Phenomenology of TMD evolution: recent progress

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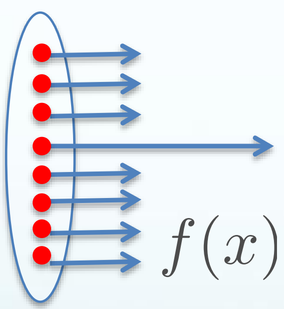
Outline

- Introduction on TMD evolution
- Phenomenology of TMD evolution
- Summary and outlook

New structure of nucleon

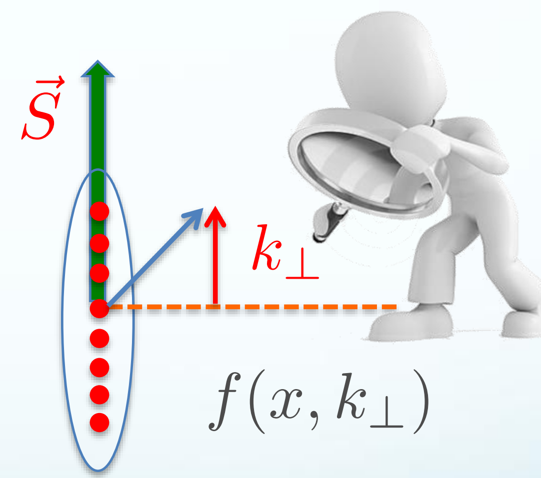
- TMDs provide new structure of nucleon – 3D structure: both longitudinal + transverse momentum dependent structure (confined motion in a nucleon)

Transverse Momentum Dependent parton distribution (TMDs)



$$p_a \approx xP_A$$

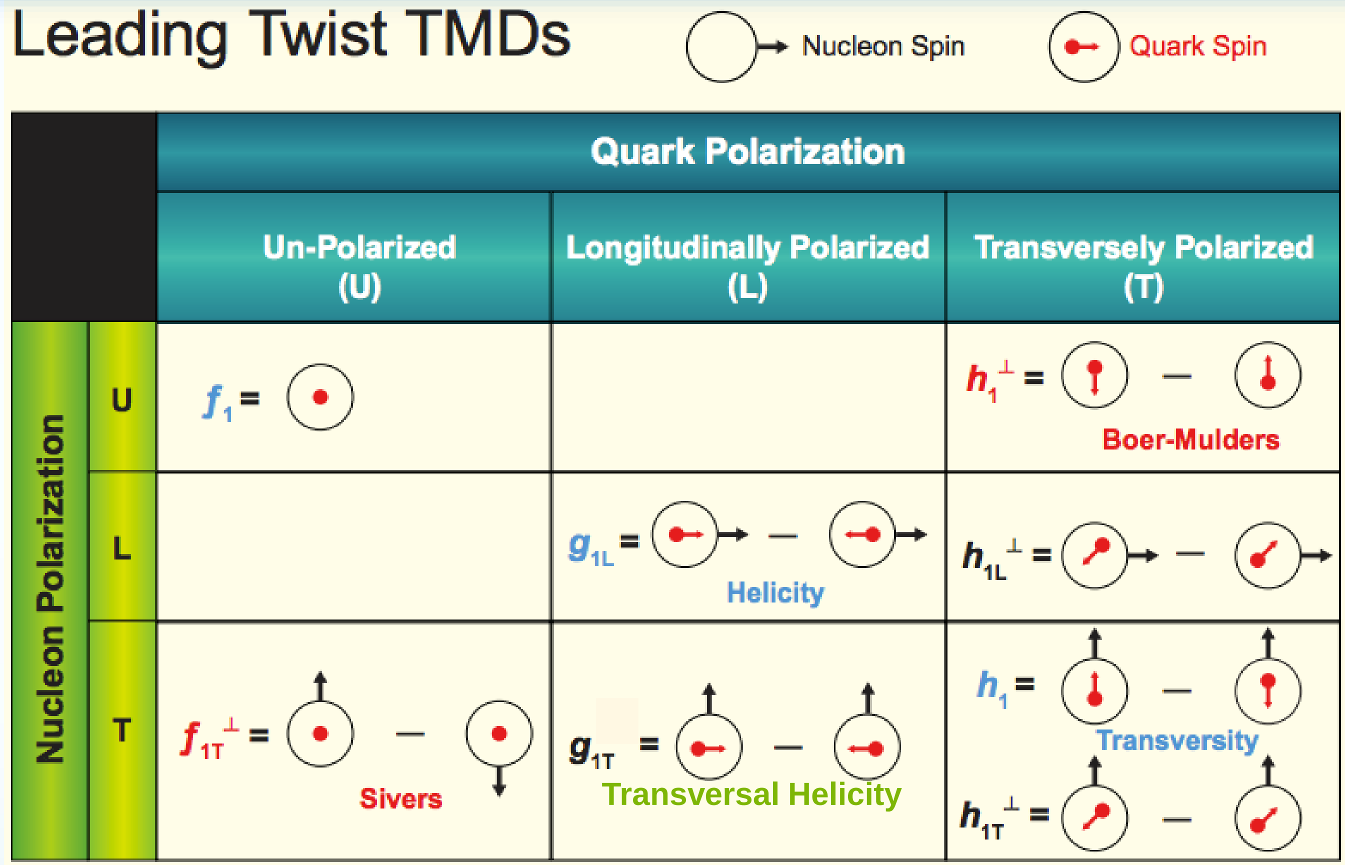
Longitudinal motion only



Longitudinal + transverse motion

TMDs: rich quantum correlations

TMD parton distribution



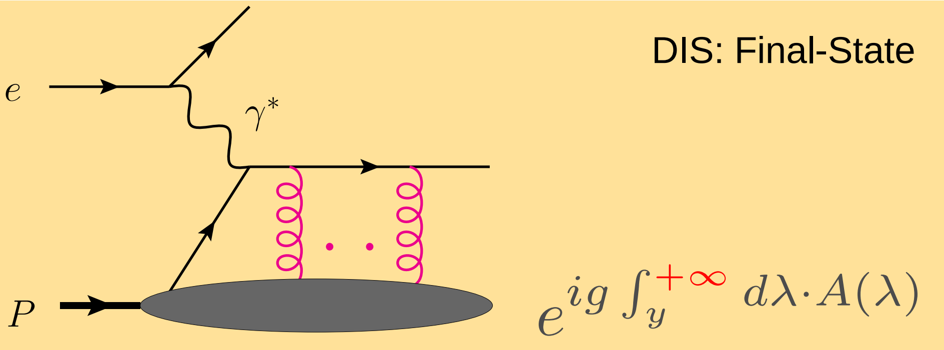
TMD fragmentation function

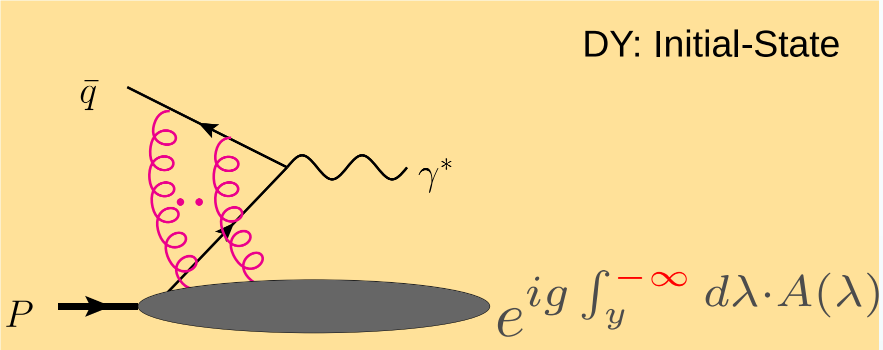
	Quark Polarization U	Quark Polarization L	Quark Polarization T
Pion	D_1		$H_1^{\perp}$ **Collins**

Universality and TMD evolution

- Two most important properties of TMDs
  - Universality: TMD *might not be universal* when probed through different hard scattering processes

Sivers function $f_{1T}^{\perp\text{DIS}}(x,k_\perp) = -f_{1T}^{\perp\text{DY}}(x,k_\perp)$



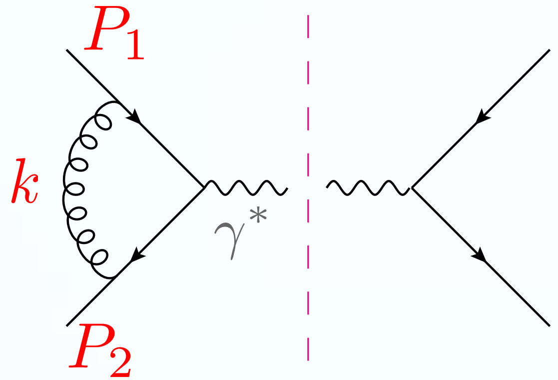


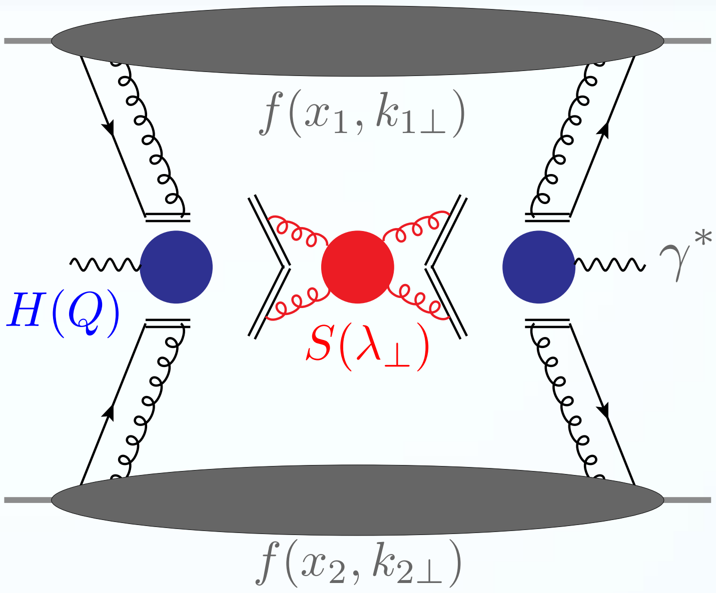
  - TMD evolves

$$F(x,k_\perp,Q_i) \longrightarrow F(x,k_\perp,Q_f)$$

TMD factorization in a nut-shell

- Drell-Yan: $p + p \to [\gamma^* \to \ell^+ \ell^-] + X$





Factorization of regions:
(1) k//P_1, (2) k//P_2, (3) k soft, (4) k hard

- Factorized form and mimic "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2k_{1\perp}\, d^2k_{2\perp}\, d^2\lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

$$F(x, b) = f(x, b)\sqrt{S(b)}$$

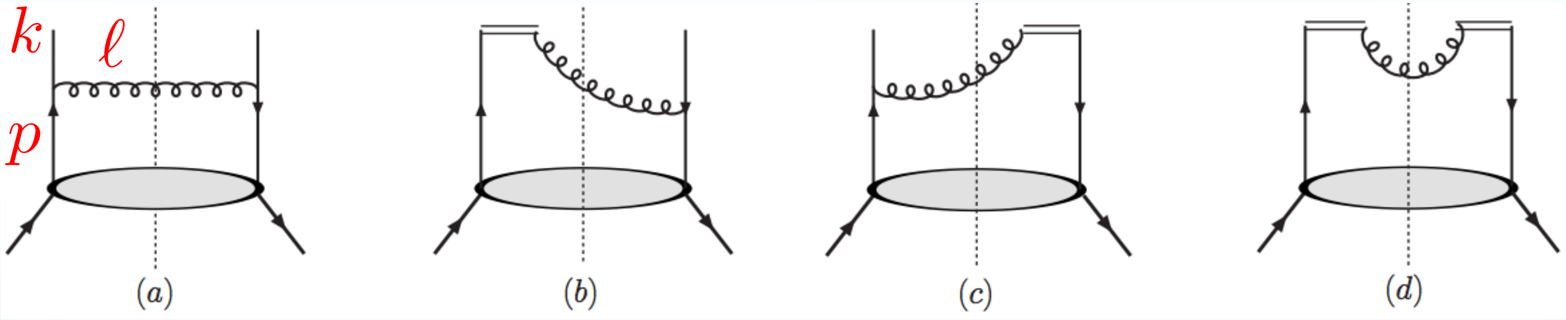
$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)$$

mimic "parton model"

Divergence and evolution

- Divergence leads to evolution
  - Ultraviolet divergence: renormalization group equation, e.g. running of coupling constant
  - Collinear divergence: DGLAP evolution of collinear parton distribution function, fragmentation function, semi-inclusive jet function

    Kang, Ringer, Vitev, arXiv:1606.06732
  - Rapidity divergence (light-cone singularity): TMD evolution
- What is rapidity divergence?



$$f_{q/q}(x, k_\perp^2) = \frac{\alpha_s}{2\pi^2}\frac{1}{k_\perp^2}\left[\frac{2z}{1-z} + (1-z)\right]$$

$$z = \frac{k^+}{p^+} \qquad z \to 1 \Leftrightarrow \ell^+ \to 0 \qquad y = \frac{1}{2}\ln\frac{\ell^+}{\ell^-} \to -\infty$$

Different ways to regularize rapidity divergences

- There are different ways to regularize rapidity divergences
  - Off-light-cone — Collins, Soper 79, …
  - δ-regulator — Chiu, Fuhrer, Hoang, Kelley, Manohar, 09, Echevarria, Idilbi, Scimemi, 11, …
  - Analytic regulator — Becher, Bell, 11, …
  - Rapidity regulator — Chiu, Jain, Neill, Rothstein, 11, 12, …
  - Exponential regulator — Li, Neill, Zhu, 16, …

- Rapidity regulator

$$W_n = \sum_{\text{perms}} \exp\left[-\frac{g w^2}{\bar{n}\cdot\mathcal{P}} \frac{|\bar{n}\cdot\mathcal{P}_g|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n\right]$$

$$S_n = \sum_{\text{perms}} \exp\left[-\frac{g w}{n\cdot\mathcal{P}} \frac{|2\mathcal{P}_{g3}|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s\right]$$

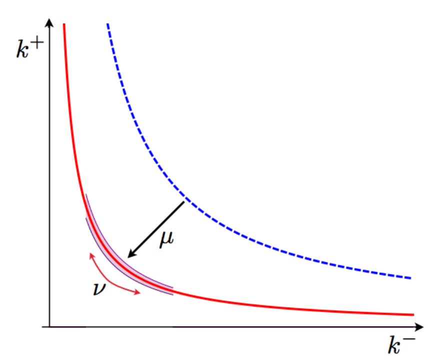
$$\int \frac{dk^+}{k^+} \to \int \frac{dk^+}{k^+} \left|\frac{\nu}{p^+}\right|^{\eta}$$

$$f_{q/q}(x, k_\perp^2) = \frac{\alpha_s}{2\pi^2}\Gamma(1+\epsilon)e^{\gamma_E \epsilon}\frac{1}{\mu^2}\left(\frac{\mu^2}{k_\perp^2}\right)^{1+\epsilon}\left[\frac{2z}{(1-z)^{1+\eta}}\left(\frac{\nu}{p^+}\right)^{\eta} + (1-z) - \epsilon(1-z)\right]$$

TMD evolution in b-space

- Quark TMD at one loop

$$f_{q/q}(x,b) = \frac{\alpha_s}{2\pi} C_F \left\{ \left[\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon} \ln\frac{\nu}{p^+} + \frac{3}{2}\frac{1}{\epsilon} \right] \delta(1-x) + \left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{\mu_b^2} \right) P_{qq}(x) + \left[2\ln\frac{\mu^2}{\mu_b^2}\ln\frac{\nu}{p^+} + \frac{3}{2}\ln\frac{\mu^2}{\mu_b^2} \right] \delta(1-x) + (1-x) \right\}$$



- Soft factor

$$S(b) = \frac{\alpha_s}{2\pi} C_F \left\{ \frac{4}{\eta} \left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln\frac{\mu^2}{\mu_b^2} - \ln\frac{\nu^2}{\mu_b^2} \right) + \left[-2\ln\frac{\mu^2}{\mu_b^2}\ln\frac{\nu^2}{\mu_b^2} + \ln^2\frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{6} \right] \right\}$$

$$\mu_b = 2e^{-\gamma_E}/b$$

- Interesting features
  - Rapidity divergence cancels in $F_{q/q}^{\rm sub}(x,b) = f_{q/q}(x,b)\sqrt{S(b)}$
  - $f_{q/q}(x, b)$ and $S(b)$ lives in the same $\mu \sim \mu_b$, but different rapidity scale $\nu \sim p^+, \mu_b$

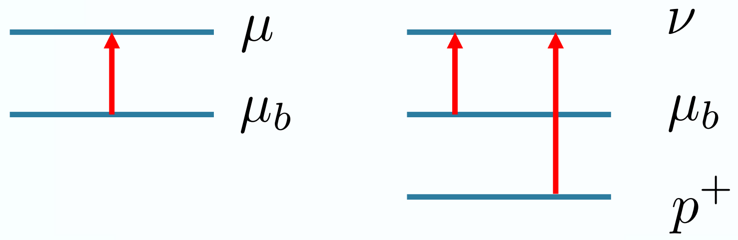
- Two evolution equations: μ-RG and ν-RG

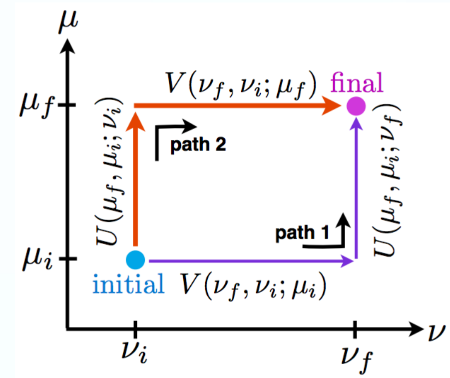
$$\mu\frac{d}{d\mu}\ln f_{q/q}(x,b) = \gamma_\mu^f \qquad \mu\frac{d}{d\mu}\ln S(b) = \gamma_\mu^S$$

$$\nu\frac{d}{d\nu}\ln f_{q/q}(x,b) = \gamma_\nu^f \qquad \nu\frac{d}{d\nu}\ln S(b) = \gamma_\nu^S$$

TMD evolution in b-space

- Solution of TMD evolution equations





  - The well-known CSS solution

$$F(x,b;Q_f) = F(x,b;Q_i)\exp\left\{-\int_{Q_i}^{Q_f}\frac{d\mu}{\mu}\left(A\ln\frac{Q_f^2}{\mu^2}+B\right)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-\int_{c/b}^{Q_i}\frac{d\mu}{\mu}A}$$

$$A = \sum_{n=1} A^{(n)}\left(\frac{\alpha_s}{\pi}\right)^n, \qquad B = \sum_{n=1} B^{(n)}\left(\frac{\alpha_s}{\pi}\right)^n$$

Only valid for small b

Collins-Sopoer-Sterman papers
Kang, Xiao, Yuan, PRL 11,
Aybat, Rogers, Collins, Qiu, 12,
Aybat, Prokudin, Rogers, 12,
Sun, Yuan, 13,
Echevarria, Idilbi, Schafer, Scimemi, 13,
Echevarria, Idilbi, Kang, Vitev, 14,
Kang, Prokudin, Sun, Yuan, 15, 16, …

TMD evolution contains non-perturbative component

- Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation
  - Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q) = \frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, …

- Eventually evolved TMDs in b-space

$$F(x,b;Q) \approx C\otimes F(x,c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

- $C\otimes F(x,c/b^*)$ → longitudinal/collinear part
- $\exp\{\ldots\}$ → transverse part
- $\exp\left(-S_{\text{non-pert}}(b,Q)\right)$ →
  - ✓ Non-perturbative: fitted from data
  - ✓ The key ingredient – ln(Q) piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract the key ingredient for the non-perturbative part

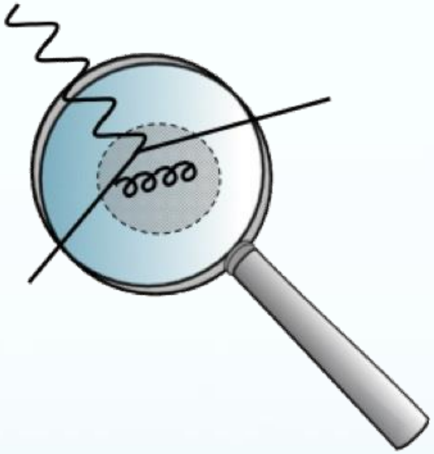
TMD evolves

- Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$F(x, Q)$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative



TMDs

$F(x, k_\perp; Q)$

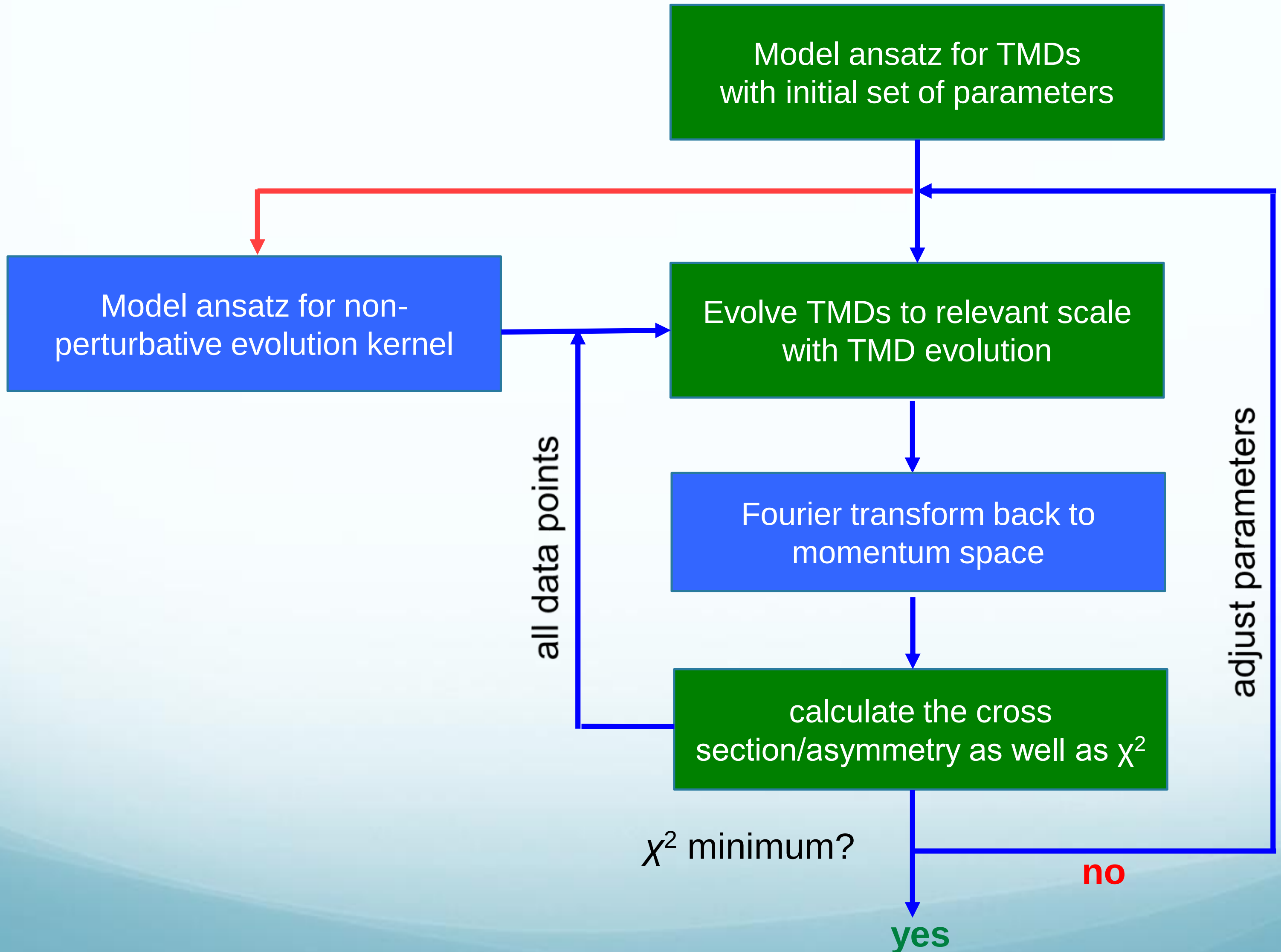
- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i)$$
$$\downarrow$$
$$R^{\rm coll}(x, Q_i, Q_f)$$
$$\downarrow$$
$$F(x, Q_f)$$

$$F(x, k_\perp, Q_i)$$
$$\downarrow$$
$$R^{\rm TMD}(x, k_\perp, Q_i, Q_f)$$
$$\downarrow$$
$$F(x, k_\perp, Q_f)$$

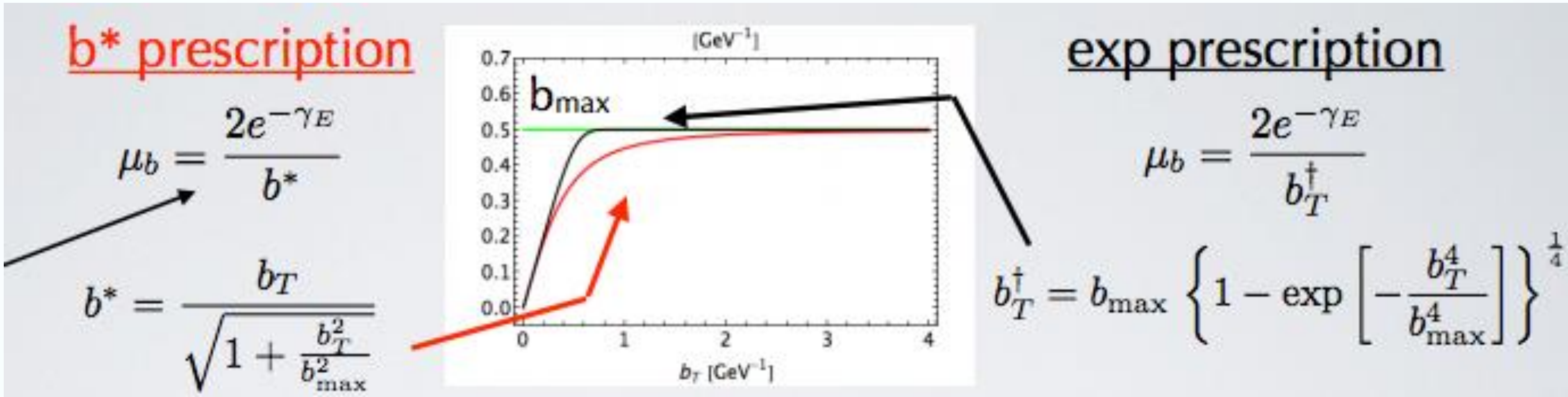
TMD global analysis

- Outline of a TMD global analysis: numerically more heavy



Different treatments at large b

- In terms of b* prescription (see also other proposals Qiu, Vogelsang)



- Non-perturbative Sudakov factor

$$\exp\left[-g_2 b^2 \ln(Q/Q_0) + \cdots\right]$$

CSS, Echevarria, Idlibi, Kang, Vitev, 14, …

$$\exp\left[-g_2 \ln(b/b^*) \ln(Q/Q_0) + \cdots\right]$$

$$\frac{1}{2}\ln\left(1+\frac{b^2}{b_{\max}^2}\right)$$

Aidala, Field, Gamberg, Rogers, 1401.2654, Sun, Isaacson, Yuan, Yuan, 1406.3073

$$\exp\left\{-g_0(b_{\max})\left[1-\exp\left(-\frac{C_F\alpha_s(\mu_{b_*})b^2}{\pi g_0(b_{\max})b_{\max}}\right)\right]\right\}$$

Collins, Rogers, 1412.3820

Different fits to date

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam,Bilbao) arXiv:1309.3507	No evo	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2016	NLL	✔	✔	✔	✔	8156

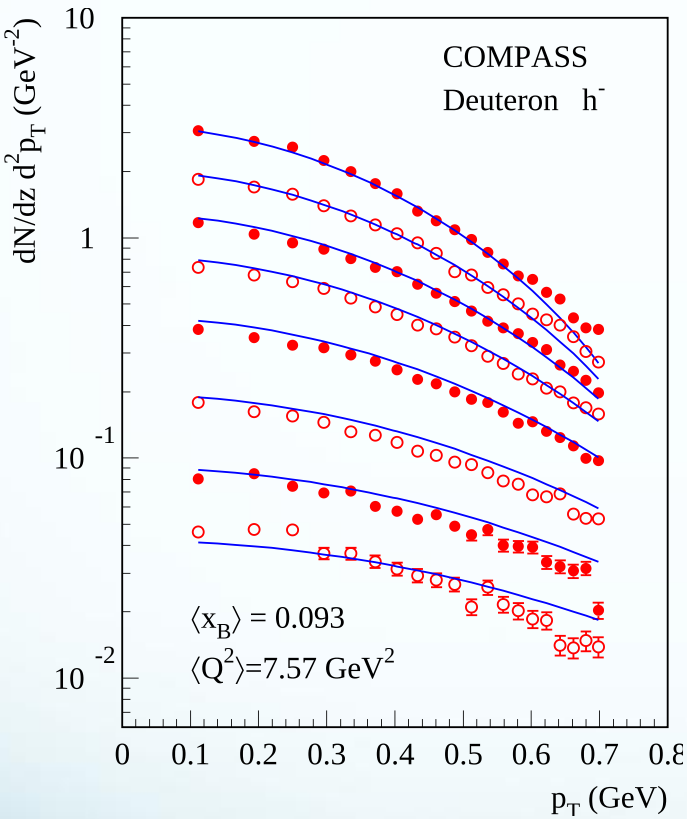
Taken from Bacchetta, QCD Evolution Workshop 2016

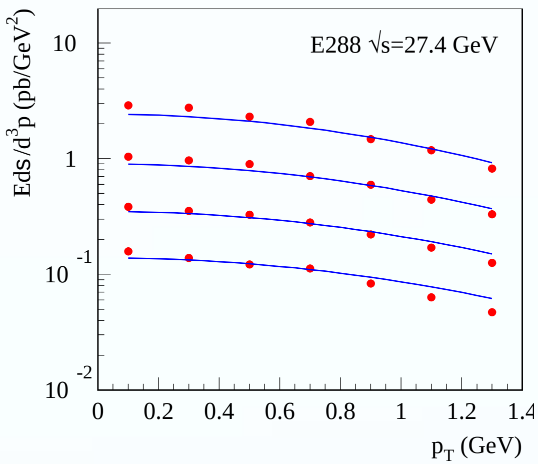
- ✓ It is easier to fit either SIDIS or DY, but quite difficult to fit both
- ✓ Two groups tried very hard, but the good quality of $\chi 2$ is achieved with scarifying the overall normalization of SIDIS cross section (has to multiply a K factor ~ 2)

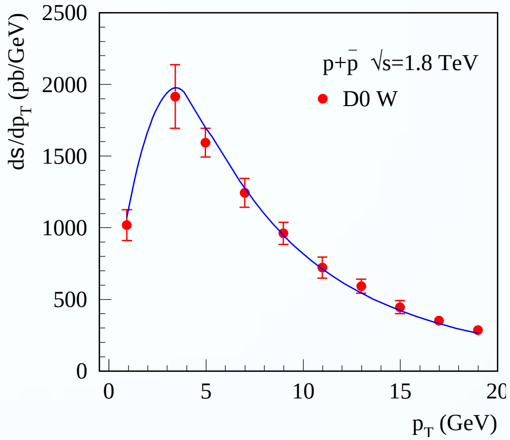
Try both SIDIS and DY/W/Z: EIKV 2014

- SIDIS, DY, and W/Z

Echevarria, Idilbi, Kang, Vitev, 14





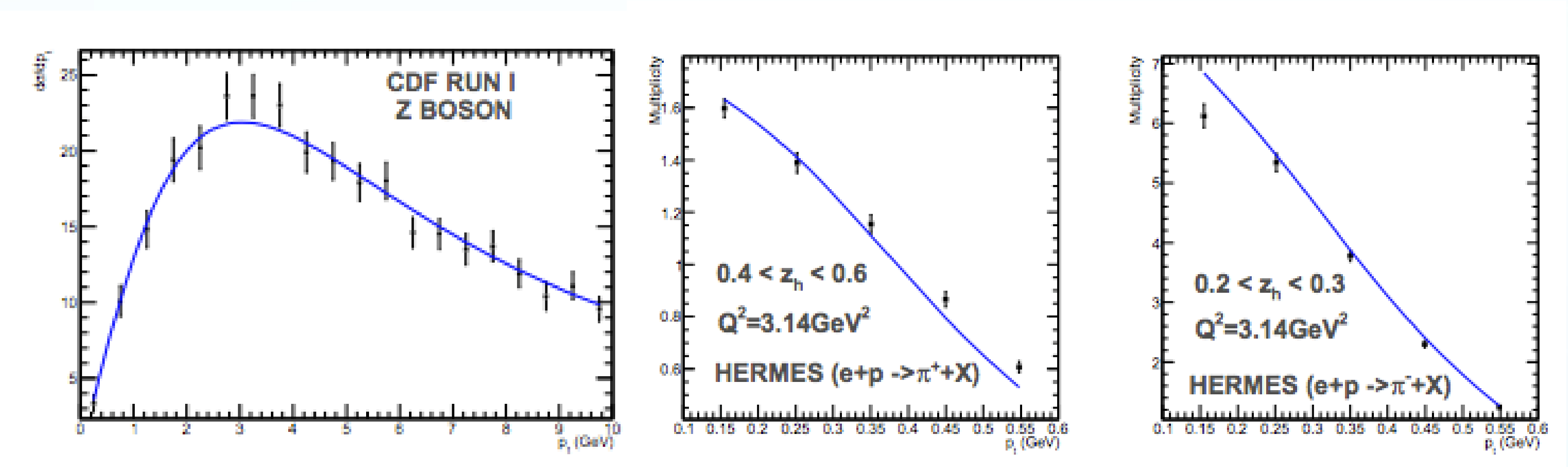


- Works reasonably for SIDIS, DY, and W/Z in all the energy ranges
- Look closer at DY, not so good

Another try: Sun-Isaacson-Yuan-Yuan 2014

- A new fit with DY, not SIDIS

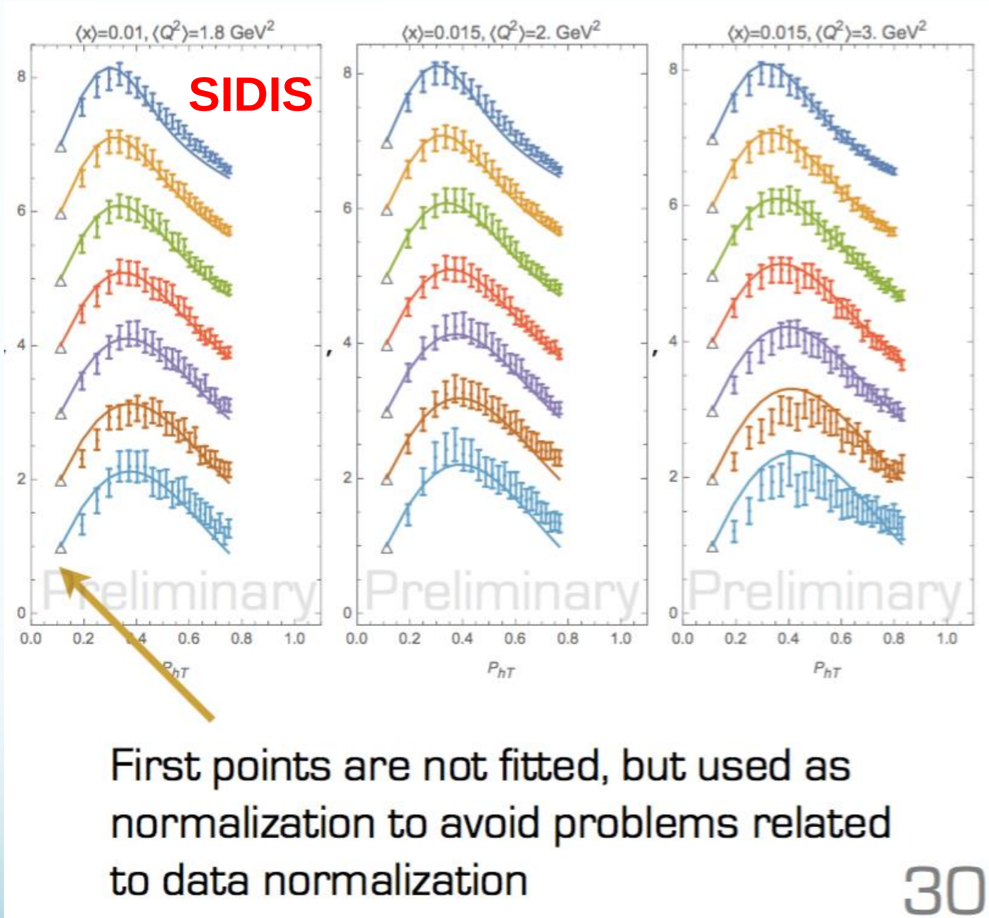
Sun, Isaacson, Yuan, Yuan, 1406.3073

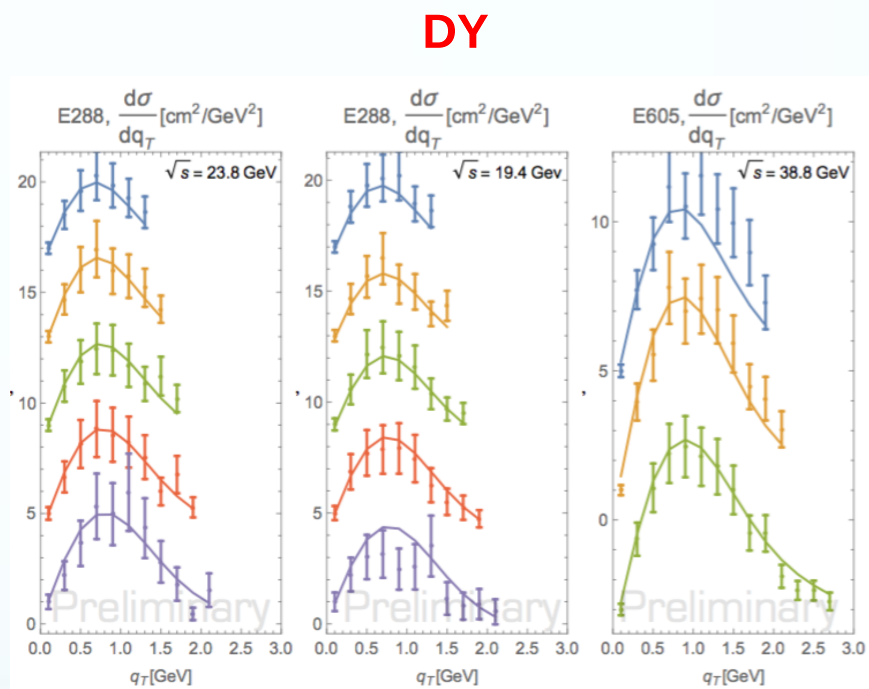


- Seems rather well for SIDIS multiplicity, though requires additional K factor ~ 2 for multiplicity distribution

New fit: Pavia group

- K-factor is needed for SIDIS

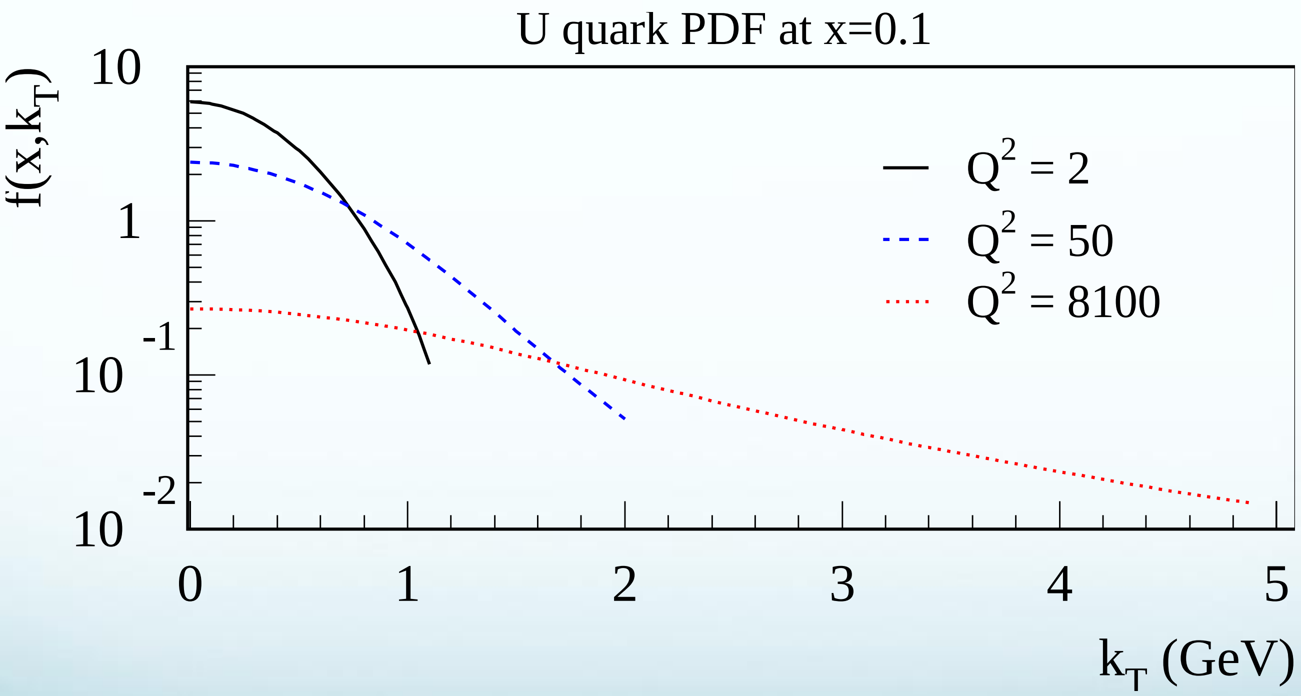




QCD evolved unpolarized TMD

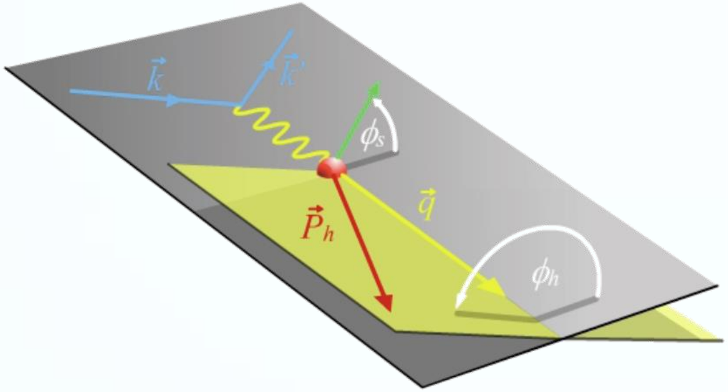
- What evolution does
  - Spread out the distribution to much larger kt
  - At low kt, the distribution decreases due to this spread

Based on Echevarria, Idilbi, Kang, Vitev, 14



Sivers asymmetry from SIDIS

- Sivers asymmetry has been measured in SIDIS process: HERMES, COMPASS, JLab



$$\ell + p^{\uparrow} \to \ell' + \pi(p_T) + X$$

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2)\left[F_{UU} + \sin(\phi_h - \phi_s)\, F_{UT}^{\sin(\phi_h - \phi_s)} + \ldots\right]$$

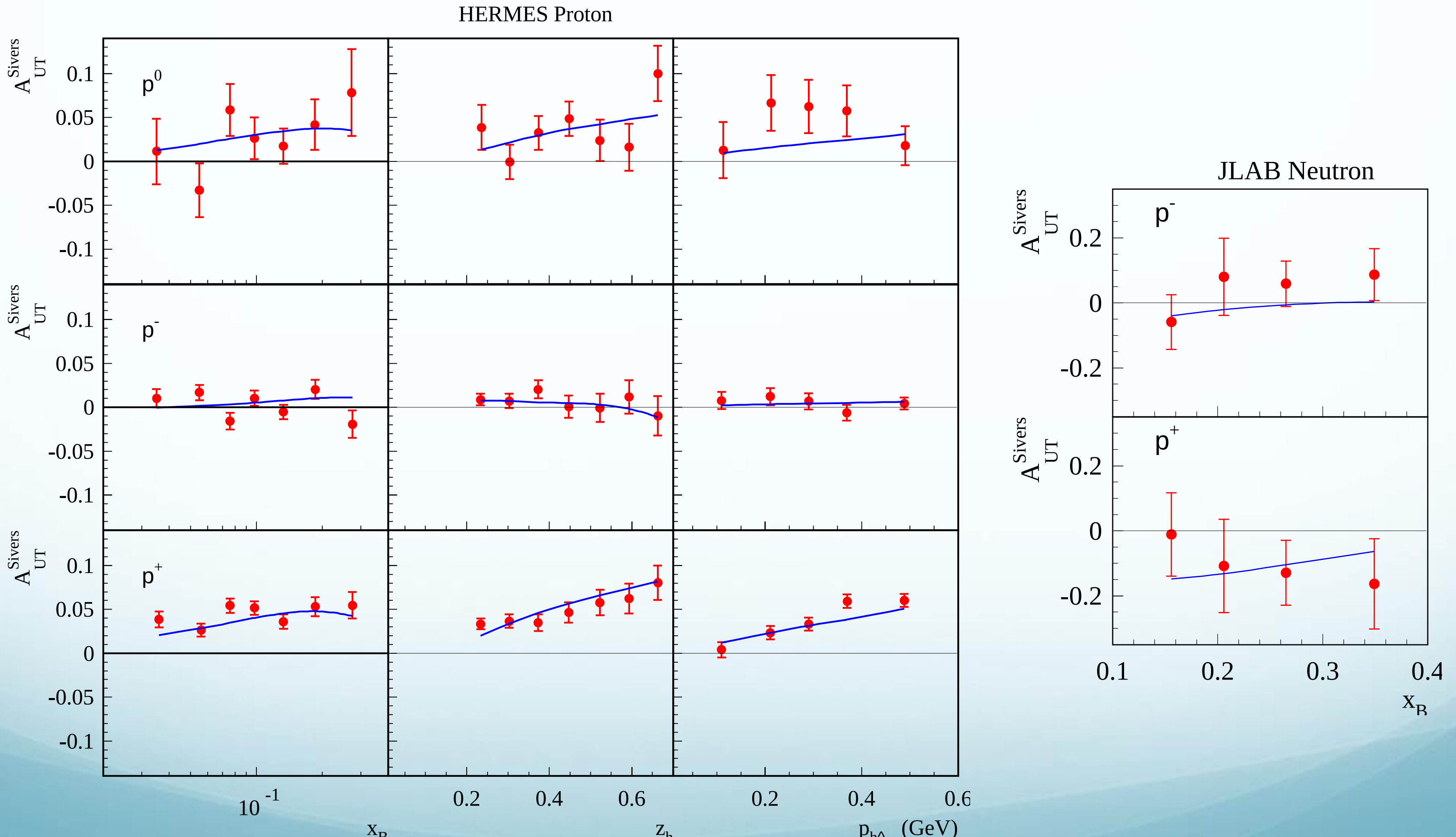
$$F_{UT}^{\sin(\phi_h - \phi_s)} \sim f_{1T}^{\perp q}(x_B, k_\perp) D_{h/q}(z_h, p_\perp)$$

Sivers function

Sivers function with energy evolution

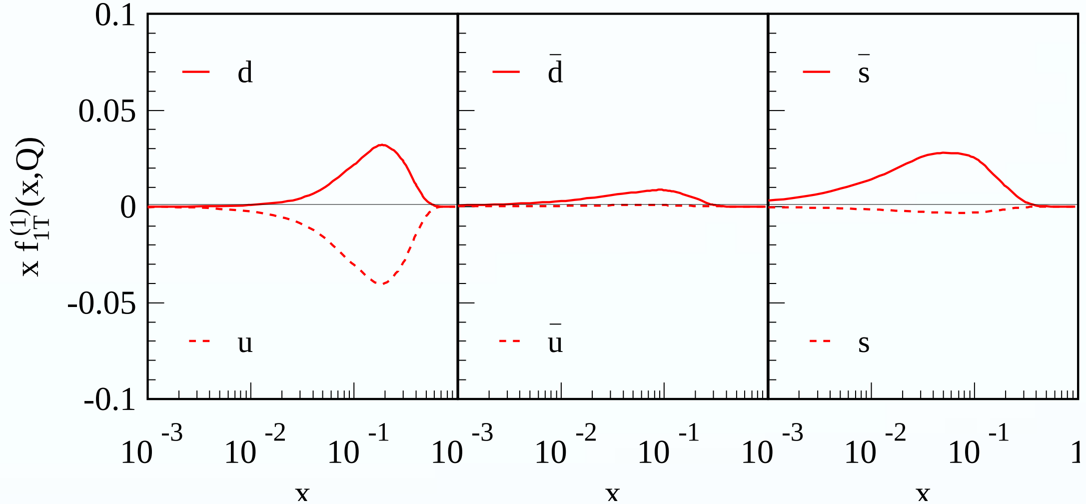
- Example of the fit: JLab, HERMES, COMPASS

Echevarria, Idilbi, Kang, Vitev, 14

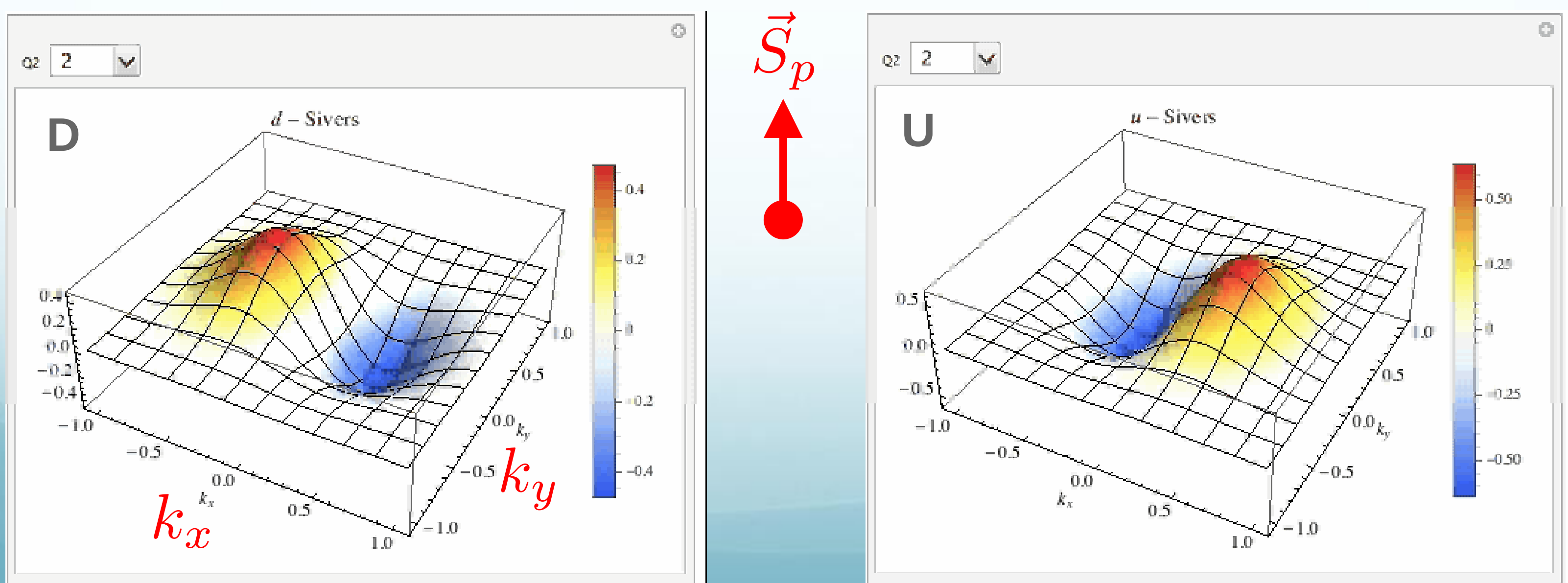


Extracted Sivers function and evolution

- χ^2/d.o.f. = 1.3, the collinear part is plotted: only u and d valence quark Sivers are constrained

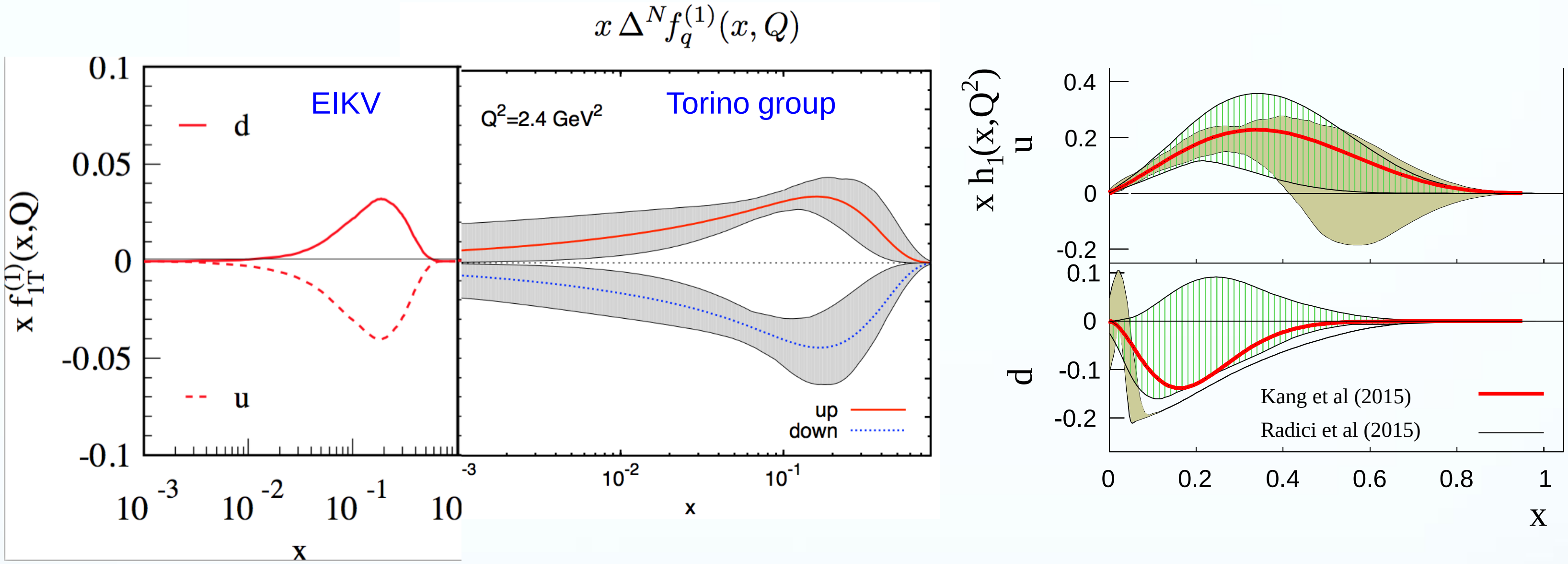


- Visualization: positive = more quark moves to the left (Q^2=2 – 100 GeV^2)



Status

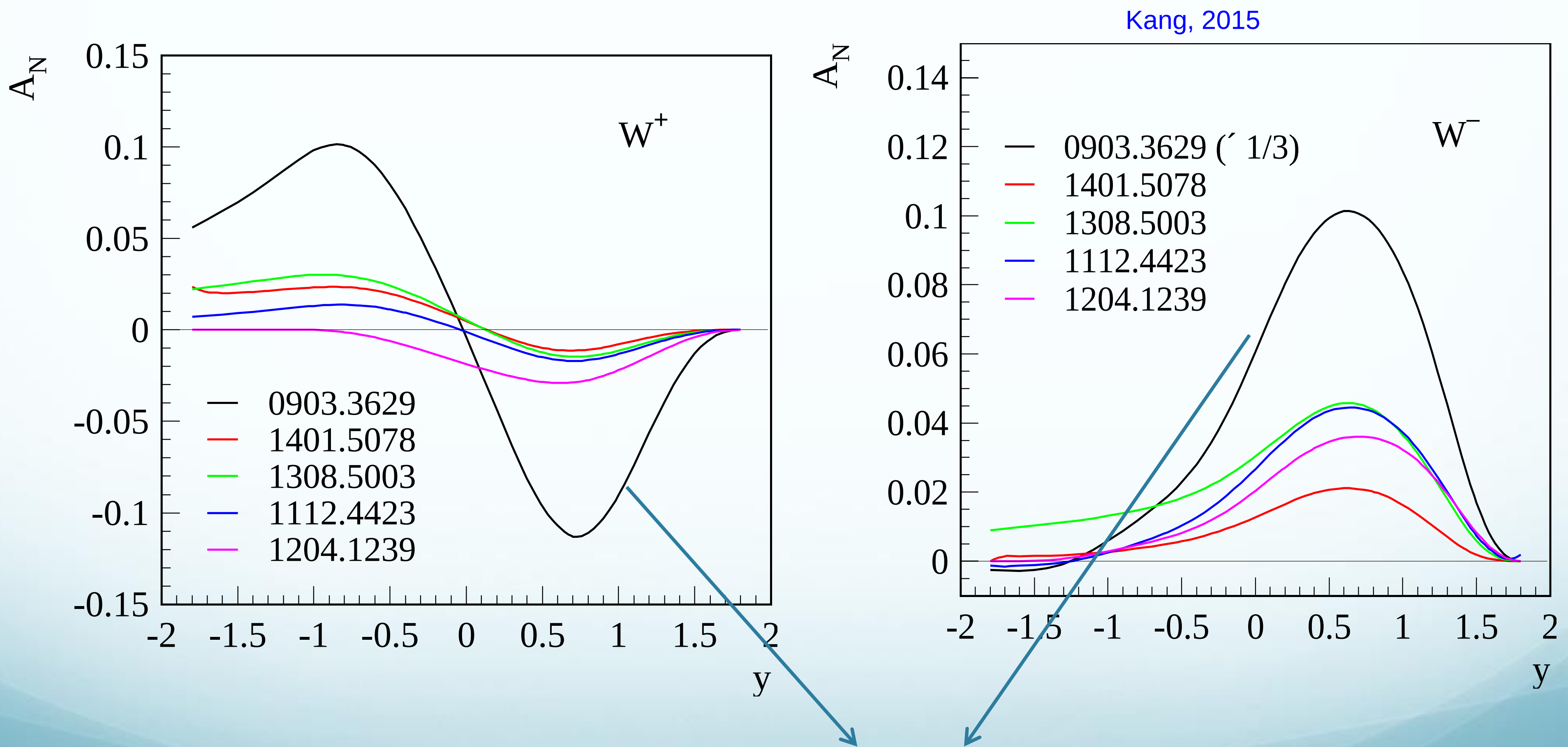
- Within the region constrained by the experimental data, the spin-dependent TMDs seem to be rather consistent among different groups



- TMD evolution cancels between the ratios??
- Need more data on the absolute cross section
- However, the extrapolations can be very different

Uncertainty in the evolution formalism

- Even the evolution formalism itself has large room to improve – non-perturbative Sudakov needs further improvement

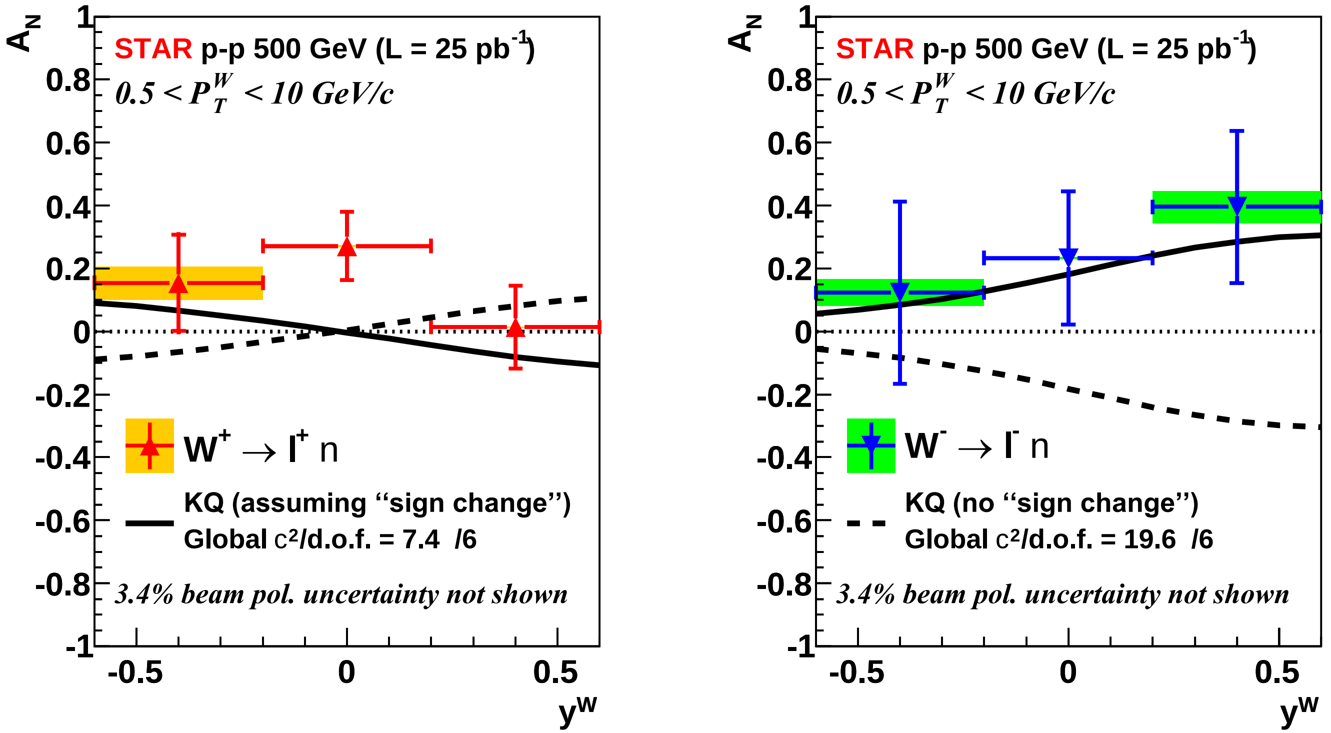


w/o TMD evolution

Experimental evidence of sign change

- STAR measurements: the data favors sign change
- Both theory and experiment has large uncertainty: hope to be improved in the near future (2017 run)

STAR, PRL 16, arXiv:1511.06003

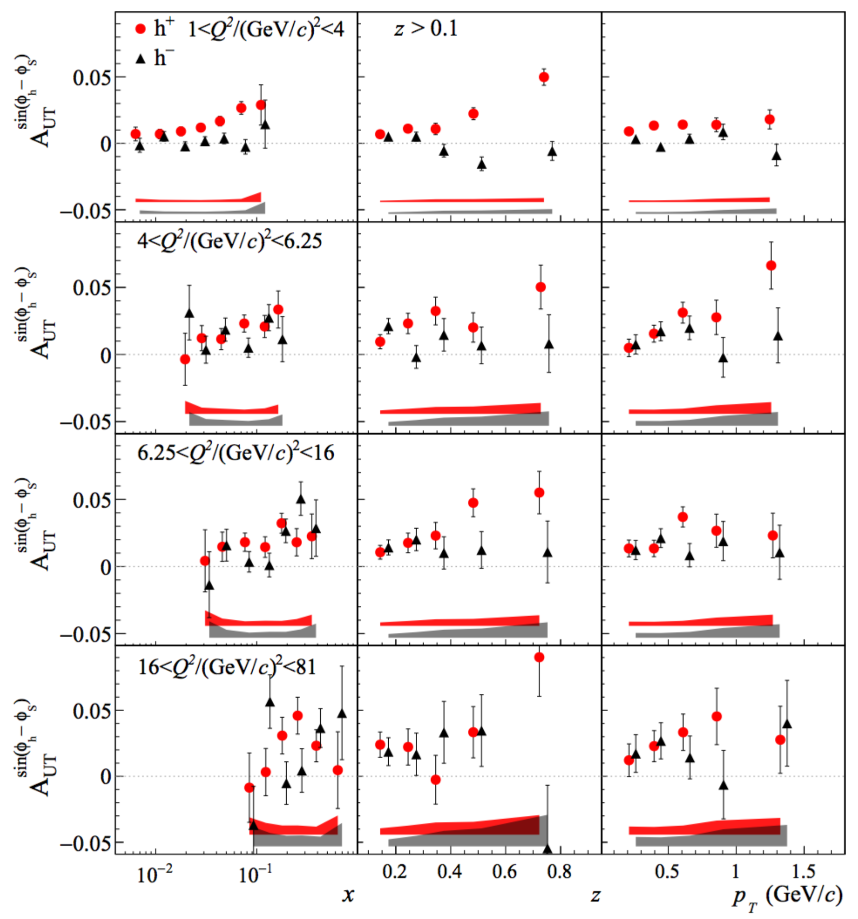


KQ = Kang, Qiu, PRL09 – **w/o evolution**

- Looking forward to see the result from COMPASS!

Another new data: COMPASS

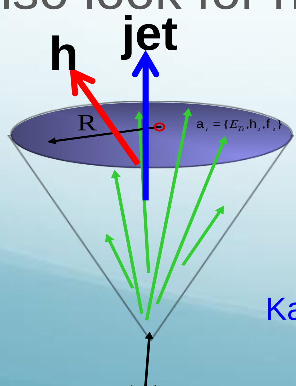
- Sivers asymmetry in DY scale region



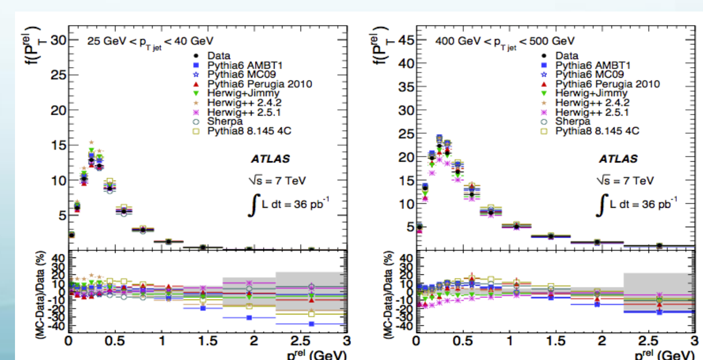
COMPASS, arXiv:1609.07374

Further constrain TMD evolution

- The absolute cross section (not the ratio) should certainly be better
- We will see how the asymmetry (ratio) at a broad range of Q can constrain TMD evolution formalism: low Q^2 SIDIS data in the past: $Q^2 \sim 1 - 10$ GeV^{2}N; new SIDIS data at DY scale: $Q^2 \sim 1 - 81$ GeV^2; W/Z asymmetry at $(80 - 90\ \text{GeV})^2$
- How to move forward
  - Perform data analysis directly in b-space — Boer, Gamberg, Prokudin, et.al.
  - Perform evolution directly in momentum space — Kang, in preparation
  - Improve the current non-perturbative model — Collins, Rogers; Kang, Qiu; Prokudin, Yuan; …
  - Of course improve the controllable perturbative part — Stewart et.al. N^{3}LL ($B^{(3)}$) See also Echevarria's talk
- Also look for new channels to probe TMDs



Kang, Ringer, Vitev, 1606.07063



Summary

- Study on TMDs are extremely active in the past few years, lots of progress made, though still large uncertainty on TMD evolution
- With great excitement, we look forward to the future experimental results from COMPASS/RHIC, as well as Jefferson Lab, of course also LHC

Proposal for a Topical Collaboration in Nuclear Theory for the Coordinated Theoretical Approach to Transverse Momentum Dependent Hadron Structure in QCD

January 1, 2016 - December 31, 2020

TMD Collaboration

Thank you!