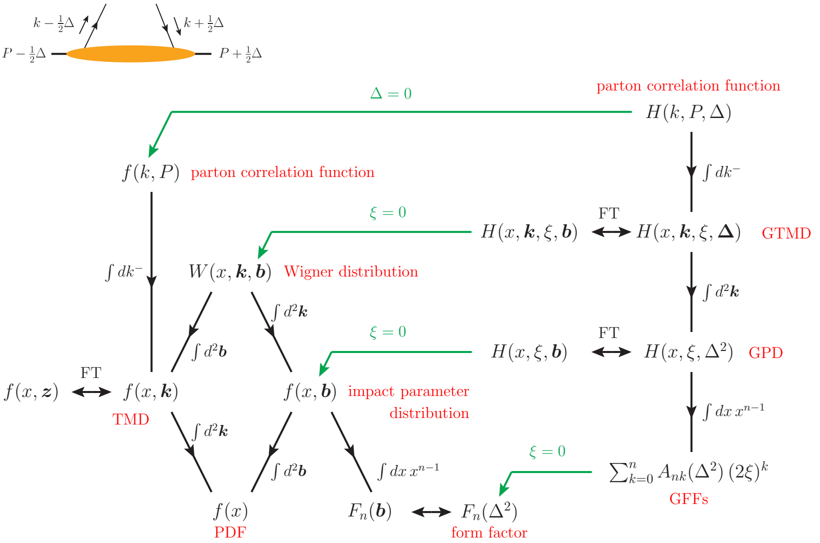
GPDs: Status and Future

Krešimir Kumerički

Physics Department
University of Zagreb, Croatia

SPIN 2016
22nd International Spin Symposium
Urbana-Champaign, IL, USA, 25–30 September 2016

Family tree of hadron structure functions



[Fig. by Markus Diehl]

($\xi \to \eta$ from now on)

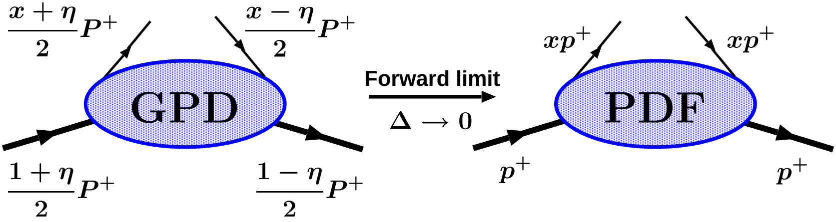
Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$\widetilde{F}^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

(and similarly for gluons F^g and $\widetilde{F}^g$).



$P = P_1 + P_2$; $\quad t = \Delta^2 = (P_2 - P_1)^2$; $\quad \eta = -\frac{\Delta^+}{P^+}$ (skewedness)

Some properties of GPDs

- Decomposing into spin-non-flip and spin-flip part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

$$\tilde{F}^a = \frac{\bar{u}(P_2)\gamma^+\gamma_5 u(P_1)}{P^+} \tilde{H}^a + \frac{\bar{u}(P_2)\gamma_5 u(P_1)\Delta^+}{2MP^+} \tilde{E}^a \qquad a = q, g$$

Some properties of GPDs

- Decomposing into spin-non-flip and spin-flip part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

$$\tilde{F}^a = \frac{\bar{u}(P_2)\gamma^+ \gamma_5 u(P_1)}{P^+} \tilde{H}^a + \frac{\bar{u}(P_2)\gamma_5 u(P_1)\Delta^+}{2MP^+} \tilde{E}^a \qquad a = q, g$$

- "Ji's sum rule" (related to proton spin problem)

$$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^q(x,\eta,t) + E^q(x,\eta,t) \Big]_{t\to 0} \qquad \text{[Ji '96]}$$

(where E is still very unconstrained by experiments).

Some properties of GPDs

- Decomposing into spin-non-flip and spin-flip part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

$$\tilde{F}^a = \frac{\bar{u}(P_2)\gamma^+\gamma_5 u(P_1)}{P^+} \tilde{H}^a + \frac{\bar{u}(P_2)\gamma_5 u(P_1)\Delta^+}{2MP^+} \tilde{E}^a \qquad a = q, g$$

- "Ji's sum rule" (related to proton spin problem)

$$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x \Big[H^q(x,\eta,t) + E^q(x,\eta,t)\Big]_{t\to 0} \qquad \text{[Ji '96]}$$

(where E is still very unconstrained by experiments).

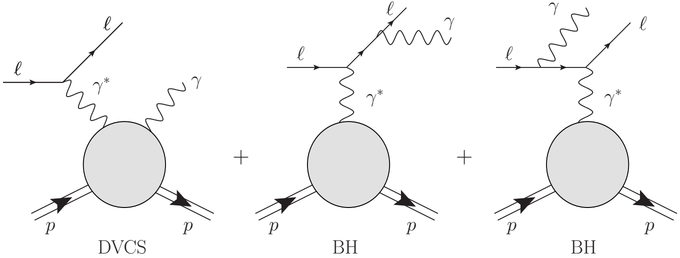
- Distribution of partons in transversal space

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H(x, 0, -\vec{\Delta}_\perp^2) \qquad \text{[Burkardt '00]}$$

(where experiments are mostly sensitive to $H(x, x, -\vec{\Delta}_\perp^2)$)

Access to GPDs via DVCS

- Deeply virtual Compton scattering (DVCS) — "gold plated" process of exclusive physics
- DVCS is measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} \propto \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\},$$

$$|\mathcal{T}_{\rm DVCS}|^2 \propto \left\{c_0^{\rm DVCS} + \sum_{n=1}^{2}\left[c_n^{\rm DVCS}\cos(n\phi) + s_n^{\rm DVCS}\sin(n\phi)\right]\right\},$$

- Choosing polarizations (and charges) we focus on particular harmonics:

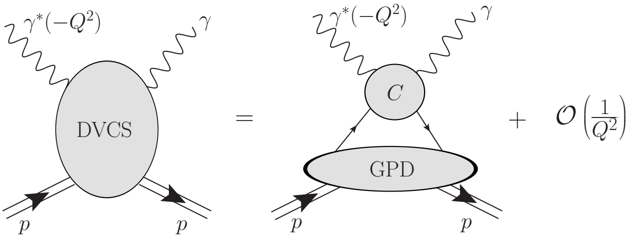
$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1\,\mathfrak{Re}\,\mathcal{H} - \frac{t}{4M_p^2}F_2\,\mathfrak{Re}\,\mathcal{E} + \frac{x_{\rm B}}{2-x_{\rm B}}(F_1+F_2)\,\mathfrak{Re}\,\widetilde{\mathcal{H}}\right]$$

[Belitsky, Müller et. al '01–'14]

- $\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2)$, ...— four complex Compton form factors (CFFs)

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- Compton form factor is a convolution:

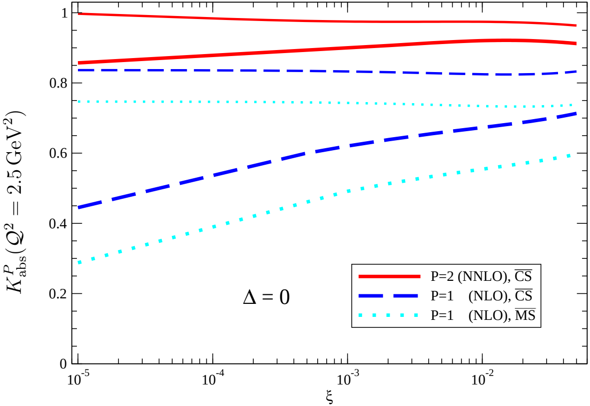
$$ {}^a\mathcal{H}(x_{\rm B}, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \frac{x_{\rm B}}{2 - x_{\rm B}}, \frac{\mathcal{Q}^2}{\mathcal{Q}_0^2}) \; H^a(x, \frac{x_{\rm B}}{2 - x_{\rm B}}, t, \mathcal{Q}_0^2) $$

$a{=}q,G$

- $H^a(x, \eta, t, \mathcal{Q}_0^2)$ — Generalized parton distribution (GPD)

(N)NLO corrections

- [K.K., Müller and Passek-K. '07]

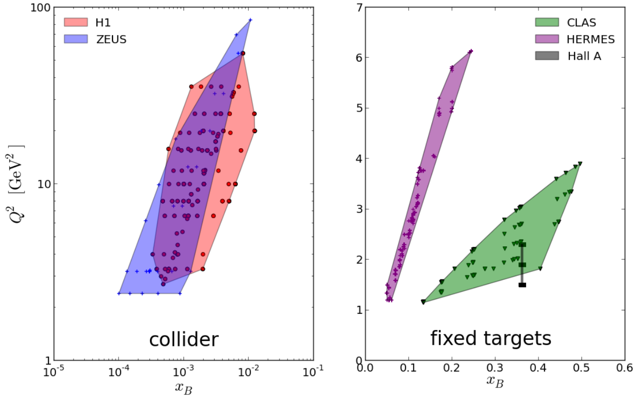


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0){=}\alpha_\Sigma(0)$
$+\ 0.05$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0){=}\alpha_\Sigma(0)$
$-\ 0.02$

$$K^P_{\rm abs} \equiv \left| \frac{\mathcal{H}^{(P)}}{\mathcal{H}^{(P-1)}} \right|$$

Experimental coverage (1/2)



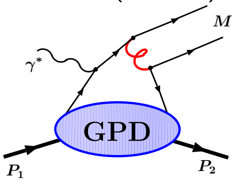
- Coming soon: COMPASS [talk by A. Ferrero], JLab12, EIC [talk by S. Fazio]
- see next [talk by C. Muñoz-Camacho] for overview

Experimental coverage (2/2)

Collab.	Year	Observables	Kinematics			No. of points	
			x_B	Q^2 [GeV^2]	$\lvert t\rvert$ [GeV^2]	total	indep.
HERMES	2001	$A_{LU}^{\sin\phi}$	0.11	2.6	0.27	1	1
CLAS	2001	$A_{LU}^{\sin\phi}$	0.19	1.25	0.19	1	1
CLAS	2006	$A_{UL}^{\sin\phi}$	0.2–0.4	1.82	0.15–0.44	6	3
HERMES	2006	$A_{C}^{\cos\phi}$	0.08–0.12	2.0–3.7	0.03–0.42	4	4
Hall A	2006	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.36	1.5–2.3	0.17–0.33	4×24+12×24	4×24+12×24
CLAS	2007	$A_{LU}(\phi)$	0.11–0.58	1.0–4.8	0.09–1.8	62×12	62×12
HERMES	2008	$A_{C}^{\cos(0,1)\phi}$, $A_{UT,DVCS}^{\sin(\phi-\phi_S)}$, $A_{UT,I}^{\sin(\phi-\phi_S)\cos(0,1)\phi}$, $A_{UT,I}^{\cos(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12*+12+12 12+12 *12*	*4*+4+4 4+4 *4*
CLAS	2008	$A_{LU}(\phi)$	0.12–0.48	1.0–2.8	0.1–0.8	66	33
HERMES	2009	$A_{LU,I}^{\sin(1,2)\phi}$, $A_{LU,DVCS}^{\sin\phi}$, $A_{C}^{\cos(0,1,2,3)\phi}$	0.05–0.24	1.2–5.75	<0.7	18+18+18 18+18+*18*+*18*	6+6+6 6+6+*6*+*6*
HERMES	2010	$A_{UL}^{\sin(1,2,3)\phi}$, $A_{LL}^{\cos(0,1,2)\phi}$	0.03–0.35	1–10	<0.7	12+12+*12* 12+*12*+12	4+4+*4* 4+*4*+4
HERMES	2011	$A_{LT,I}^{\cos(\phi-\phi_S)\cos(0,1,2)\phi}$, $A_{LT,I}^{\sin(\phi-\phi_S)\sin(1,2)\phi}$, $A_{LT,BH+DVCS}^{\cos(\phi-\phi_S)\cos(0,1)\phi}$, $A_{LT,BH+DVCS}^{\sin(\phi-\phi_S)\sin\phi}$	0.03–0.35	1–10	<0.7	*12+12+12* *12+12* *12*+12 12	*4+4+4* *4+4* *4*+4 4
HERMES	2012	$A_{LU,I}^{\sin(1,2)\phi}$, $A_{LU,DVCS}^{\sin\phi}$, $A_{C}^{\cos(0,1,2,3)\phi}$	0.03–0.35	1–10	<0.7	18+*18+18* 18+18+*18+18*	6+*6+6* 6+6+*6+6*
CLAS	2015	$A_{LU}(\phi)$, $A_{UL}(\phi)$, $A_{LL}(\phi)$	0.17–0.47	1.3–3.5	0.1–1.4	166+166+166	166+166+166
CLAS	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.1–0.58	1–4.6	0.09–0.52	2640+2640	2640+2640
Hall A	2015	$\sigma(\phi)$, $\Delta\sigma(\phi)$	0.33–0.40	1.5–2.6	0.17–0.37	480+600	240+360

Alternative processes for GPD access

- Deeply virtual meson production (DVMP) $\gamma^* p \to Mp$.



  - Theory more "dirty" than for DVCS (second "soft" function appears: meson distribution amplitude)
  - Different mesons enable access to different flavours of GPDs [Goloskokov, Kroll]

- double DVCS $\gamma^* p \to \gamma^* p$, timelike DVCS [talk by P. Nadel-Turonski], ... (see Trento workshop October 2016) — direct access to GPDs away from $\eta = x$ line

Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to *PDFs*(x), it is very difficult to perform truly model-independent extraction of *GPDs*(x, η, t)

Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to *PDFs*(x), it is very difficult to perform truly model-independent extraction of *GPDs*(x, η, t)
- When the dimensionality of domain space increases, the available data becomes sparse very fast. (*"Curse of dimensionality"*)

Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to *PDFs*(x), it is very difficult to perform truly model-independent extraction of *GPDs*(x, η, t)
- When the dimensionality of domain space increases, the available data becomes sparse very fast. (*"Curse of dimensionality"*)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.

Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to $PDFs(x)$, it is very difficult to perform truly model-independent extraction of $GPDs(x, \eta, t)$
- When the dimensionality of domain space increases, the available data becomes sparse very fast. (*"Curse of dimensionality"*)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of $CFFs(x_B, t)$.

Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to *PDFs*(x), it is very difficult to perform truly model-independent extraction of *GPDs*(x, η, t)
- When the dimensionality of domain space increases, the available data becomes sparse very fast. (*"Curse of dimensionality"*)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of *CFFs*(x_B, t).
- Instead of functions *CFFs*(x_B, t) one can extract CFFs as numbers for fixed x_B and t — local fits (hope of a model-independent procedure)

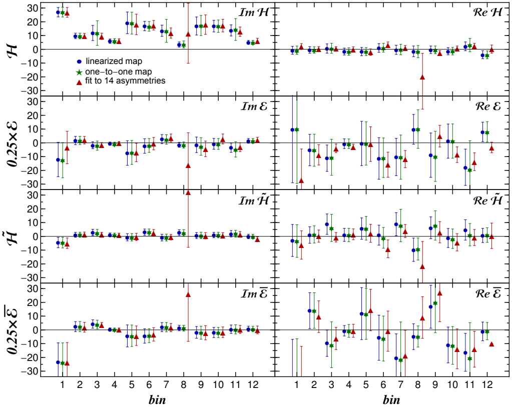
Extraction of GPDs/CFFs by fits to DVCS data

- In contrast to $PDFs(x)$, it is very difficult to perform truly model-independent extraction of $GPDs(x, \eta, t)$
- When the dimensionality of domain space increases, the available data becomes sparse very fast. (*"Curse of dimensionality"*)
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of $CFFs(x_B, t)$.
- Instead of functions $CFFs(x_B, t)$ one can extract CFFs as numbers for fixed x_B and t — local fits (hope of a model-independent procedure)

- (Dependence on additional variable, photon virtuality $\mathcal{Q}^2$, is in principle known — given by evolution equations.)

Local fits — some results

- [K.K., Müller, Murray '13]: using HERMES set of 10+ observables measured at same 12 kinematical points and extracting 8 leading "CFFs"



- Many CFFs are not very constrained.

Hybrid GPD models for global fits

- Sea quarks and gluons modelled using SO(3) partial wave expansion in conformal GPD moment space + $\mathcal{Q}^2$ evolution.
- Valence quarks — model CFFs directly (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi, t) = \pi \left[\frac{4}{9} H^{u_{\rm val}}(\xi, \xi, t) + \frac{1}{9} H^{d_{\rm val}}(\xi, \xi, t) + \frac{2}{9} H^{\rm sea}(\xi, \xi, t)\right]$$

$$H(x, x, t) = n\, r\, 2^{\alpha} \left(\frac{2x}{1+x}\right)^{-\alpha(t)} \left(\frac{1-x}{1+x}\right)^{b} \frac{1}{\left(1 - \frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}} .$$

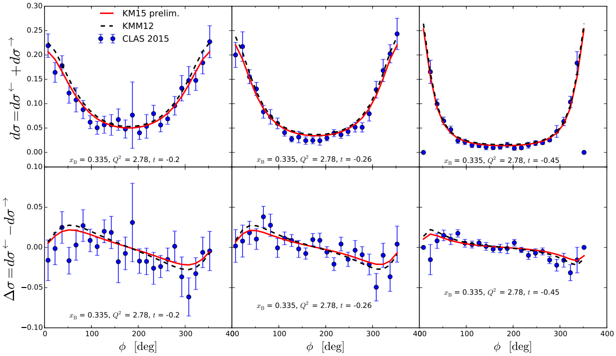
- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations
- 15 free parameters in total for H, $\tilde{H}$, E, $\tilde{E}$.

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12	KM15
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15	{3}+15
χ^2/d.o.f.	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80	240./275
F_2	{85}	{85}	{85}	{85}	{85}		{85}	{85}
$\sigma_{\rm DVCS}$	(45)	(45)	51	51	45		11	11
$d\sigma_{\rm DVCS}/dt$	(56)	(56)	56	56	56		24	24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4	13
$A_{LU,I}^{\sin\phi}$			18	18		18	6	6
$A_{C}^{\cos 0\phi}$							6	6
$A_{C}^{\cos\phi}$	12	12	18	18	12	18	6	6
$\Delta\sigma^{\sin\phi,w}$			12				12	63
$\sigma^{\cos 0\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}/\sigma^{\cos 0\phi,w}$		4			4			
$A_{UL}^{\sin\phi}$							10	17
$A_{LL}^{\cos 0\phi}$							4	14
$A_{LL}^{\cos\phi}$								10
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4	4

- [K.K., Müller, et al. '09–'15]
- These models are publicly available (google for "gpd page")
- Other approaches: [Goldstein, Gonzalez, Liuti '11] (see [talk by G. Goldstein]), French PARTONS project, ...

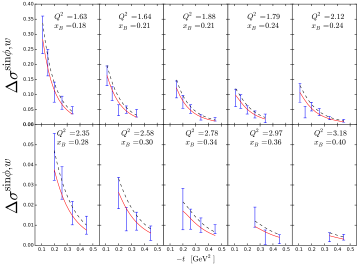
2015 CLAS cross-sections (1/2)

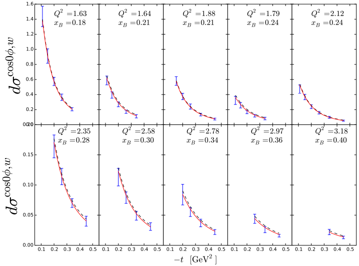
- Restriction to kinematics where leading-order framework should be valid: $-t/Q^2 < 0.25$ with $Q^2 > 1.5\,\mathrm{GeV}^2$, means using 48 out of measured 110 x_B–Q^2–t bins.

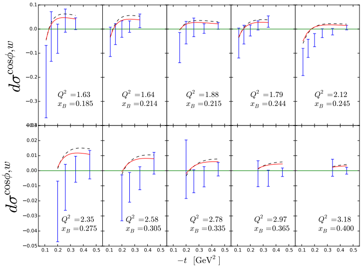


- $\chi^2/\mathrm{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$

2015 CLAS cross-sections (2/2)



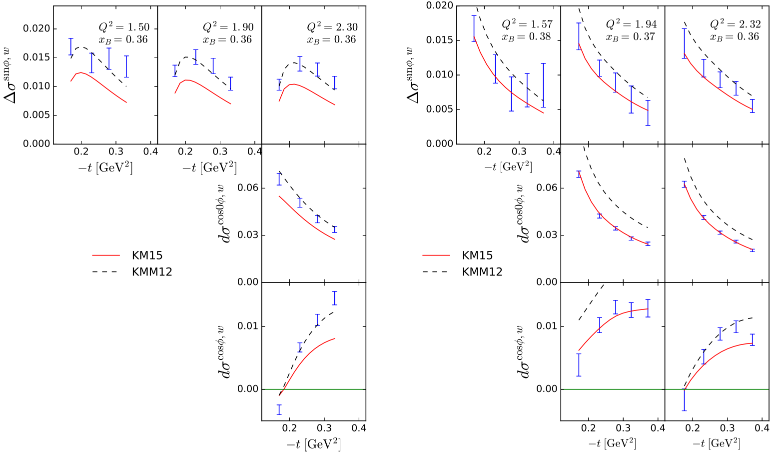




- $\chi^2/\mathrm{npts} = 62.2/48$ for $d\sigma^{\cos\phi,w}$

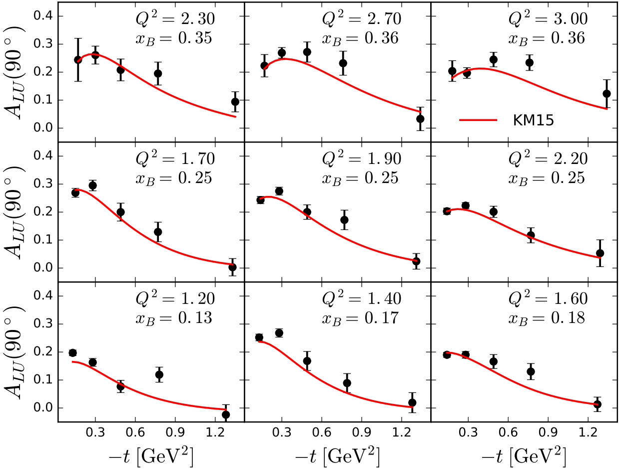
(O.K. but not so perfect as in ϕ-space)

2006 vs 2015 Hall A cross-sections



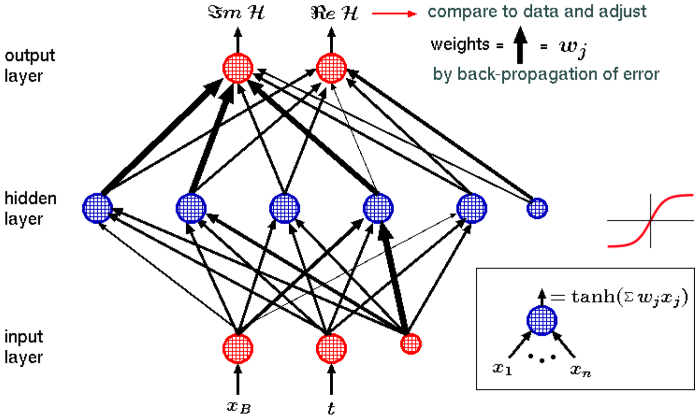
- Improvement of global $\chi^2/\mathrm{d.o.f.}$ 123.5/80 → 240./275

2007 CLAS beam spin asymmetry



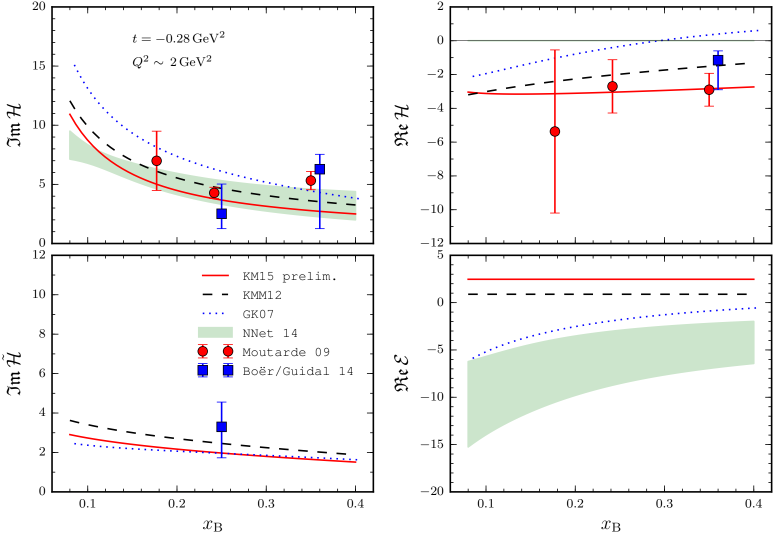
- Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.

Neural network fits



- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias
- preliminary fit just to HERMES data [KK, Müller, Schäfer '11]

CFFs from various fits



Going beyond first approximations

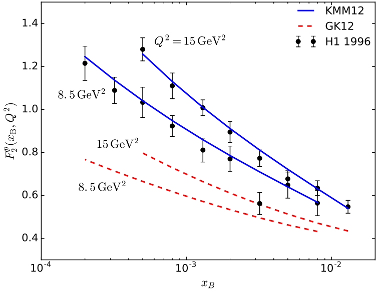
Published DVCS data is well described within leading order, leading twist and other approximations. Some available corrections:

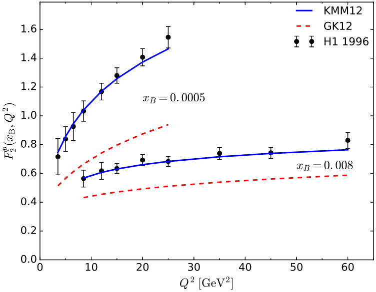
- NLO evolution and NLO coefficient functions
- twist-3 GPDs
- massive charm [Noritzsch '03]
- Finite-t and target mass corrections [Braun, Manashov, Pirnay, Müller '14] (See also [talk by M. Defurne].)
- Small-x ressumation [Ivanov et al. '16]

can presently be absorbed in redefinition of GPDs (think "DIS scheme" factorization), but have to be taken into account when working with more processes (striving towards universal GPDs) and with more precise future data.

Testing universality

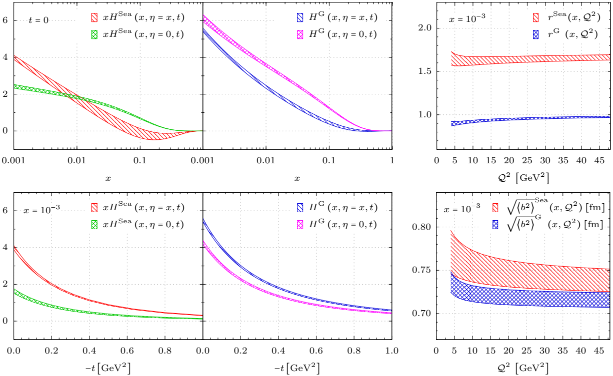
GK model GPDs (extracted from DVMP data) describe also DVCS reasonably well [Kroll, Moutarde, Sabatie '12], but low-x DIS is not working well





Testing universality II

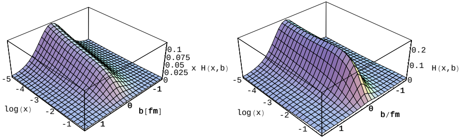
First KM-like model fits to DIS+DVCS+DVMP [Lautenschlager, Müller, Schäfer '13] show promise, and prefer NLO:



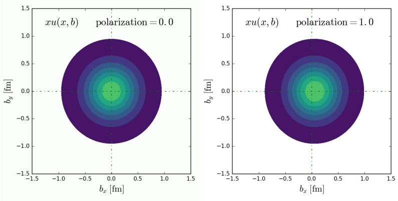
(Here one needs relaxed model with different Regge trajectories for each SO(3) partial wave of conformal space GPD.)

Tomography

- Quark and gluon sea 2D distributions $H(x, \vec{b}_\perp)$ ([KM] model)



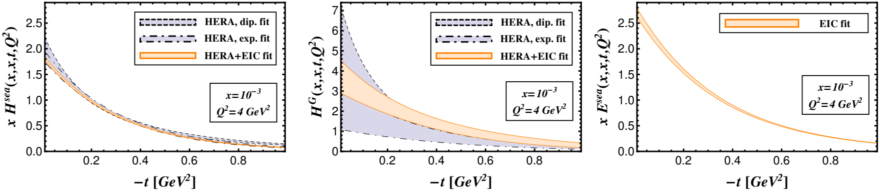
- Sivers effect for valence quarks ([GK] model)



- See also [Dupré, Guidal, Vanderhaeghen '16]
- Tomography is still very much model-dependent; e.g. some extrapolation from $H(x, x, t)$ to $H(x, 0, t)$ is needed. (see [talk by M. Burkardt])

DVCS at EIC

- Future polarized electron-ion collider (EIC) will provide unique insight into sea GPDs.
- [Aschenauer, Fazio, K.K., Müller '13] fit to simulated DVCS data at 20 GeV $\times$ 250 GeV taking $E_{\rm sea}(x, \eta, t) = \kappa_{\rm sea} H_{\rm sea}(x, \eta, t)$



- Improved knowledge of low-t quark and gluon GPDs H ($\Longrightarrow$ 3D parton imaging)
- Improved knowledge of sea quark GPD E

Summary

- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape . (New 2015 JLab data relieve some old tensions.)
- Data clearly restrict $H(x, x, t)$, and to some extent $\tilde{H}$, but E is still very model-dependent
- Truly global DVCS+DVMP fits should be possible soon.

Summary

- Global fits of all proton DVCS data using flexible hybrid models are in healthy shape . (New 2015 JLab data relieve some old tensions.)
- Data clearly restrict $H(x, x, t)$, and to some extent $\tilde{H}$, but E is still very model-dependent
- Truly global DVCS+DVMP fits should be possible soon.

The End