Flavor Structure of the Nucleon Sea from Lattice QCD

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arXiv: 1402.1462 + 1603.06664

Parton Distribution Functions (PDF)---an Infinite-Body Problem

- The number of quark anti-quark pairs diverges (manifestation of non-perturbative nature of the problem)

One Possible Solution…

- Lattice QCD: making the degree of freedom finite by discretizing the space time

- Goal: Computing the x-dependence of PDF’s from first principles (QCD).

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Past Limitation

- Traditional approach: can only calculate lower moments PDFs.
- Still first principle, carried out successfully: close to using physical parameters---highly non-trivial and demanding in computing power.
- However, it also means the community has reached the limit on what one can learn from the lower moments.

New Hopes

- Smeared sources: Davoudi & Savage
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x -dependence directly.

Lattice PDF: from Moments to the Sea

- Quark PDF in a proton: $(\lambda^2 = 0)$

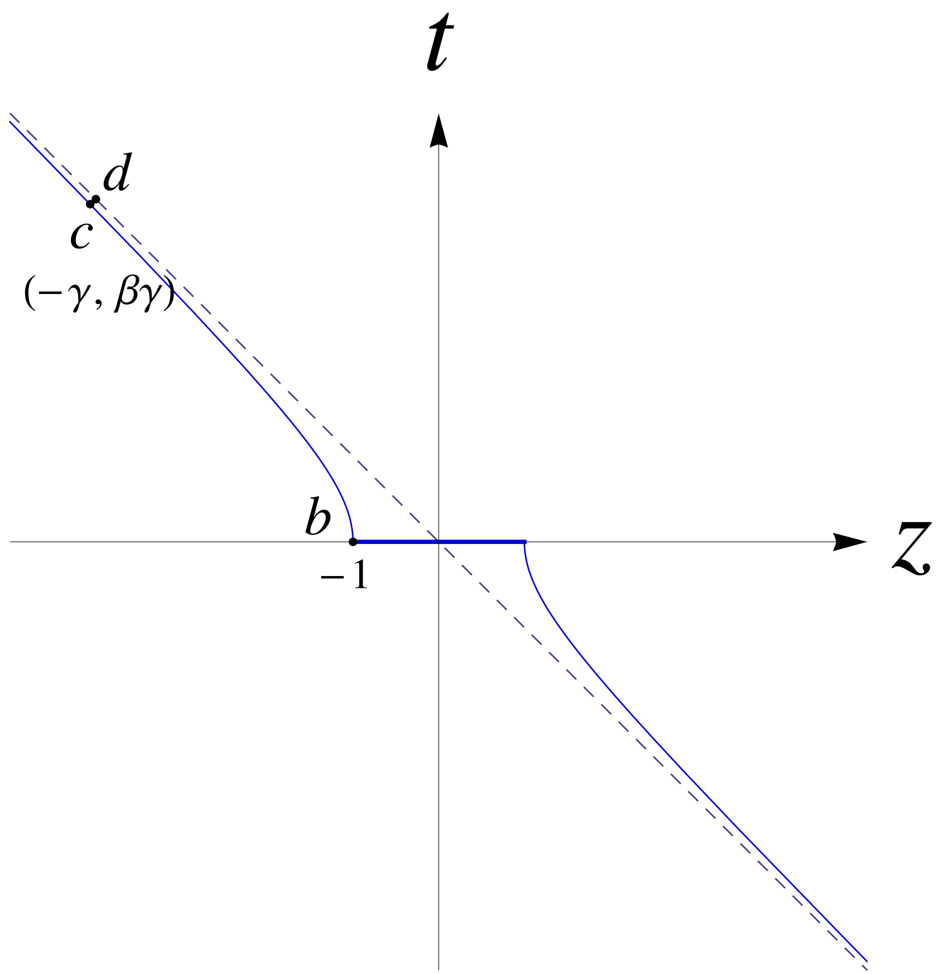
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \rangle$$

- Euclidean lattice: light cone operators cannot be distinguished from local operators
- Moments of PDF given by local twist-2 operators; LPDF limited to first few moments; Sea quarks cannot be isolated

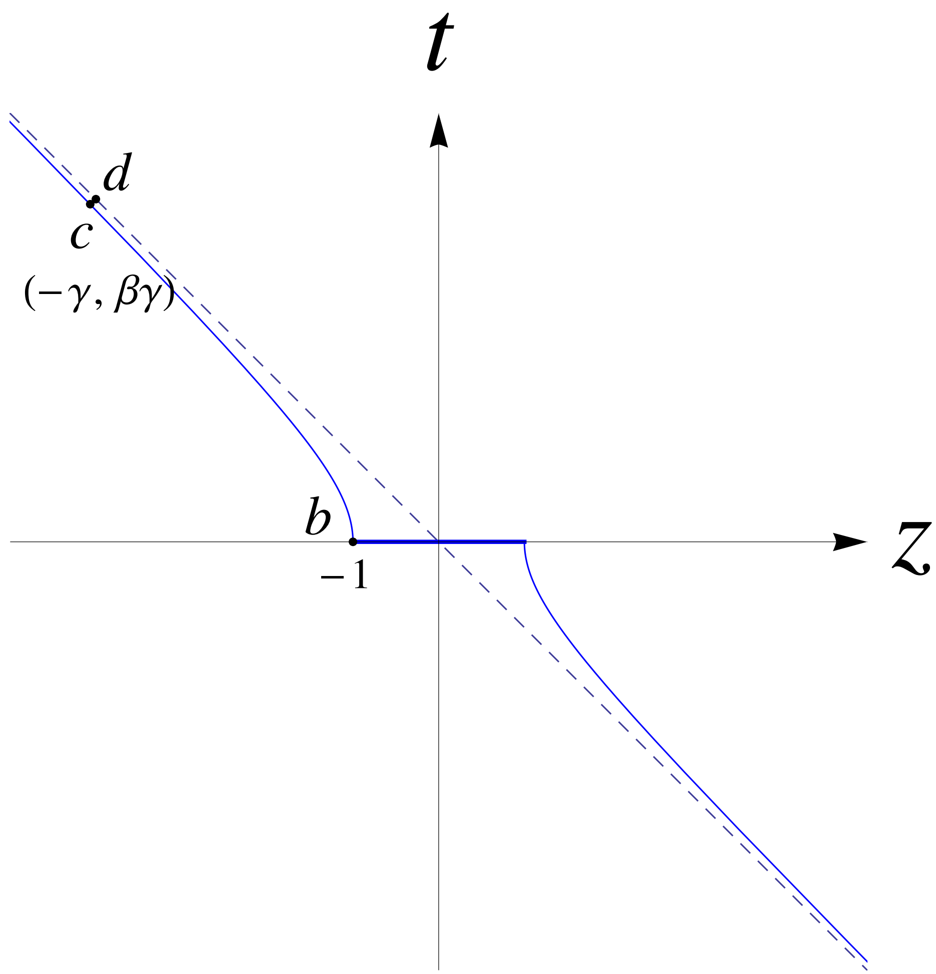
- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \,|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda)|\, P\rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?



t
d
c
(−γ, βγ)
b
−1
Z



- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

Review: Ji's LPDF

$$\begin{aligned}\widetilde{q}(x,\mu^2,P^z) &= \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda)\right| P\right\rangle \\ &\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z)P^z\end{aligned}$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^n\psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

op. symmetric but not traceless

Review: Ji's LPDF

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

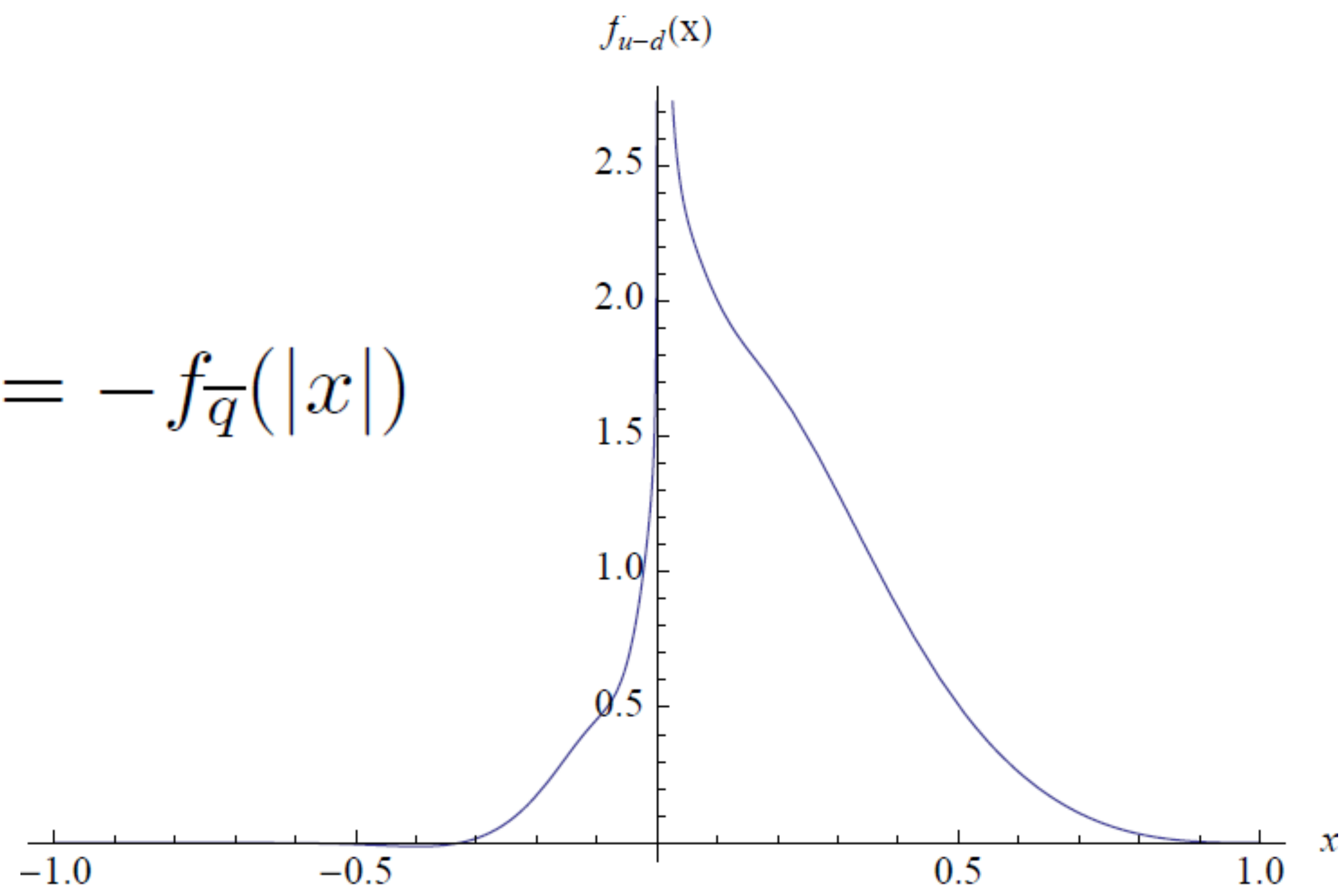
- LHS: trace, twist-4 $\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots$$

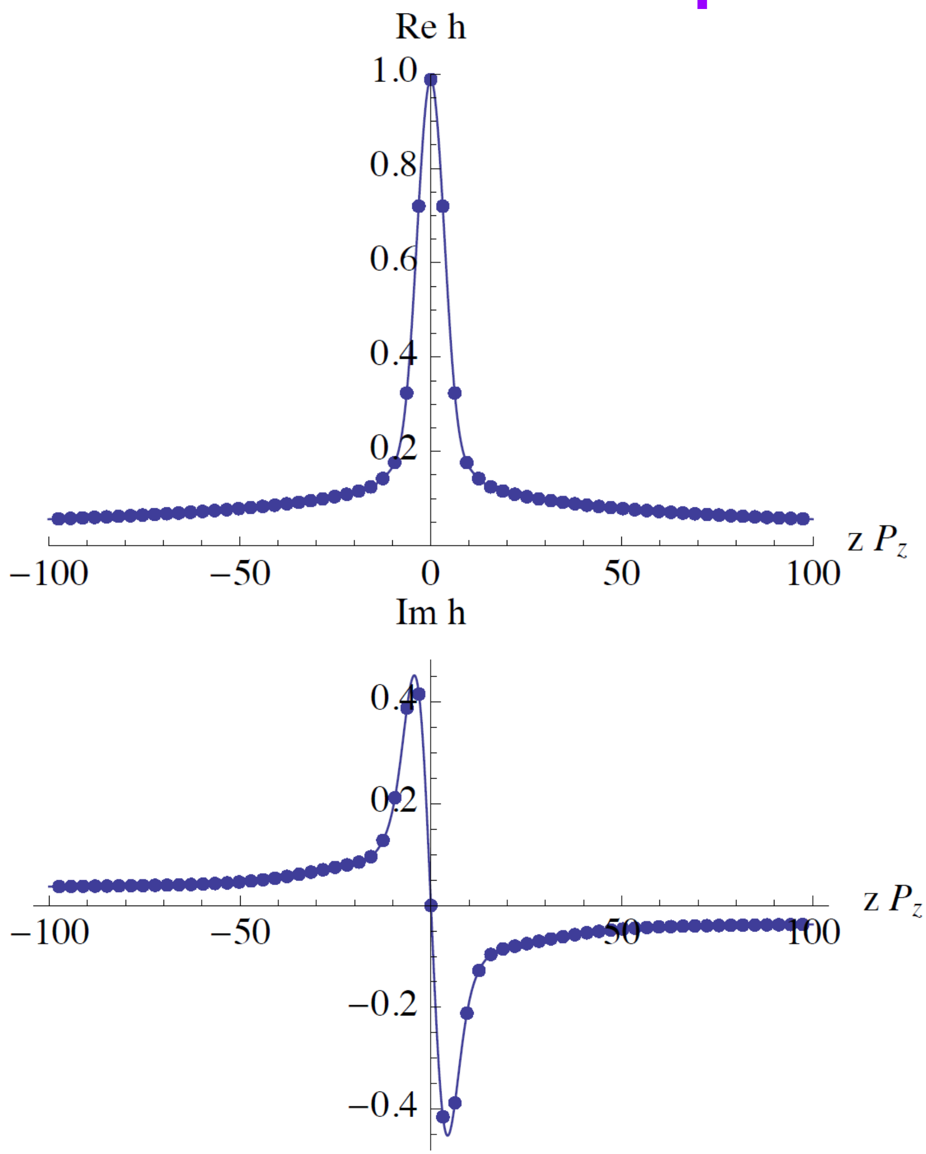
What do we expect to see on the lattice?

- Suppose LPDF is the CTEQ PDF at $P^z \to \infty$

$$f_q(-|x|) = -f_{\overline{q}}(|x|)$$



in the Fourier Space

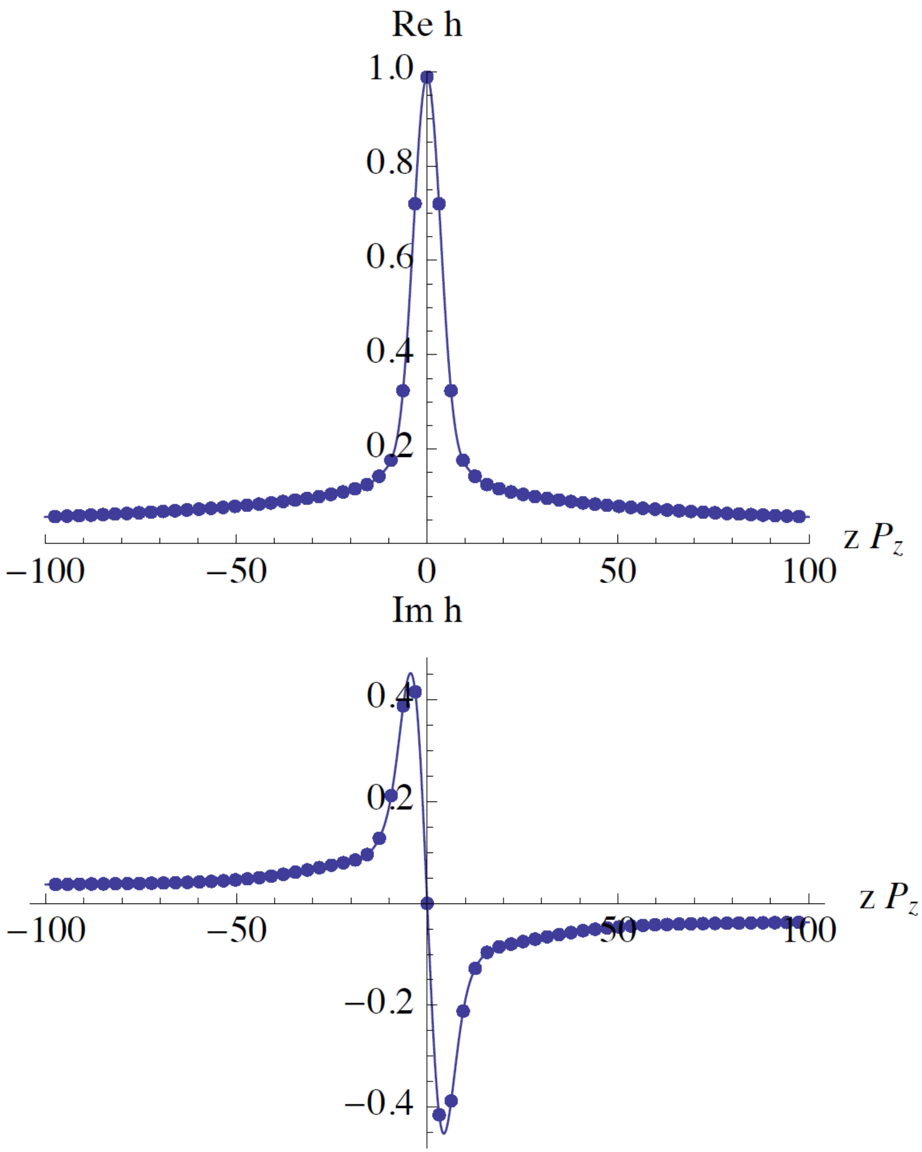


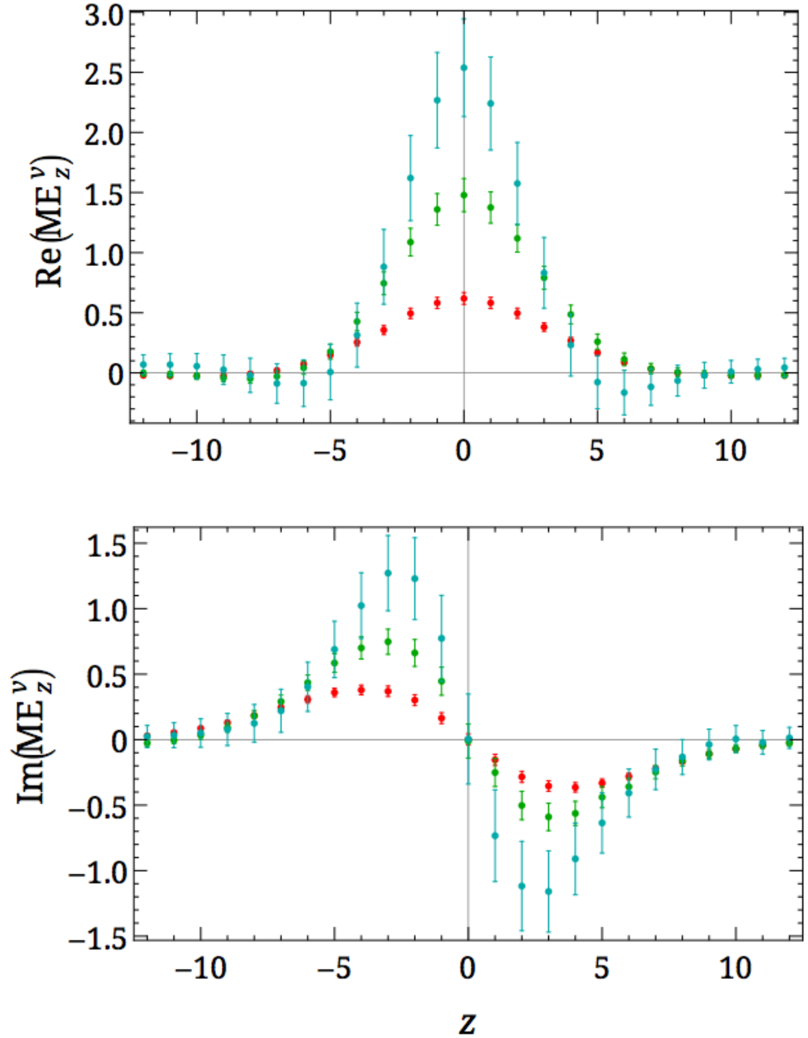
First (isovector) LPDF Computation

- Lattice: $24^3 \times 64$

  $a \approx 0.12 \text{ fm} \quad L \approx 3 \text{ fm}$
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

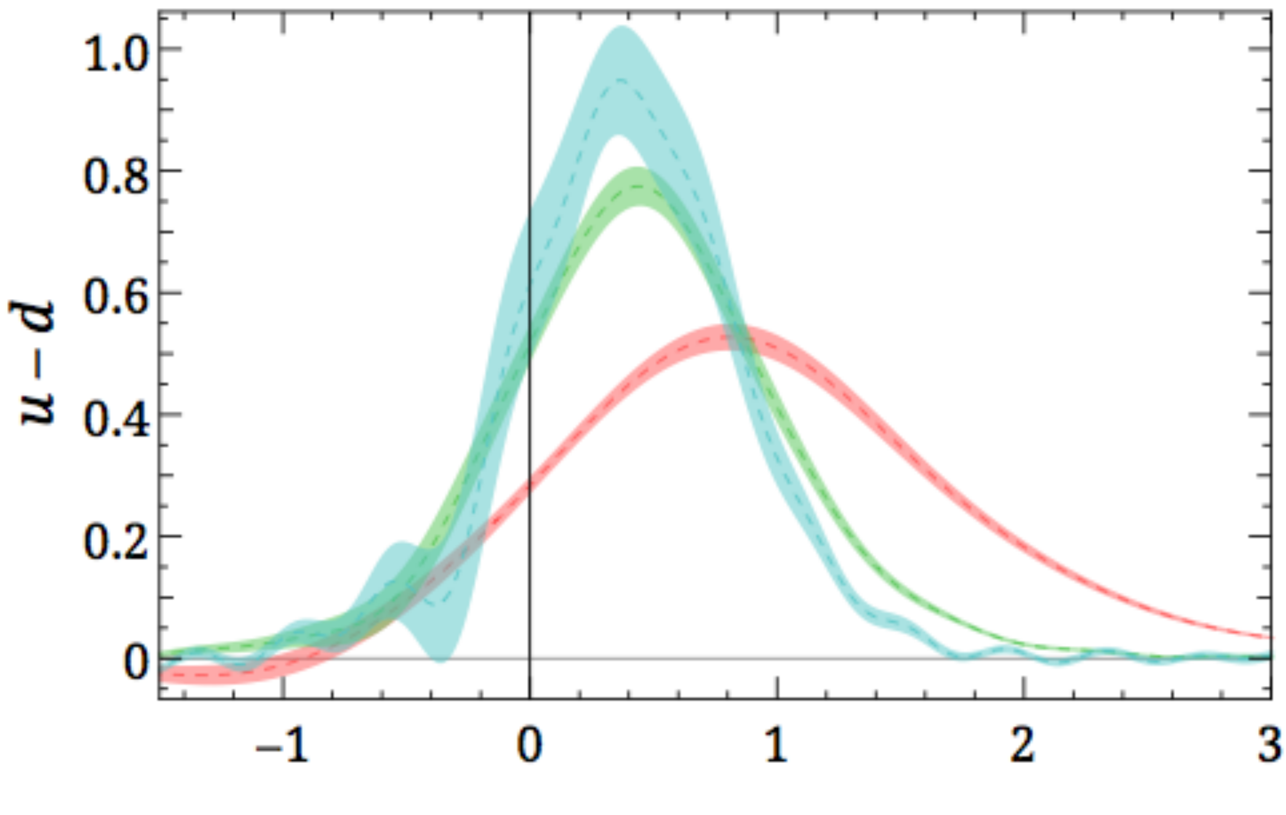
  $N_f = 2 + 1 + 1 \qquad M_\pi \approx 310 \text{ MeV}$
- Gauge fields/links: hypercubic (HYP) smearing, 461 config.
- $P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$ n = 1,2,3…





$$P^z = \frac{0.26n}{a},\ n = 1, 2, 3$$

Quasi-PDF (unpolarized)



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV \quad \text{n} = 1, 2, 3.$$

RG of Wilson Coefficient

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}, \frac{M_N^2}{P_z^2}\right) + \ldots .$$

Xiong, Ji, Zhang, Zhao

(GPD: Ji, Schfer, Xiong, Zhang; Xiong, Zhang)

factorization, Linear divergence , lattice PT

(Ma, Qiu; Ishikawa, Ma, Qiu, Yoshida; H.N. Li)

$\mathcal{O}(M^2/(P^z)^2)$ Corrections

$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

- Computed to all orders in $\mathcal{O}(M^2/(P^z)^2)$.

$$q(x) = \sqrt{1+4c} \sum_{n=0}^{\infty} \frac{f_-^n}{f_+^{n+1}} \left[(1+(-1)^n)\tilde{q}\Big(\frac{f_+^{n+1} x}{2f_-^n}\Big) + (1-(-1)^n)\tilde{q}\Big(\frac{-f_+^{n+1} x}{2f_-^n}\Big) \right]$$

$$f_\pm = \sqrt{1+4c} \pm 1 \qquad c = M^2/4P_z^2$$

$\mathcal{O}(\Lambda_{QCD}^2/(P^z)^2)$ Corrections

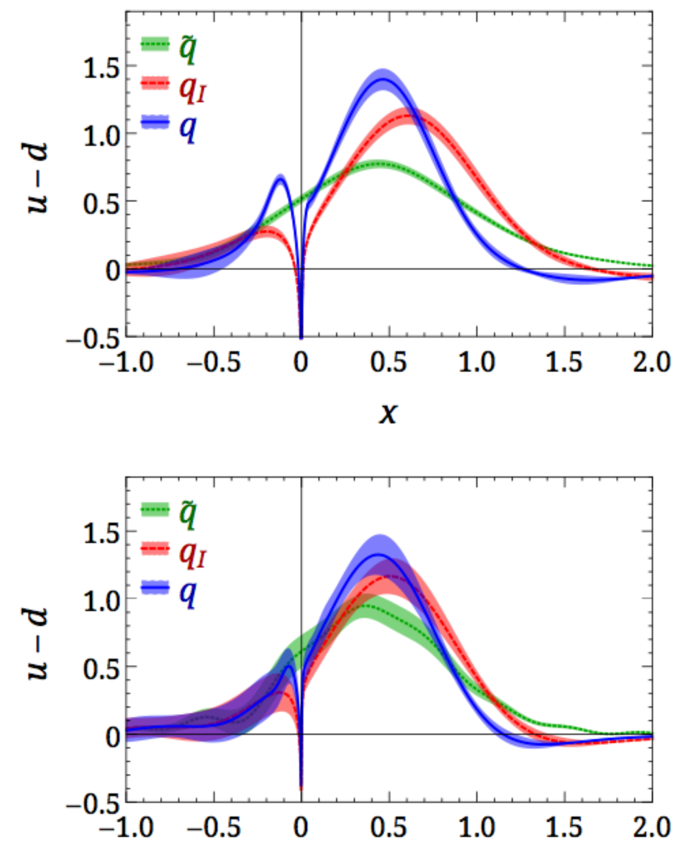
- Twist-4:

$$q_{tr}(x,\mu^2,P^z) = \frac{\lambda^2}{8\pi}\int_{-\infty}^{\infty} dz \int_0^1 \frac{dt}{t} e^{i\frac{zk^z}{t}} \left\langle P \left| \widetilde{\mathcal{O}}_{tr}(z) \right| P \right\rangle$$

$$\widetilde{\mathcal{O}}_{tr}(z) = \int_0^z dz_1 \bar{\psi}(0) \Big[\gamma^\nu \Gamma(0,z_1) D_\nu \Gamma(z_1,z) + \int_0^{z_1} dz_2 \lambda\cdot\gamma\Gamma(0,z_2) D^\nu \Gamma(z_2,z_1) D_\nu \Gamma(z_1,z) \Big] \psi(z\lambda)$$

⇨ Parameterized

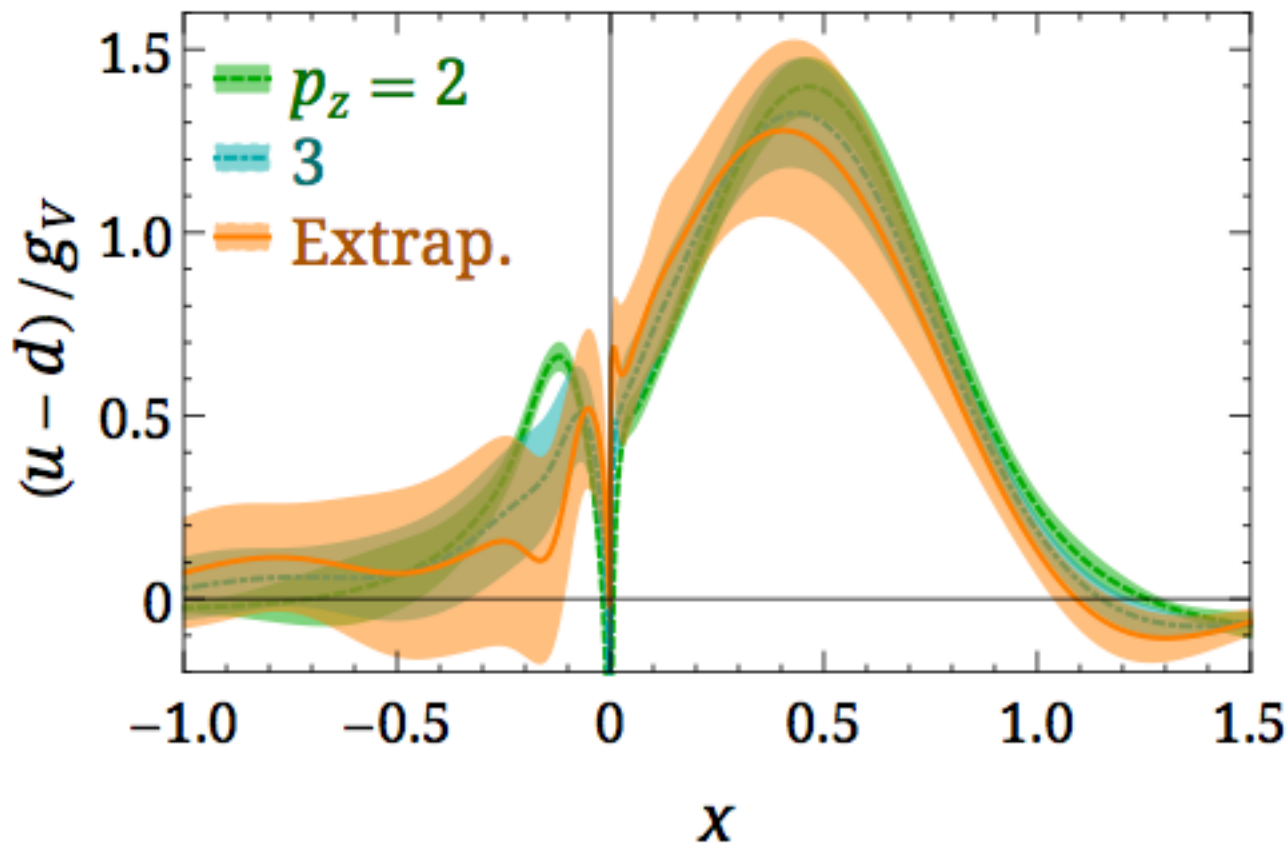
Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)

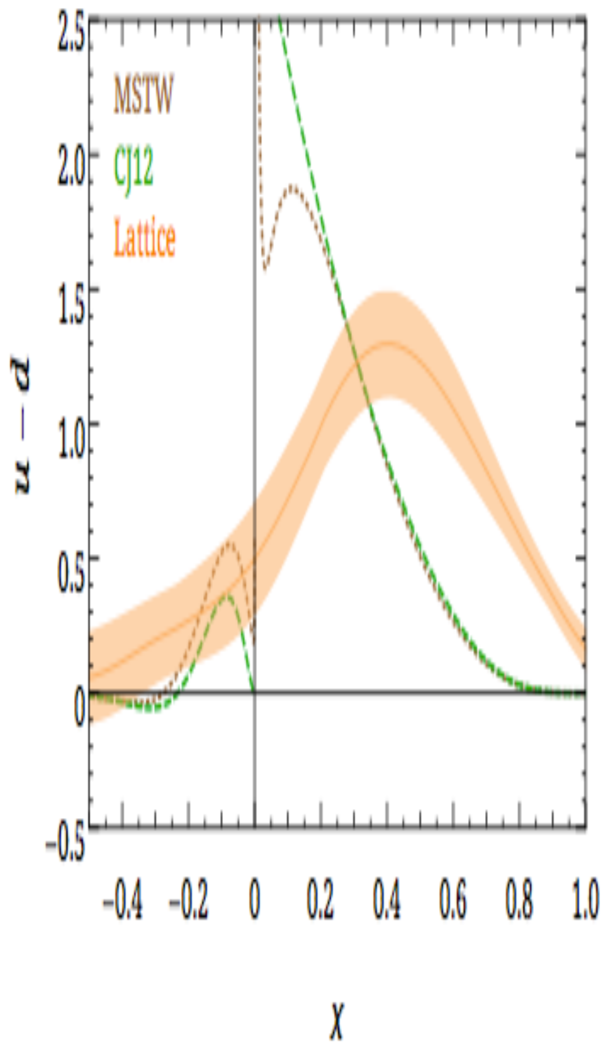


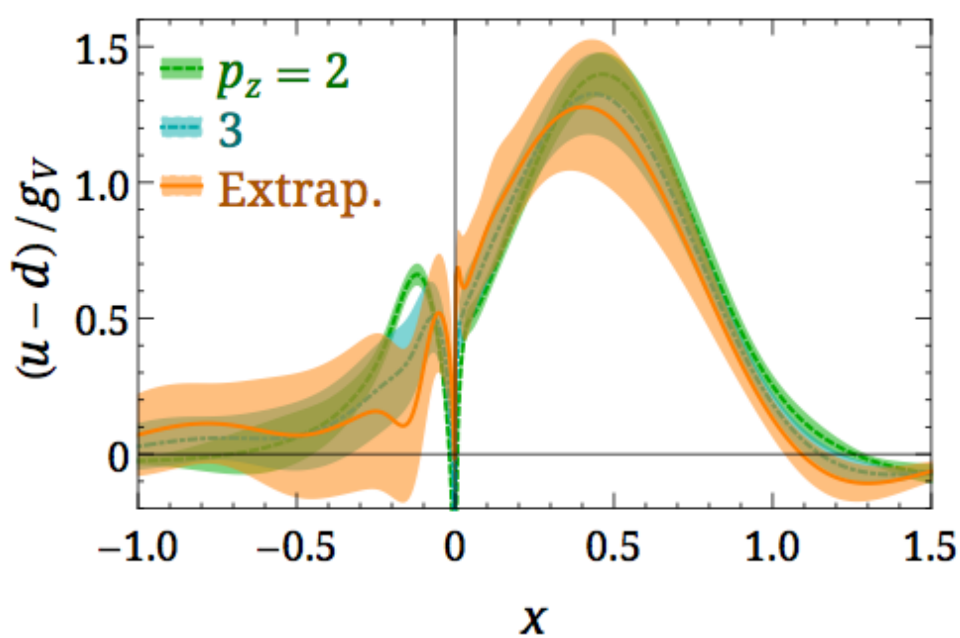
$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

n = 2 (upper) & 3

Unpolarized Isovector Proton PDF



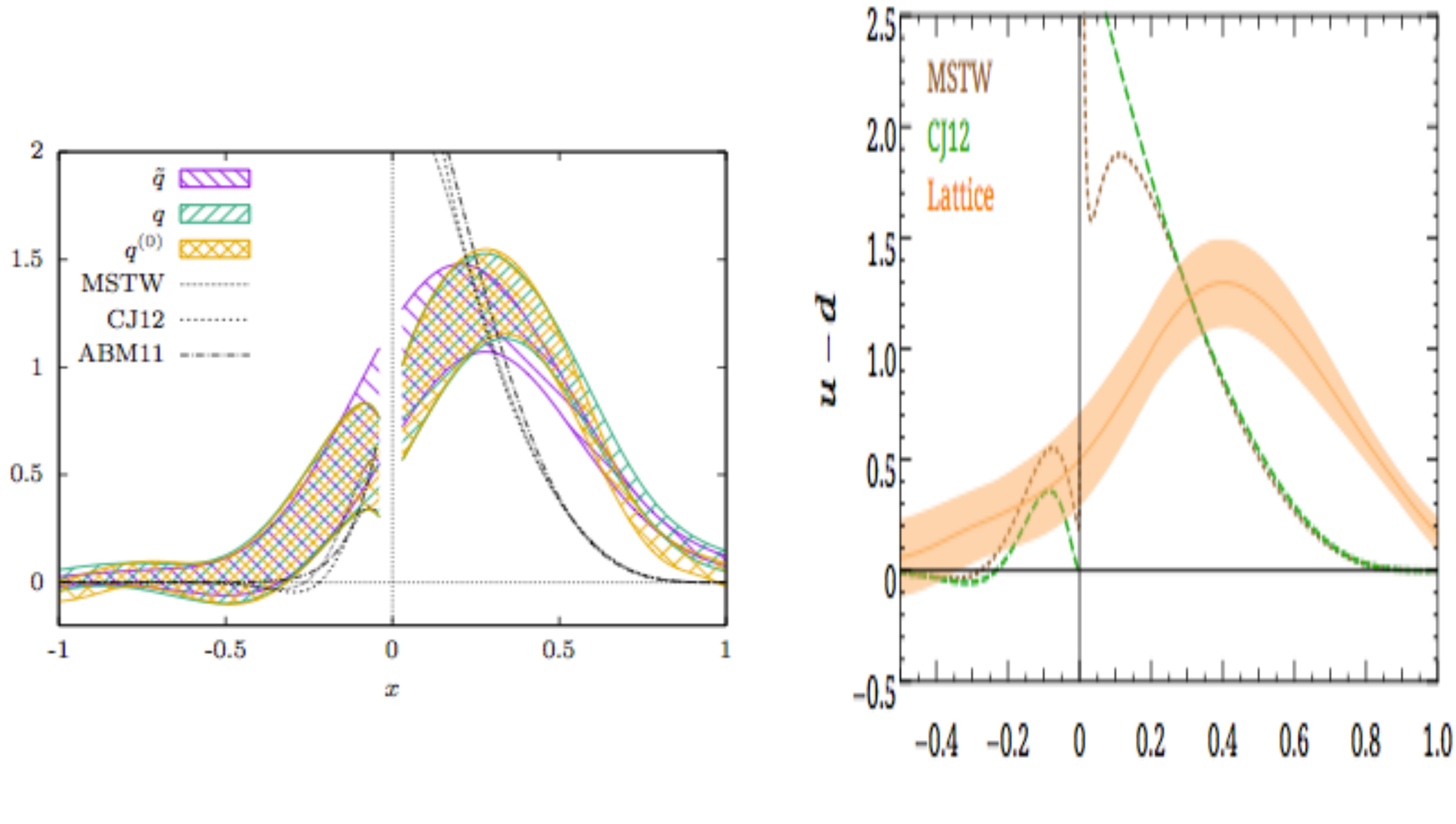




Quark mass effect!

A follow-up work

(Alexandrou et. al.1504.07455)



A follow-up work

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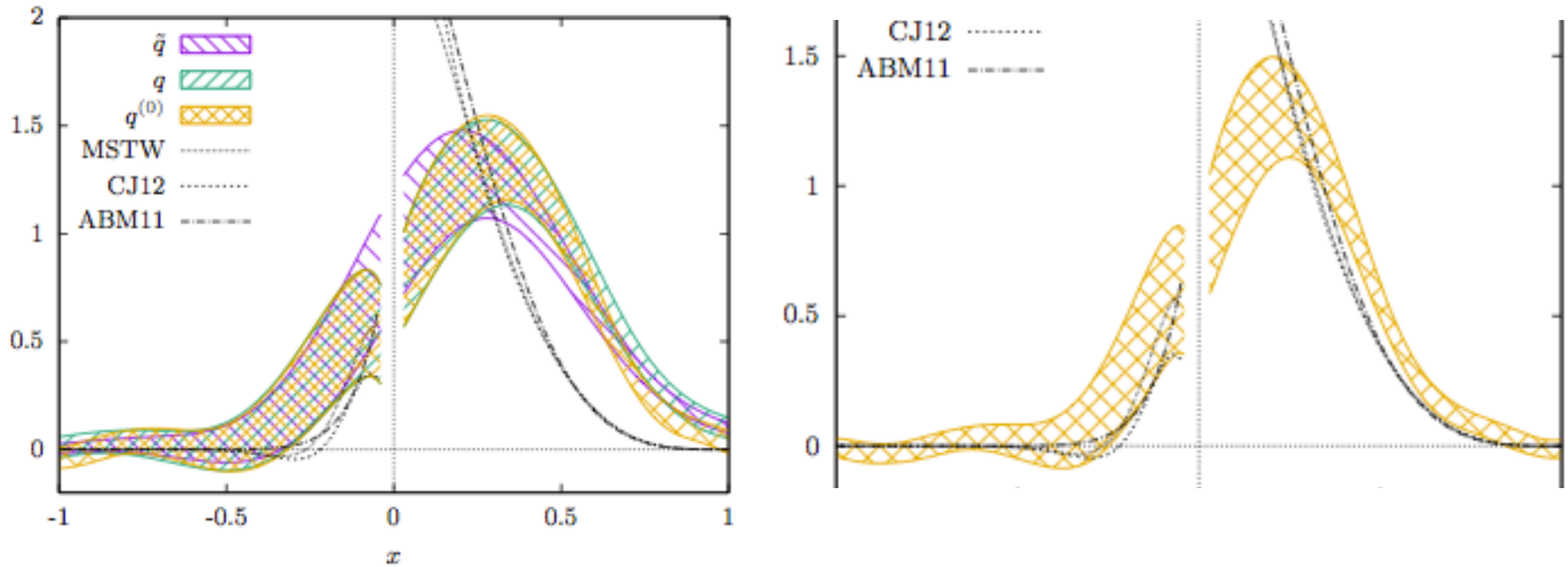
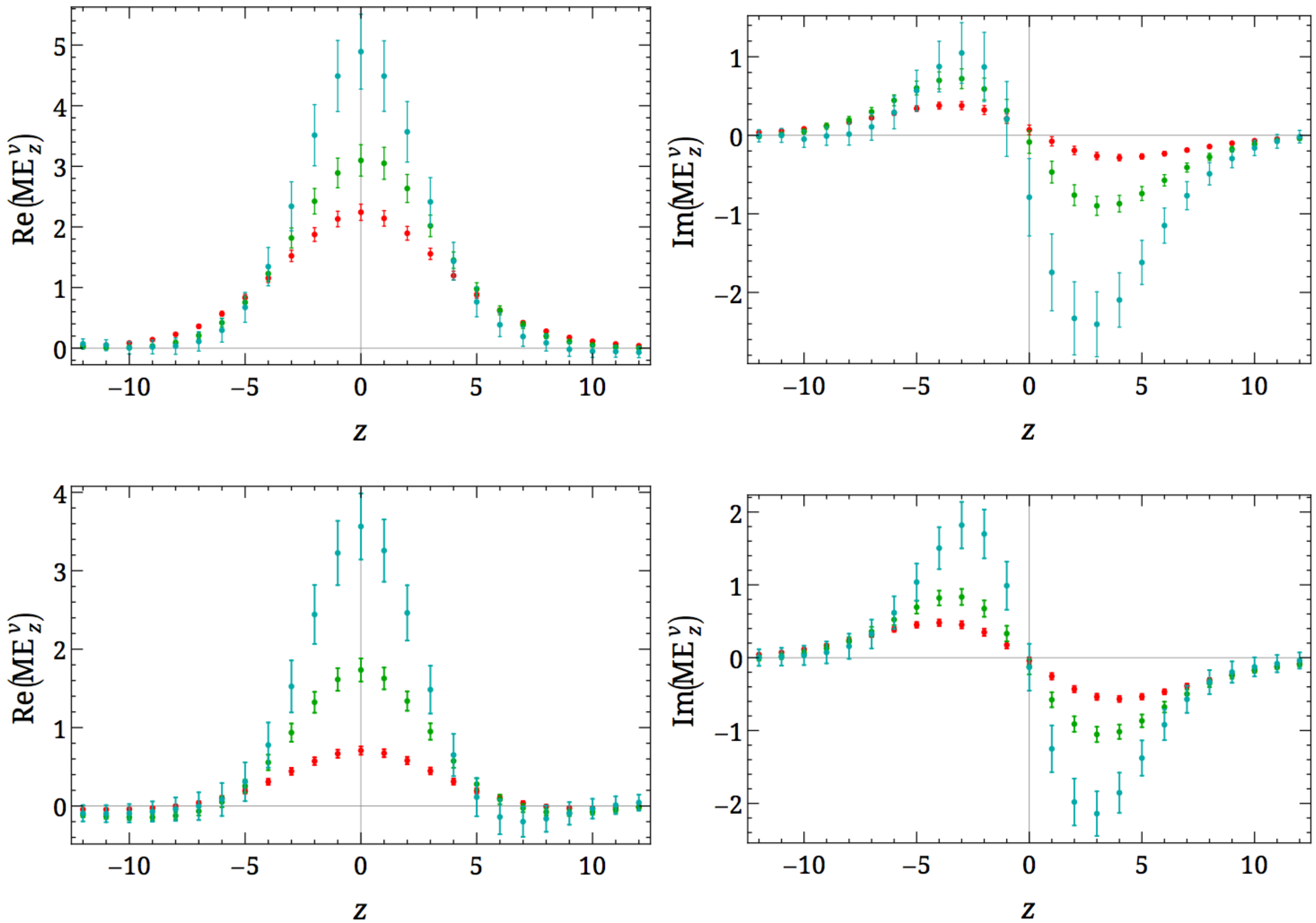
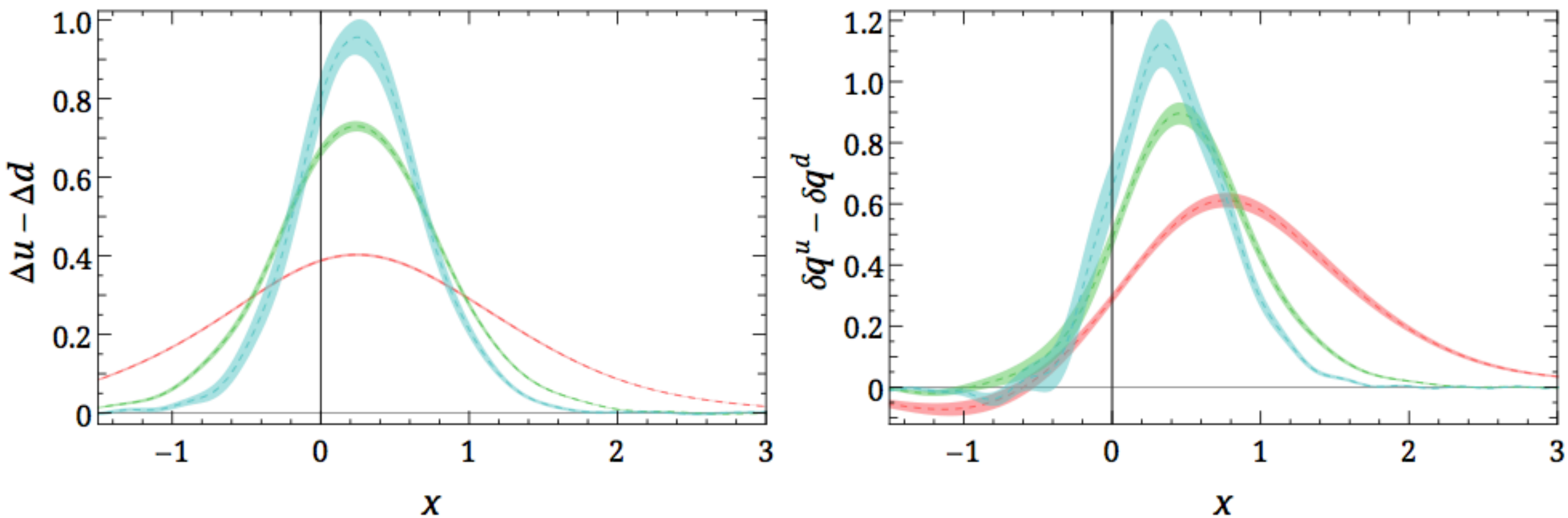


Helicity and Transversity (isovector)

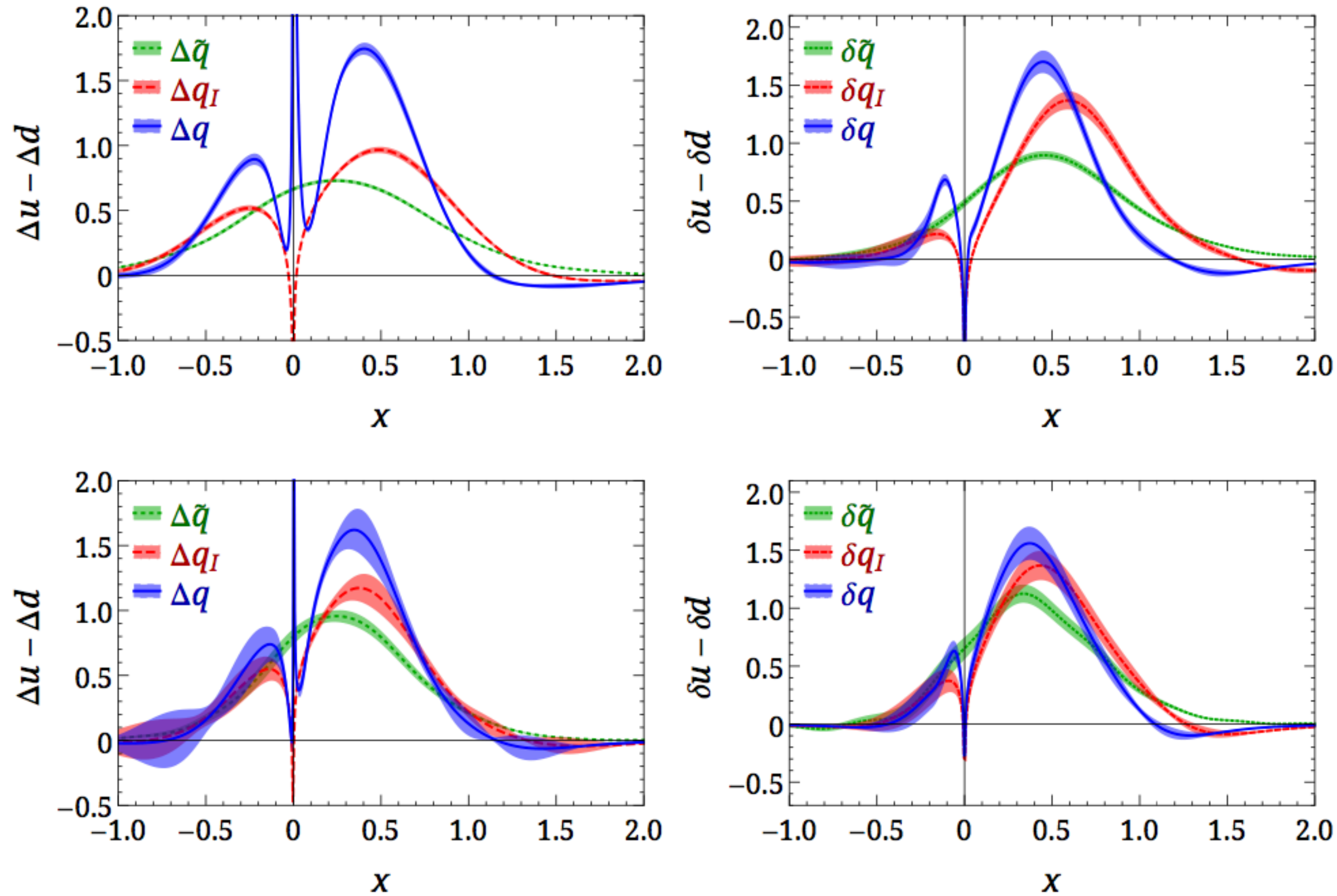


Quasi-PDF (Helicity and Transversity)



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV \quad \text{n = 1, 2, 3.}$$

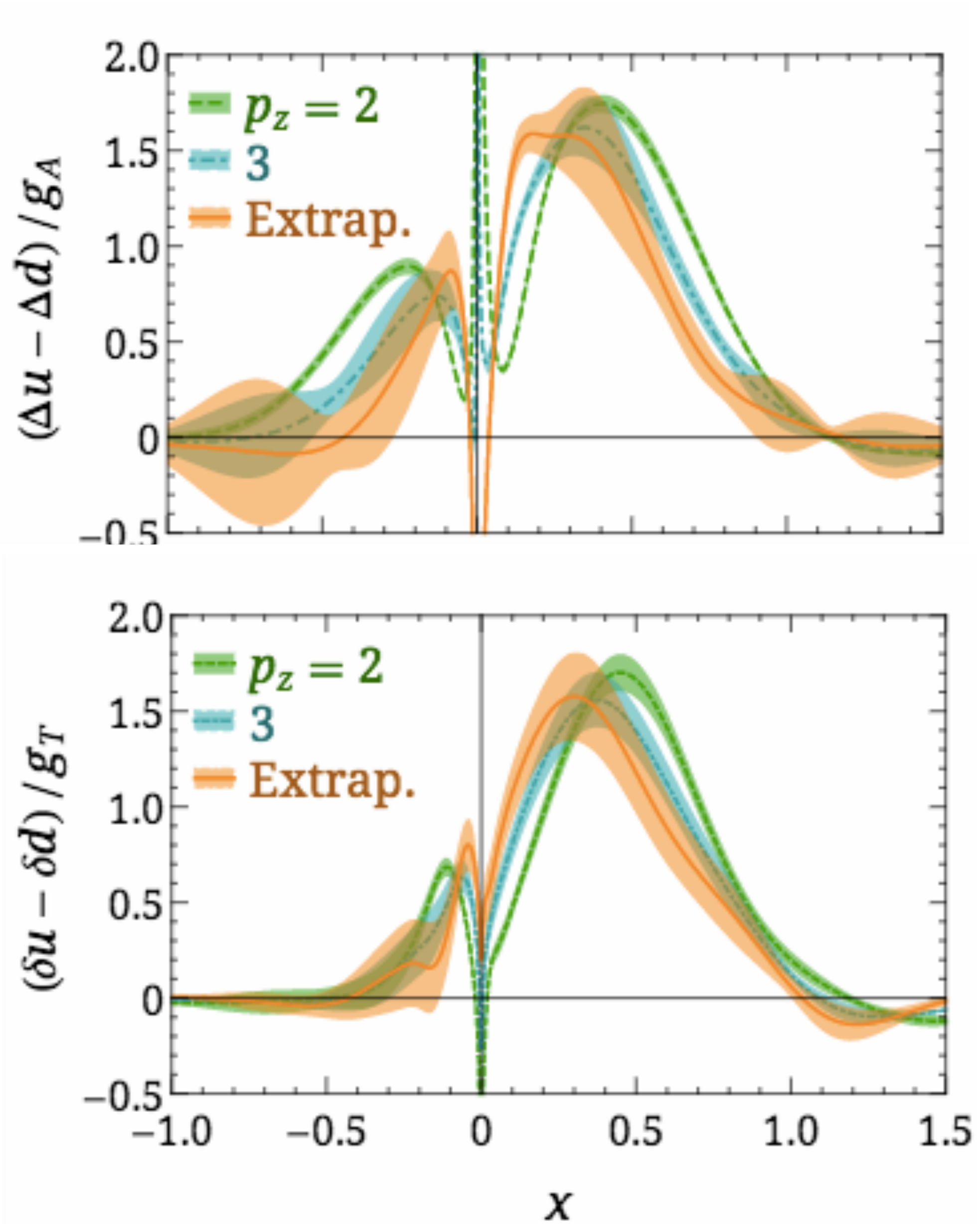
Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)

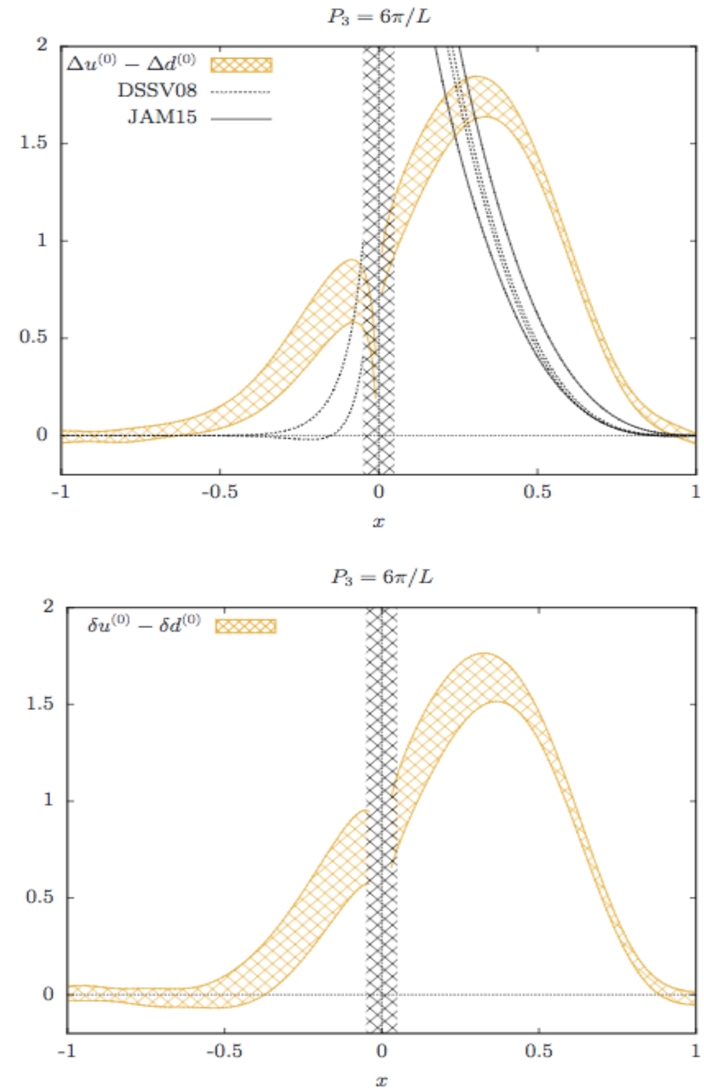


$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

n = 2 (upper) & 3

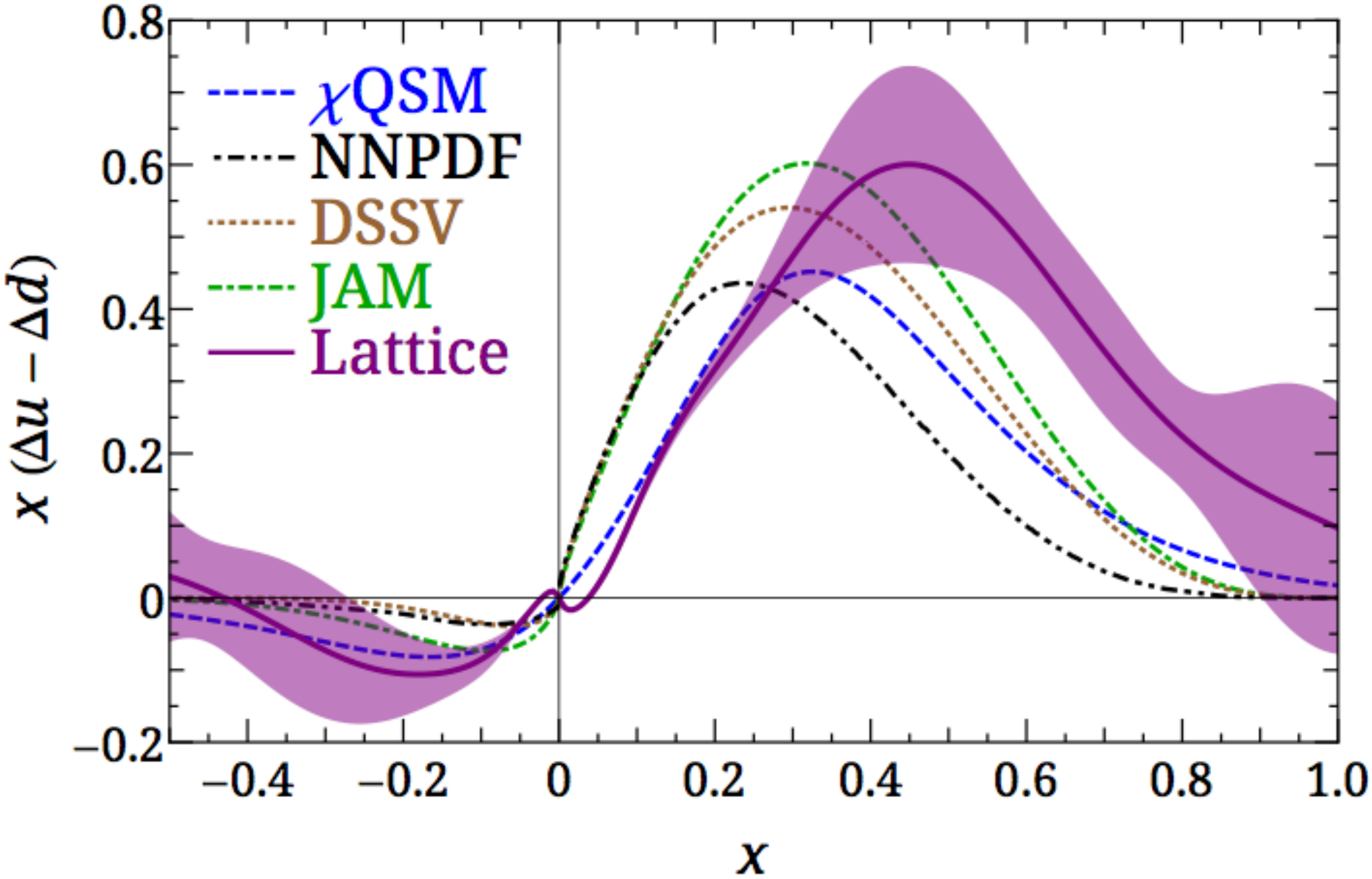
Isovector Proton Helicity and Transversity



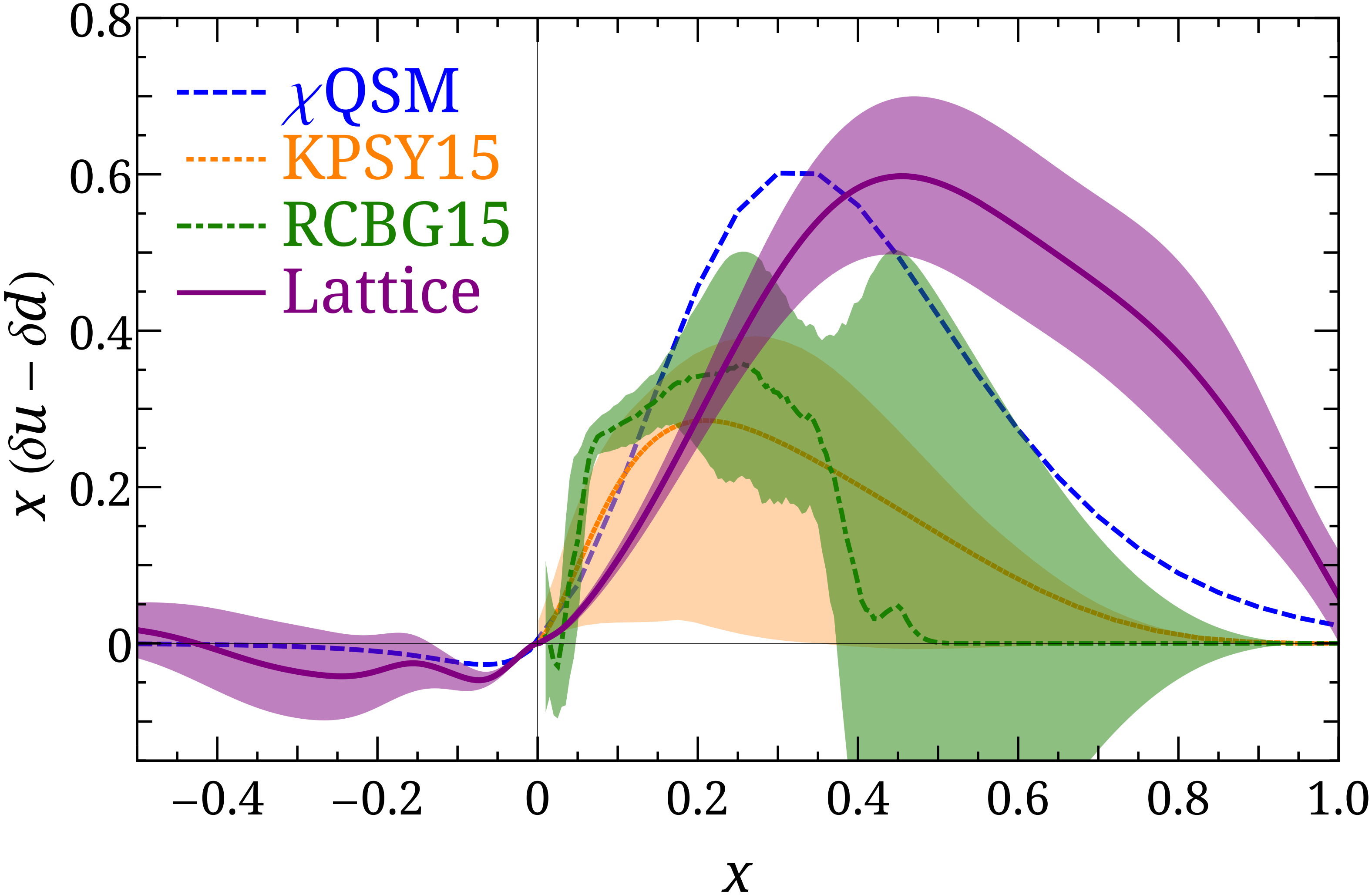


(Alexandrou et.al., 1609.00172)

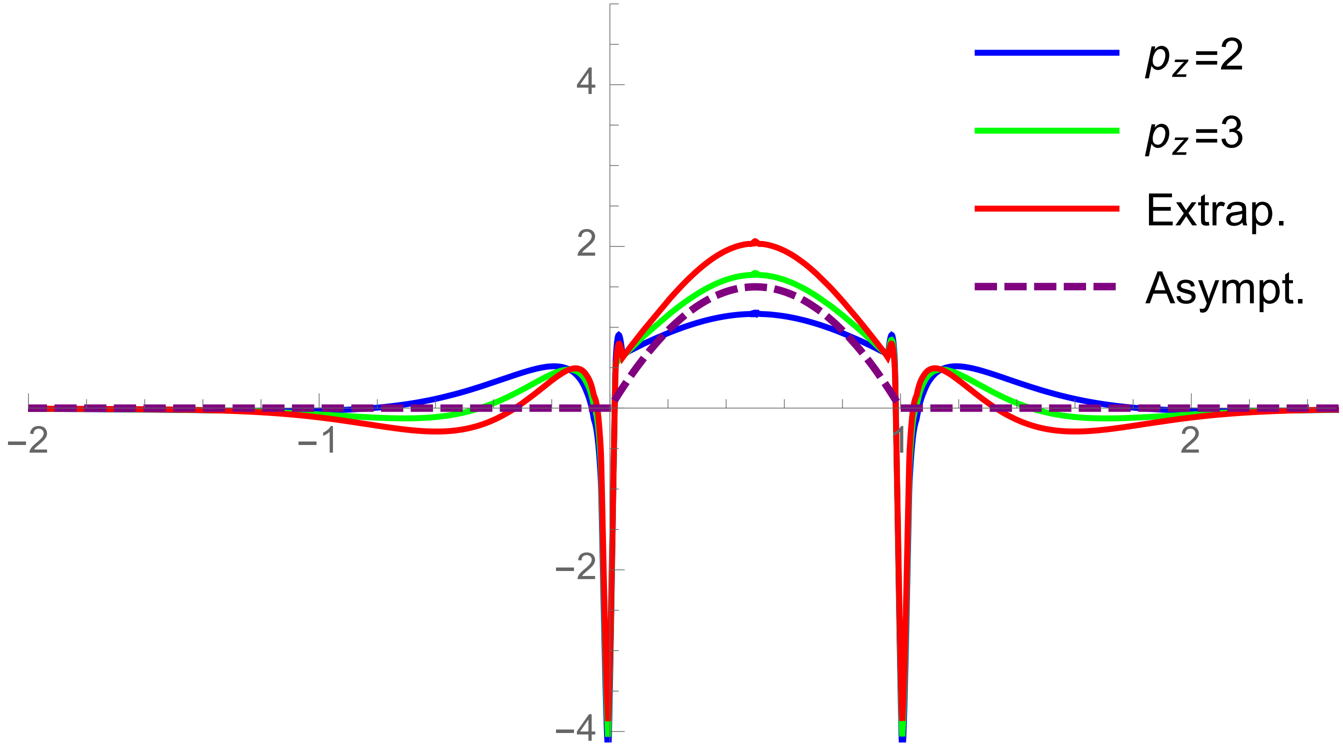
Isovector Proton Helicity



Isovector Proton Transversity



Pion Light Cone Wave Function
(preliminary)



$$\phi_\pi(x,\mu) + 3\phi_\eta(x,\mu) = 2[\phi_{K^+}(x,\mu) + \phi_{K^-}(x,\mu)]$$
$$= 2[\phi_{K^0}(x,\mu) + \phi_{\overline{K}^0}(x,\mu)],$$

JWC, Iain W. Stewart, Phys.Rev.Lett. 92 (2004) 202001

Outlook

- Wee partons, a good test to this approach---smaller quark mass
- Next: linear divergence in the matching kernel, lattice perturbation theory, proof of factorization

- If it works, lots of things to do: LCW, PDF, GPD, TMD …

Backup slides

Linear Divergence in Matching

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