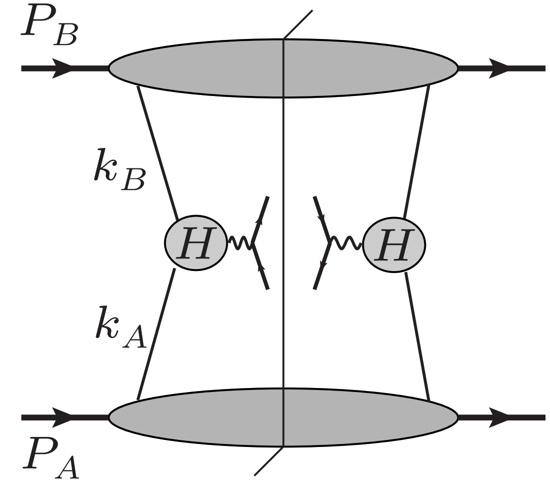
Combining TMD factorization and collinear factorization

John Collins (Penn State)

Based on J. Collins, L. Gamberg, A. Prokudin, T.C. Rogers, N. Sato, and B. Wang, "Relating Transverse Momentum Dependent and Collinear Factorization Theorems in a Generalized Formalism", Phys. Rev. D94, 034014 (2016); arXiv:1605.00671.

Key approximations to get TMD factorization

Collinear quark and antiquark in:

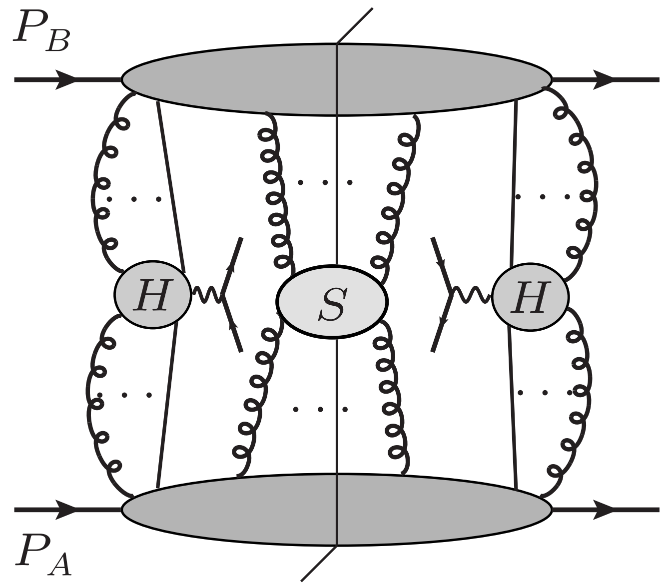


- Light-front coordinates: $k_A = (x_A P_A^+, k_A^-, \boldsymbol{k}_{A\,\mathsf{T}})$, $k_B = (k_B^+, x_B P_B^-, \boldsymbol{k}_{B\,\mathsf{T}})$.
- In H, replace k_A and k_B by on-shell values
- In kinematics, replace q by $(x_A P_A^+, x_B P_B^-, \boldsymbol{k}_{A\,\mathsf{T}} + \boldsymbol{k}_{B\,\mathsf{T}})$.

Hence

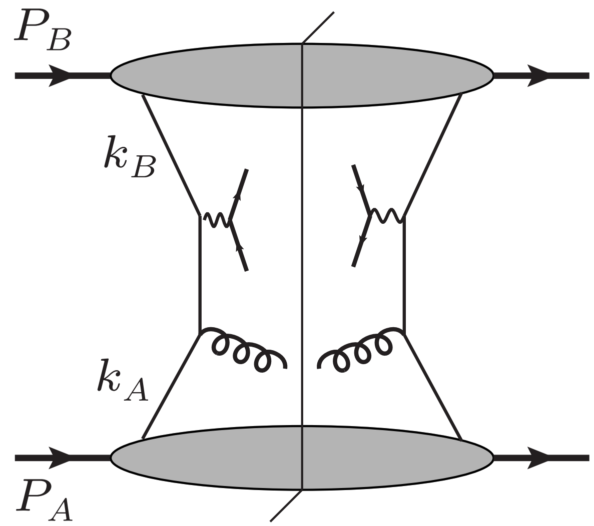
- Integrate k_A^- within A, etc $\Longrightarrow$ usual definition of TMD pdfs.
- $k_A^- = O(q_{\mathsf{T}}^2/q^+)$, etc.
- So approximation gets bad when q_{T} increases to roughly order Q and up.

Some other graphical structures are treated similarly



Key approximation to get collinear factorization for large q_T

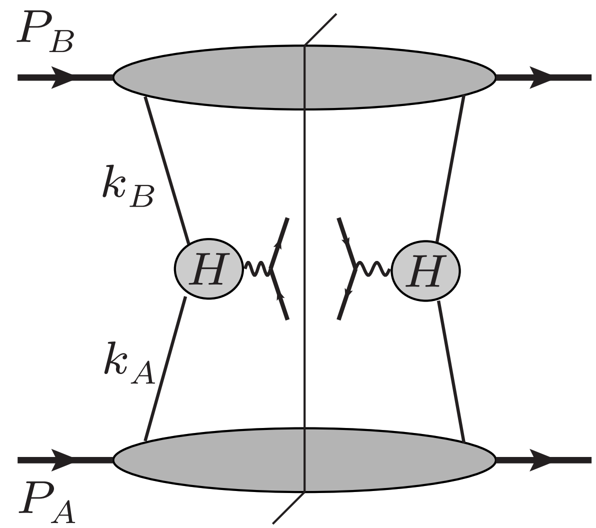
When q_T of order Q, generate q_T by hard scattering:



- In hard scattering: neglect *both* virtuality *and* transverse momentum of incoming partons.
- Leads to use of collinear parton densities $f(x; \dots)$.
- Breaks down once q_T is comparable to typical parton k_T.

And key approximation for collinear factorization also applies to cross section integrated over q_T

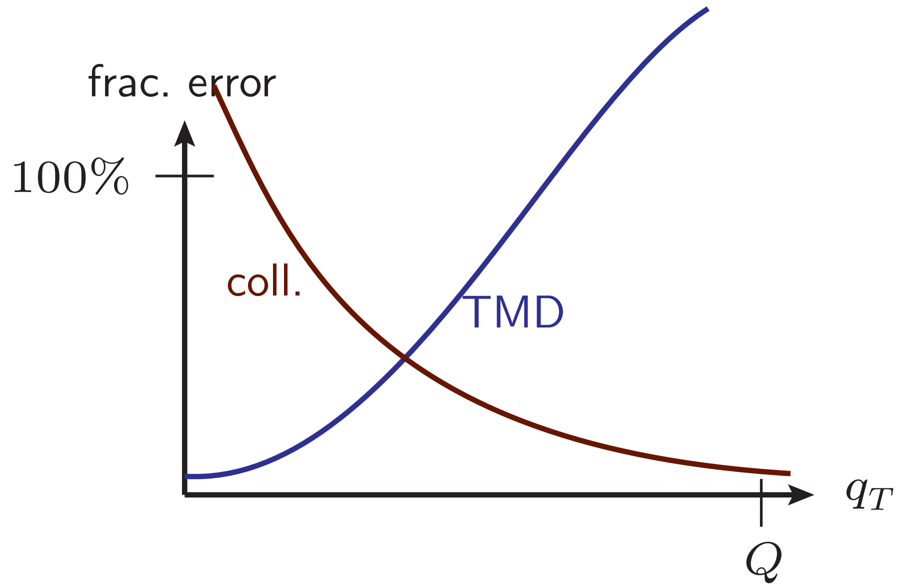
- In



  integral over $\boldsymbol{q}_\mathsf{T}$ gives independent integrals over $\boldsymbol{k}_{A\,\mathsf{T}}$ & $\boldsymbol{k}_{B\,\mathsf{T}}$.

- Collinear approximation replaces q_T by zero. Shift leaves integrated cross section unchanged.

Error sizes



CSS's $W+Y$ method to combine TMD & collinear factorizations

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} = W + Y + \text{error}$$

with TMD term

$$W = \sigma_0 H(Q/\mu) \int \mathrm{d}^2 \boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_\mathsf{T}\cdot\boldsymbol{b}_\mathsf{T}} \tilde{f}(x_A, \boldsymbol{b}_\mathsf{T}; \mu, Q) \tilde{f}(x_B, \boldsymbol{b}_\mathsf{T}; \mu, Q)$$

and collinear correction term

$$Y = \text{collinear approx. to} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} - W \right)$$

Errors (to which are to be added truncation errors!):

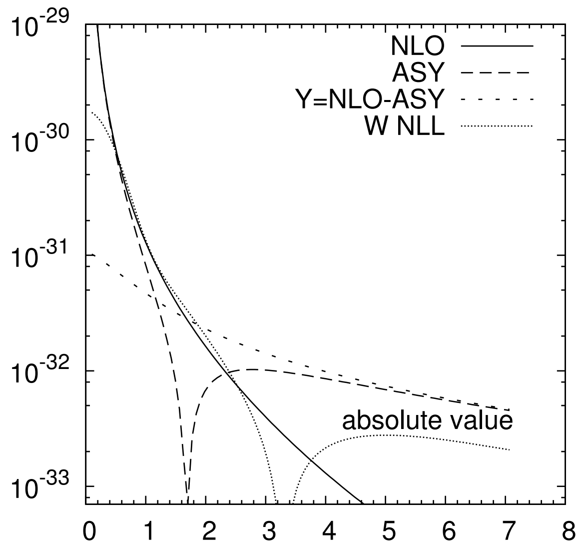
$$W = \frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \left\{ 1 + O\left[\left(\frac{\Lambda}{Q} \right)^a \right] + O\left[\left(\frac{q_\mathsf{T}}{Q} \right)^a \right] \right\} \qquad \text{when } q_\mathsf{T} \lesssim Q$$

$$Y = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} - W \right) \left\{ 1 + O\left[\left(\frac{\Lambda}{q_\mathsf{T}} \right)^a \right] + O\left[\left(\frac{\Lambda}{Q} \right)^a \right] \right\} \qquad \text{when } \Lambda \lesssim q_\mathsf{T} \lesssim Q$$

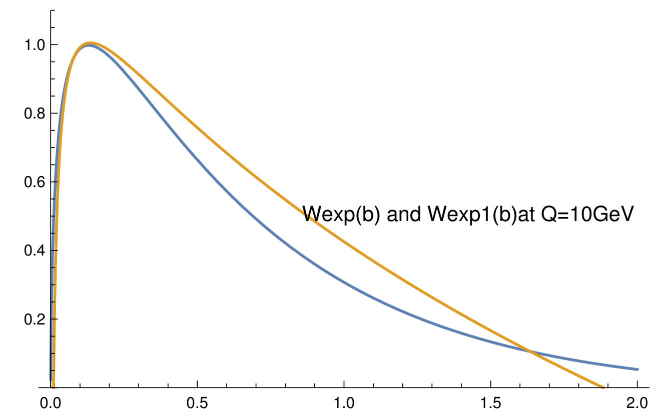
Hence

$$\text{error} = \frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} - W - Y = \frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \times O\left[\left(\frac{\Lambda}{Q} \right)^a \right] \qquad \text{when } \Lambda \lesssim q_\mathsf{T} \lesssim Q$$

What is problematic with the original $W+Y$ formulation?



(Boglione et al.,
arXiv:1412.6927
$Q = 10\,\mathrm{GeV}$)



- When q_{T} is above some small fraction of Q: W deviates a lot from $\mathrm{d}\sigma/\mathrm{d}^4q$.
- Then it becomes negative, and asymptotes to $1/q_{\mathrm{T}}^2$ times logarithms.
- Hence at large enough q_{T}, $W+Y$ is a difference of larger terms: Truncation errors etc are magnified.
- $\int \mathrm{d}^2\boldsymbol{q}_{\mathrm{T}}\, W = 0$ since $\tilde{W}(b_{\mathrm{T}}=0)=0$.
  But in $\mathrm{d}\sigma/\mathrm{d}^4q = W + Y +$ error:
  - $\int \mathrm{d}^2\boldsymbol{q}_{\mathrm{T}}\, \mathrm{d}\sigma/\mathrm{d}^4q$ given by collinear factorization starting at LO (i.e., α_s^0)
  - Y is used with collinear factorization starting at NLO (i.e., α_s^1)
- At small q_{T}, Y is bad approximation to $\mathrm{d}\sigma/\mathrm{d}^4q - W$

Our new proposal

Modify W to

$$W_{\rm New} = \Xi\left(\frac{q_{\sf T}}{Q}\right) \int \frac{{\rm d}^2\boldsymbol{b}_{\sf T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\sf T}\cdot\boldsymbol{b}_{\sf T}} \tilde{W}(b_c(b_{\sf T}), Q)$$

with small $b_{\sf T}$ cutoff, to avoid problems in $\int {\rm d}^2\boldsymbol{q}_{\sf T}\, W_{\rm New}$:

$$b_c(b_{\sf T}) = \sqrt{b_{\sf T}^2 + \text{const}/Q^2}$$

and with large $q_{\sf T}$ cutoff, sharp or smooth:

$$\Xi\left(\frac{q_{\sf T}}{Q}\right) = \exp\left[-\left(\frac{q_T}{\text{const.}Q}\right)^{\rm const.}\right]$$

Modify Y to

$$Y_{\rm New} = X(q_{\sf T}) \times \text{collinear approx. to} \left(\frac{{\rm d}\sigma}{{\rm d}^4 q} - W_{\rm New}\right)$$

with small $q_{\sf T}$ cutoff:

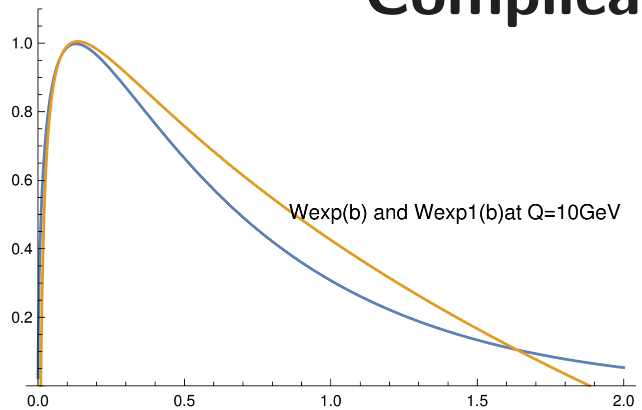
$$X(q_{\sf T}/\lambda) = 1 - \exp\left\{-(q_{\sf T}/\text{const.})^{\text{const.}}\right\} \qquad (1)$$

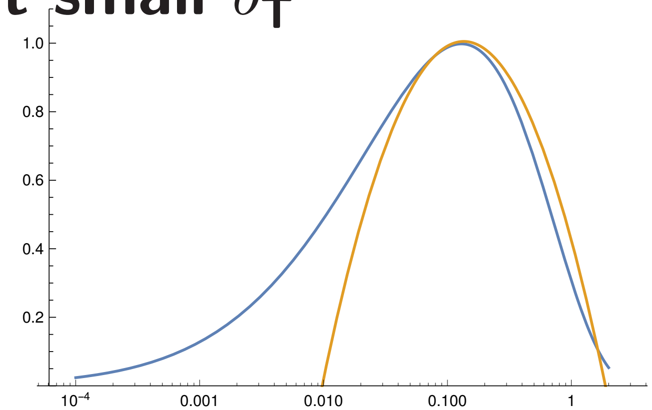
Conclusions

- Modified $W + Y$ so that
  - $W_{\rm New}$ is $\int \mathrm{d}^2\boldsymbol{b}_{\mathsf T}\, e^{i\boldsymbol{q}_{\mathsf T}\cdot\boldsymbol{b}_{\mathsf T}} \tilde{W}(b_c(b_{\mathsf T}); \dots)$, with a $b_c(0)$ of order $1/Q$
  - $Y_{\rm New}$ has cutoff at low $q_{\mathsf T}$.
- Further improvements possible. (See appendix for constraints.)
- Generally: Need to "look under hood":
  - What are the nature of the approximations giving factorization (TMD and collinear)?
  - How much do they fail, with proper account of non-perturbative properties?
- Importance for spin physics: Improved behavior at moderate Q.
- In SIDIS, for determining what is large and small transverse momentum relative to Q, is the appropriate variable $q_{\mathsf T}$ or $P_{\mathsf T}$? (This concerns quark transverse momentum relative to hadron or quark relative to hadron.) Was that the right question?
- Need hard scattering coefficient for Y for SIDIS at $O(\alpha_s^2)$.

APPENDIX

Complications at small b_{T}





$$W = \sum_j \sigma_0 H \int \frac{d^2\boldsymbol{b}_{\mathsf{T}}}{(2\pi)^2} e^{i\boldsymbol{q}_{\mathsf{T}}\cdot\boldsymbol{b}_{\mathsf{T}}} \, (C\otimes f)_A \, (C\otimes f)_B \times e^{-g_K(b_{\mathsf{T}}) \ln\frac{Q^2}{Q_0^2} - g_{j/A}(x_A,b_{\mathsf{T}}) - g_{\bar{j}/B}(x_B,b_{\mathsf{T}})}$$

$$\times \exp\left\{ -\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu}{\mu} \left[\ln\frac{Q^2}{(\mu')^2} \gamma_K(\alpha_s(\mu)) - 2\gamma_j(\alpha_s(\mu)) \right] + \tilde{K}(b_*;\mu_{b_*}) \ln\frac{Q^2}{\mu_{b_*}^2} \right\}$$

$$Y = \text{collinear fixed-order approx. to} \left(\frac{d\sigma}{d^4q} - W \right)$$

- $W = 0$ at $b_{\mathsf{T}} = 0$, so $\int d^2\boldsymbol{q}_{\mathsf{T}}\, W = 0$.
- W good approximation to $d\sigma/d^4q$ at small q_{T}, but is negative at large q_{T}.
- Small-b_{T} singularity not given by fixed-order P.T.

Constraints on improved $W + Y$ formulation

Given that

- $W_{\rm New}$ is given by (modified) TMD/resummed formula.
- $Y_{\rm New}$ is given by fixed order collinear factorization

Then for $W_{\rm New} + Y_{\rm New}$, we require:

- $W_{\rm New}$ uses normal operator definitions of TMD densities (for which we know evolution equations).

(Next items refer to errors beyond non-logarithmic truncation errors.)

- Error relative to $\mathrm{d}\sigma/\mathrm{d}^4 q$ is suppressed by a power of Q over whole range of q_{T}.
- The power suppression also applies to the cross section integrated over $\boldsymbol{q}_{\mathsf{T}}$.
- $\int \mathrm{d}^2\boldsymbol{q}_{\mathsf{T}}\, W_{\rm New}$ gives LO collinear factorization plus well-behaved NLO terms (and beyond).