

*SPIN 2016, September 26, 2016, Urbana-Champaign, USA*

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# Overview on gluon TMDs

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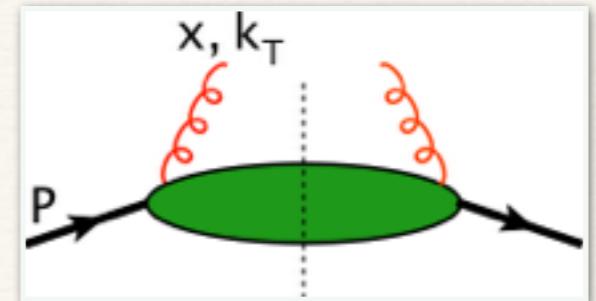
Marc Schlegel  
Institute for Theoretical Physics  
University of Tübingen

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**Gluon TMDs = Transverse Momentum Dependent Parton Distributions  
of gluons in nucleon**

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## TMD gluonic matrix element

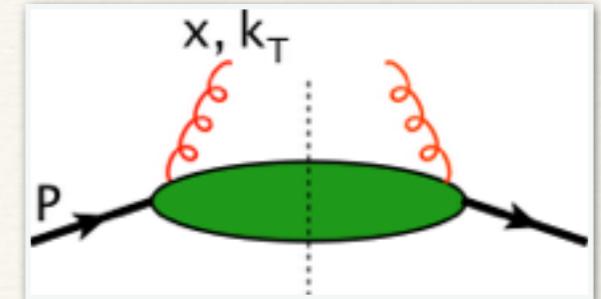


$$\Gamma^{\alpha\beta; [\mathcal{W}, \mathcal{W}']}(x, \mathbf{k}_T) = \int \frac{d\lambda d^2 z_T}{(2\pi)^3} e^{ix\lambda + i\mathbf{k}_T \cdot \mathbf{z}_T} \langle P, S | F^{n\alpha}(0) \mathcal{W} F^{n\beta}(\lambda n + z_T) \mathcal{W}' | P, S \rangle$$

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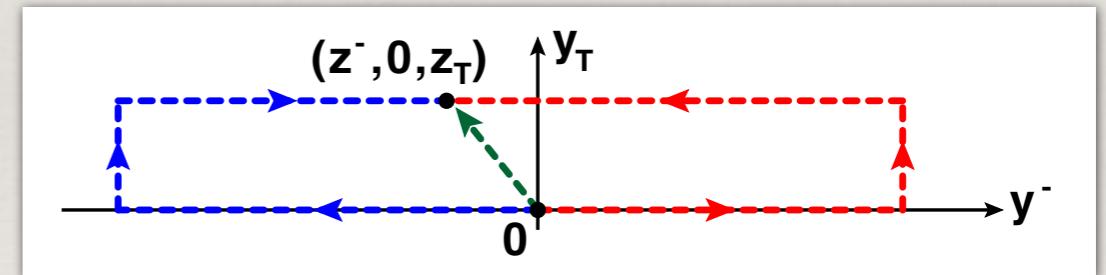


## Wilson line in fundamental representation

Color Gauge invariant definition of TMDs → Wilson line

$$\mathcal{W}[a, b] = \mathcal{P}e^{-ig \int_a^b ds \cdot A^a(s)} t^a$$

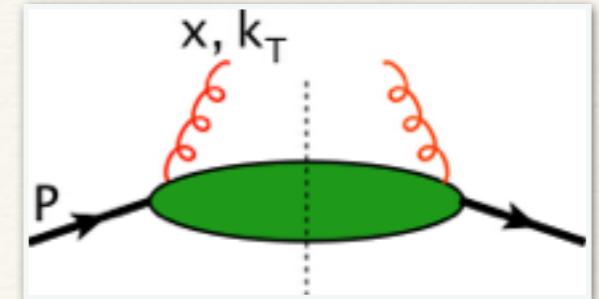
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nontrivial, process dependent:



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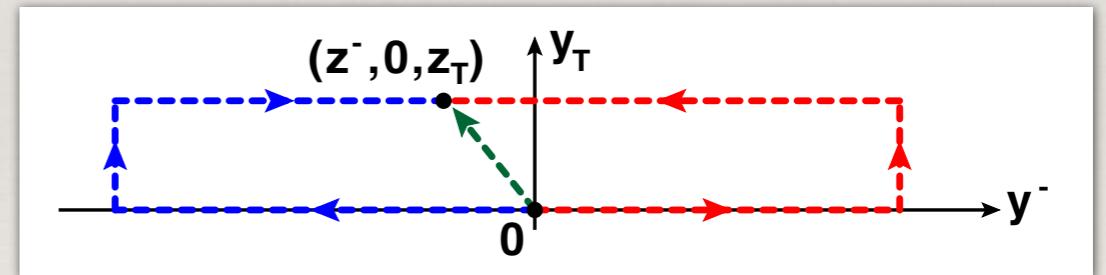


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past-pointing WL →  
Initial State Interactions:  
pp → color singlet + X (DY-type)

$$\Gamma^{\alpha\beta; [-, -]}$$

future-pointing WLs →  
Final State Interactions:  
ep → jets + X (SIDIS-type)

$$\Gamma^{\alpha\beta; [+, +]}$$

more complicated Wilson lines → Buffing, Mulders, Mukherjee, ...

# Proper Definition & Soft function

[Collins; Sun, Xiao Yuan; Aybat, Rogers; recent work: Echevarria, Kasemats, Mulders, Pisano, JHEP 07, 158 (2015)]

Inclusion of Soft Function  $\implies$

$$S(z_T) = \frac{\text{Tr}_c}{N_c} \langle 0 | \mathcal{W}_n^\dagger(-z_T/2) \mathcal{W}_{\bar{n}}(-z_T/2) \mathcal{W}_{\bar{n}}^\dagger(z_T/2) \mathcal{W}_n(z_T/2) | 0 \rangle$$

$$\tilde{\Gamma}^{\alpha\beta}(x, \mathbf{b}_T) \rightarrow \frac{\tilde{\Gamma}^{\alpha\beta}(x, \mathbf{b}_T)}{\sqrt{S(\mathbf{b}_T)}}$$

Modification needed in order to have:

- 1) Renormalizable Matrix Element
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## Parameterization

$\Gamma^{[T-even]}(x, \vec{k}_T)$		$\Gamma^{[T-odd]}(x, \vec{k}_T)$	
	<b>flip</b>		<b>flip</b>
<b>U</b>	$f_1^g$	$h_1^{\perp g}$	
<b>L</b>	$g_{1L}^{\perp g}$		$h_{1L}^{\perp g}$
<b>T</b>	$g_{1T}^{\perp g}$	$f_{1T}^{\perp g}$	$h_1^g \ h_{1T}^{\perp g}$

\* unpolarized & linearly polarized gluons :  
helicity flip TMDs  $\rightarrow$  azimuthal modulations

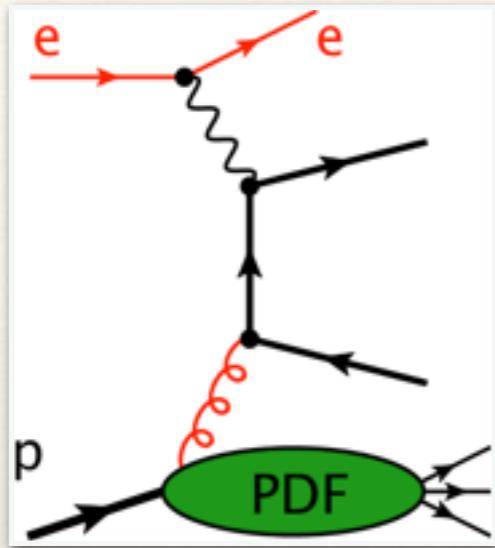
$$\Gamma^{\alpha\beta}(x, k_T) = \frac{1}{2x} \left[ -g_T^{\alpha\beta} f_1^g(x, k_T^2) + \frac{k_T^\alpha k_T^\beta - \frac{1}{2} k_T^2 g_T^{\alpha\beta}}{M^2} h_1^{\perp g}(x, k_T^2) \right]$$

\* unpolarized gluons in transversely pol.  
proton: gluon Sivers function (T-odd)

$$\Gamma_T^{\alpha\beta}(x, k_T) = \frac{1}{2x} \left[ g_T^{\alpha\beta} \frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp g}(x, k_T^2) + \dots \right]$$

# Processes sensitive to gluon TMDs

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## Heavy Quark production in $e + p -$ collisions

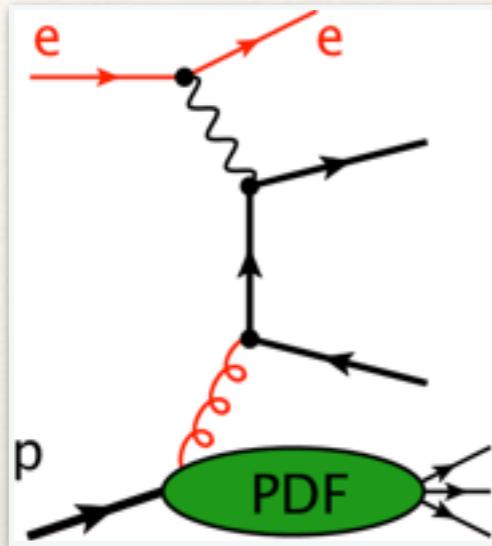
[Boer, Brodsky, Mulders, Pisano, PRL 2011]

$$e + p^{(\uparrow)} \rightarrow e' + \text{jet}(c, b) + \text{jet}(\bar{c}, \bar{b}) + X$$

TMD factorization ok!

Spin dependent (+independent...) gluon TMDs: EIC would be ideal!

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unpolarized cross section:

$$\frac{d\sigma_{UU}}{dq_T} \propto (F_1 + F_2 \cos(2\phi))$$

azimuthally independent term:

$$F_1 \propto f_1^g(x, q_T)$$

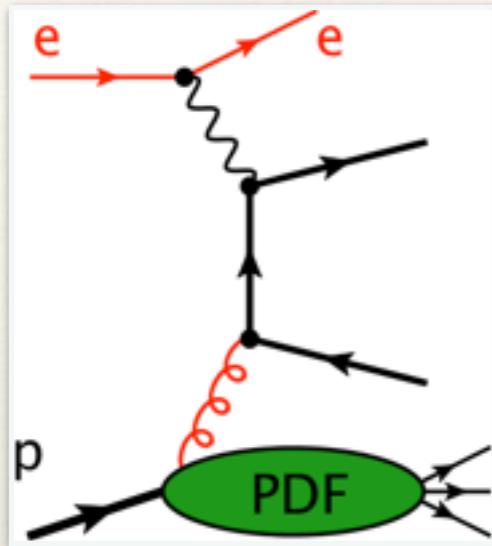
→ unpol. gluon distribution

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→ linearly pol. gluons

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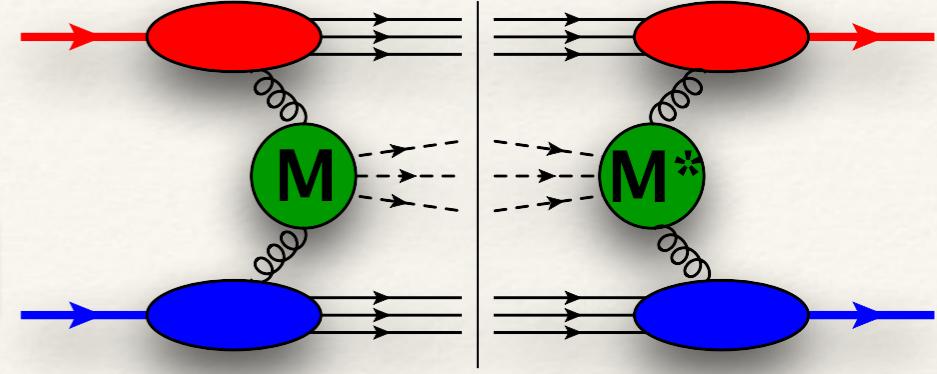
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gluon TMDs from (pol.)  $e + p \rightarrow$  more talks by van Daal, Silva, Sangem, ...

## General TMD expression for gluon fusion:

$$\frac{d\sigma}{d^4q...}(q_T \ll Q) \propto \mathcal{C}[\Gamma^{\alpha\alpha'}(x_a, k_{aT}) \Gamma^{\beta\beta'}(x_b, k_{bT})] (\mathcal{M}_{\alpha\beta} (\mathcal{M}_{\alpha'\beta'})^*)$$



$$\tilde{C}[w \ f \ g] \equiv \int d^2k_{aT} \int d^2k_{bT} \delta^{(2)}(k_{aT} + k_{bT} - q_T) w(k_{aT}, k_{bT}) f(x_a, k_{aT}^2) g(x_b, k_{bT}^2)$$

Calculation of partonic amplitudes

- convenient to use gluon polarization vectors
- helicity amplitudes

$$\varepsilon_\lambda^\mu(k_a) = (0, 1, i\lambda, 0)/\sqrt{2}$$

$$\delta_T^{\mu\nu} = \sum_\lambda \varepsilon_\lambda^\mu(k_{a/b})(\varepsilon_\lambda^\nu)^*(k_{a/b})$$

## Helicity correlator:

$$\Gamma_{\lambda_1\lambda_2}(x, \vec{k}_T) \equiv \Gamma^{ij}(x, \vec{k}_T) (\varepsilon_{\lambda_1}^i(k))^* (\varepsilon_{\lambda_2}^j(k))^* =$$

$$\frac{1}{2}(\delta_{\lambda_1,\lambda_2} f_1^g(x, \vec{k}_T^2) + \delta_{\lambda_1,-\lambda_2} \frac{(k_x - i\lambda_1 k_y)^2}{2M^2} h_1^{\perp g}(x, \vec{k}_T^2))$$

helicity non-flip

helicity flip

# General gluon fusion - structures

General TMD cross section in the helicity formalism

$$\frac{d\sigma}{d^4q_{...}}(q_T \ll Q) \propto \mathcal{C}[\Gamma_{\lambda_a \lambda_{a'}}(x_a, k_{aT}) \Gamma_{\lambda_b \lambda_{b'}}(x_b, k_{bT})] (\mathcal{M}_{\lambda_a \lambda_b} (\mathcal{M}_{\lambda_{a'} \lambda_{b'}})^*)$$

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Decomposition into four structures

$$\mathcal{C}[f_1^g f_1^g] (\mathcal{M}_{\lambda_a \lambda_b} (\mathcal{M}_{\lambda_a \lambda_b})^*) \rightarrow F_1(Q, \Omega, \dots) \rightarrow \text{helicity non-flip, } \phi \text{- independent, survives } q_T\text{-integration}$$

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$$\mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] (\mathcal{M}_{\lambda_a, \lambda_a} (\mathcal{M}_{-\lambda_a, -\lambda_a})^*) \rightarrow F_2(Q, \Omega, \dots) \rightarrow \text{double helicity flip, } \phi \text{- independent}$$

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$$\mathcal{C}[w_{3a} h_1^{\perp g} f_1^g] (\mathcal{M}_{\lambda_a, \lambda_b} (\mathcal{M}_{-\lambda_a, \lambda_b})^*) + \{a \leftrightarrow b\} \rightarrow F_{3,a}(Q, \Omega, \dots) \rightarrow \text{single helicity flip, } \cos(2\phi) / \sin(2\phi) \text{- mode}$$

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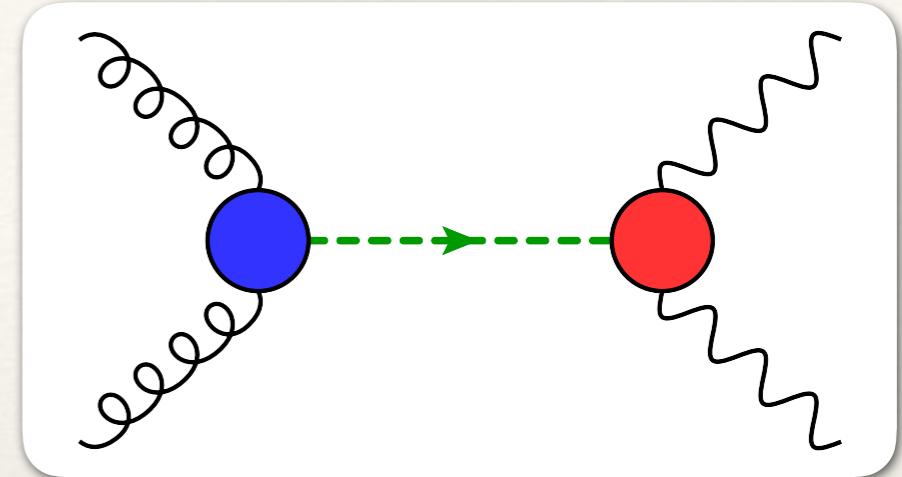
$$\frac{d\sigma^{gg}}{d^4q \ d\Omega} \Big|_{q_T \ll Q} = \hat{F}_1 [f_1^g \otimes f_1^g] + \hat{F}_2 [h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi) \hat{F}_3 [h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi) \hat{F}_4 [h_1^{\perp g} \otimes h_1^{\perp g}]$$

# Linearly polarized gluons as analysis tool

Example: Higgs spin / parity in the  $\gamma\gamma$  - decay channel

[Boer, den Dunnen, Pisano, M.S., Vogelsang; PRL 2012, 2013]

most general parameterization of the  
 $gg \rightarrow X_{0/2}, X_{0/2} \rightarrow \gamma\gamma$  vertices  
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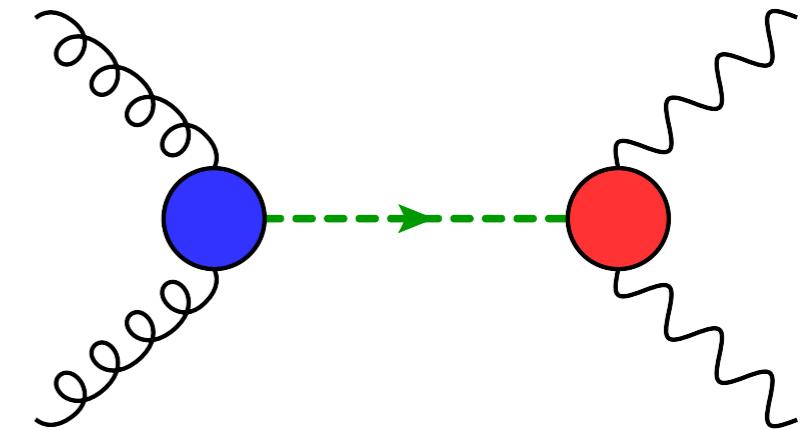


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Spin 0:

$$V_{X_0 \rightarrow V^\mu(q_1)V^\nu(q_2)} = a_1 q^2 g^{\mu\nu} + a_3 \epsilon^{q_1 q_2 \mu\nu}$$

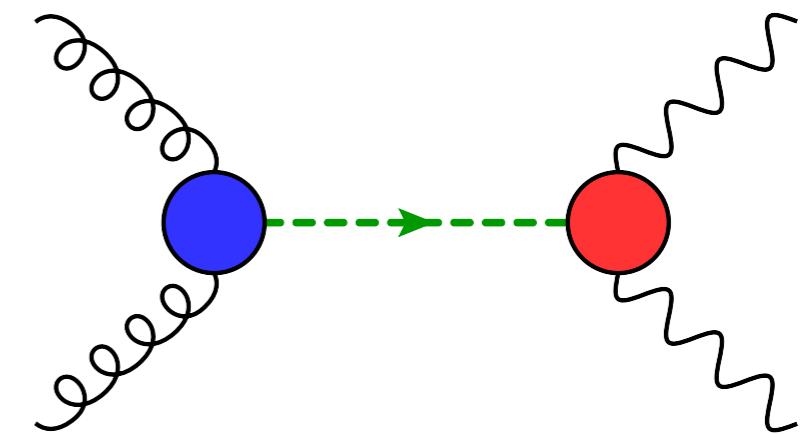
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Spin 2:

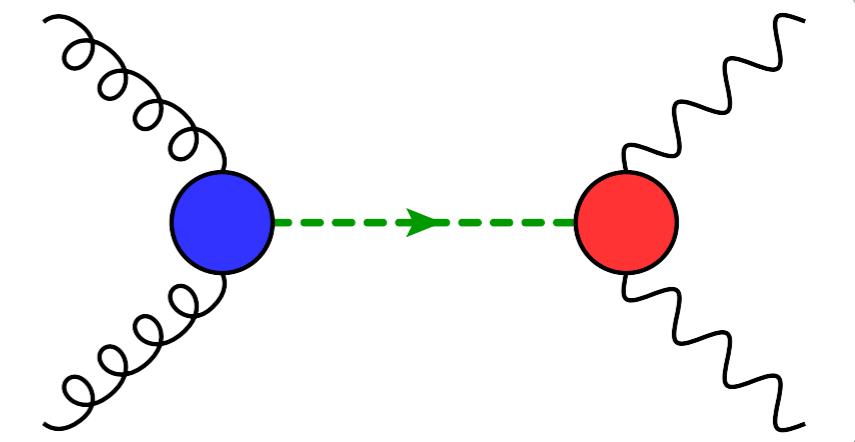
$$\begin{aligned} V_{X_2^{\alpha\beta} \rightarrow V^\mu(q_1)V^\nu(q_2)} &= c_1 q^2 g^{\mu\alpha} g^{\mu\beta} + c_2 g^{\mu\nu} (q_1 - q_2)^\alpha (q_1 - q_2)^\beta \\ &+ c_5 \epsilon^{q_1 q_2 \mu\nu} \frac{1}{Q^2} (q_1 - q_2)^\alpha (q_1 - q_2)^\beta \end{aligned}$$

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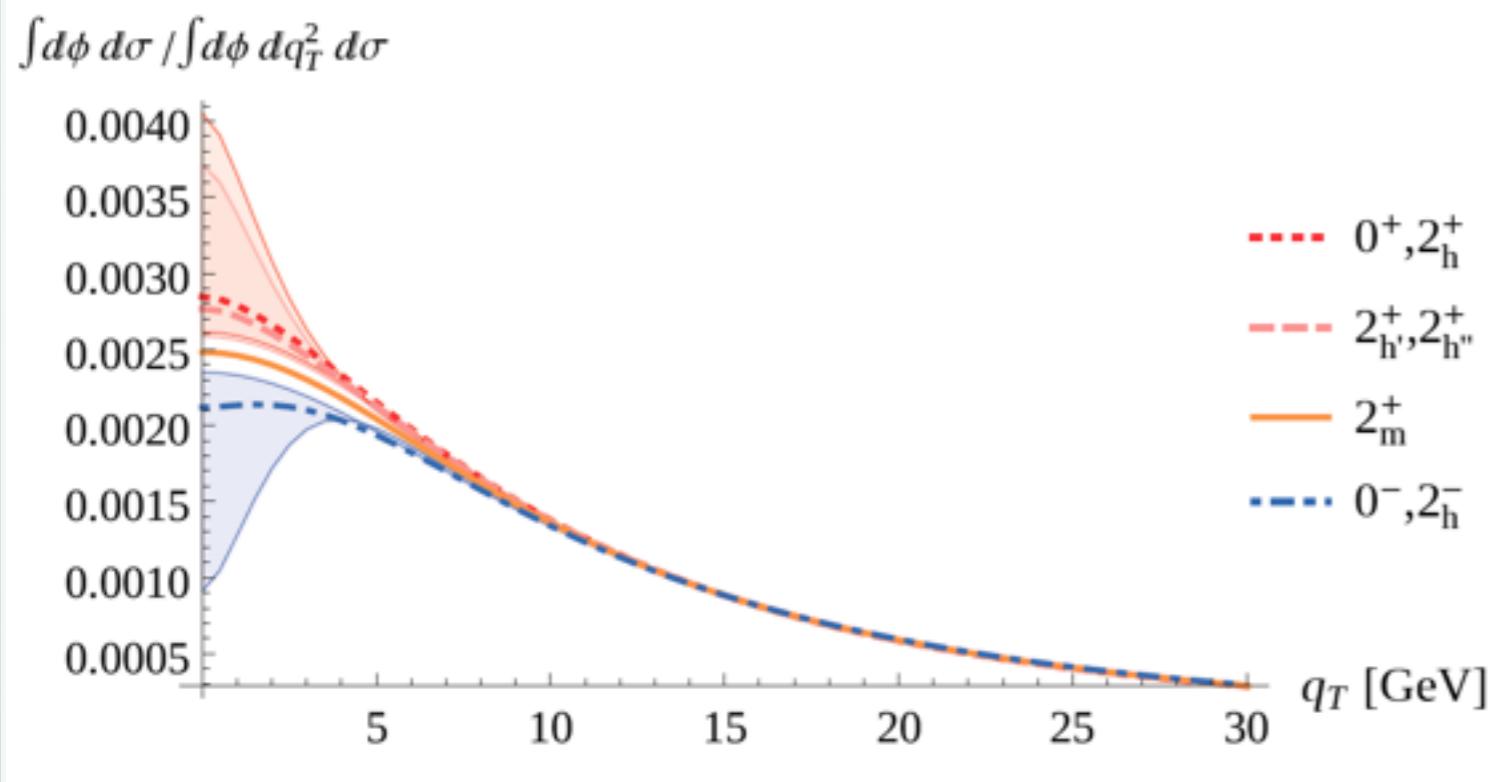
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benchmark scenarios

scenario	0 <sup>+</sup>	0 <sup>-</sup>	2 <sub>m</sub> <sup>+</sup>	2 <sub>h</sub> <sup>+</sup>	2 <sub>h'</sub> <sup>+</sup>	2 <sub>h''</sub> <sup>+</sup>	2 <sub>h</sub> <sup>-</sup>
$a_1$	1	0	-	-	-	-	-
$a_3$	0	1	-	-	-	-	-
$c_1$	-	-	1	0	1	1	0
$c_2$	-	-	$-\frac{1}{4}$	1	1	$-\frac{3}{2}$	0
$c_5$	-	-	0	0	0	0	1

azimuthal modulations due to  
 linear polarized gluons!  
 $\sin(2\phi) \rightarrow$  signal for CP-violation

# Parity from $q_T$ -dependence

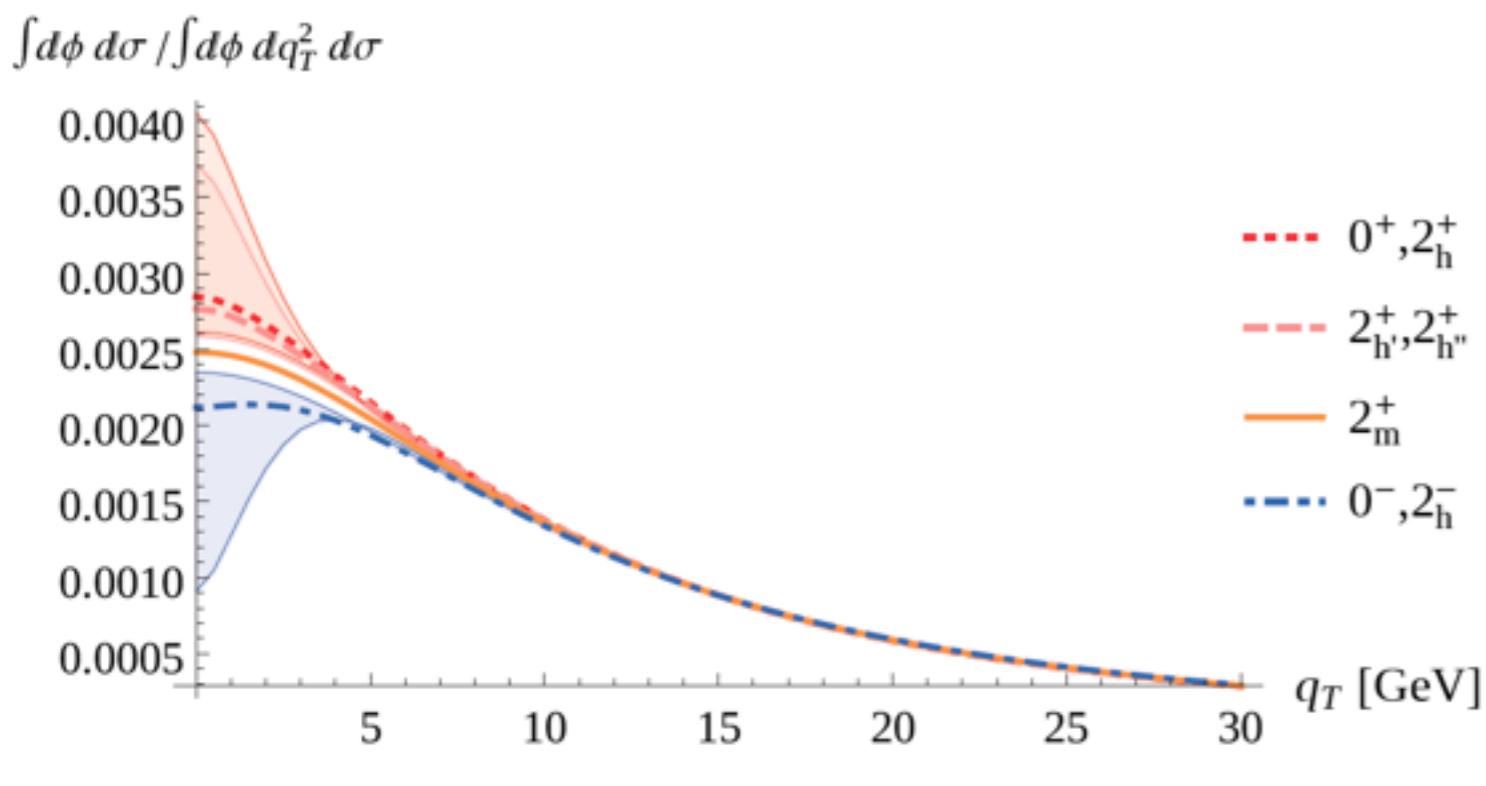


$$\frac{d\sigma}{d^2 q_T} \propto F_1 \ C[f_1^g \ f_1^g] \pm F_2 \ C[w_2 \ h_1^{\perp g} \ h_1^{\perp g}]$$

P-even: increase of at low  $q_T$ ,  
P-odd: decrease (also for Spin-2)

Evolution:  
effect only  $\sim 1\%$  at  $Q = m_H$   
[Boer, den Dunnen, NPB 2014]

# Parity from $q_T$ -dependence

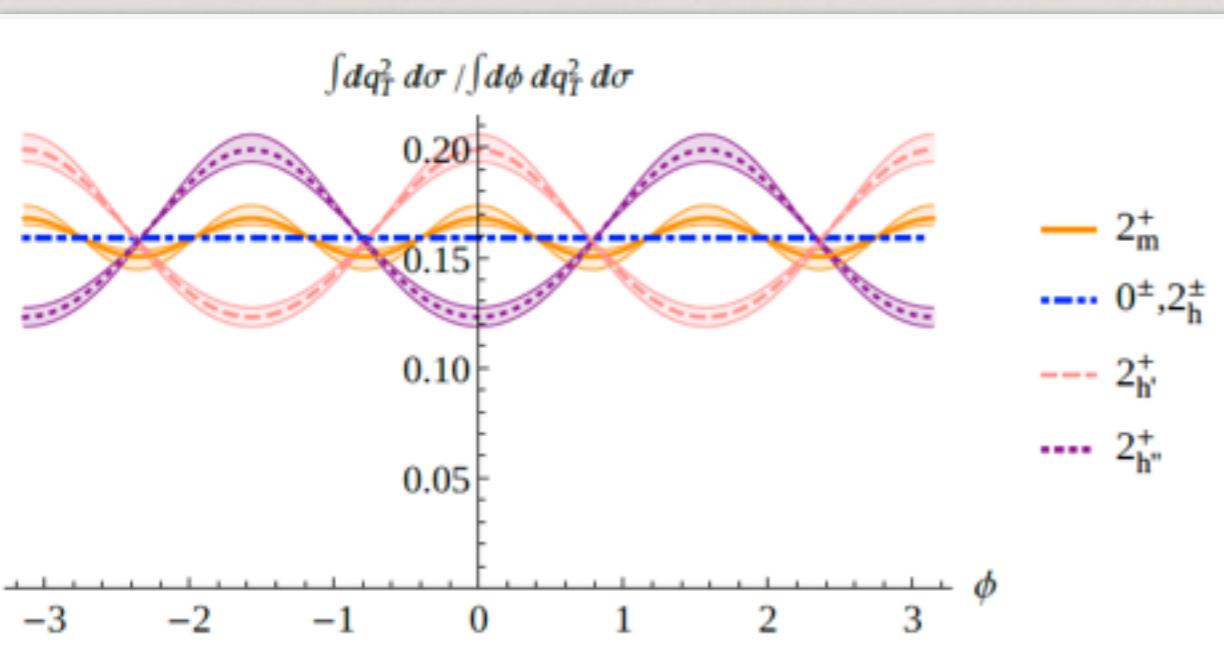


$$\frac{d\sigma}{d^2 q_T} \propto F_1 C[f_1^g f_1^g] \pm F_2 C[w_2 h_1^{\perp g} h_1^{\perp g}]$$

P-even: increase of at low  $q_T$ ,  
P-odd: decrease (also for Spin-2)

Evolution:  
 effect only  $\sim 1\%$  at  $Q = m_H$   
 [Boer, den Dunnen, NPB 2014]

# Spin from $\phi$ -dependence

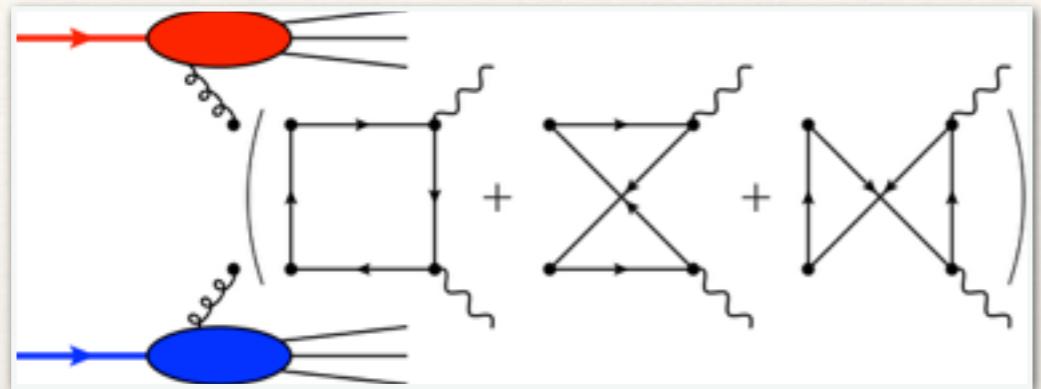


(pseudo-)scalar,  $2_h$  isotropic  
 $2_m$  - hyp.  $\rightarrow \cos(4\phi)$ -modulation:  $\sim 5\%$  ,  
 $2_{h'}, h''$  - hyp.  $\rightarrow \cos(2\phi)$ -modulation:  $\sim 20\%$   
 $h'$  and  $h''$  distinguishable!  
 (unlike in collinear factorization)

# Other final states in pp - collisions

$gg \rightarrow \gamma\gamma$  at  $Q \neq m_H$ : QCD background process

[Qiu, M.S., Vogelsang, PRL 2011]

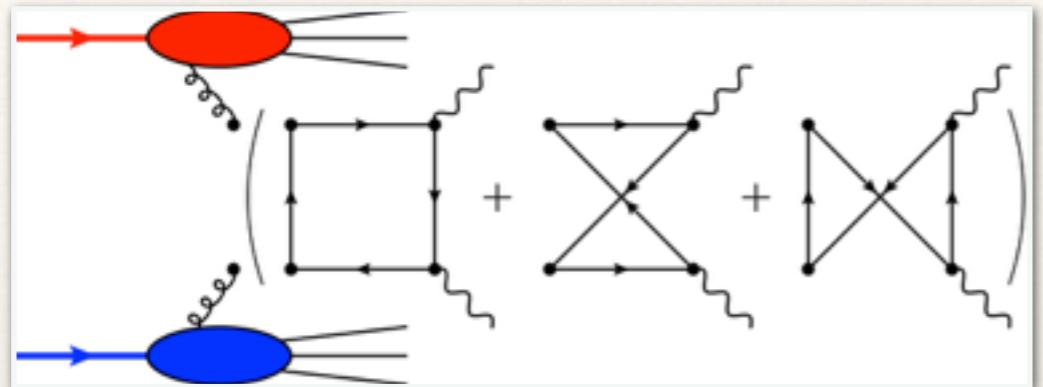


- contaminations from quark contributions:  
only  $\cos(4\phi)$ -modulation purely gluonic
- $\gamma\gamma$  - production: background from  $\pi^0$  - decays

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## Single Quarkonium production

[LO: Boer, Pisano, PRD 2012; NLO: Ma, Wang, Zhao, PRD 2012]

(Exclusive) production of a heavy Quarkonium state (color singlet mode):

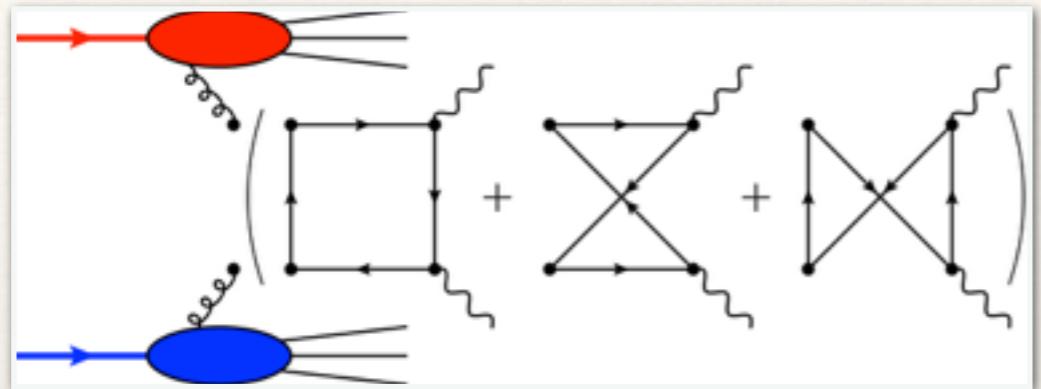
$$p + p \rightarrow (\eta, \chi, \dots) + X$$

S-waves:  $L=0, J=0:$        $\eta : {}^1S_0^{(1)}$   
P-waves:  $L=1, J=0, 2:$        $\chi_{0,2} : {}^3P_{0,2}^{(1)}$        $2S+1 L_J$

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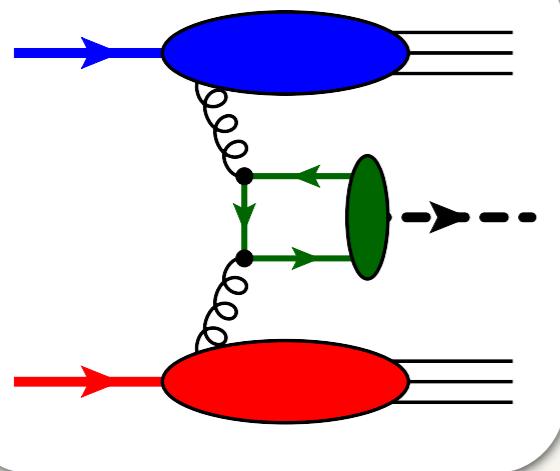
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## TMD - formalism ( $Q = M_Q \gg q_T$ ):



$$\frac{d\sigma(\eta)}{dy d^2 q_T} = C_\eta ([f_1^g \otimes f_1^g] - [h_1^g \otimes h_1^g])$$

$$\frac{d\sigma(\chi_0)}{dy d^2 q_T} = C_{\chi_0} ([f_1^g \otimes f_1^g] + [h_1^g \otimes h_1^g])$$

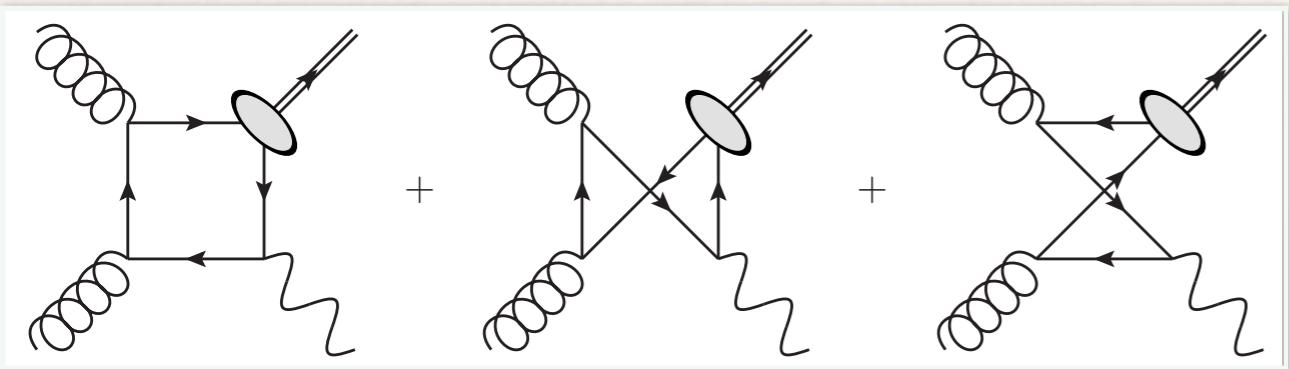
$$\frac{d\sigma(\chi_2)}{dy d^2 q_T} = C_{\chi_2} ([f_1^g \otimes f_1^g])$$

- no contamination from quark channel (at LO)
- $q_T$  very small, difficult to measure

# $\Upsilon(\text{J}/\psi) + \gamma$ - production at the LHC

[den Dunnen, Lansberg, Pisano, M.S., PRL 2014]

$$\frac{d\sigma}{d^4q d\Omega} \Big|_{q_T \ll Q} = \hat{F}_1 \mathcal{C}[f_1^g f_1^g] + \cos(2\phi) \hat{F}_3 (\mathcal{C}[w_{3a} h_1^{\perp g} f_1^g] + \{x_a \leftrightarrow x_b\}) + \cos(4\phi) \hat{F}_4 \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]$$

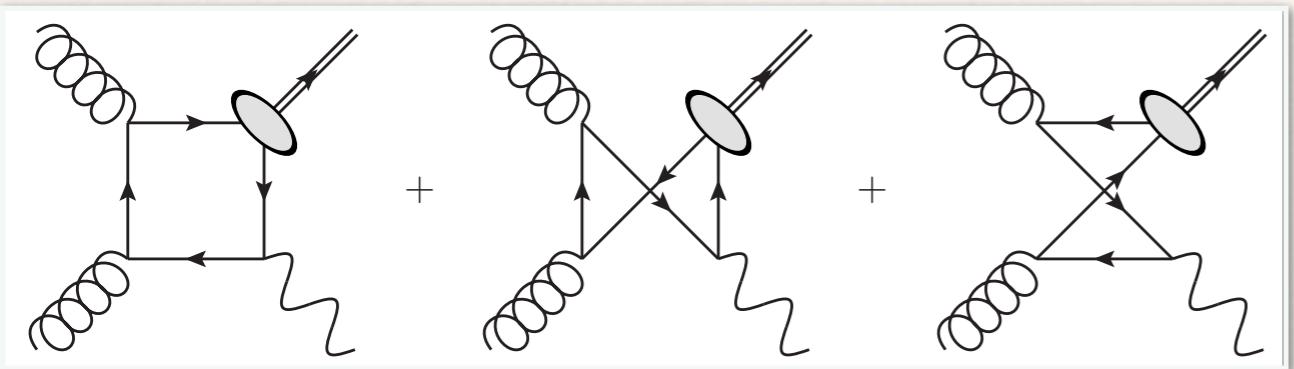


- Hard scale Q adjustable (back-to-back pair)
- main contribution from gluon fusion
- $\Upsilon$  - production:
- Color Octet (Fragm.)  $\ll$  Color Singlet ✓
- $F_2 = 0, \cos(2\phi) \sim 3\%, \cos(4\phi) \sim 1\%$ !
- Cross section large enough at the LHC

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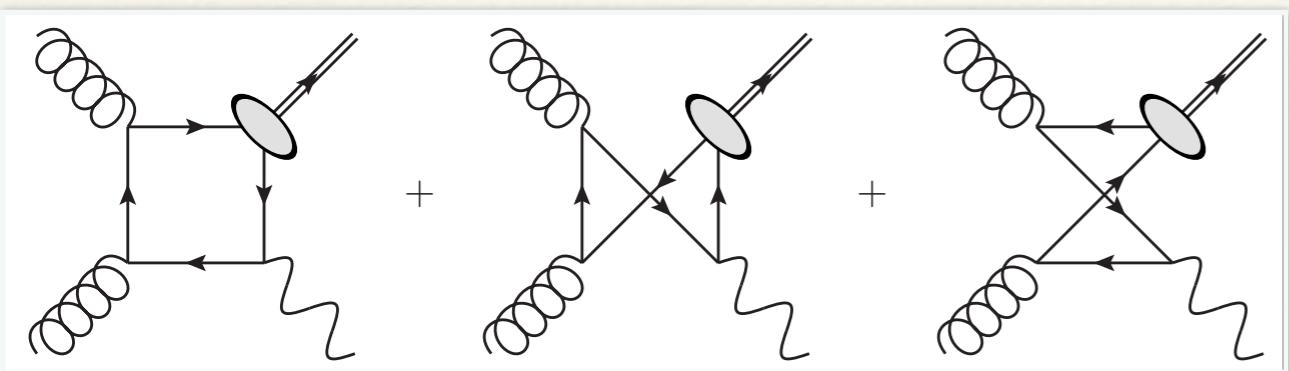
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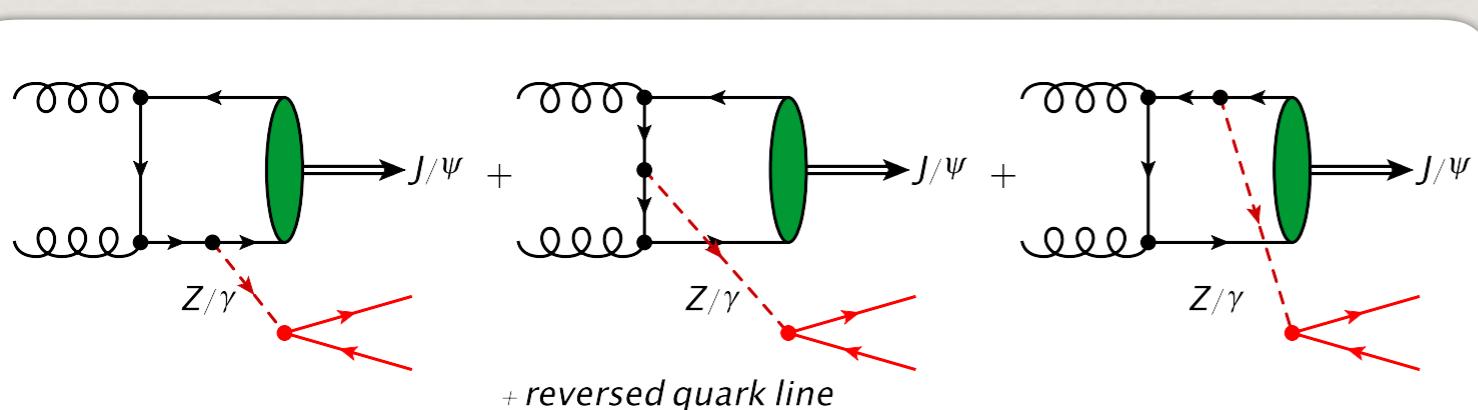


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# $\Upsilon + Z$ - production at the LHC

[Lansberg, Pisano, M.S., in preparation]



$$F_1 \gg F_2, F_3, F_4$$

$$\frac{d\sigma}{d^4q} \Big|_{q_T \ll Q} = \hat{F}_1 \mathcal{C}[f_1^g f_1^g]$$

ATLAS data exists for  $q_T$  - integrated cross section

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# Summary

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- ❖ Gluon TMDs → new aspects on the 3D gluonic structure of the nucleon → linear gluon polarization
- ❖ Several final states in pp - collisions: Leptons, Photons, Heavy Quarkonium states
- ❖ “Golden” Experiment: Lepton - Nucleon collisions at EIC
- ❖ Theory: Evolution of gluon TMDs and higher order calculations become available, need for data fits
- ❖ small-x physics: talks by Xiao, van Daal, Hatta, ...

# “Color Entanglement”

TMD factorization problematic in pp - collisions with a colored final state!

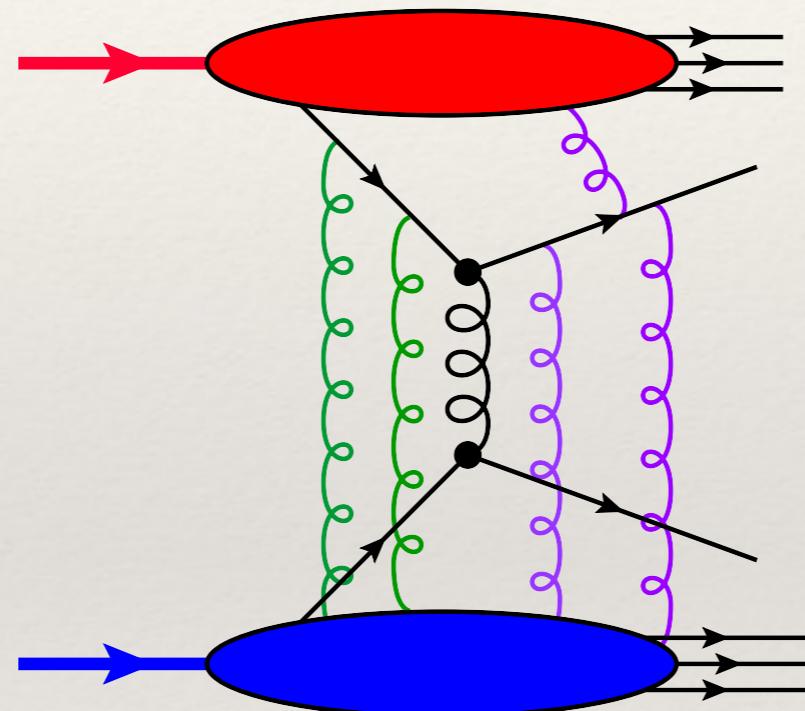
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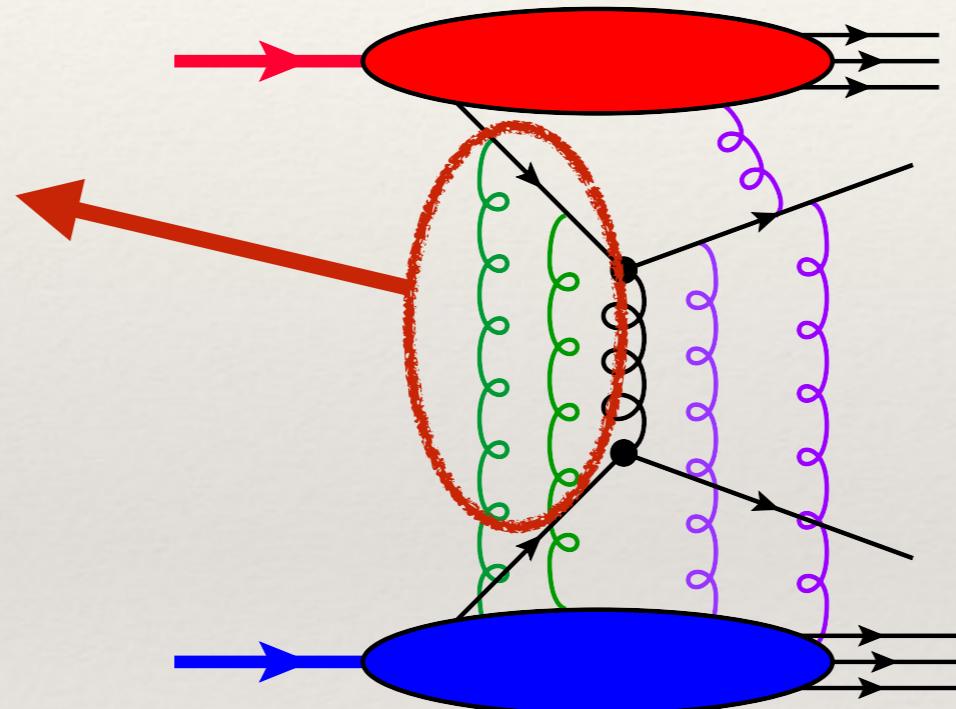
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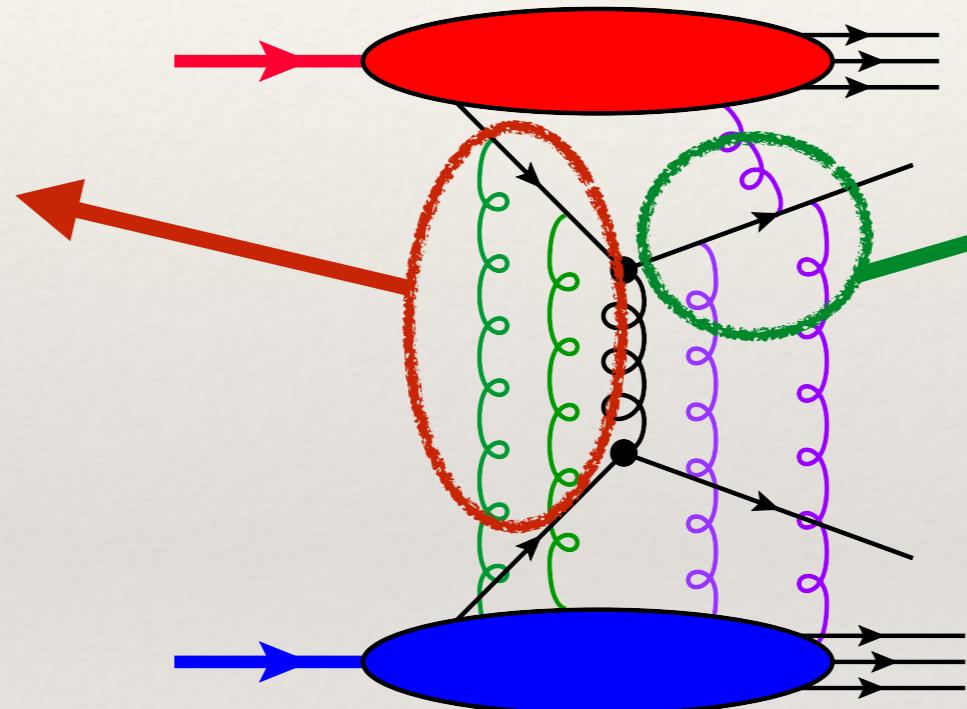
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from both nucleons are  
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→ cannot define Wilson lines  
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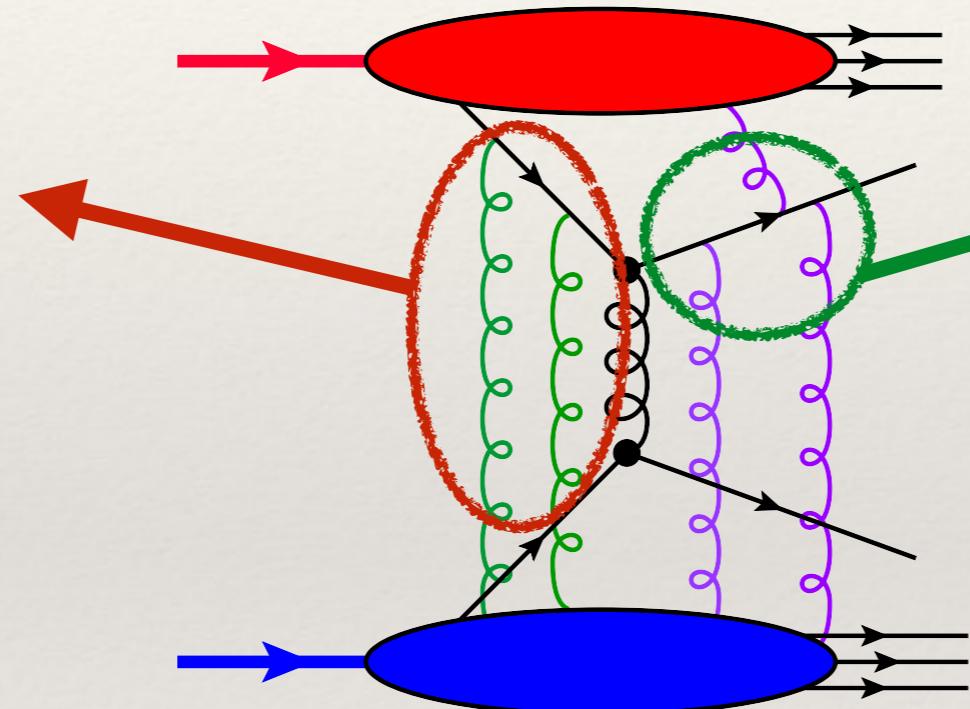
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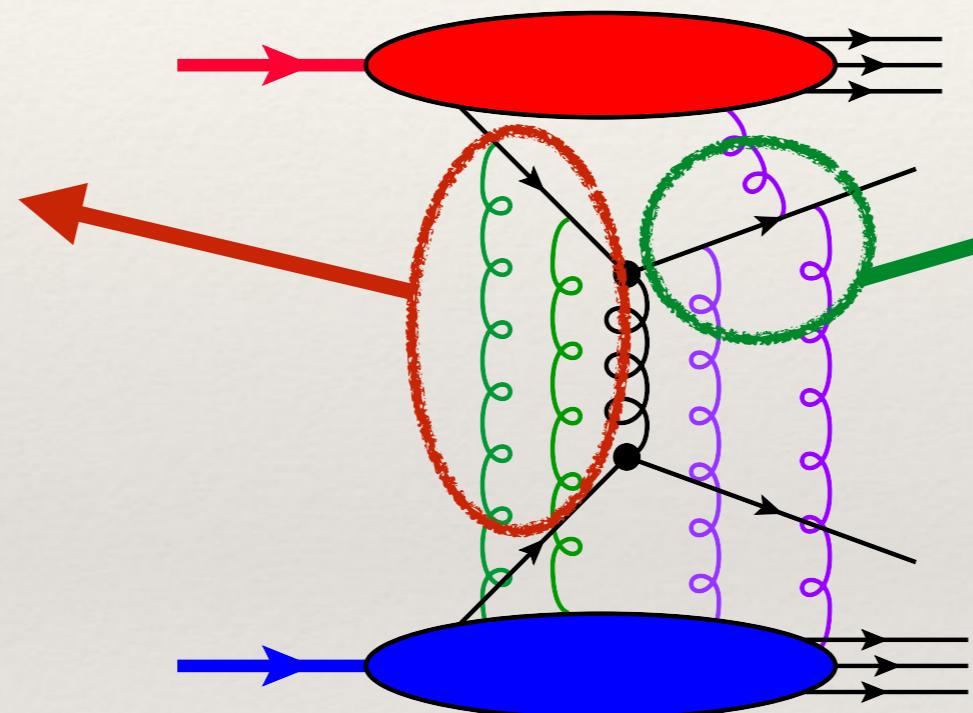
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TMD factorization breaking effects large? small? and why...?