.35 \times 10^{-10} \end{aligned}$$

# Point source method in pQED (L. Jin) [Blum et al., 2015c]



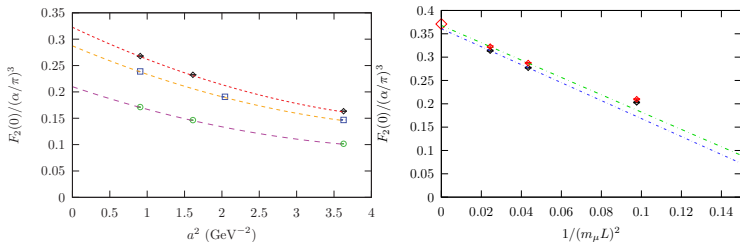
Compute quark loop non-perturbatively using lattice QCD

Photons, muon on lattice, but use (exact) tree-level props

Amplitude is obtained by integrating (summing) over all vertices (QED loop integrals) in FV

- Do QED loop integrals ( $O(V^2)$ ) stochastically by randomly choosing pairs of points,  $r = |x - y|$
- Quark loop exponentially suppressed with separation  $r$ . Concentrate on “short distance” ( $r \lesssim \pi \text{ Compton } \lambda$ ) using importance sampling!

QED systematics large,  $O(a^4)$ ,  $O(1/L^2)$ , but under control



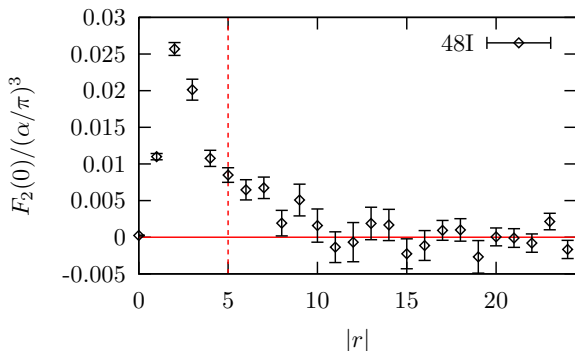
$a$  set using physical muon mass, *i.e.*, input parameter  $am_\mu$

Limits quite consistent with well known PT result

Very good check on method/code

# Physical point cHLbL contribution (L. Jin)

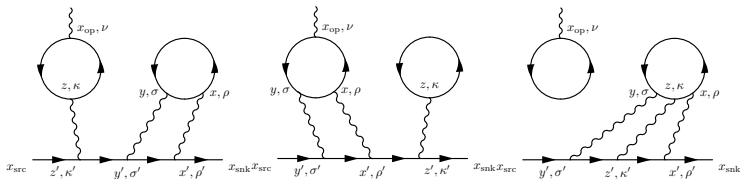
ALCC award on MIRA (100 PF BG/Q) at ANL ALCF  
Physical mass 2+1f Möbius DWF ensemble ( $_{\text{RBC/UKQCD}}$ ),  
(5.5 fm)<sup>3</sup> QCD box,  $a = 0.114$  fm ( $a^{-1} = 1.7848$  GeV)  
Uses AMA with 2000 low-modes of the Dirac operator and  
 $\sim 2200$  sloppy propagators per configuration (65 total)



$$a_{\mu}^{\text{cHLbL}} = 11.60 \pm 0.96 \times 10^{-10}$$

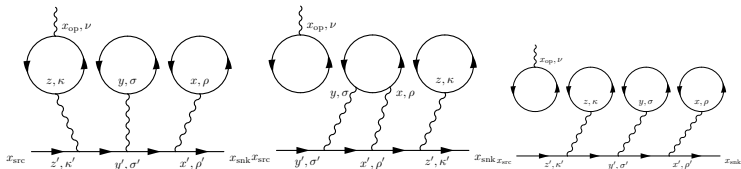
# Disconnected contributions

SU(3) flavor:



Leading

$O(m_s - m_{u,d})$

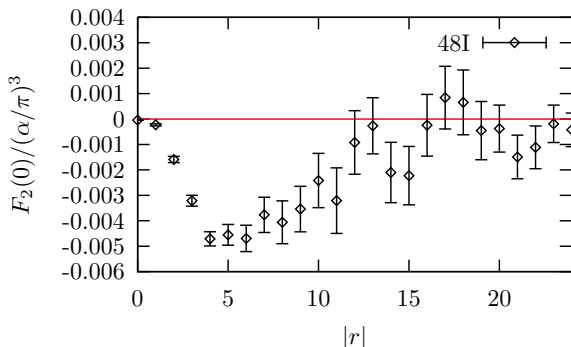


$O(m_s - m_{u,d})^2$  and higher



# Physical point dHLbL contribution (L. Jin)

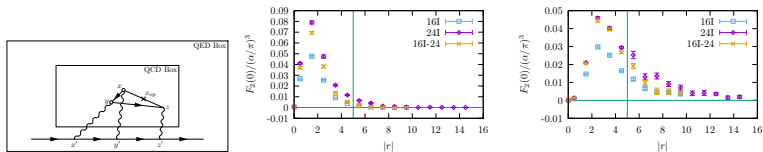
ALCC award on MIRA (100 PF BG/Q) at ANL ALCF  
Physical mass 2+1f Möbius DWF ensemble ( $_{\text{RBC/UKQCD}}$ ),  
(5.5 fm)<sup>3</sup> QCD box,  $a = 0.114$  fm ( $a^{-1} = 1.7848$  GeV)  
Uses AMA with 2000 low-modes of the Dirac operator and  
(1024 + 512)<sup>2</sup> measurements per configuration (65 total)



$$a_{\mu}^{\text{cHLbL}} = -6.25 \pm 0.80 \times 10^{-10}$$

# Eliminating discretization and (QED) FV effects

- Systematic errors could be 20% or more
- Ongoing calculation at  $a = 0.08$  fm for continuum limit  
Integrand exponentially suppressed with distance between any pair of points on the quark loop. **QCD FV effects small.**  
Amplitude *not* suppressed with distance between points on muon line and quark loop. **QED FV effects large.**
- Use larger QED box for QED FV effect



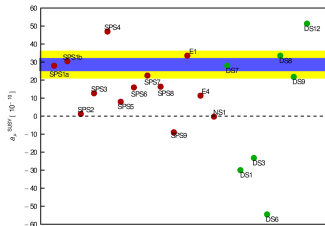
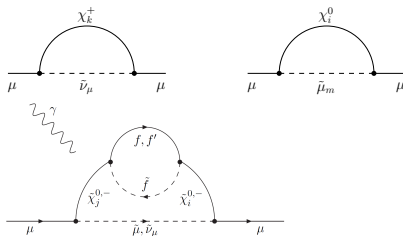
- Or, continuum,  $\infty$  volume QED calculation (Mainz group, Lattice 2016) analogous to HVP computation (two-loop integrals still done stochastically)

# Physics beyond the SM

If there really is a discrepancy, where does it come from?

Most likely scenario is still SUSY

[Bach et al., 2015, Athron et al., 2016, Belyaev et al., 2016], . . .



SUSY signatures at LHC

But there are other models too: 2HDM [Crivellin et al., 2016, Cherchiglia et al., 2016],  
Dark Matter [Kobakhidze et al., 2016], . . . , LFV [Altmannshofer et al., 2016]

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# Summary

- The muon anomalous magnetic moment provides a stringent test of the SM:  $\sim 3$  standard deviation difference at the level of 0.5 ppm
- Lattice QCD(+QED) calculations with physical masses, large boxes + improved measurement algorithms are powerful
- Physical point calculations nearly complete at  $a = 0.114$  fm,  $a = 0.086$  fm calculations begun
- Lattice QCD calculations will reduce and solidify current theory errors in time for
- Upcoming E989 measurement at Fermilab (goal 0.14 ppm)
- Good opportunity to test the SM

# Acknowledgments

- This research is supported in part by the US DOE
- Computational resources provided by the RIKEN BNL Research Center, RIKEN, USQCD Collaboration, and the ALCF at Argonne National Lab

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Interaction of particle with static magnetic field

$$V(\vec{x}) = -\vec{\mu} \cdot \vec{B}_{\text{ext}}$$

The magnetic moment  $\vec{\mu}$  is proportional to its spin ( $c = \hbar = 1$ )

$$\vec{\mu} = g \left( \frac{e}{2m} \right) \vec{S}$$

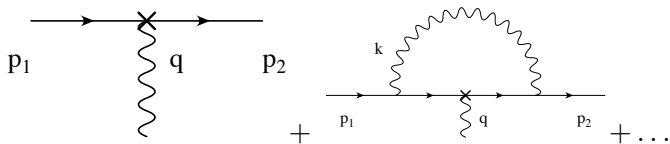
The Landé ***g*-factor** is predicted from the **free Dirac eq.** to be

$$g = 2$$

for elementary fermions



In interacting quantum field theory  $g$  gets corrections



$$\langle \mu(p') | J^\mu | \mu(p) \rangle = \bar{u}(p') \left( \gamma^\mu F_1(q^2) + i \frac{[\gamma^\mu, \gamma^\nu] q^\nu}{2} \frac{F_2(q^2)}{2m} \right) u(p)$$

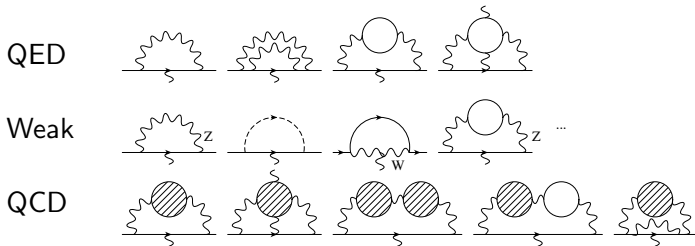
which results from Lorentz invariance and charge conservation when the muon is on-mass-shell and where  $q = p' - p$

$$F_2(0) = \frac{g-2}{2} \equiv a_\mu \quad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

Compute these corrections order-by-order in perturbation theory by expanding  $\Gamma^\mu(q^2)$  in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \dots$$



Corrections begin at  $\mathcal{O}(\alpha)$ ; Schwinger term =  $\frac{\alpha}{2\pi} = 0.0011614\dots$   
hadronic contributions  $\sim 6 \times 10^{-5}$  smaller, **dominate theory error**.

# Dispersive method [Bouchiat and Michel (1961); Durand (1962); ...]

The vacuum polarization (blob) is an *analytic* function.

$$\begin{aligned}\Pi(q^2) &= \frac{1}{\pi} \int_0^\infty ds \frac{\Im \Pi(s)}{(s - q^2)} \\ \sigma_{\text{total}}(e^+ e^- \rightarrow \text{hadrons}) &= \frac{4\pi^2 \alpha}{s} \frac{1}{\pi} \Im \Pi(s)\end{aligned}$$

(by the optical theorem) which leads to

$$a_\mu(\text{HVP}) = \frac{1}{4\pi^2} \int_{4m_\pi^2}^\infty ds K(s) \sigma_{\text{total}}(s)$$

- $a_\mu(\text{HVP}) \sim 693(4)$  (0.6% error, but largest contribution to SM value)
- $\sigma_{\text{total}}(S)$  also from  $\tau \rightarrow \pi^\pm \pi^0 \nu$  (needs isospin correction)

# Simulation details

Gauge field ensembles generated by RBC/UKQCD collaborations

Domain wall fermions: chiral symmetry at finite  $a$

Iwasaki Gauge action (gluons)

- Range of pion (quark) masses  $m_\pi = 140, 170, 330, 420$  MeV
- Range of lattice spacings,  $a = 0.144, 0.114, 0.086$  fm
- Range of lattice sizes,  $L/a = 16, 24, 32, 48, 64$
- Range of lattice volumes,  $(1.8)^3, (2.7)^3, (4.6)^3, (5.5)^3$  fm<sup>3</sup>

Use all-mode-averaging technique [Izubuchi et al., 2013]

## Brief aside: Lattice setup

- Compute correlation functions (e.g.  $\langle j^\mu(x)j^\nu(y) \rangle$ ,  $j^\mu = \bar{\psi}\gamma_\mu\psi$ ) in Feynman path integral formalism
- 4(5)D hypercubic lattice regularization, non-zero lattice spacing  $a$  and finite volume  $V$  (extrap  $a \rightarrow 0$ ,  $V \rightarrow \infty$ )
- Handle fermion integrals analytically. Propagators inverse of large sparse matrix  $M$ , lattice Dirac operator (domain wall, staggered, Wilson, ...). Costliest part of calculation
- Do path integrals over gauge fields stochastically by Monte Carlo importance sampling: generate ensemble of gauge field configurations  $\{U(x)\}$  with weight  $\det M(U) \exp -S_g$ , then  $\langle \dots \rangle$  simple average over ensemble
- Gauge field configurations represent fluctuations (virtual particles) of the vacuum
- Statistical errors  $O(1/\sqrt{N_{\text{meas}}})$

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