Phenomenology of TMD evolution: recent progress

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Outline

- Introduction on TMD evolution
- Phenomenology of TMD evolution
- Summary and outlook
New structure of nucleon

- TMDs provide new structure of nucleon – 3D structure: both longitudinal + transverse momentum dependent structure (confined motion in a nucleon)

**Transverse Momentum Dependent parton distribution (TMDs)**

\[ p_a \approx x P_A \]

Longitudinal motion only

Longitudinal + transverse motion
TMDs: rich quantum correlations

### Leading Twist TMDs

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>Quark Polarization</th>
<th>Un-Polarized (U)</th>
<th>Longitudinally Polarized (L)</th>
<th>Transversely Polarized (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1 = \bullet$</td>
<td></td>
<td></td>
<td>$h_1^\perp = \bullet - \bullet$</td>
</tr>
<tr>
<td>L</td>
<td>$g_{1L} = \bullet \rightarrow - \bullet$</td>
<td>Helicity</td>
<td></td>
<td>$h_{1L}^\perp = \bullet - \bullet$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T} = \bullet - \bullet$</td>
<td>Sivers</td>
<td>$g_{1T} = \bullet - \bullet$</td>
<td>$h_{1T}^\perp = \bullet - \bullet$</td>
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</table>

### Quark Polarization

<table>
<thead>
<tr>
<th>Pion</th>
<th>$D_1$</th>
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<tbody>
<tr>
<td></td>
<td>$H_1^\perp$</td>
</tr>
</tbody>
</table>

**TMD parton distribution**

**TMD fragmentation function**
Universality and TMD evolution

- Two most important properties of TMDs
  - Universality: TMD might not be universal when probed through different hard scattering processes

\[ f_{1T}^{\text{DIS}}(x, k_\perp) = - f_{1T}^{\text{DY}}(x, k_\perp) \]

- TMD evolves

\[ F(x, k_\perp, Q_i) \rightarrow F(x, k_\perp, Q_f) \]
TMD factorization in a nut-shell

- **Drell-Yan:** \( p + p \to [\gamma^* \to \ell^+ \ell^-] + X \)

  \[ H(Q) \]

  \[ S(\lambda_{\perp}) \]

  \[ f(x_1, k_{1\perp}) \]

  \[ f(x_2, k_{2\perp}) \]

  \[ \frac{d\sigma}{dQ^2 dy d^2 q_{\perp}} \propto \int d^2 k_{1\perp} d^2 k_{2\perp} d^2 \lambda_{\perp} H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_{\perp}) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_{\perp} - q_{\perp}) \]

  \[ = \int \frac{d^2 b}{(2\pi)^2} e^{i q_{\perp} \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b) \]

  \[ F(x, b) = f(x, b) \sqrt{S(b)} \]

  \[ = \int \frac{d^2 b}{(2\pi)^2} e^{i q_{\perp} \cdot b} H(Q) F(x_1, b) F(x_2, b) \]

  mimic “parton model”

- Factorized form and mimic “parton model”

  - Factorization of regions:
    1. \( k//P_1 \)
    2. \( k//P_2 \)
    3. \( k \) soft
    4. \( k \) hard
Divergence and evolution

- Divergence leads to evolution
  - Ultraviolet divergence: renormalization group equation, e.g. running of coupling constant
  - Collinear divergence: DGLAP evolution of collinear parton distribution function, fragmentation function, semi-inclusive jet function
- Rapidity divergence (light-cone singularity): TMD evolution

What is rapidity divergence?

\[
f_{q/q}(x, k^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{k^2} \left[ \frac{2z}{1 - z} + (1 - z) \right]
\]

\[
z = \frac{k^+}{p^+}
\]

\[
z \to 1 \Leftrightarrow \ell^+ \to 0\]

\[
y = \frac{1}{2} \ln \frac{\ell^+}{\ell^-} \to -\infty
\]
Different ways to regularize rapidity divergences

- Off-light-cone
- $\delta$-regulator
- Analytic regulator
- Rapidity regulator
- Exponential regulator

Rapidity regulator

$$f_{q/q}(x, k_{\perp}^2) = \frac{\alpha_s}{2\pi^2} \frac{\Gamma(1 + \epsilon)}{\mu^2} \left( \frac{\mu^2}{k_{\perp}^2} \right)^{1+\epsilon} \left[ \frac{2z}{(1-z)^{1+\eta}} \left( \frac{\nu}{p^+} \right)^\eta \right] + (1-z) - \epsilon(1-z)$$
TMD evolution in b-space

- **Quark TMD at one loop**
  \[
  f_{q/q}(x, b) = \frac{\alpha_s}{2\pi} C_F \left\{ \left( \frac{2}{\eta} \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu_b^2} \right) \right) + \frac{2}{\epsilon} \ln \frac{\nu}{p^+} + \frac{3}{2} \frac{1}{\epsilon} \right\} \delta(1 - x)
  \]
  \[+ \left( -\frac{1}{\epsilon} \ln \frac{\mu^2}{\mu_b^2} \right) P_{qq}(x)
  \]
  \[+ \left[ 2 \ln \frac{\mu^2}{\mu_b^2} \ln \frac{\nu}{p^+} + \frac{3}{2} \ln \frac{\mu^2}{\mu_b^2} \right] \delta(1 - x) + (1 - x) \}

- **Soft factor**
  \[
  S(b) = \frac{\alpha_s}{2\pi} C_F \left\{ \frac{4}{\eta} \left( -\frac{1}{\epsilon} \ln \frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon} \ln \frac{\mu^2}{\mu_b^2} - \frac{\nu^2}{\epsilon} \right\}
  \]
  \[+ \left[ -2 \ln \frac{\mu^2}{\mu_b^2} \ln \frac{\nu^2}{\mu_b^2} + \ln \frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{6} \right] \}

- **Interesting features**
  - Rapidity divergence cancels in \( F_{q/q}^{\text{sub}}(x, b) = f_{q/q}(x, b) \sqrt{S(b)} \)
  - \( f_{q/q}(x, b) \) and \( S(b) \) lives in the same \( \mu \sim \mu_b \), but different rapidity scale \( \nu \sim p^+, \mu_b \)

- **Two evolution equations: \( \mu \text{-RG and } \nu \text{-RG}**
  \[
  \mu \frac{d}{d\mu} \ln f_{q/q}(x, b) = \gamma^f_{\mu}
  \]
  \[
  \nu \frac{d}{d\nu} \ln f_{q/q}(x, b) = \gamma^f_{\nu}
  \]
  \[
  \mu \frac{d}{d\mu} \ln S(b) = \gamma^S_{\mu}
  \]
  \[
  \nu \frac{d}{d\nu} \ln S(b) = \gamma^S_{\nu}
  \]
TMD evolution in b-space

- Solution of TMD evolution equations

\[ F(x, b; Q_f) = F(x, b; Q_i) \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left( \frac{Q_f^2}{Q_i^2} \right)^{-\int_{c/b}^{Q_f} \frac{d\mu}{\mu} A} \]

\[
A = \sum_{n=1}^{\infty} A^{(n)} \left( \frac{\alpha_s}{\pi} \right)^n, \quad B = \sum_{n=1}^{\infty} B^{(n)} \left( \frac{\alpha_s}{\pi} \right)^n
\]

- The well-known CSS solution

Only valid for small b

Collins-Sopoer-Sterman papers
Kang, Xiao, Yuan, PRL 11,
Aybat, Rogers, Collins, Qiu, 12,
Aybat, Prokudin, Rogers, 12,
Sun, Yuan, 13,
Echevarria, Idilbi, Schafer, Scimemi, 13,
Echevarria, Idilbi, Kang, Vitev, 14,
Kang, Prokudin, Sun, Yuan, 15, 16, …
Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation

- Many different methods/proposals to model this non-perturbative part

\[ F(x, k_\perp; Q) = \frac{1}{(2\pi)^2} \int d^2 b e^{ik_\perp \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^\infty db b J_0(k_\perp b) F(x, b; Q) \]

Eventually evolved TMDs in b-space

\[ F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp \left\{ -\int_{c/b^*}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp \left( -S_{\text{non-pert}}(b, Q) \right) \]

- Non-perturbative: fitted from data
- The key ingredient – ln(Q) piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract the key ingredient for the non-perturbative part.
Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

\[ F(x, Q) \]

- DGLAP evolution
- Resum \[ \left[ \alpha_s \ln \left( \frac{Q^2}{\mu^2} \right) \right]^n \]
- Kernel: purely perturbative

TMDs

\[ F(x, k_\perp; Q) \]

- Collins-Soper/rapidity evolution equation
- Resum \[ \left[ \alpha_s \ln^2 \left( \frac{Q^2}{k_\perp^2} \right) \right]^n \]
- Kernel: can be non-perturbative when \[ k_\perp \sim \Lambda_{QCD} \]
TMD global analysis

- Outline of a TMD global analysis: numerically more heavy

  Model ansatz for TMDs with initial set of parameters

  Evolve TMDs to relevant scale with TMD evolution

  Fourier transform back to momentum space

  calculate the cross section/asymmetry as well as $\chi^2$

  $\chi^2$ minimum?

  yes

  no

  adjust parameters

  all data points

  Model ansatz for non-perturbative evolution kernel
Different treatments at large $b$

- In terms of $b^*$ prescription (see also other proposals Qiu, Vogelsang)

\[ \mu_b = \frac{2e^{-\gamma_E}}{b^*} \]

\[ b^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\max}^2}}} \]

- Non-perturbative Sudakov factor

\[
\begin{align*}
\exp \left[ -g_2 b^2 \ln \left( \frac{Q}{Q_0} \right) + \cdots \right] \\
\exp \left[ -g_2 \ln \left( \frac{b}{b^*} \right) \ln \left( \frac{Q}{Q_0} \right) + \cdots \right] \\
\exp \left\{ -g_0 \left( b_{\max} \right) \left[ 1 - \exp \left( -\frac{C_F \alpha_s \left( \mu_{b^*} \right) b^2}{\pi g_0 \left( b_{\max} \right) b_{\max}} \right) \right] \right\}
\end{align*}
\]

CSS, Echevarria, Idlibi, Kang, Vitev, 14, …
Aidala, Field, Gamberg, Rogers, 1401.2654, Sun, Isaacson, Yuan, Yuan, 1406.3073
Collins, Rogers, 1412.3820
Different fits to date

<table>
<thead>
<tr>
<th>Framework</th>
<th>HERMES</th>
<th>COMPASS</th>
<th>DY</th>
<th>$Z$ production</th>
<th>N of points</th>
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Taken from Bacchetta, QCD Evolution Workshop 2016

- It is easier to fit either SIDIS or DY, but quite difficult to fit both
- Two groups tried very hard, but the good quality of $\chi^2$ is achieved with scarifying the overall normalization of SIDIS cross section (has to multiply a K factor ~ 2)
Try both SIDIS and DY/W/Z: EIKV 2014

- **SIDIS, DY, and W/Z**

  - Works reasonably for SIDIS, DY, and W/Z in all the energy ranges
  - Look closer at DY, not so good
Another try: Sun-Isaacson-Yuan-Yuan 2014

- A new fit with DY, not SIDIS
  Sun, Isaacson, Yuan, Yuan, 1406.3073

- Seems rather well for SIDIS multiplicity, though requires additional K factor ~ 2 for multiplicity distribution
New fit: Pavia group

- K-factor is needed for SIDIS

First points are not fitted, but used as normalization to avoid problems related to data normalization.
QCD evolved unpolarized TMD

- What evolution does
  - Spread out the distribution to much larger $k_T$
  - At low $k_T$, the distribution decreases due to this spread

Based on Echevarria, Idilbi, Kang, Vitev, 14

U quark PDF at $x=0.1$

$Q^2 = 2$
$Q^2 = 50$
$Q^2 = 8100$
Sivers asymmetry from SIDIS

- Sivers asymmetry has been measured in SIDIS process: HERMES, COMPASS, JLab

\[ \ell + p^\uparrow \rightarrow \ell' + \pi(p_T) + X \]

\[
\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2) \left[ F_{UU} + \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)} + \ldots \right]
\]

\[
F_{UT}^{\sin(\phi_h - \phi_s)} \sim f_{1T}(x_B, k_\perp) D_{h/q}(z_h, p_\perp)
\]

Sivers function
Sivers function with energy evolution

- Example of the fit: JLab, HERMES, COMPASS

Echevarria, Idilbi, Kang, Vitev, 14
Extracted Sivers function and evolution

- $\chi^2$/d.o.f. = 1.3, the collinear part is plotted: only $u$ and $d$ valence quark Sivers are constrained

- Visualization: positive = more quark moves to the left ($Q^2 = 2 - 100$ GeV$^2$)
Within the region constrained by the experimental data, the spin-dependent TMDs seem to be rather consistent among different groups.

TMD evolution cancels between the ratios??

Need more data on the absolute cross section

However, the extrapolations can be very different

\[ x \Delta_f^{N_q(1)}(x, Q) \]
Uncertainty in the evolution formalism

- Even the evolution formalism itself has large room to improve – non-perturbative Sudakov needs further improvement

\[ A_N \]

\[ W^+ \]

\[ W^- \]

w/o TMD evolution

Kang, 2015
Experimental evidence of sign change

- STAR measurements: the data favors sign change
- Both theory and experiment has large uncertainty: hope to be improved in the near future (2017 run)

- Looking forward to see the result from COMPASS!
Another new data: COMPASS

- Sivers asymmetry in DY scale region

COMPASS, arXiv:1609.07374
Further constrain TMD evolution

- The absolute cross section (not the ratio) should certainly be better.
- We will see how the asymmetry (ratio) at a broad range of $Q$ can constrain TMD evolution formalism: low $Q^2$ SIDIS data in the past: $Q^2 \sim 1 - 10$ GeV$^2$; new SIDIS data at DY scale: $Q^2 \sim 1 - 81$ GeV$^2$; W/Z asymmetry at $(80 - 90$ GeV$)^2$

How to move forward

- Perform data analysis directly in $b$-space
- Perform evolution directly in momentum space
- Improve the current non-perturbative model
- Of course improve the controllable perturbative part

Also look for new channels to probe TMDs

- Boer, Gamberg, Prokudin, et.al.
- Kang, in preparation
- Collins, Rogers; Kang, Qiu; Prokudin, Yuan; …
- Stewart et.al. N$^3$LL (B$^{(3)}$)
- See also Echevarria’s talk

Kang, Ringer, Vitev, 1606.07063
Summary

- Study on TMDs are extremely active in the past few years, lots of progress made, though still large uncertainty on TMD evolution.

- With great excitement, we look forward to the future experimental results from COMPASS/RHIC, as well as Jefferson Lab, of course also LHC.

Thank you!