TMDs: *entering the precision era*

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SPIN 2016

University of Illinois (Urbana-Champaign, US)

25-30 September 2016

In collaboration with

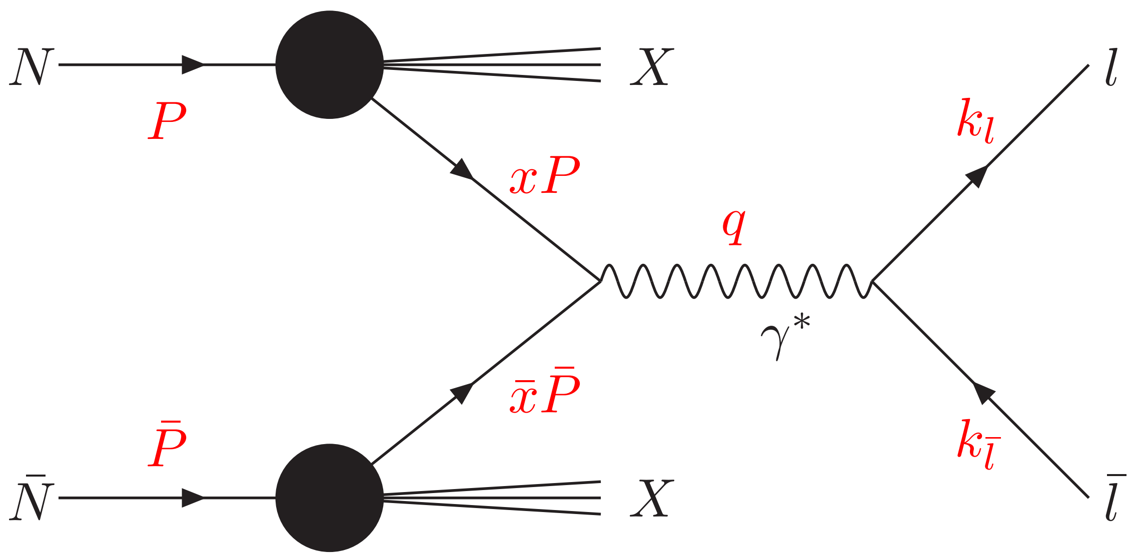
I. Scimemi and A. Vladimirov

[MGE, Scimemi, Vladimirov, 1509.06392]
[MGE, Scimemi, Vladimirov, 1511.05590]
[MGE, Scimemi, Vladimirov, 1604.07869]

1. *Definition of TMDs*
2. *Evolution of TMDs at NNLL'*
3. *Refactorization of TMDs*
4. *Soft function at NNLO*
5. *Unpolarized TMDs at NNLO*
6. *Conclusions & Outlook*

TMD factorization

- *Take Drell-Yan as a benchmark process:*



$$q^2 = Q^2 \gg q_T^2$$

q_T large: perturbative origin
q_T small: non-perturbative origin

- *Same story applies to all processes with "at most two hadrons" (for the moment):*

$$H_1 + H_2 \to h + X$$
$$H_1 + H_2 \to [Q\bar{Q}] + X$$
$$e^- + H_1 \to e^- + H_2 + X$$
$$e^- + H_1 \to e^- + Q + \bar{Q} + X$$
$$e^+ + e^- \to H_1 + H_2 + X$$
$$e^+ + e^- \to [Q\bar{Q}] + H_1 + X$$

etc...

- *It's important to have a correct definition in order to properly connect different processes*

TMD factorization: soft and collinear

- The cross-section is given in terms of collinear and soft matrix elements:

$$d\sigma = \sigma_0(\mu)\, H(Q^2,\mu)\, dy\, \frac{d^2q_\perp}{(2\pi)^2} \int d^2y_\perp\, e^{-iq_\perp \cdot y_\perp}\, J_n(x_A, y_\perp, \mu)\, S(y_\perp,\mu)\, J_{\bar{n}}(x_B, y_\perp, \mu)$$

$$J_n(0^+, y^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_1} \langle N_1(P,\sigma_1)|\, \bar{\chi}_n(0^+, y^-, \vec{y}_\perp)\, \frac{\bar{n}\!\!\!/}{2}\, \chi_n(0)\, |N_1(P,\sigma_1)\rangle|_{\text{zb subtracted}}$$

$$S(0^+, 0^-, \vec{y}_\perp) = \langle 0|\, Tr\, \bar{T}[S_n^{T\dagger} S_{\bar{n}}^{T}](0^+, 0^-, \vec{y}_\perp) T[S_{\bar{n}}^{T\dagger} S_n^{T}](0)\, |0\rangle$$

$$J_{\bar{n}}(y^+, 0^-, \vec{y}_\perp) = \frac{1}{2}\sum_{\sigma_2} \langle N_2(\bar{P},\sigma_2)|\, \bar{\chi}_{\bar{n}}(0)\, \frac{n\!\!\!/}{2}\, \chi_{\bar{n}}(y^+, 0^-, \vec{y}_\perp)\, |N_2(\bar{P},\sigma_2)\rangle|_{\text{zb subtracted}}$$

But these matrix elements individually are ill-defined!!
They contain mixed UV/Rapidity divergences...

Definition of TMDs

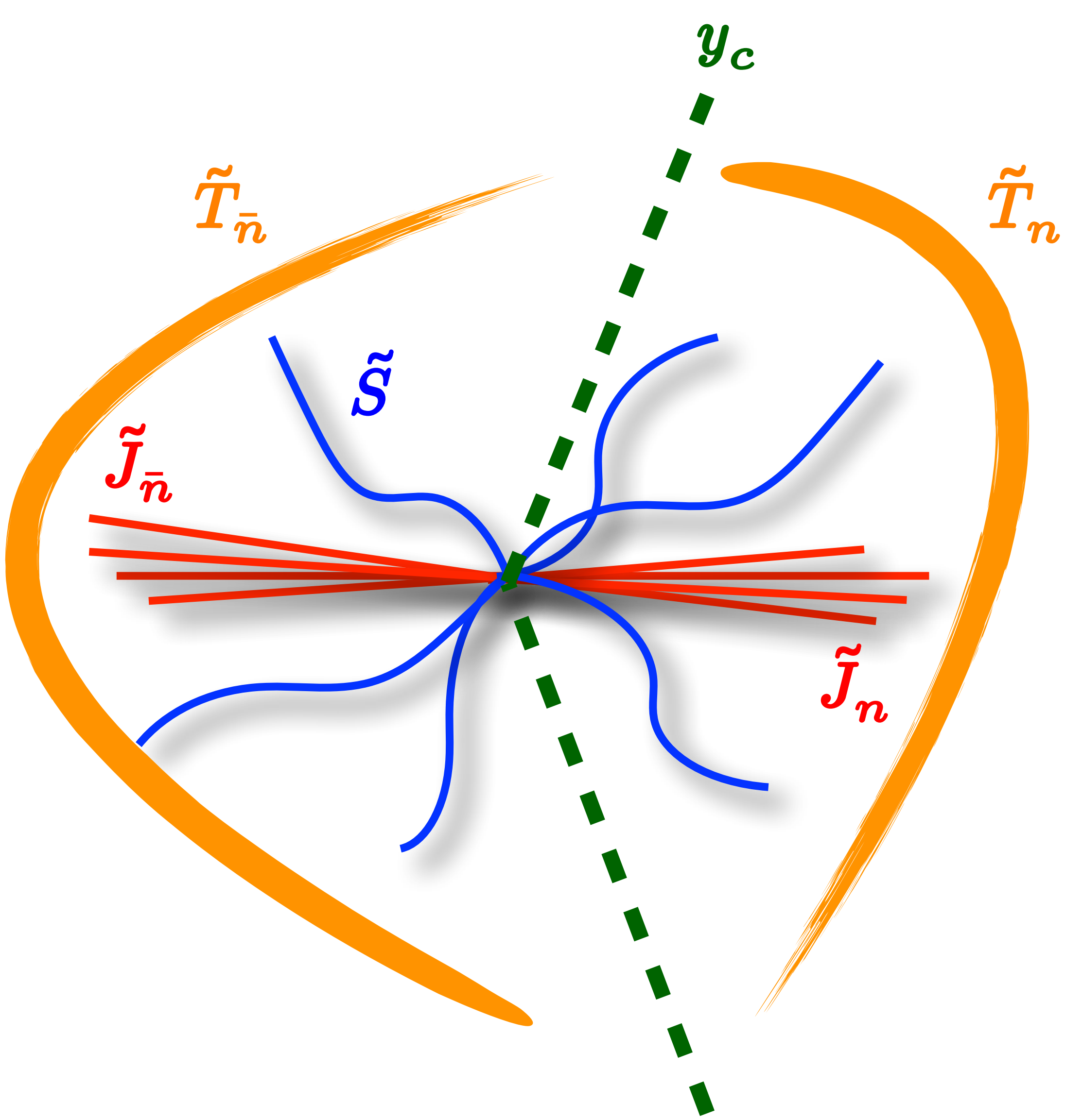
- *Their definition is a bit tricky:*

$$k_n \sim Q(1,\lambda^2,\lambda) \qquad k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim Q^2\lambda^2$$

Same invariant mass!

$$k_{\bar{n}} \sim Q(\lambda^2,1,\lambda)$$
$$k_s \sim Q(\lambda,\lambda,\lambda) \qquad y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

Different rapidities (mixed under boosts)



Cancel spurious rapidity divergences

$$\zeta_A = (p^+)^2 e^{-2y_c}$$

$$\tilde{T}_n(x_A, \vec{k}_{n\perp}, S_A; \zeta_A, \mu) = \tilde{J}_n \sqrt{\tilde{S}}$$

$$\tilde{T}_{\bar{n}}(x_B, \vec{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu) = \tilde{J}_{\bar{n}} \sqrt{\tilde{S}}$$

$$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

[MGE, Idilbi, Scimemi 1111.4996, 1211.1947, 1402.0869]
[MGE, Kasemets, Mulders, Pisano 1502.05354]
[Collins' book '11]

Universal TMD evolution kernel (1/3)

- *TMDs depend on two scales:* ***renormalization*** *and* ***rapidity*** *scales*
- *We know the evolution of all (un)polarized quark/gluon TMDPDFs and TMDFFs*

$$\tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_{A,f}, \mu_f) = \tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_{A,i}, \mu_i)\, \tilde{R}^j\left(b_T; \zeta_{A,i}, \mu_i, \zeta_{A,f}, \mu_f\right)$$

- *The dependence on the renormalization scale is:*

$$\frac{d}{d\ln\mu}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x, b_\perp, S_A; \zeta_A, \mu) = \gamma_j\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops both for quark and gluon TMDs
[Moch, Vermaseren, Vogt '04, '05]

$$\gamma_j\left(\alpha_s(\mu), \ln\frac{\zeta_A}{\mu^2}\right) = -\Gamma^j_{cusp}(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2} - \gamma^j_{nc}(\alpha_s(\mu))$$

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Universal TMD evolution kernel (2/3)

- *The dependence on the **rapidity** scale is:*

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}^{[pol]}_{j\leftrightarrow A}(x,b_\perp,S_A;\zeta_A,\mu) = -D_j(b_T;\mu)$$

Known at NLO. Recently at NNLO.

Indirect: [Becher, Neubert 1007.4005]
Direct: [MGE, Scimemi, Vladimirov 1511.05590]

[Li, Zhu 1604.01404]

$$\frac{dD_j}{d\ln\mu} = \Gamma^j_{cusp}(\alpha_s(\mu))$$

Cusp anomalous dimension does __not__ completely determine D_j. One needs the soft function (or cross-section)

- *Combining the evolution in both scales:*

$$\tilde{R}^j(b_T;\zeta_{A,i},\mu_i,\zeta_{A,f},\mu_f) = \exp\left[\int_{\mu_i}^{\mu_f}\frac{d\hat{\mu}}{\hat{\mu}}\gamma_j\left(\alpha_s(\hat{\mu}),\ln\frac{\zeta_{A,f}}{\hat{\mu}^2}\right)\right]\left(\frac{\zeta_{A,f}}{\zeta_{A,i}}\right)^{-D_j(b_T;\mu_i)}$$

The evolution itself contains some __non-perturbative input__ (in the D_j term at large b_T)

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Universal TMD evolution kernel (3/3)

- *Currently known perturbative ingredients allow* ***NNLL'*** *evolution:*

$$D = C_F \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n \sum_{k=0}^{n} \mathbf{L}_\mu^k d^{(n,k)}$$

Order	Γ^j_{cusp}	γ^j_{nc}	D_j
LL	α_s	α_s^0	α_s^0
NLL	α_s^2	α_s	α_s
NNLL	α_s^3	α_s^2	α_s^2
NNNLL	α_s^4	α_s^3	α_s^3

??

[Li, Zhu 1604.01404]

$$\begin{aligned} d^{(3,0)} =& \frac{-1}{2} C_A^2 \left(-\frac{176}{3}\zeta_3\zeta_2 + \frac{6392\zeta_2}{81} + \frac{12328\zeta_3}{27} + \frac{154\zeta_4}{3} - 192\zeta_5 - \frac{297029}{729}\right) \\ & - C_A T_r N_f \left(-\frac{824\zeta_2}{81} - \frac{904\zeta_3}{27} + \frac{20\zeta_4}{3} + \frac{62626}{729}\right) - 2T_r^2 N_f^2 \left(-\frac{32\zeta_3}{9} - \frac{1856}{729}\right) \\ & - C_F T_r N_f \left(\frac{-304\zeta_3}{9} - 16\zeta_4 + \frac{1711}{27}\right) \end{aligned}$$

3. Refactorization of TMDs

- k_T dependence is perturbatively calculable when k_T is large:

$$\tilde{T}_{i\leftrightarrow A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^{T}_{i\leftrightarrow j}(x, b_T; \zeta, \mu) \otimes t_{j\leftrightarrow A}(x; \mu) + O(b_T \Lambda_{QCD})$$

- For each TMD we have a different OPE. For example:

$$\tilde{f}_1^{q/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^{f}_{q/j}(\bar{x}, b_T; \zeta, \mu) f_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{h}_1^{\perp g/A\,(2)}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^{h}_{g/j}(\bar{x}, b_T; \zeta, \mu) f_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{g}_{1L}^{g/A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}^{g}_{g/j}(\bar{x}, b_T; \zeta, \mu) g_{j/A}(x/\bar{x}; \mu)$$

$$\tilde{f}_{1T}^{\perp g/A\,(1)}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}_1}{\bar{x}_1} \frac{d\bar{x}_2}{\bar{x}_2} \tilde{C}^{sivers}_{g/j}(\bar{x}_1, \bar{x}_2, b_T; \zeta, \mu) T_{F\,j/A}(x_1/\bar{x}_1, x_2/\bar{x}_2; \mu)$$

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Resummation of large logs

- This is how a resummed TMD looks like:

$$\tilde{T}_{i\leftrightarrow A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x,\hat{b}_T;\mu_b^2,\mu_b) \otimes t_{j\leftrightarrow A}(x;\mu_b)$$
$$\times \exp\left[\int_{\mu_b}^{\mu} \frac{d\hat{\mu}}{\hat{\mu}} \gamma_j\left(\alpha_s(\hat{\mu}), \ln\frac{\zeta}{\hat{\mu}^2}\right)\right] \left(\frac{\zeta}{\mu_b^2}\right)^{-D_j(\hat{b}_T;\mu_b)}$$
$$\times \tilde{T}^{NP}_{i\leftrightarrow A}(x,b_T;\zeta)$$

- General philosophy: only parametrize what **cannot** be calculated
- The non-perturbative part of D_j is **universal** (for all (un)polarized TMD PDFs and FFs)
- The non-perturbative part of D_j seems not well-constrained by current data [MGE, D'Alesio, Melis, Scimemi 1407.3311]
- Higher-order calculations allow better determination of non-perturbative ingredients
- At low b_T the TMDs are neither supposed to be correct (q_T>Q region)
- The determination of non-perturbative pieces is not easy (Fourier transform mixes regions, overlap of perturbative and non-perturbative)

Modified δ-Regulator

[MGE, Scimemi, Vladimirov, 1511.05590]

- *The old-fashion δ-regulator…* [MGE, Idilbi, Scimemi '11]

$$\frac{1}{k^- + i0} \longrightarrow \frac{1}{k^- + i\delta}$$

At NNLO and beyond:

- *Violates non-abelian exponentiation*
- *Zero-bin ≠ Soft Function*

- *Modified δ-regulator:*

TMDPDF: $$P\exp\left[ig\int_0^\infty d\sigma n\cdot A(\sigma n)\right] \to P\exp\left[ig\int_0^\infty d\sigma n\cdot A(\sigma n)e^{-\delta\sigma x}\right]$$

TMDFF: $$P\exp\left[ig\int_0^\infty d\sigma n\cdot A(\sigma n)\right] \to P\exp\left[ig\int_0^\infty d\sigma n\cdot A(\sigma n)e^{-\delta\sigma/z}\right]$$

Soft Function: $$P\exp\left[ig\int_0^\infty d\sigma n\cdot A(\sigma n)\right] \to P\exp\left[ig\int_0^\infty d\sigma n\cdot A(\sigma n)e^{-\delta\sigma}\right]$$

δ-regulator violates gauge properties of Wilson lines. Only δ=0 makes sense.
Note: careful with power divergent integrals!

Of course physics is independent of the regulator!!

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4. TMD Soft Function at NNLO

[MGE, Scimemi, Vladimirov, 1511.05590]

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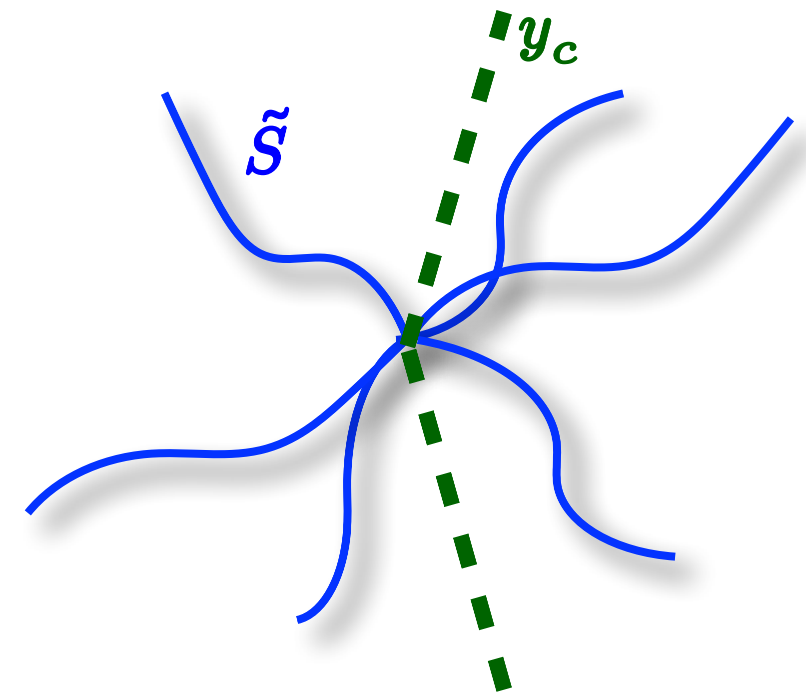
TMD soft function: splitting

$$\tilde{S}(b_T;\mu) = \langle 0| \left[T(S_n^\dagger S_{\bar{n}})\right](b_\perp) \left[\bar{T}(S_{\bar{n}}^\dagger S_n)\right](0) |0\rangle$$

$$\tilde{S}(b_T) = e^{a_s C_a (S^{[1]} + a_s S^{[2]} + \dots)}$$



- *The most important property:*

$$\tilde{S}(b_T) = \exp\left(A(b_T,\epsilon)\ln|\delta^+\delta^-| + B(b_T,\epsilon)\right)$$

- *In general one expects for a diagram that (at finite epsilon):*

$$\text{Diagram} = \mu^{4\epsilon}\left(A_0\delta^{-2\epsilon} + A_1\delta^{-\epsilon}B^{\epsilon} + A_2 B^{2\epsilon}\right) + \mathcal{O}(\delta)$$

$$\delta = \pm\delta^+\delta^- \qquad B = \frac{b_T^2}{4}$$

Virtual diagrams are irrelevant…

$$\lim_{b_T\to 0}\tilde{S} = 0 \qquad A_0 = 0$$

$$\left(\lim_{b_T\to 0}\tilde{S}\right)\Big|_{\delta/b_T\ fixed} = 0 \qquad A_1 = 0$$

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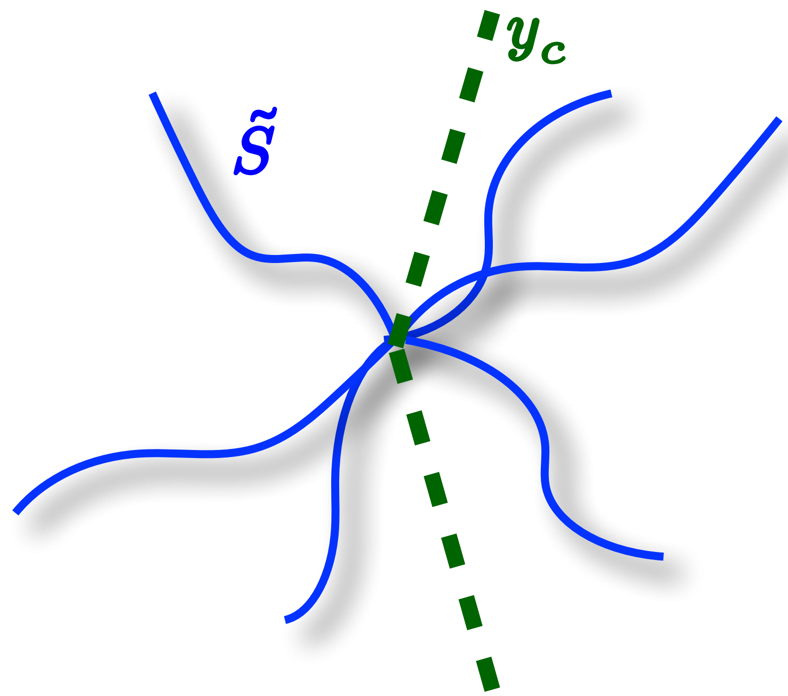

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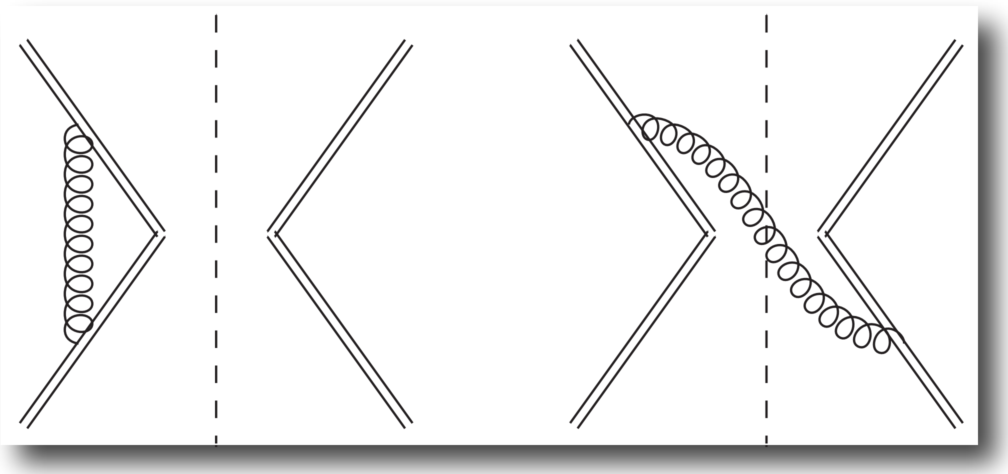
Splitting at NLO

- *The SF relevant for gluon TMDs:*

$$\tilde{S}(b_T;\mu) = \frac{1}{N_c^2-1}\sum_{X_s}\langle 0|\,(S_n^\dagger S_{\bar n})^{ab}(b_\perp)\,|X_s\rangle\,\langle X_s|\,(S_{\bar n}^\dagger S_n)^{ba}(0)\,|0\rangle$$



$$L_T = \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

$$\tilde{S}\left(b_T;\mu;\delta^+,\delta^-\right) = 1 + \frac{\alpha_s C_A}{2\pi}\left[-\frac{2}{\varepsilon_{UV}^2} + \frac{2}{\varepsilon_{UV}}\ln\frac{\delta^+\delta^-}{\mu^2} + L_T^2 + 2L_T\ln\frac{\delta^+\delta^-}{\mu^2} + \frac{\pi^2}{6}\right]$$

$$\tilde{S}\left(b_T;\mu;\delta^+,\delta^-\right) = \tilde{S}_-\left(b_T;\nu,\mu;\delta^-\right)\,\tilde{S}_+\left(b_T;1/\nu,\mu;\delta^+\right)$$

$$\tilde{S}_- = 1 + \frac{\alpha_s C_A}{2\pi}\left[-\frac{1}{\varepsilon_{UV}^2} + \frac{1}{\varepsilon_{UV}}\ln\frac{\nu(\delta^-)^2}{\mu^2} + \frac{1}{2}L_T^2 + \frac{1}{\varepsilon_{UV}}L_T\ln\frac{\nu(\delta^-)^2}{\mu^2} + \frac{\pi^2}{12}\right]$$

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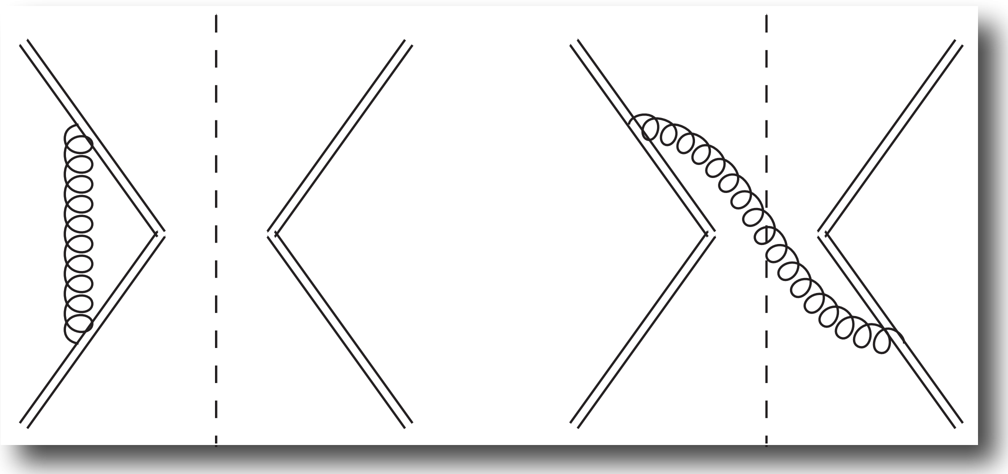

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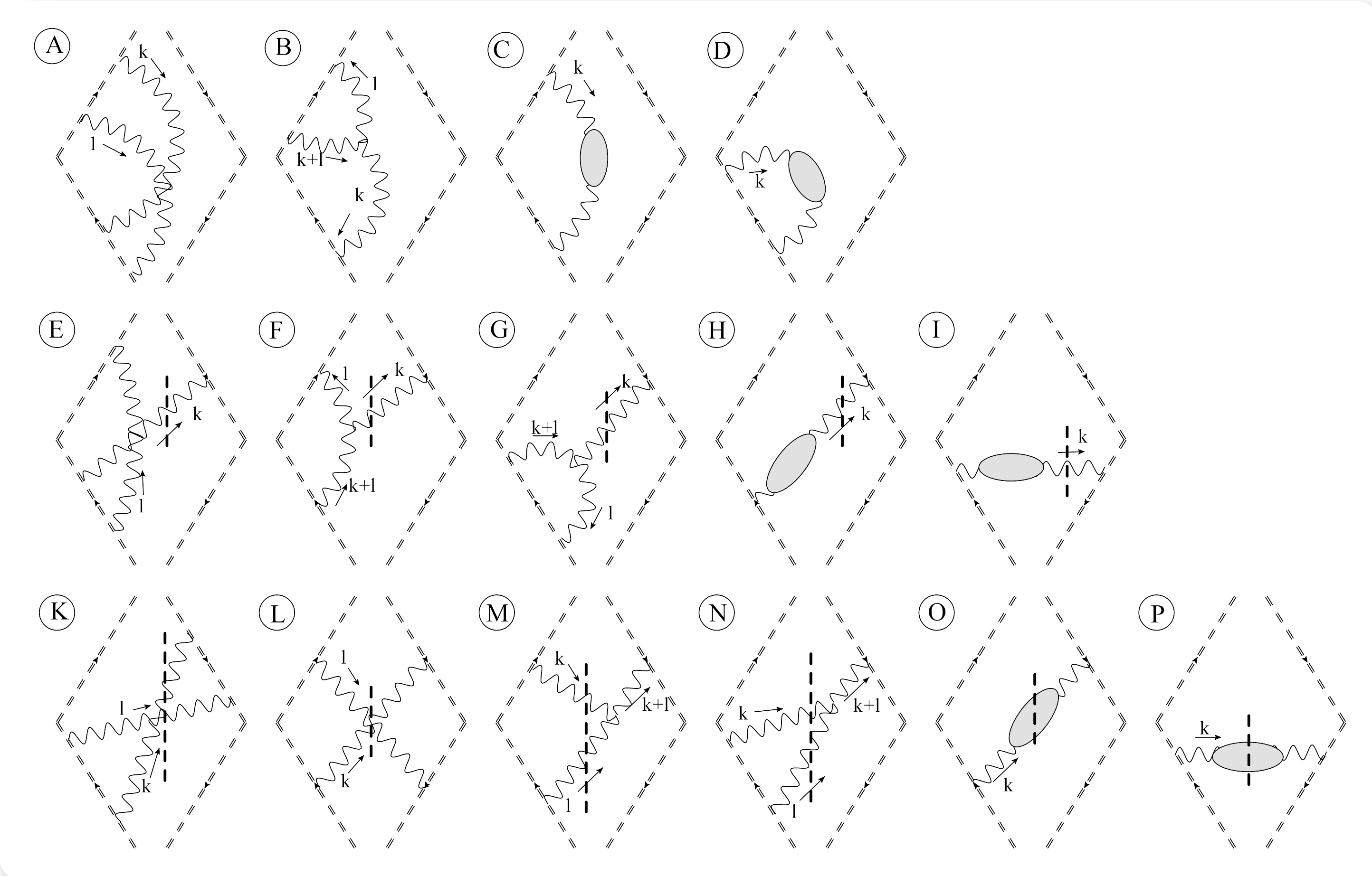
Splitting at NNLO



[MGE, Scimemi, Vladimirov 1511.05590]

- *SF calculated at NNLO*
- *All cancellations shown explicitly*
- *Depends on $|\boldsymbol{\delta}|$: process independent*

$$\mathbf{l}_\delta = \ln\left(\mu^2/|\boldsymbol{\delta}|\right)$$

$$\begin{aligned} S^{[2]} = &\frac{d^{(2,2)}}{C_F}\left(\frac{3}{\epsilon^3}+\frac{2\mathbf{l}_\delta}{\epsilon^2}+\frac{\pi^2}{6\epsilon}+\frac{4}{3}\mathbf{L}_\mu^3-2\mathbf{L}_\mu^2\mathbf{l}_\delta+\frac{2\pi^2}{3}\mathbf{L}_\mu+\frac{14}{3}\zeta_3\right)-\frac{d^{(2,1)}}{C_F}\left(\frac{1}{2\epsilon^2}+\frac{\mathbf{l}_\delta}{\epsilon}-\mathbf{L}_\mu^2+2\mathbf{L}_\mu\mathbf{l}_\delta-\frac{\pi^2}{4}\right) \\ &-\frac{d^{(2,0)}}{C_F}\left(\frac{1}{\epsilon}+2\mathbf{l}_\delta\right)+C_A\left(\frac{\pi^2}{3}+4\ln 2\right)\left(\frac{1}{\epsilon^2}+\frac{2\mathbf{L}_\mu}{\epsilon}+2\mathbf{L}_\mu^2+\frac{\pi^2}{6}\right)+C_A\left(8\ln 2-9\zeta_3\right)\left(\frac{1}{\epsilon}+2\mathbf{L}_\mu\right) \\ &+\frac{656}{81}T_R N_f+C_A\left(-\frac{2428}{81}+16\ln 2-\frac{7\pi^4}{18}-28\ln 2\,\zeta_3+\frac{4}{3}\pi^2\ln^2 2-\frac{4}{3}\ln^4 2-32\,\mathrm{Li}_4\left(\frac{1}{2}\right)\right)+\mathcal{O}(\epsilon) \end{aligned}$$

5. Unpolarized TMDs at NNLO

[MGE, Scimemi, Vladimirov, 1509.06392]
[MGE, Scimemi, Vladimirov, 1604.07869]

- We calculated at NNLO the quark/gluon TMD PDFs and FFs:

$$F_{f/A}(x,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\bar{x}}{\bar{x}} \tilde{C}_{f/j}(x/\bar{x},b_T;\zeta,\mu)\, f_{j/A}(\bar{x};\mu)$$

$$D_{A/f}(z,b_T;\zeta,\mu) = \sum_{j=q,\bar{q},g} \int_z^1 \frac{d\bar{z}}{\bar{z}^{3-2\epsilon}} \mathbb{C}_{j/f}(z/\bar{z},b_T;\zeta,\mu)\, d_{A/j}(\bar{z};\mu)$$

All flavor channels!

- With the modified δ-regulator the virtual diagrams of TMDs are zero
- Integrated PDFs/FFs also vanish, so we just have their renormalization:

$$f^{[1]}_{f/f'} = \frac{-1}{\epsilon} P^{(1)}_{f/f'}$$

$$d^{[1]}_{f'/f} = \frac{-1}{\epsilon} \mathbb{P}^{(1)}_{f'/f}$$

$$f^{[2]}_{f\leftarrow f'} = \frac{1}{2\epsilon^2}\left(\sum_r P^{(1)}_{f\leftarrow r}\otimes P^{(1)}_{r\leftarrow f'} + \beta^{(1)} P^{(1)}_{f\leftarrow f'}\right) - \frac{P^{(2)}_{f\leftarrow f'}}{2\epsilon}$$

$$d^{[2]}_{f\to f'} = \frac{1}{2\epsilon^2}\left(\sum_r \mathbb{P}^{(1)}_{f\to r}\otimes \mathbb{P}^{(1)}_{r\to f'} + \beta^{(1)} \mathbb{P}^{(1)}_{f\to f'}\right) - \frac{\mathbb{P}^{(2)}_{f\to f'}}{2\epsilon}$$

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Matching coefficients for all channels

- *~100 non-zero diagrams*
- *~20 master integrals (calculated at finite ϵ)*
- *Checks: RGEs, crossing TMDPDF-TMDFF, cancellation of IR divergencies by topology,…*
- *Algebra done with* Mathematica

- *At LO:*

$$C^{[0]}_{f\leftarrow f'} = \delta_{ff'}\delta(\bar{x}) \qquad \mathbb{C}^{[0]}_{f\to f'} = \delta_{ff'}\delta(\bar{z})$$

- *At NLO:*

$$C^{[1]}_{f\leftarrow f'} = F^{[1]}_{f\leftarrow f'} - f^{[1]}_{f\leftarrow f'} \qquad \mathbb{C}^{[1]}_{f\to f'} = D^{[1]}_{f\to f'} - \frac{d^{[1]}_{f\to f'}}{z^{2-2\epsilon}}$$

- *At NNLO:*

$$C^{[2]}_{f\leftarrow f'} = F^{[2]}_{f\leftarrow f'} - \sum_r C^{[1]}_{f\leftarrow r}\otimes f^{[1]}_{r\leftarrow f'} - f^{[2]}_{f\leftarrow f'}$$

$$\mathbb{C}^{[2]}_{f\to f'} = D^{[2]}_{f\to f'} - \sum_r \mathbb{C}^{[1]}_{f\to r}\otimes \frac{d^{[1]}_{r\to f'}}{z^{2-2\epsilon}} - \frac{d^{[2]}_{f\to f'}}{z^{2-2\epsilon}}$$

Results

- *We calculated both quark/gluon TMDPDF and TMDFF, all flavor channels.*
- *And their matching coefficients onto their integrated counterparts*
- *Coefficients for TMDPDFs coincide with* [Catani et.al., Gehrmann et.al.]
- *For TMDFFs the results are new*
- *Gluon TMDFF is considered for the first time, even at one loop*

$$\begin{aligned}
z^2\mathbb{C}^{(2,0)}_{g\to q}(z) &= T_rC_F\Big\{p_{qg}(z)\Big[32\mathrm{Li}_3(z)+\frac{2}{3}\ln^3\bar z-6\ln^2\bar z\ln z+18\ln^2 z\ln\bar z+3\ln^2\bar z-18\ln\bar z\ln z\\
&+16\ln\bar z-2\pi^2\ln\bar z+\frac{2\pi^2}{3}\ln z-3\pi^2-32\zeta_3\Big]+z\bar z\Big[32\mathrm{Li}_2(z)-4\ln^2\bar z+24\ln z\ln\bar z\\
&-4\ln\bar z-\frac{4\pi^2}{3}\Big]-\frac{11}{3}(1-2z+4z^2)\ln^3 z-\left(\frac{7}{2}+26z-34z^2\right)\ln^2 z\\
&-(8-73z+76z^2)\ln z+63-101z+56z^2\Big\}\\
&+T_rC_A\Big\{p_{qg}(-z)\Big[4\mathrm{Li}_3\left(\frac{1}{1+z}\right)-4\mathrm{Li}_3\left(\frac{z}{1+z}\right)-2\mathrm{Li}_3(z^2)-2\ln z\mathrm{Li}_2(z^2)\\
&-6\ln^2 z\ln(1+z)-2\ln z\ln^2(1+z)+2\zeta_3\Big]+p_{qg}(z)\Big[20\mathrm{Li}_3(z)-16\ln z\mathrm{Li}_2(z)\\
&-\frac{2}{3}\ln^3\bar z+4\ln^2\bar z\ln z-4\ln^2 z\ln\bar z-\frac{11}{3}\ln^2\bar z+14\ln z\ln\bar z-\frac{152}{9}\ln\bar z+2\pi^2\ln\bar z-4\pi^2\ln z\\
&-6\zeta_3+\frac{19\pi^2}{3}\Big]-4z(1+z)\mathrm{Li}_2(1-z^2)+4z\bar z\Big[\ln^2\bar z+\frac{5}{3}\ln\bar z\Big]+32z\ln z\mathrm{Li}_2(z)\\
&+\frac{8(2-3z+15z^2-8z^3)}{3z}\mathrm{Li}_2(\bar z)+\frac{2(11+62z)}{3}\ln^3 z+\frac{2(16-22z+35z^2-59z^3)}{3z}\ln^2 z\\
&+8\ln z\ln\bar z+\frac{2(24-165z-699z^2+38z^3)}{9z}\ln z-\frac{8\pi^2}{3}\\
&-\frac{2(148+1223z-139z^2-774z^3)}{27z}\Big\}+T_r^2N_f\Big\{\frac{4}{3}p_{qg}(z)\Big[\ln^2 z+\ln^2\bar z-6\ln\bar z\ln z\\
&-10\ln z+\frac{10}{3}\ln\bar z-\pi^2+\frac{56}{9}\Big]-\frac{16}{3}z\bar z\Big[\ln z+\ln\bar z+\frac{2}{3}\Big]\Big\},
\end{aligned}$$

$$\begin{aligned}
z^2\mathbb{C}^{(2,0)}_{g\to g}(z) &= C_A^2\Big\{p_{gg}(-z)\Big[8\mathrm{Li}_3\left(\frac{1}{1+z}\right)-8\mathrm{Li}_3\left(\frac{z}{1+z}\right)-4\mathrm{Li}_3(z^2)+8\ln z\mathrm{Li}_2(z)\\
&-8\ln z\mathrm{Li}_2(-z)+6\ln^3 z-12\ln^2 z\ln(1+z)-4\ln z\ln^2(1+z)+4\zeta_3\Big]\\
&+p_{gg}(z)\Big[104\mathrm{Li}_3(z)-48\ln z\mathrm{Li}_2(z)-\frac{62}{3}\ln^3 z+28\ln\bar z\ln^2 z-4\ln^2\bar z\ln z+\frac{44}{3}\ln^2 z\\
&+\frac{268}{9}\ln z-\frac{20\pi^2}{3}\ln z-\frac{808}{27}-76\zeta_3\Big]+\frac{8}{3}\bar z\left(1-\frac{11}{z}-11z\right)\mathrm{Li}_2(\bar z)-\frac{88}{3}(1+z)\ln^3 z\\
&+\frac{44z^3-173z^2+103z-264}{3z}\ln^2 z+\frac{1340z^3+397z^2+1927z+268}{9z}\ln z-\frac{2}{3}\ln\bar z\\
&+\frac{4(-1064z^3+450z^2-414z+1019)}{27z}+\delta(\bar z)\left(\frac{1214}{81}-\frac{67\pi^2}{36}-\frac{77}{9}\zeta_3+\frac{53\pi^4}{72}\right)\Big\}\\
&+C_AT_rN_f\Big\{p_{gg}(z)\Big[-\frac{16}{3}\ln^2 z-\frac{80}{9}\ln z+\frac{224}{27}\Big]-\frac{20}{3}(1+z)\ln^2 z+\frac{4}{3}\ln\bar z\\
&+\frac{4(26z^3-5z^2+25z-26)}{9z}\ln z+\frac{4(-65z^3+54z^2-54z+83)}{27z}\\
&+\delta(\bar z)\left(-\frac{328}{81}+\frac{5\pi^2}{9}+\frac{28}{9}\zeta_3\right)\Big\}\\
&+C_FT_rN_f\Big\{\frac{44}{3}(1+z)\ln^3 z+\frac{2(16z^3+15z^2+21z+16)}{3z}\ln^2 z\\
&-\frac{8(82z^3+81z^2+135z-6)}{9z}\ln z+\frac{8(301z^3+108z^2-270z-139)}{27z}\Big\}
\end{aligned}$$

Conclusions & Outlook

- *TMD **evolution is universal**, and currently known at NNLL'*
- ***Matching coefficients** for all unpolarized TMDs calculated at NNLO (for TMDPDFs coincide with* **[Catani et.al., Gehrmann et.al.]**). *Ready to be used!*
- ***Gluon TMDFF** is considered for the first time (even at one-loop)*
- *We only want to parametrize what cannot be calculated!*
- ***TMD pheno is a mess**: non-perturbative ingredients, different regions mixed under Fourier transform, need to match TMD and collinear regions,…*
- *The more accurately we know the perturbative pieces, the better we will constrain the non-perturbative ones*

★ *We need **more experimental data**: unpolarized e+e-, more unpolarized Drell-Yan, etc*

★ ***Push the pheno**: perform global fits exploiting <u>**all**</u> available perturbative information*

★ *Calculate **polarized TMDs** at NNLO*

Thank you!