

Extraction of unpolarized TMDs from SIDIS and Drell-Yan processes



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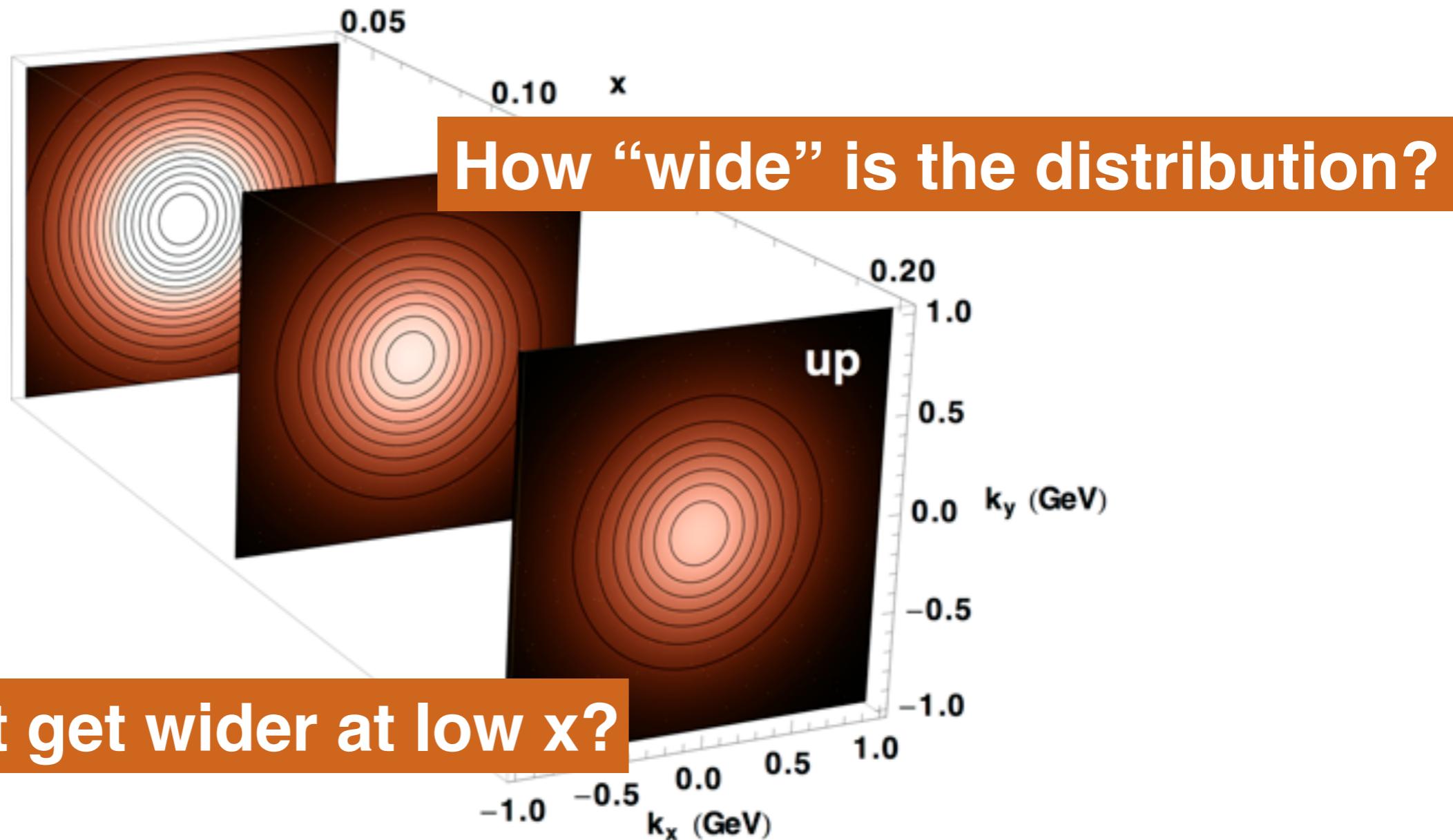


In collaboration with

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- Cristian Pisano (University of Pavia and INFN Pavia)
- Marco Radici (INFN Pavia)
- Andrea Signori (Vrije Universiteit Amsterdam and NIKHEF)

3DSPIN'16: structure of the nucleon

Difference between flavors?



TMD distributions

		quark pol.		
		U	L	T
nucleon pol.	U	f_1		h_1^\perp
	L		g_{1L}	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

quark pol.		
U	L	T
D_1		H_1^\perp

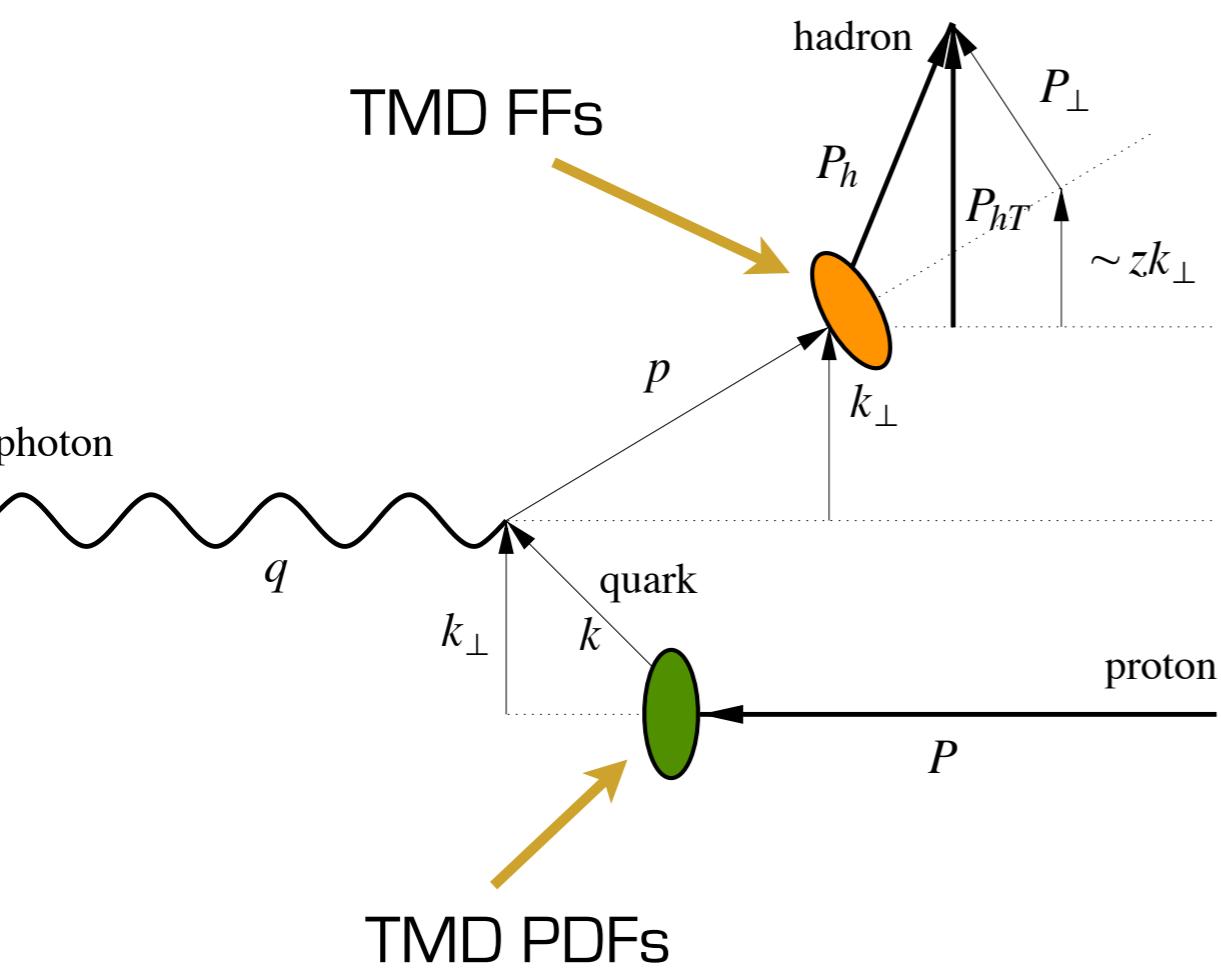
unpolarized

TMD Parton Distribution Functions
(TMD PDFs)

TMD Fragmentation Functions
(TMD FFs)

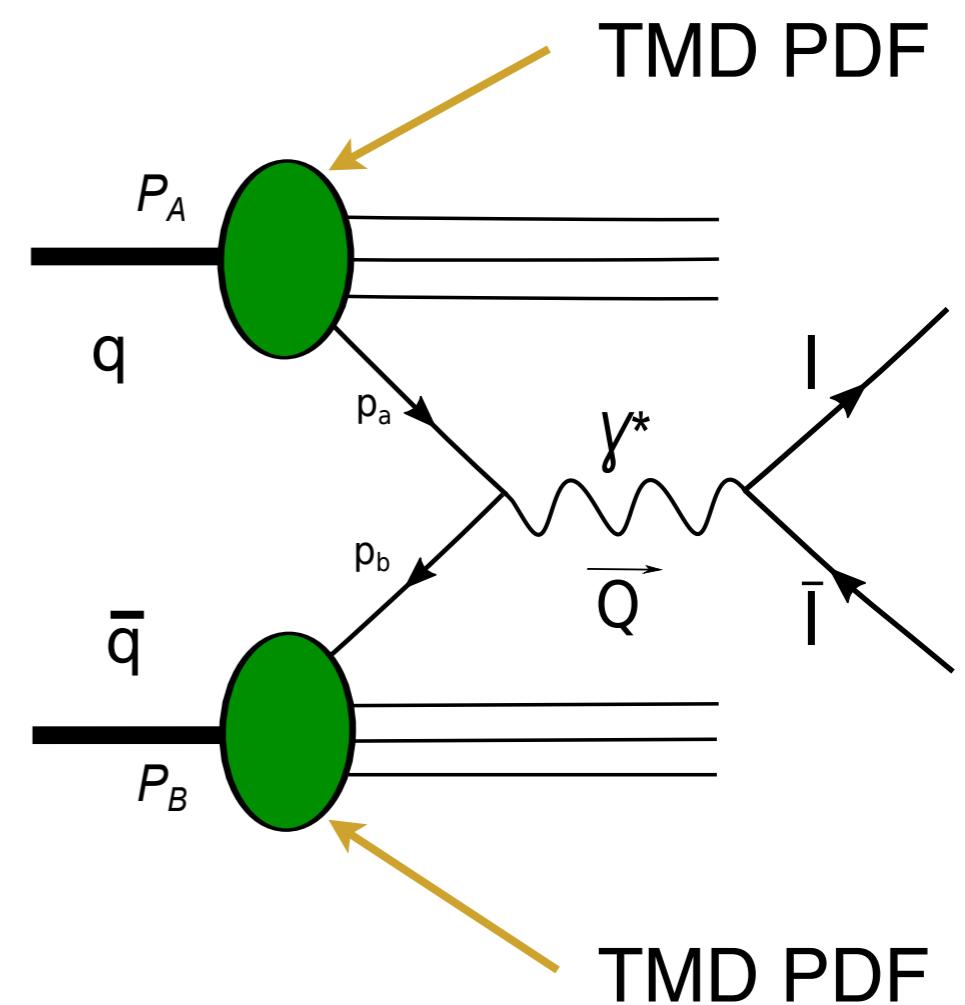
extraction from SIDIS & Drell-Yan

Semi-inclusive DIS



Drell-Yan \ Z production

$$A + B \rightarrow \gamma^* \rightarrow l^+ l^-$$
$$A + B \rightarrow Z \rightarrow l^+ l^-$$

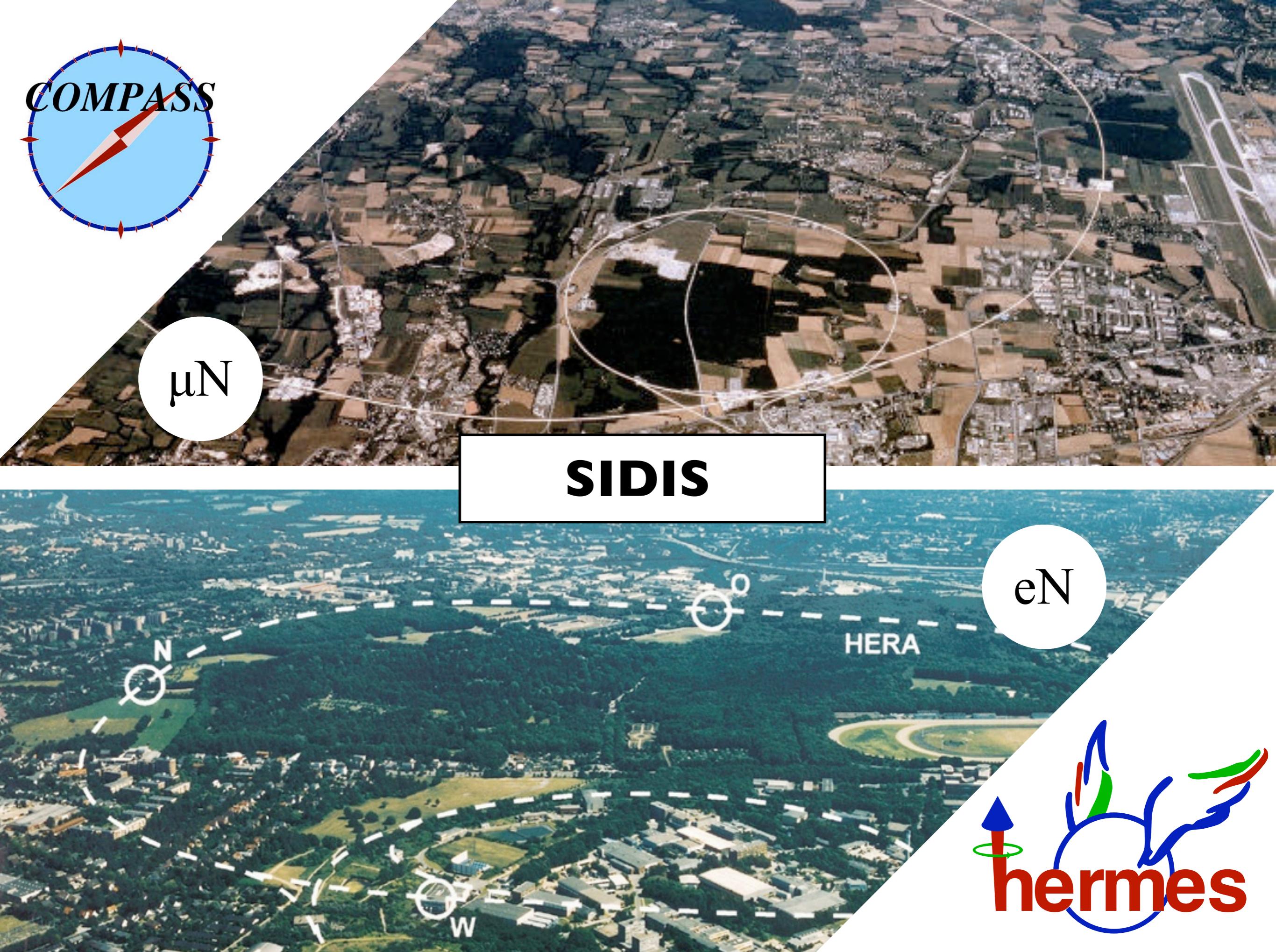


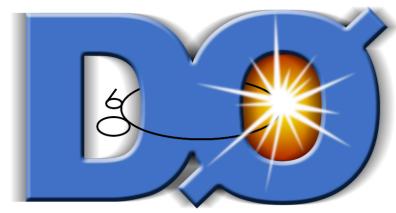


μN

SIDIS

eN





Z

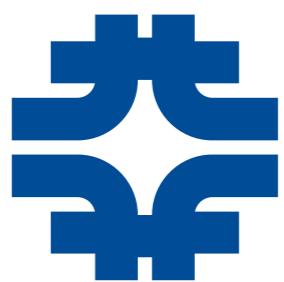


DRELL-YAN

$\bar{p}p$

pN

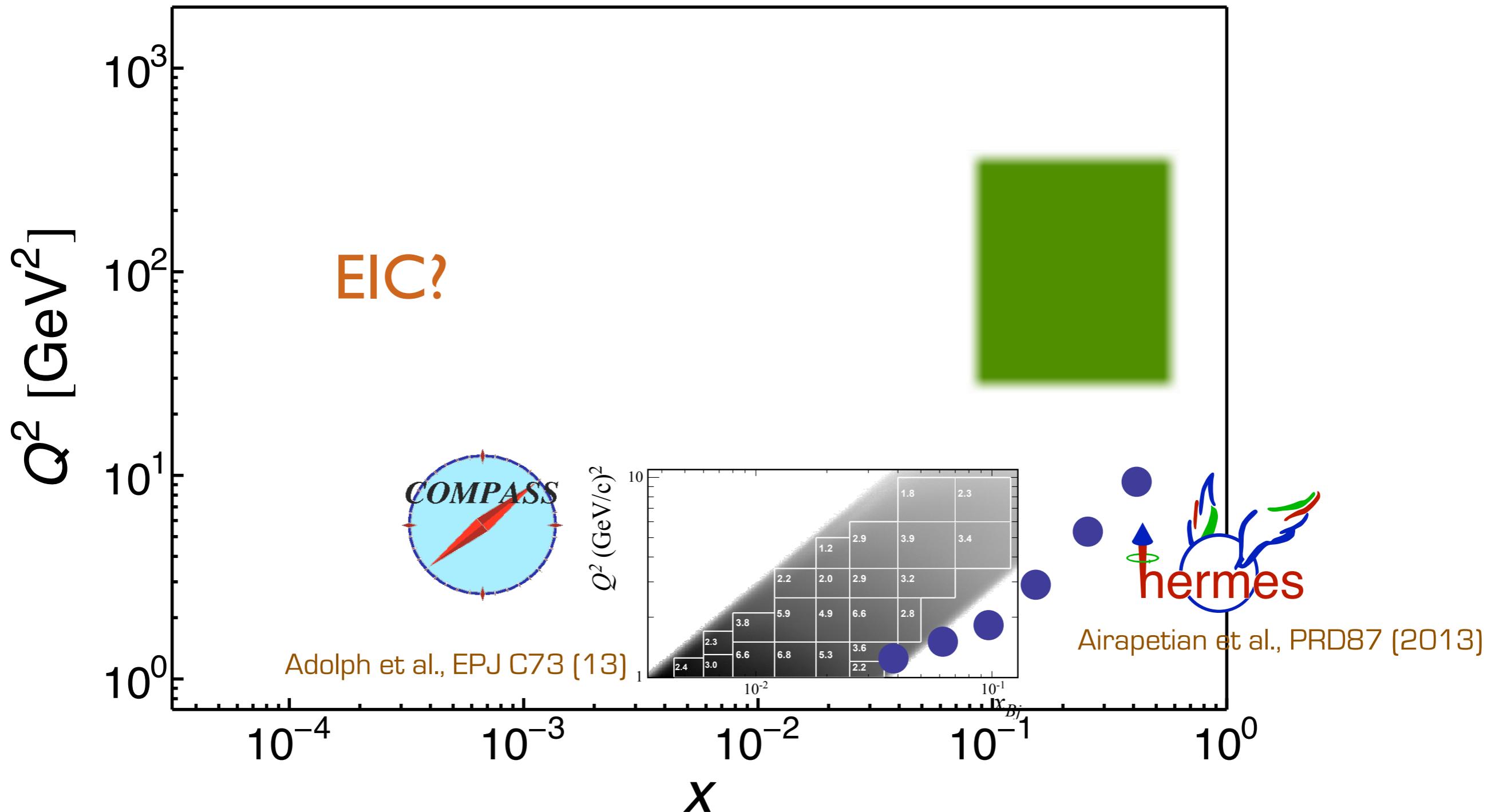
γ^*



E288
E605

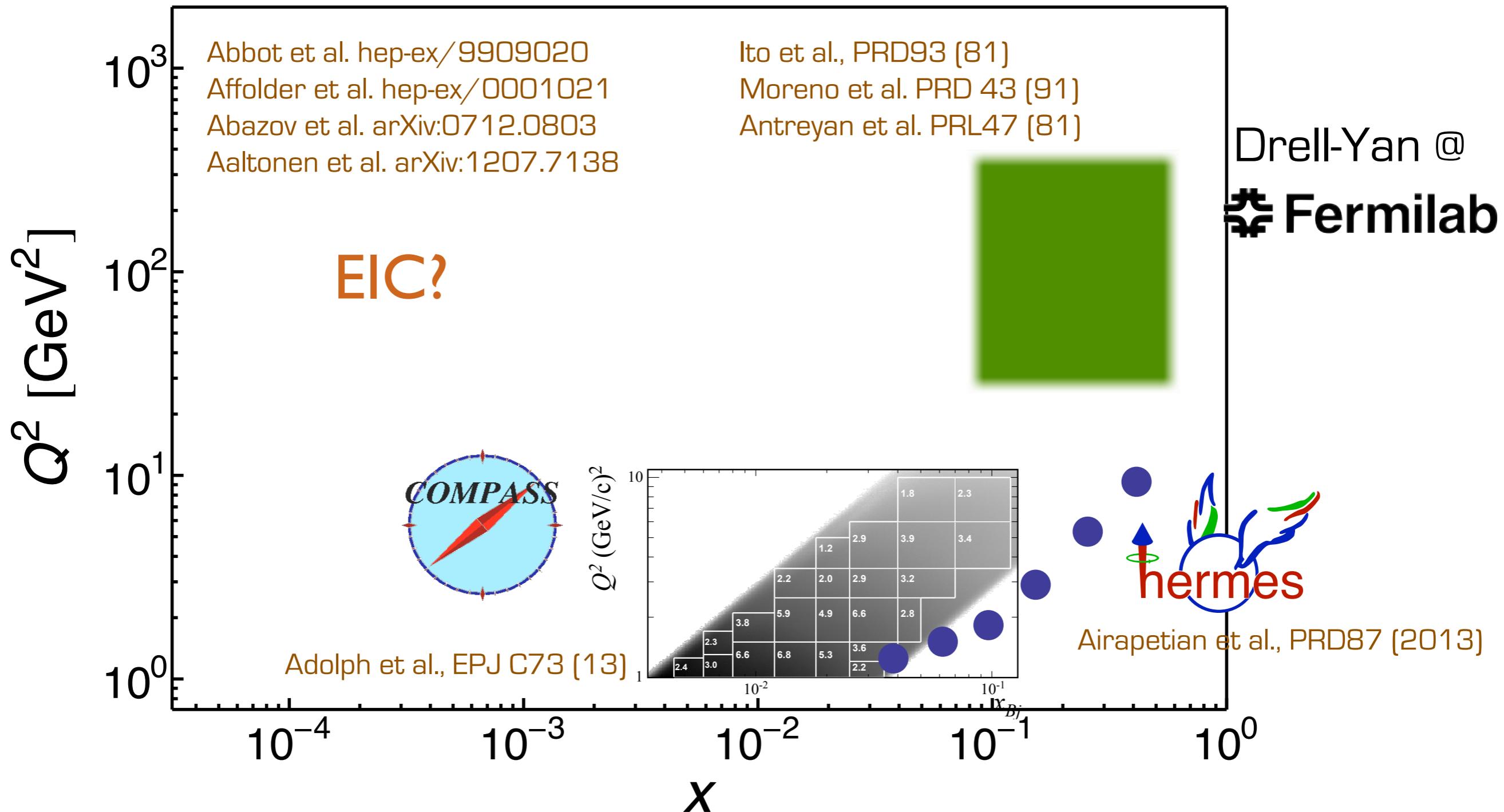


Data region



Data region

Z production



Current state fits

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✗	✗	✓	✓	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✓	✗	✗	✗	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x, Q^2) bin	1 (x, Q^2) bin	✓	✓	500 (?)
Pavia 2016 (+Amsterdam)	LO-NLL	✓	✓	✓	✓	8156

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2016 (+Amsterdam)	LO-NLL	✓	✓	✓	✓	8156

Features

heading towards a **global fit** of quark **unpolarized TMDs**

Flexible functional form (beyond gaussians)

includes TMD evolution

replica methodology

kinematic and flavor dependence (preliminary)

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2016 (+Amsterdam)	LO-NLL	✓	✓	✓	✓	8156

Cons

no “pure” info on TMD FFs
(would require e+e- annihilation)

TMD accuracy: not the state of the art

long computation time

Data selection

SIDIS proton-target data		HERMES $p \rightarrow \pi^+$	HERMES $p \rightarrow \pi^-$	HERMES $p \rightarrow K^+$	HERMES $p \rightarrow K^-$
Reference					
Cuts		$Q^2 > 1.4 \text{ GeV}^2$	$0.2 < z < 0.7$	$P_{hT} < \text{Min}[0.2 Q, 0.6 Qz] + 0.5 \text{ GeV}$	
Points		188	186	187	185
Max. Q^2			9.2 GeV^2		
x range			$0.06 < x < 0.4$		
Notes					

Motivations behind kinematical cuts

TMD factorization ($P_{hT}/z \ll Q^2$)

Avoid target fragmentation (low z)
and exclusive contributions (high z)

Data selection

SIDIS
deuteron-target
data

	HERMES $D \rightarrow \pi^+$	HERMES $D \rightarrow \pi^-$	HERMES $D \rightarrow K^+$	HERMES $D \rightarrow K^-$	COMPASS $D \rightarrow h^+$	COMPASS $D \rightarrow h^-$
Cuts					$Q^2 > 1.4 \text{ GeV}^2$ $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2 Q, 0.6 Qz] + 0.5 \text{ GeV}$	
Points	188	188	186	187	3024	3021
Max. Q^2		9.2 GeV^2				10 GeV^2
x range		0.06 < x < 0.4				0.006 < x < 0.12
Notes					Observable: $\frac{m_N^h(x, z, P_{hT}^2, Q^2)}{m_N^h(x, z, \text{Min}[P_{hT}^2], Q^2)}$	

to avoid problems
with Compass data normalization

Data selection

	E288 200	E288 300	E288 400	E605
Cuts	$q_T < 0.2 Q + 0.5 \text{ GeV}$			
Points	45	45	78	35
\sqrt{s}	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV
Q range	4-9 GeV	4-9 GeV	5-9, 11-14 GeV	7-9, 10.5-18 GeV
Kin. var.	$y=0.4$	$y=0.21$	$y=0.03$	$-0.1 < x_F < 0.2$

Drell-Yan
data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Cuts	$q_T < 0.2 Q + 0.5 \text{ GeV} = 18.7 \text{ GeV}$			
Points	31	14	37	8
\sqrt{s}	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
Normalization	1.114	0.992	1.049	1.048

fixed from DEMS fit,
different from exp.

(not really relevant for TMD
parametrizations)

Model: TMD evolution and Fourier transform

$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) = \sum_a \int d\mathbf{k}_\perp d\mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2) \delta(z\mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) + \mathcal{O}(M^2/Q^2)$$

convolution

$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi} \int d^2 b_\perp e^{-ib_\perp \cdot k_\perp} \tilde{f}_1^a(x, b_\perp; \mu^2)$$

Fourier transform: b_T space

Model: TMD evolution and Fourier transform

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Fourier transform: b_T space

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

Model: TMD evolution and Fourier transform

$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) = \sum_a \int d\mathbf{k}_\perp d\mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2) \delta(z\mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) + \mathcal{O}(M^2/Q^2)$$

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$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi} \int d^2 b_\perp e^{-ib_\perp \cdot k_\perp} \tilde{f}_1^a(x, b_\perp; \mu^2)$$

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The diagram shows a horizontal line labeled "collinear PDF" with an arrow pointing upwards. This line meets a curve labeled "pQCD" at a point. From this point, two arrows emerge: one pointing upwards labeled "nonperturbative part of evolution", and another pointing upwards labeled "nonperturbative part of TMD".

Model: non perturbative elements

$$\tilde{F}_{i,NP}(x, b_T) = \frac{\langle \mathbf{k}_\perp^2 \rangle_i e^{-\langle \mathbf{k}_\perp^2 \rangle_i b_T^2/4} + \lambda \langle \mathbf{k}'_\perp^2 \rangle_i \left(1 - \langle \mathbf{k}'_\perp^2 \rangle_i \frac{b_T^2}{4}\right) e^{-\langle \mathbf{k}'_\perp^2 \rangle_i b_T^2/4}}{\langle \mathbf{k}_\perp^2 \rangle_i + \lambda \langle \mathbf{k}'_\perp^2 \rangle_i}$$

sum of two different gaussians

both for parton distributions (not fundamental) and fragmentation
with kinematic dependence on transverse momenta

$$\langle \mathbf{k}_{\perp,a}^2 \rangle(x) = \langle \hat{\mathbf{k}}_{\perp,a}^2 \rangle \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma},$$

where $\langle \hat{\mathbf{k}}_{\perp,a}^2 \rangle \equiv \langle \mathbf{k}_{\perp,a}^2 \rangle(\hat{x})$, and $\hat{x} = 0.1$.

$$\langle \mathbf{P}_{\perp,a \rightarrow h}^2 \rangle(z) = \langle \hat{\mathbf{P}}_{\perp,a \rightarrow h}^2 \rangle \frac{(z^\beta + \delta) (1-z)^\gamma}{(\hat{z}^\beta + \delta) (1-\hat{z})^\gamma}$$

where $\langle \hat{\mathbf{P}}_{\perp,a \rightarrow h}^2 \rangle \equiv \langle \mathbf{P}_{\perp,a \rightarrow h}^2 \rangle(\hat{z})$, and $\hat{z} = 0.5$.

Model: non perturbative elements

$$\langle \mathbf{k}_{\perp,a}^2 \rangle(x) = \langle \hat{\mathbf{k}}_{\perp,a}^2 \rangle \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}, \quad \langle \mathbf{P}_{\perp,a \rightarrow h}^2 \rangle(z) = \langle \hat{\mathbf{P}}_{\perp,a \rightarrow h}^2 \rangle \frac{(z^\beta + \delta) (1-z)^\gamma}{(\hat{z}^\beta + \delta) (1-\hat{z})^\gamma}$$

$$g_K = -g_2 \frac{b_T^2}{2}$$

$$g_2 = 0.13 \pm 0.02 \text{ GeV}^2$$

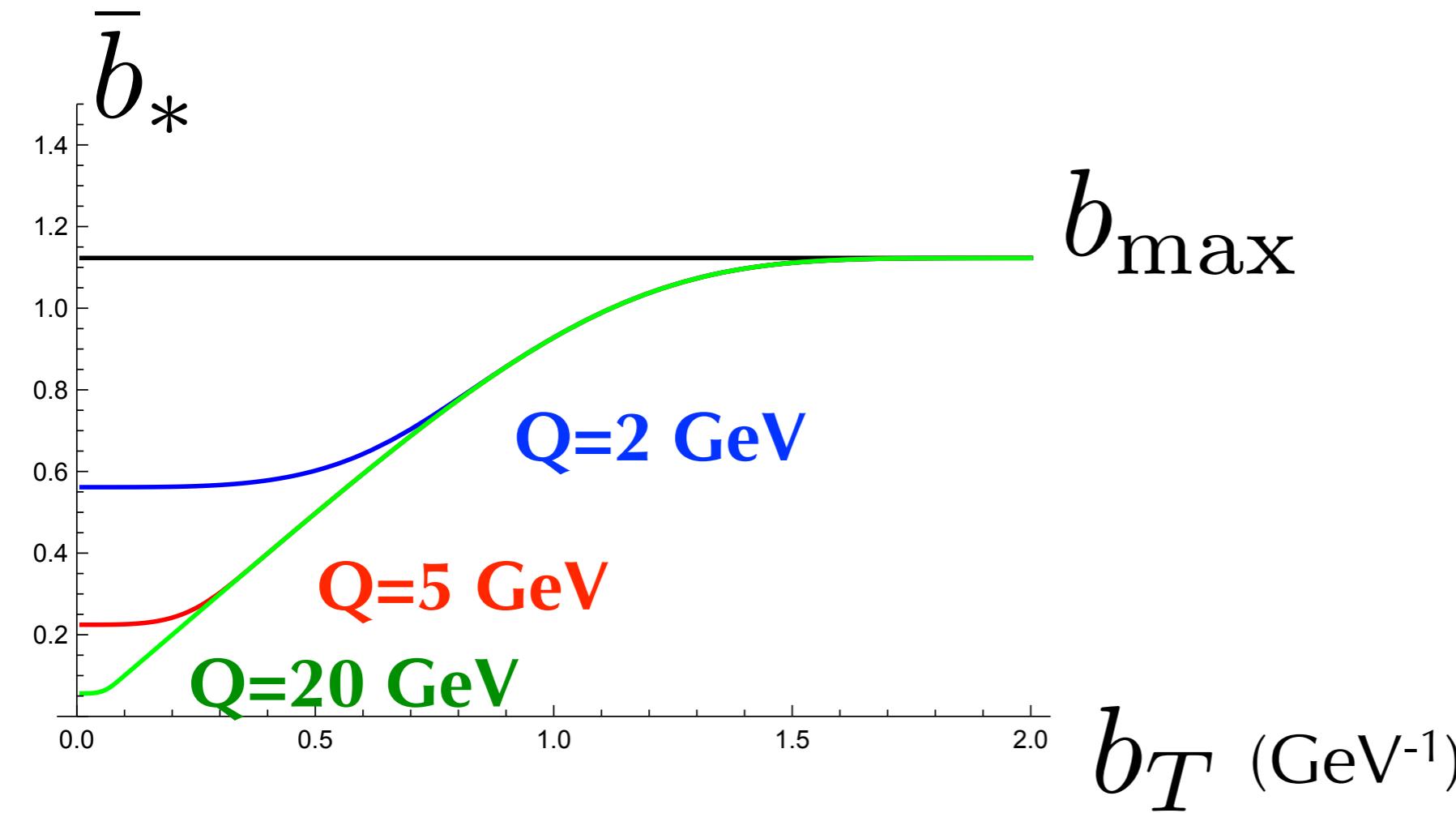
In total we have **11 parameters**, for **intrinsic** transverse momentum (4 PDFs, 6 FFs) and **evolution** (g_2)

b_T prescription

b_T-star

$$\bar{b}_*(b_T; b_{\min}, b_{\max}) = b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)$$

$\xrightarrow{b_{\max}, \quad b_T \rightarrow +\infty}$
 $\xrightarrow{b_{\min}, \quad b_T \rightarrow 0}$



$b_{\max} = 2e^{-\gamma_E}$
 $b_{\min} = 2e^{-\gamma_E}/Q$

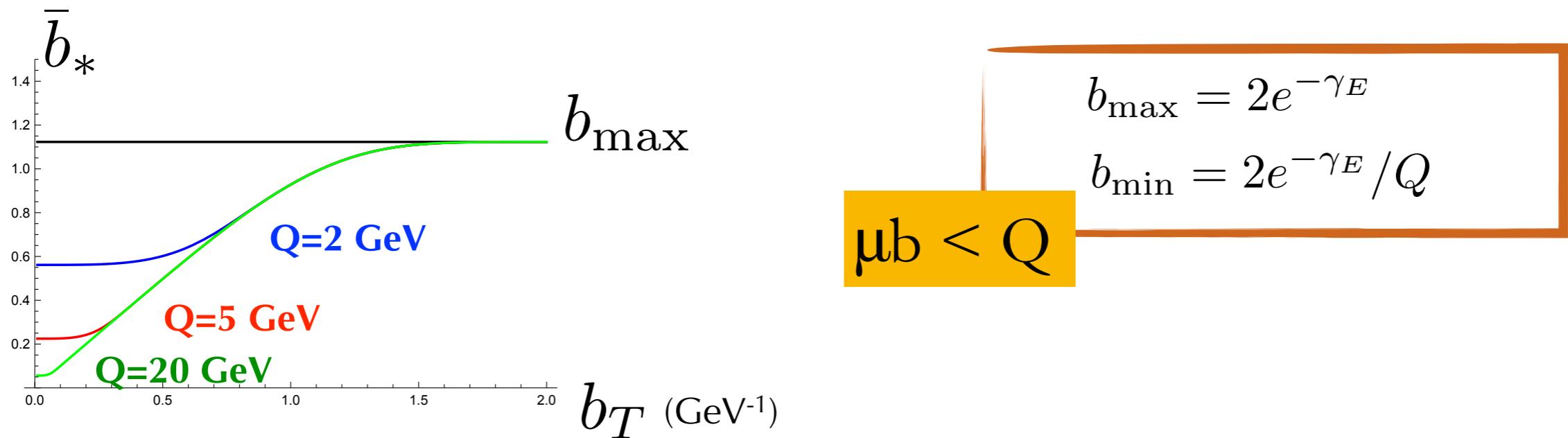
These choices guarantee that for $Q=1 \text{ GeV}$ the TMD coincides with the NP model

b_T prescription

\bar{b}_\star -star

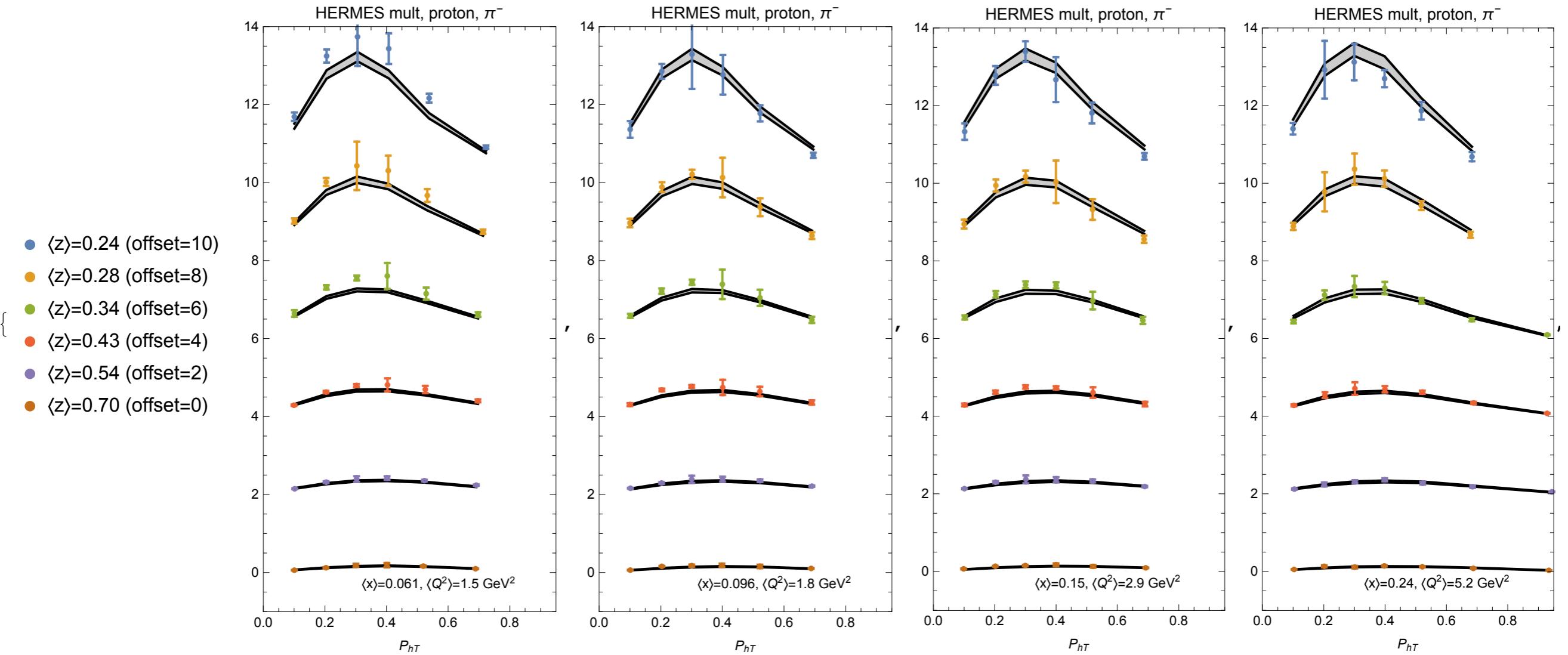
$$\bar{b}_\star(b_T; b_{\min}, b_{\max}) = b_{\max} \left(\frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)$$

$\xrightarrow{b_{\max}, \quad b_T \rightarrow +\infty}$
 $\xrightarrow{b_{\min}, \quad b_T \rightarrow 0}$

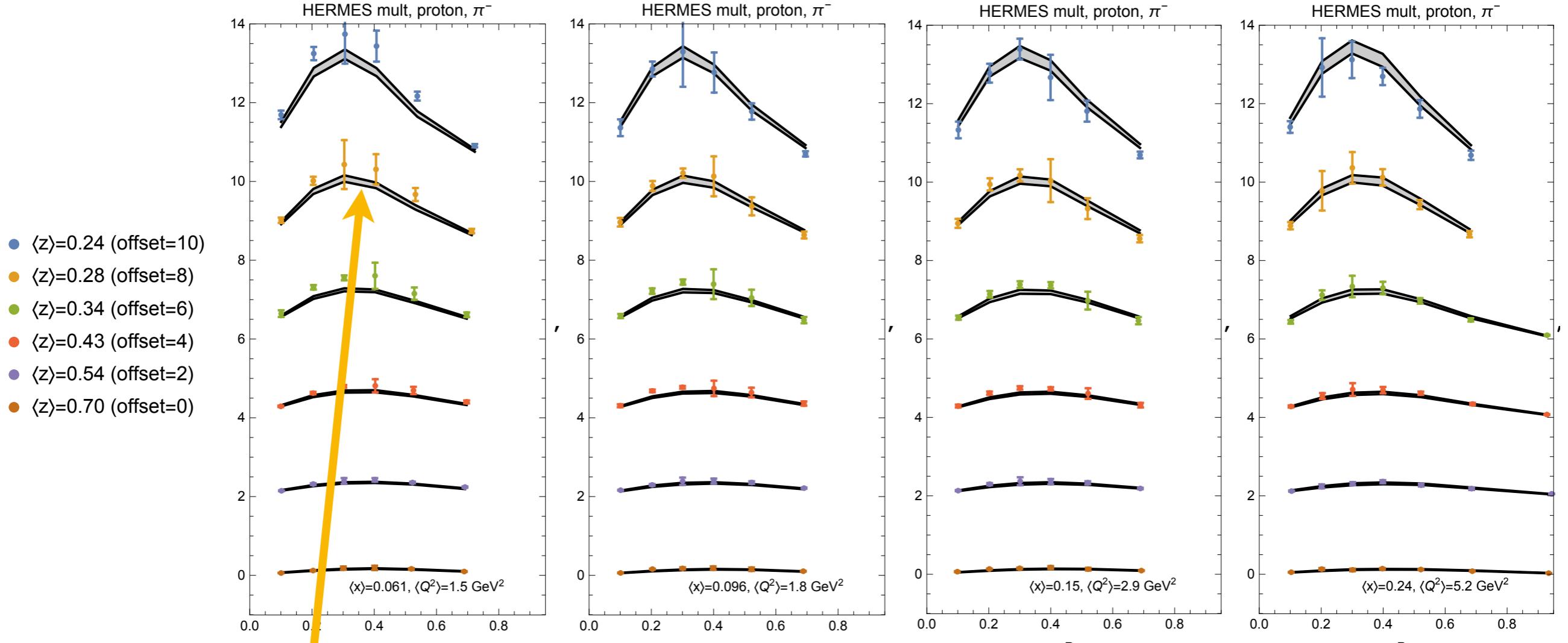


The phenomenological importance of b_{\min} is a signal that, especially in SIDIS data at **low Q** , we are exiting the proper TMD region and approaching the region of collinear factorization

Hermes results (some bins)

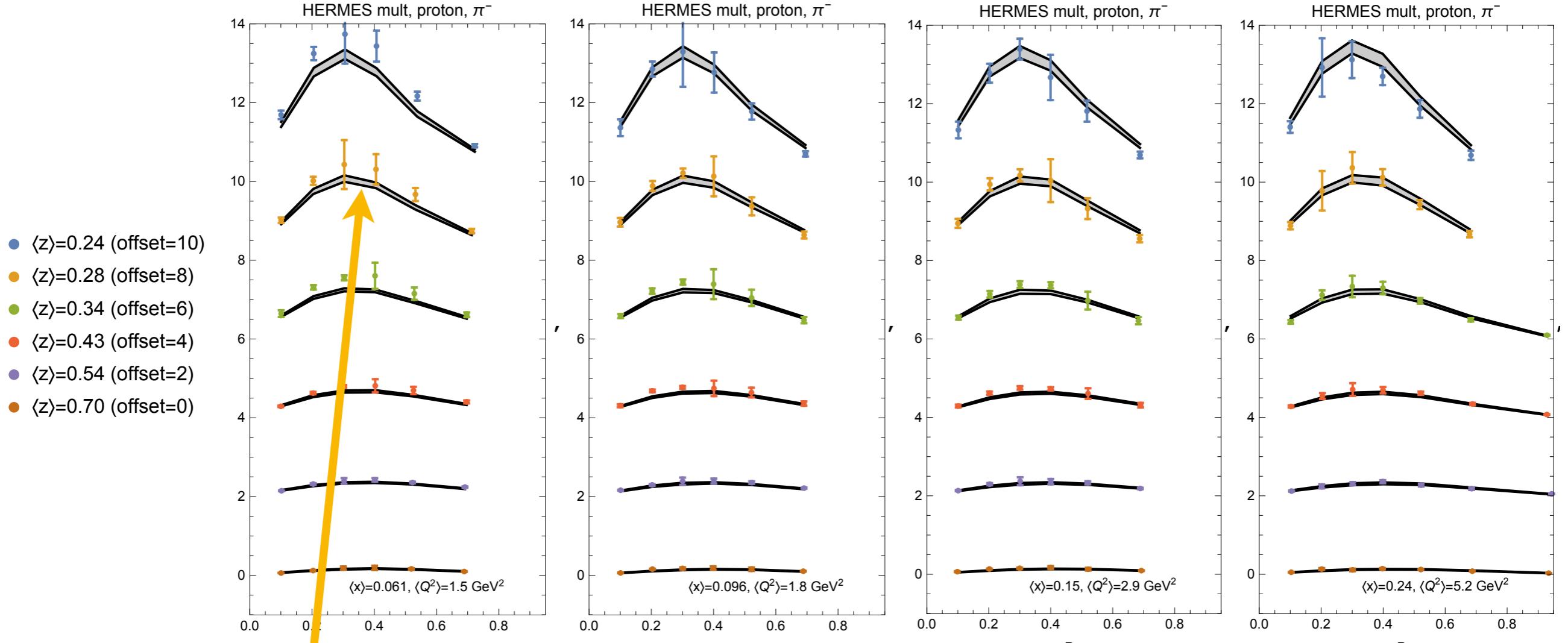


Hermes results (some bins)



grey bands represent 68% c.l. of
replica results

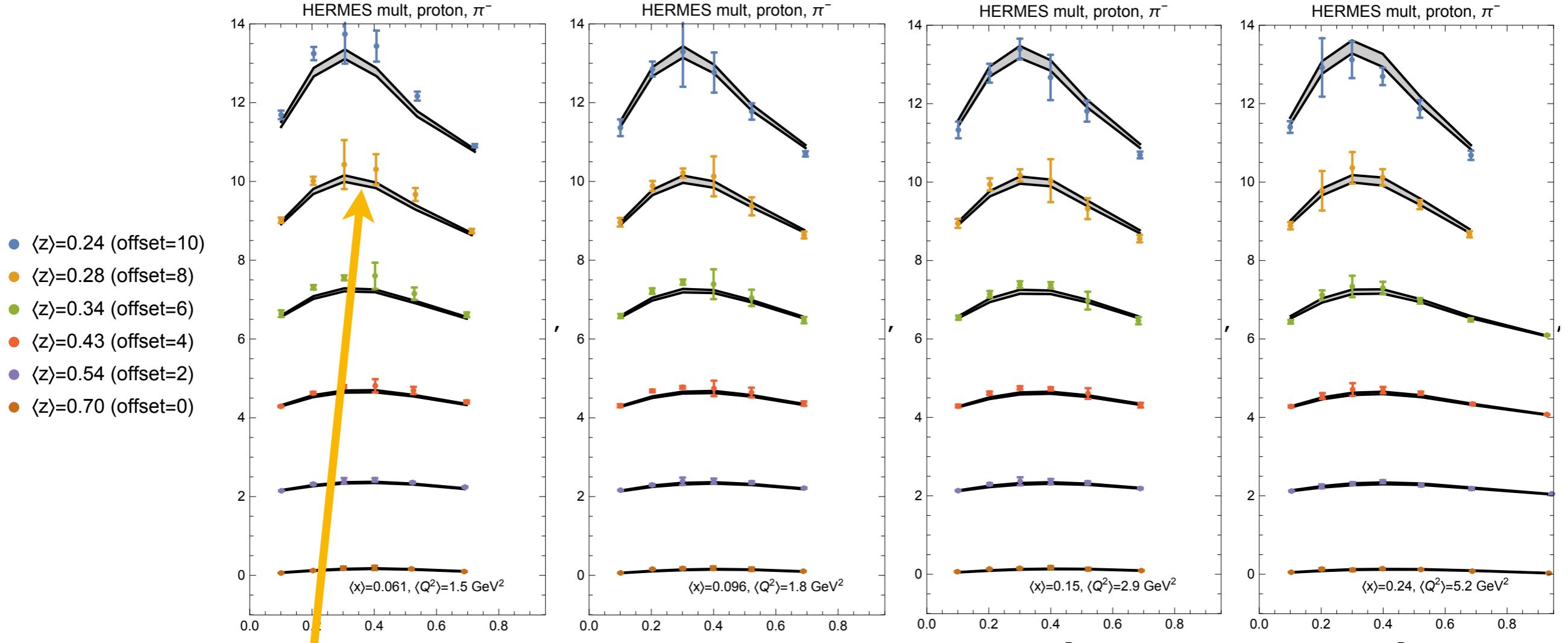
Hermes results (some bins)



grey bands represent 68% c.l. of
replica results

$\chi^2/\text{dof} = 3.30$ for proton π^-

Hermes results (some bins)

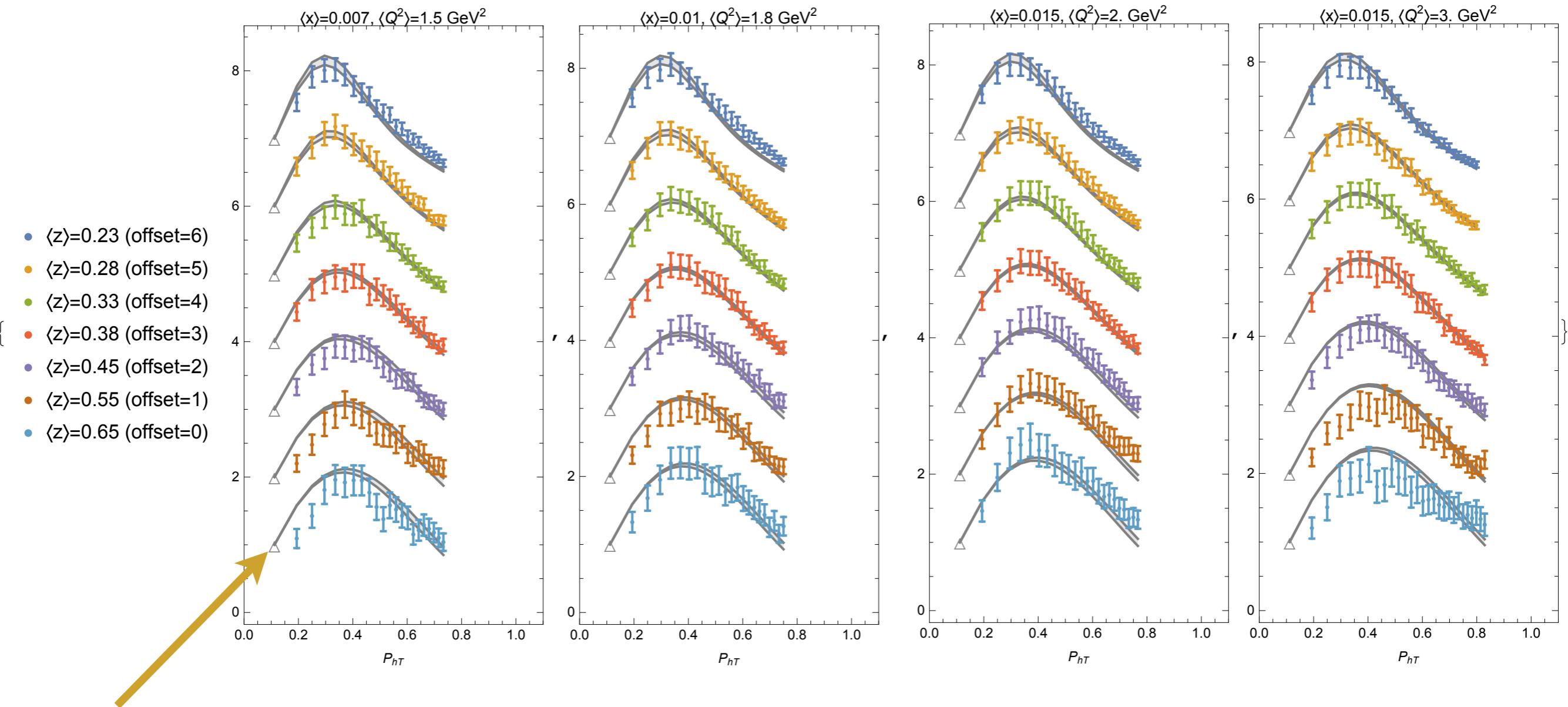


grey bands represent 68% c.l. of replica results

$$\chi^2/\text{dof} = 3.30 \text{ for proton } \pi^-$$

proton π^+ worse, however normalizing the theory curves to the first bin, without changing the parameters of the fit, χ^2/dof improves

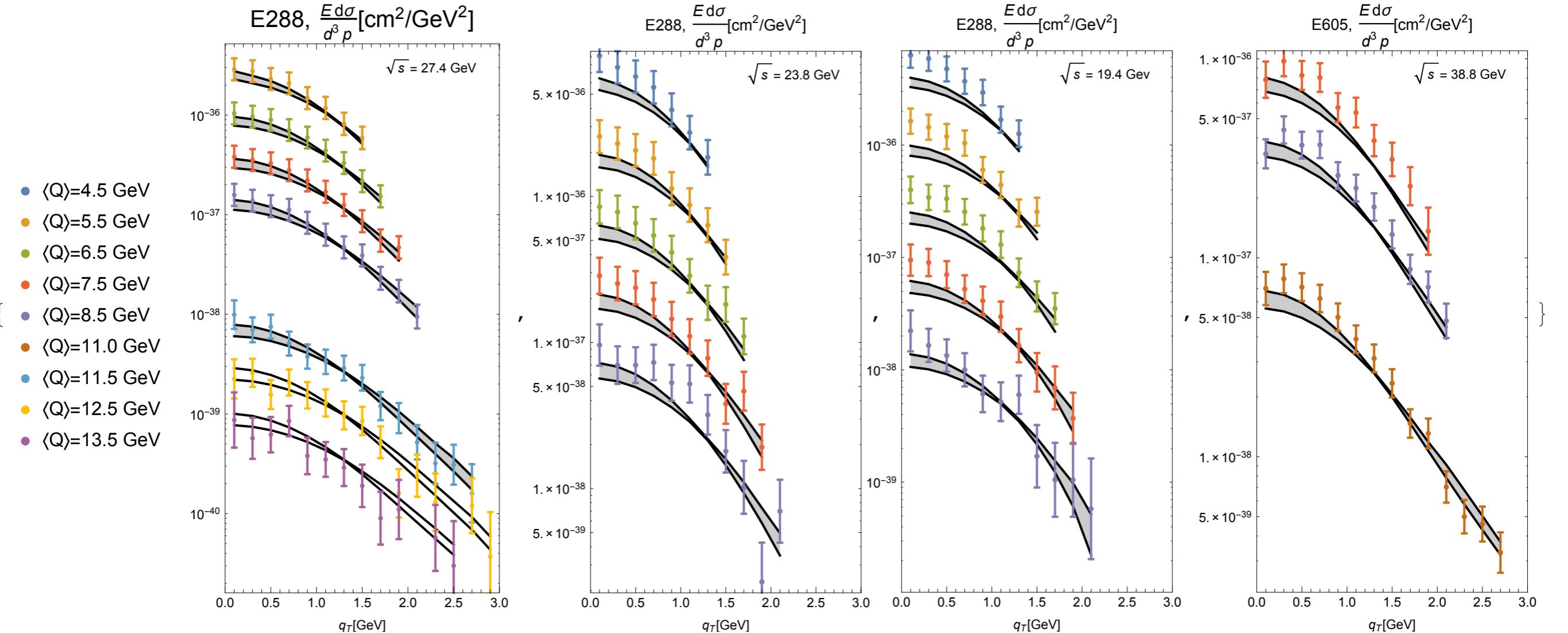
Compass (some bins)



First points are not fitted, but used as normalization to avoid problems related to data normalization (considered as fixed parameters)

Compass deuteron h^+
 $\chi^2/\text{dof} = 1.49$

Drell-Yan data



$\chi^2/\text{dof} = 1.57$

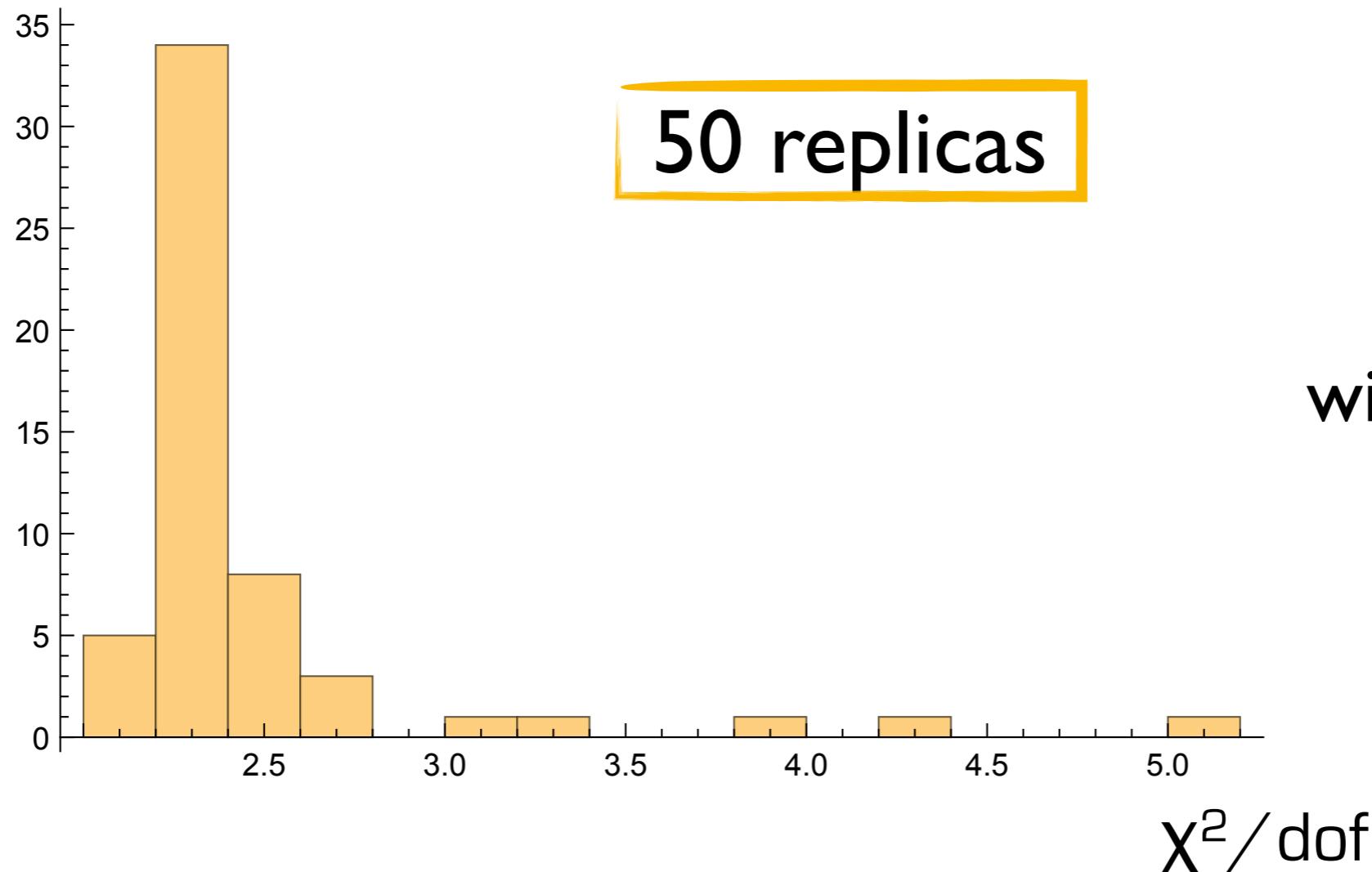
$\chi^2/\text{dof} = 0.48$

$\chi^2/\text{dof} = 0.42$

$\chi^2/\text{dof} = 0.97$

χ^2/dof distribution

n. replicas

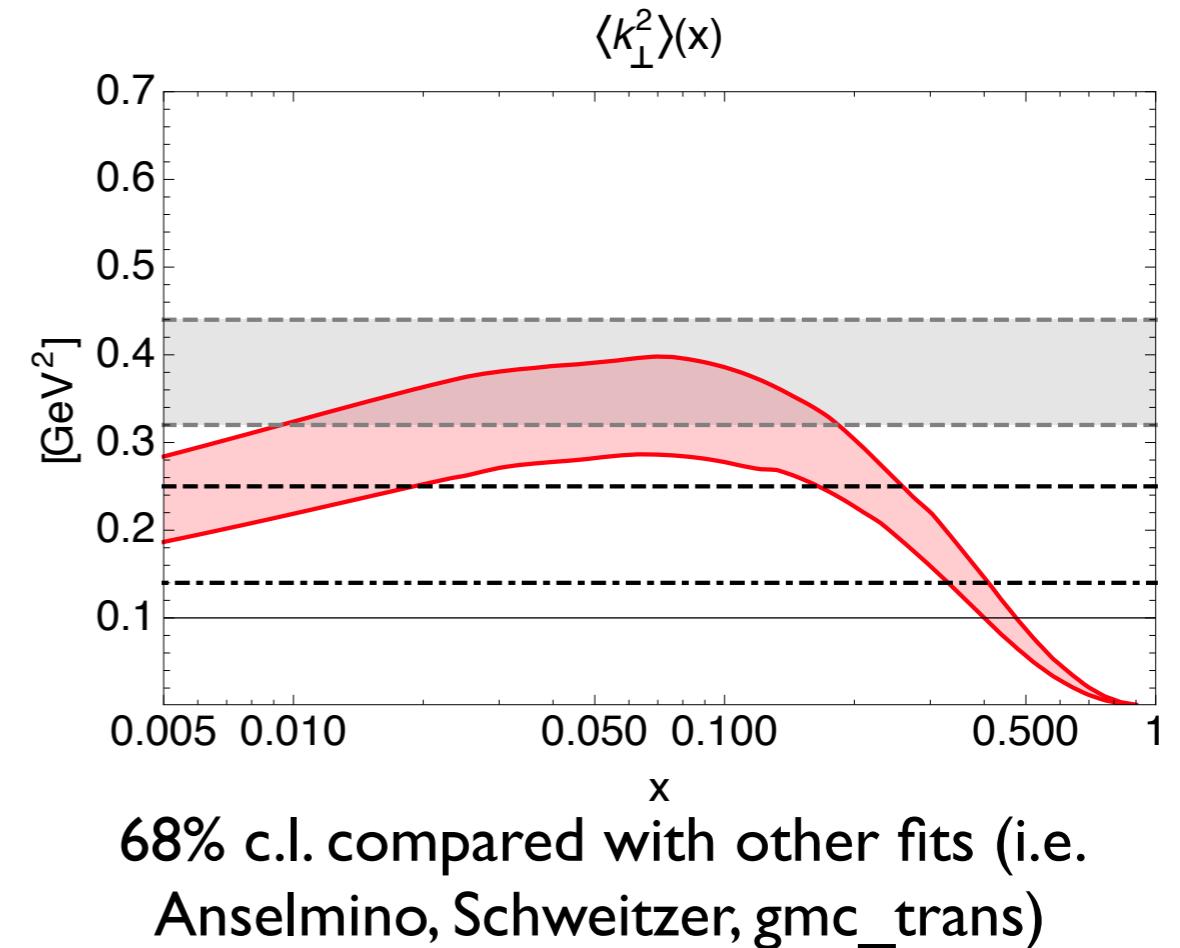
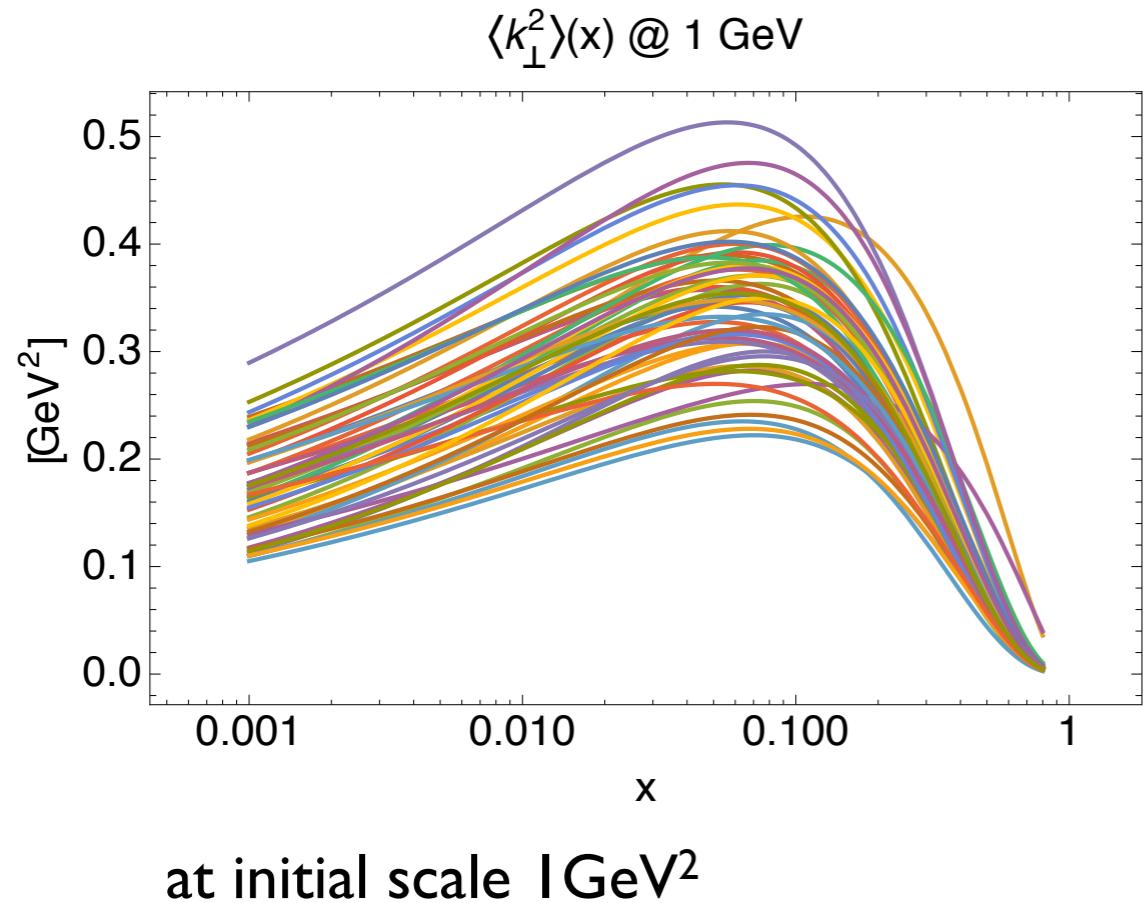


χ^2/dof greater w.r.t original data

minimization convergence more difficult

TMD PDF parameter

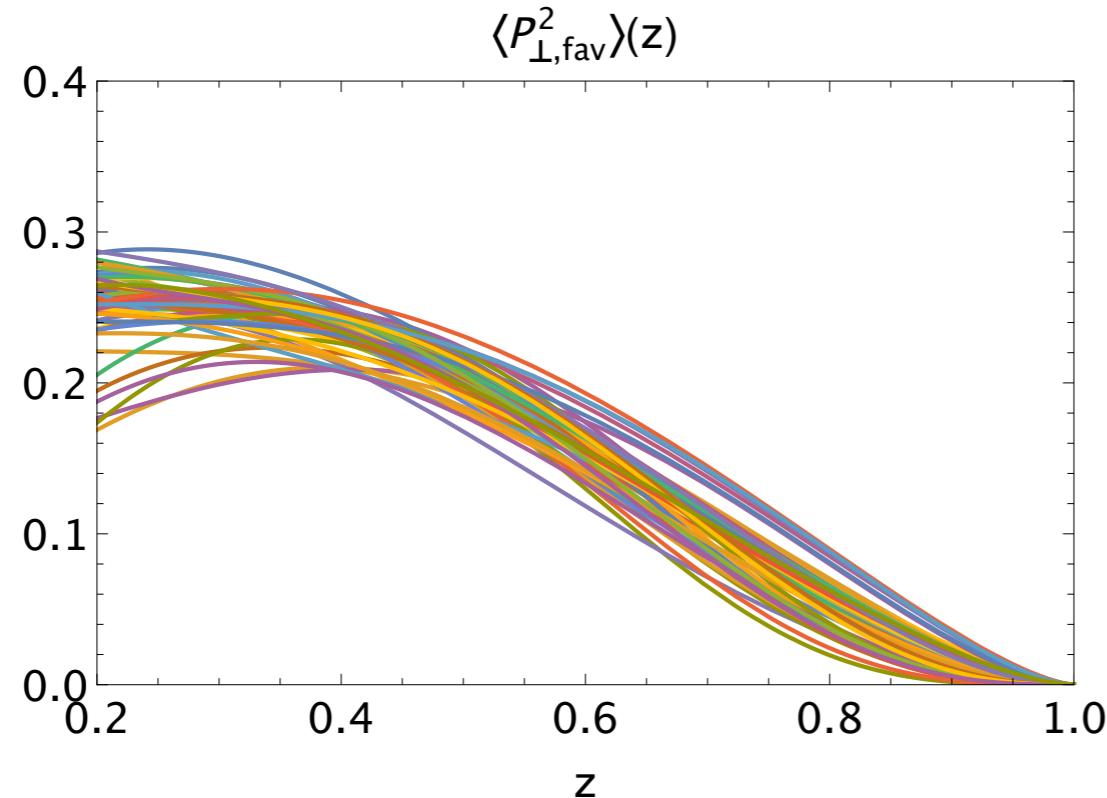
$$\langle k_{\perp,a}^2 \rangle(x) = \langle \hat{k}_{\perp,a}^2 \rangle \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma},$$



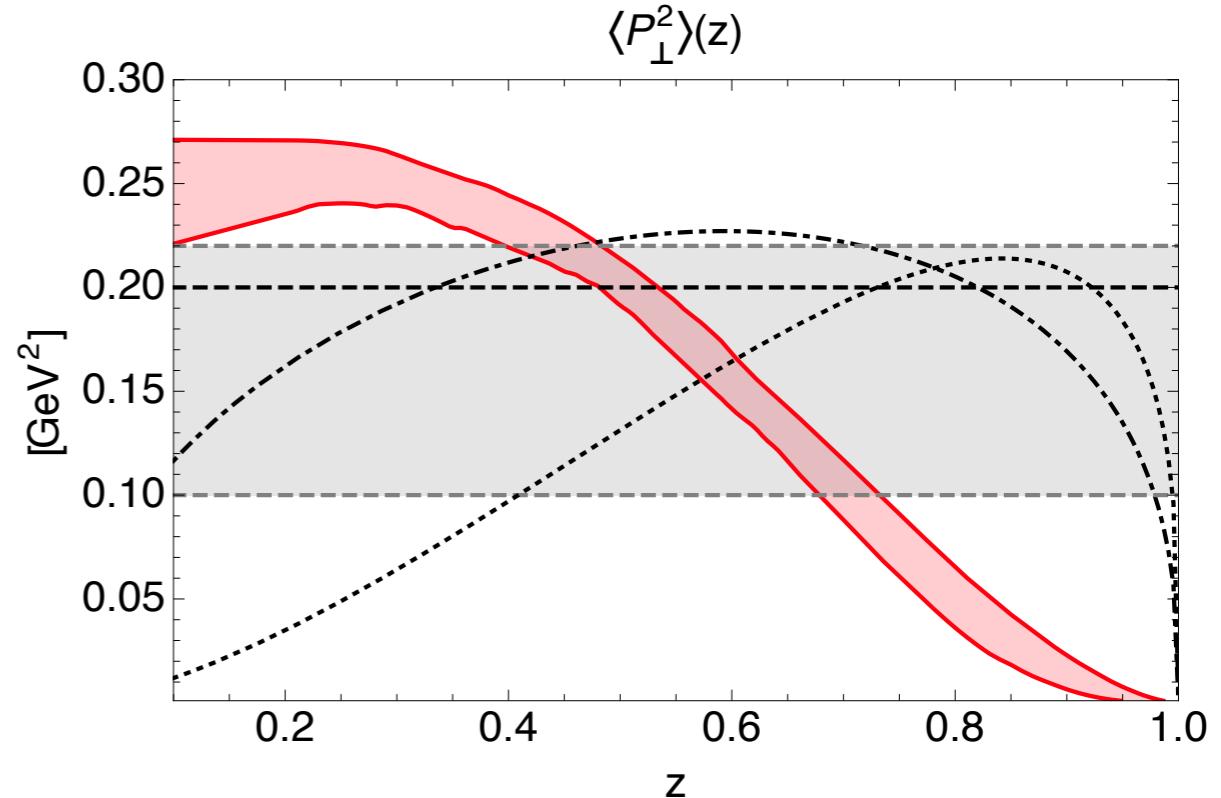
$\langle \hat{k}_{\perp,d}^2 \rangle$	$\alpha_{(\text{random})}$	σ	λ_F
0.28 ± 0.07 [GeV 2]	2.99 ± 0.01	0.20 ± 0.03	1.74 ± 1.73

TMD FF parameters

$$\langle \mathbf{P}_{\perp,a \rightarrow h}^2 \rangle(z) = \langle \hat{\mathbf{P}}_{\perp,a \rightarrow h}^2 \rangle \frac{(z^\beta + \delta) (1-z)^\gamma}{(\hat{z}^\beta + \delta) (1-\hat{z})^\gamma}$$



at initial scale 1 GeV^2



$\langle \hat{P}_{\perp,fav}^2 \rangle$	$\langle \hat{P}'_{\perp,fav}^2 \rangle$	β	γ	δ	λ_F
0.21 ± 0.01 [GeV^2]	0.03 ± 0.001 [GeV^2]	1.69 ± 0.81	2.32 ± 0.66	0.12 ± 0.09	4.31 ± 1.34

Preliminary: flavor dependence

New cuts are probably necessary:

$$Q^2 > 1.5 \text{ GeV}^2, \quad 0.25 < z < 0.6$$

Reduced number of bins

5533

improved χ^2/dof

~ 1.10

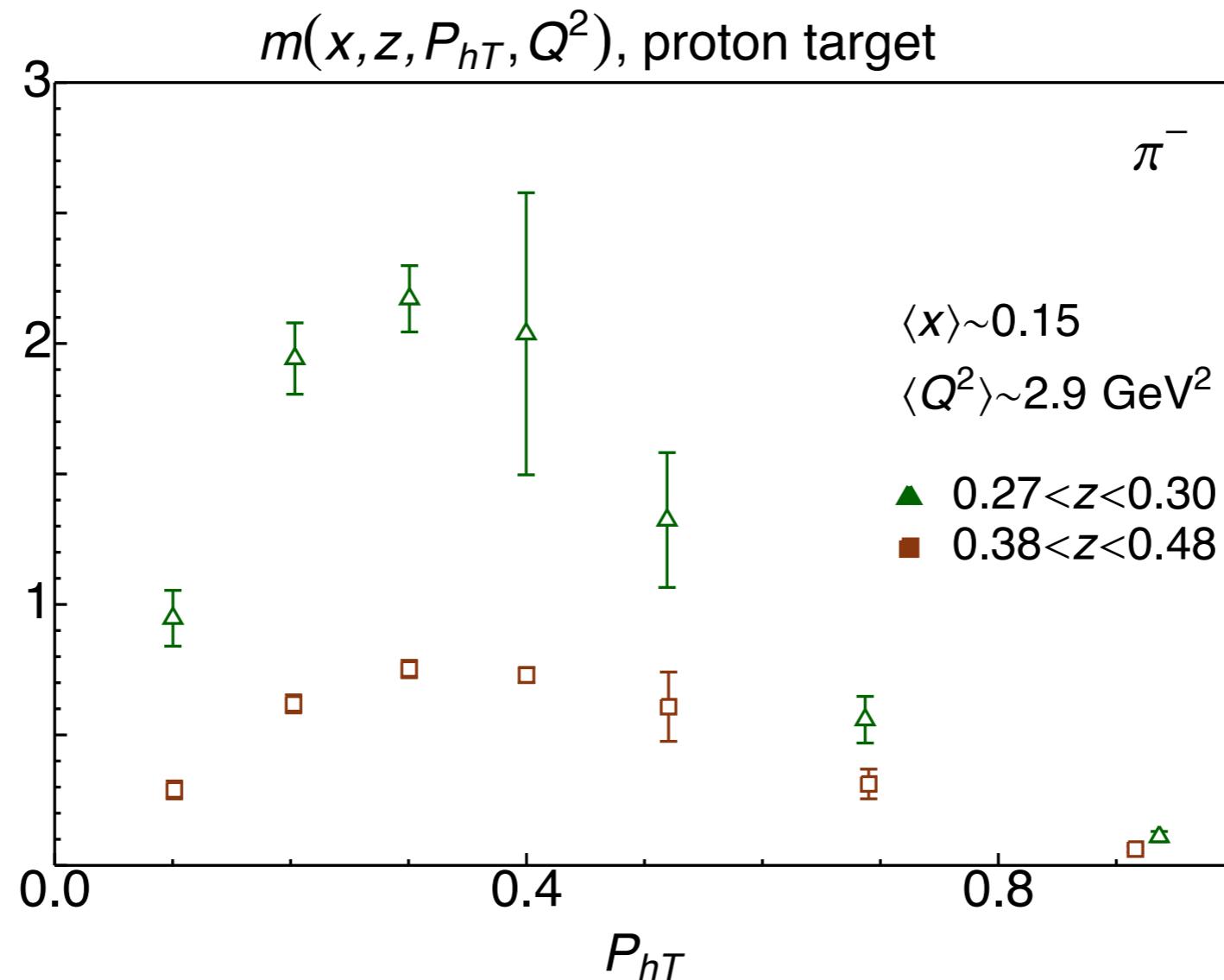
work in progress!

Conclusions

- ▶ We extracted TMD PDFs on more than 8000 data points on different data sets, multidimensional and at different energy scales
- ▶ We used the replica method to extract the parameter values
- ▶ This work is a first attempt toward a global fit of unpolarized TMDs, once the analysis of unpolarized structures will be complete, we can rely on a solid baseline to address polarized structures
- ▶ Working on improving the analysis of flavor dependence

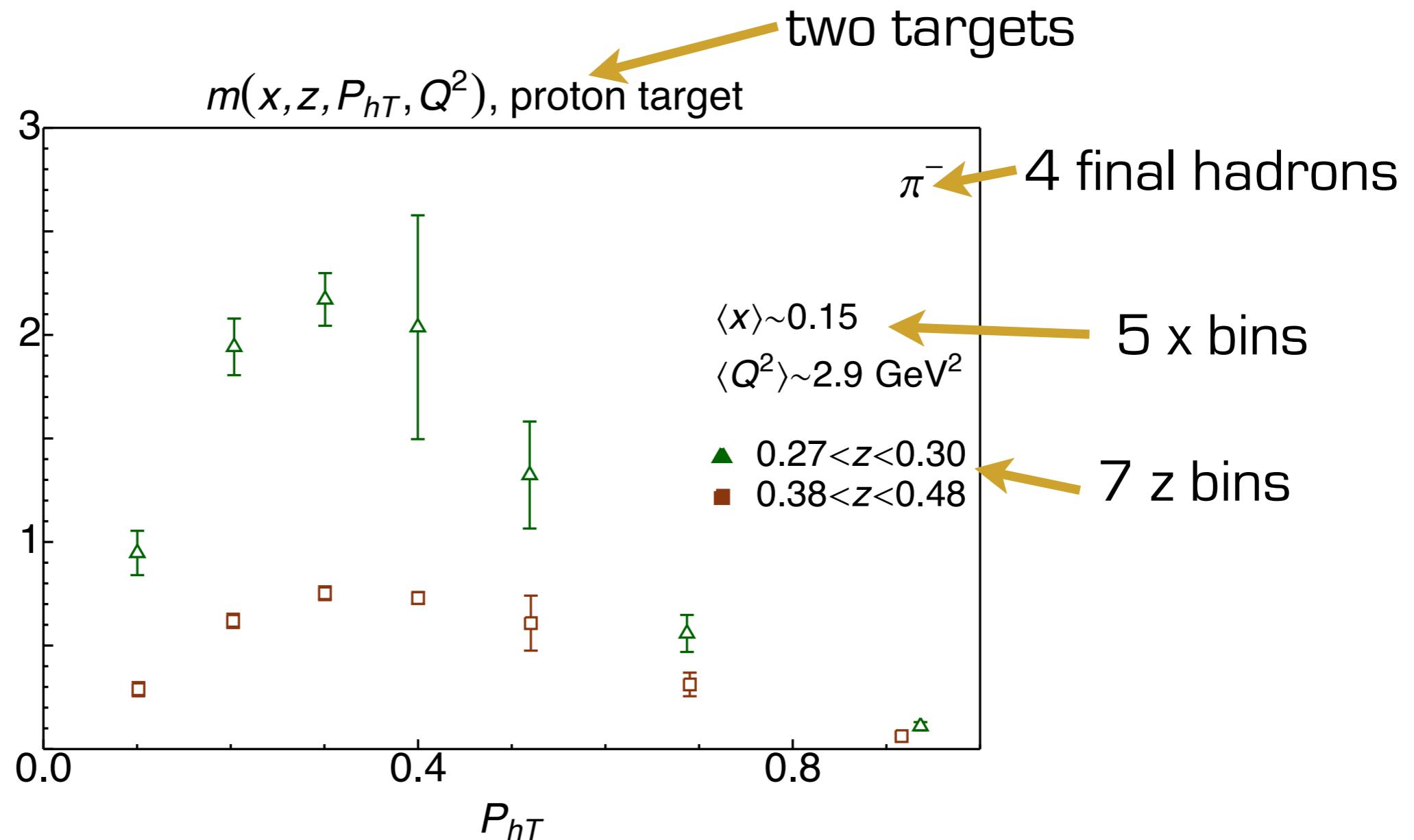
BACKUP

The replica method



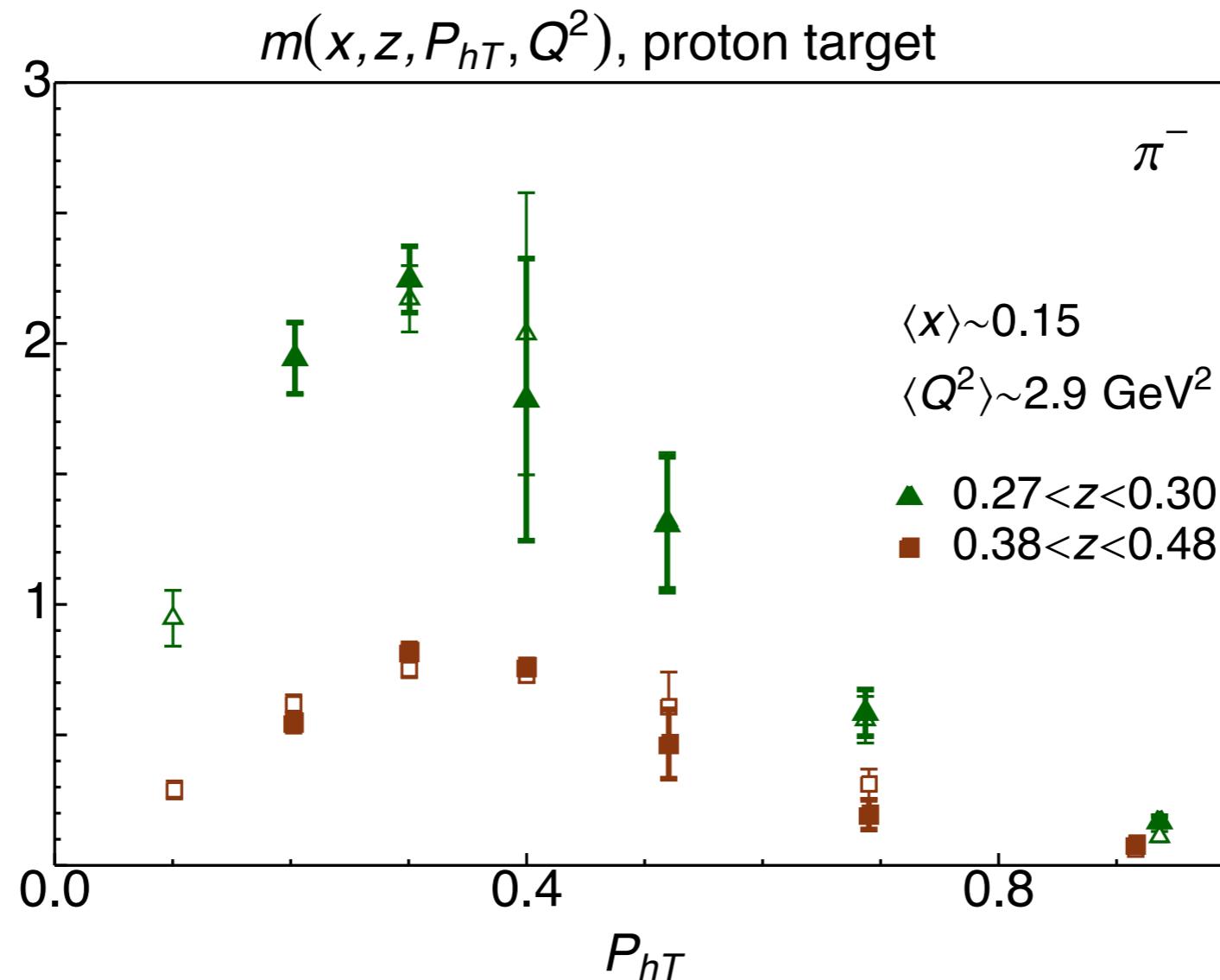
Example of original data

The replica method



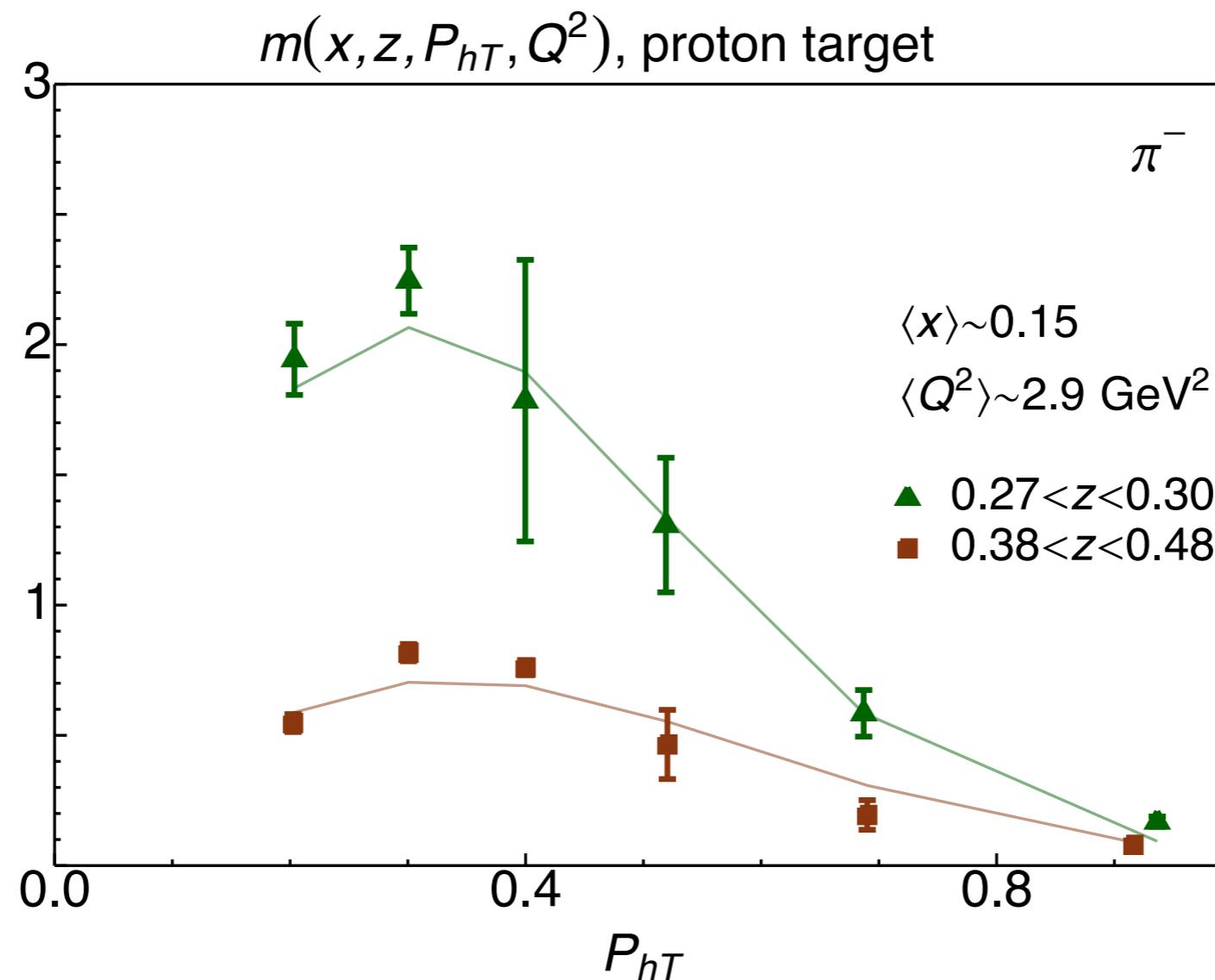
Example of original data

The replica method



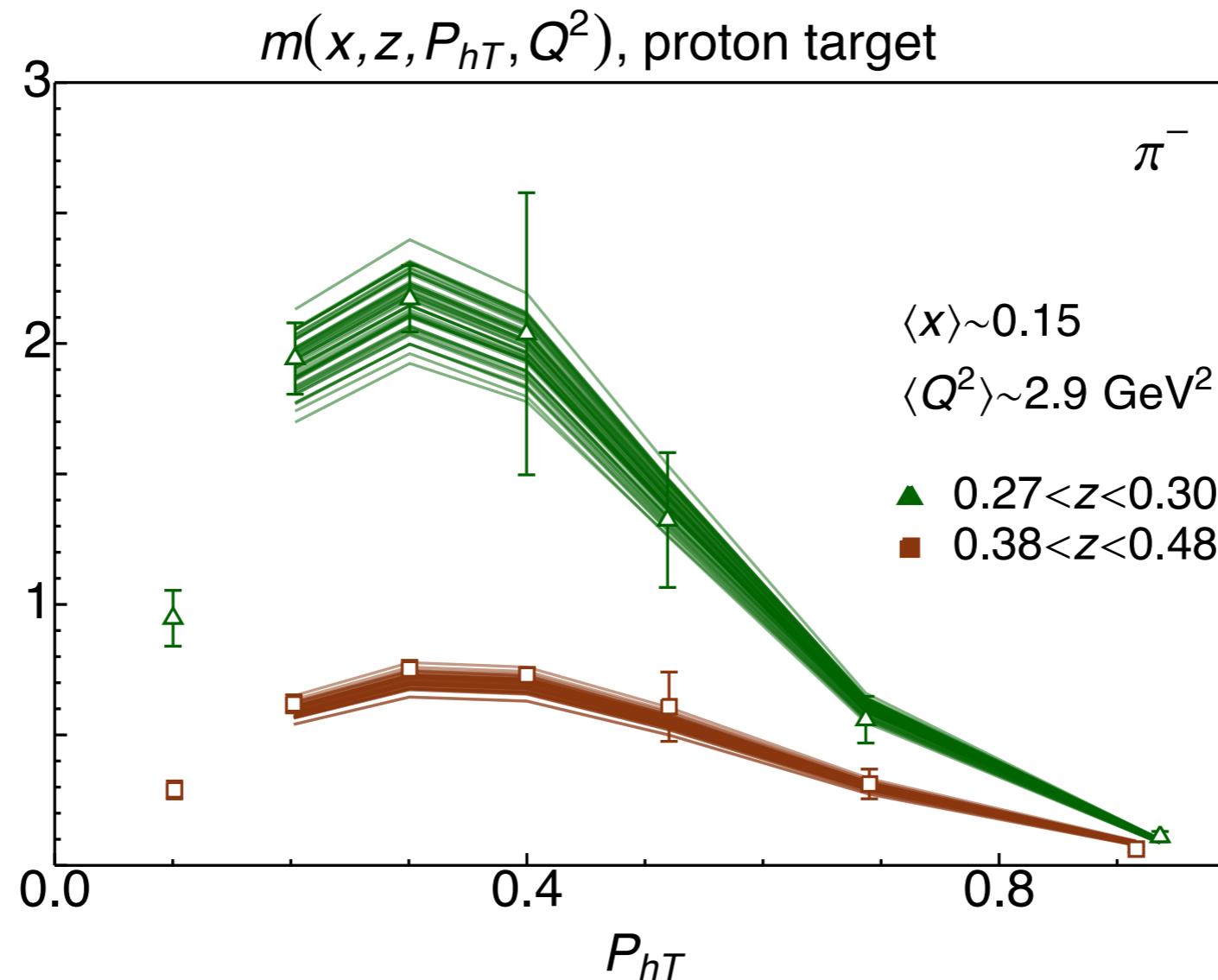
Data are replicated (with Gaussian distribution)

The replica method



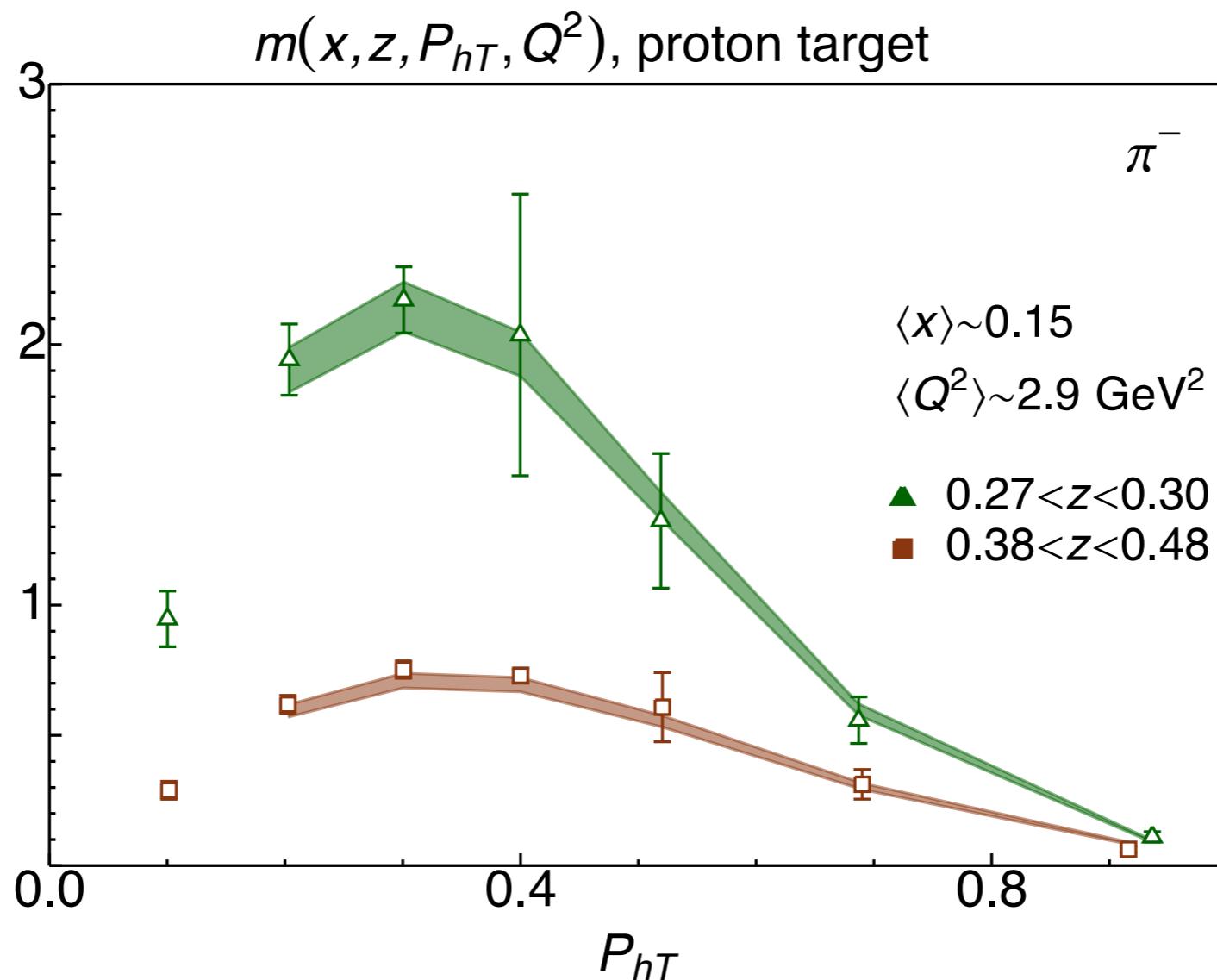
The fit is performed on the replicated data

The replica method



The procedure is repeated 200 times

The replica method



For each point, a central 68% confidence interval is identified

μ and b_* prescriptions

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

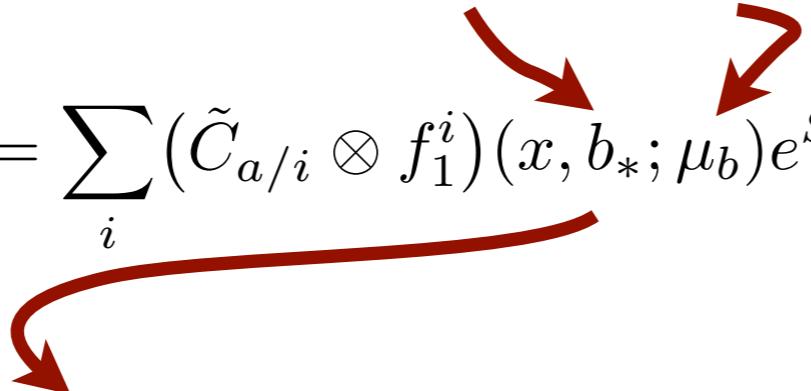
μ and b_* prescriptions

Choice Choice

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$


μ and b_* prescriptions

Choice Choice

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$


$$\mu_b = 2e^{-\gamma_E}/b_* \quad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \quad b_* \equiv b_{\max} \left(1 - e^{-\frac{b_T^4}{b_{\max}^4}} \right)^{1/4}$$

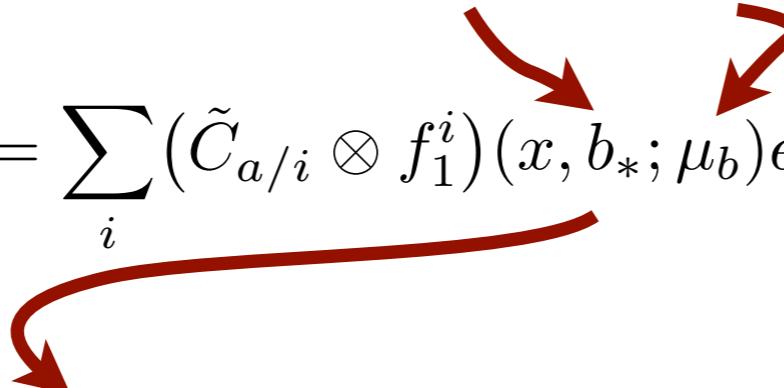
Bacchetta, Echevarria, Mulders, Radici, Signori
[arXiv:1508.00402](https://arxiv.org/abs/1508.00402)

$$\mu_b = Q_0 + q_T \quad b_* = b_T$$

DEMS 2014

μ and b_* prescriptions

Choice Choice

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$


$$\mu_b = 2e^{-\gamma_E}/b_* \quad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \quad b_* \equiv b_{\max} \left(1 - e^{-\frac{b_T^4}{b_{\max}^4}} \right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
[arXiv:1508.00402](https://arxiv.org/abs/1508.00402)

$$\mu_b = Q_0 + q_T$$

$$b_* = b_T$$

DEMS 2014

Complex-b prescription

Laenen, Sterman, Vogelsang, PRL 84 (00)

Nonperturbative ingredients 1

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

Nonperturbative ingredients 1

Choice



$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

Nonperturbative ingredients 1

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Choice



$$e^{-\frac{b_T^2}{\langle b_T^2 \rangle}}$$

almost everybody

$$e^{-\frac{b_T^2}{\langle b_T^2(x) \rangle_a}}$$

Pavia 2013, KN 2006

$$e^{-\lambda_1 b_T} (1 + \lambda_2 b_T^2)$$

DEMS 2014

Low- b_T modifications

$$\log(Q^2 b_T^2) \rightarrow \log(Q^2 b_T^2 + 1)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
[hep-ph/0302104](#)

see talks by Collins, Boglione, (Rogers?)

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$$b_*(b_c(b_T)) = \sqrt{\frac{b_T^2 + b_0^2/(C_5^2 Q^2)}{1 + b_T^2/b_{\max}^2 + b_0^2/(C_5^2 Q^2 b_{\max}^2)}}$$

$$b_{\min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q} \sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_{\max}^2)}}$$

Collins et al.
[arXiv:1605.00671](#)

see talks by Collins, Boglione, (Rogers?)

Data selection

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < 0.2 Q + 0.5 \text{ GeV}$$

$$P_{hT} < 0.8 \text{ GeV} \text{ (if } z < 0.3\text{)}$$

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Total number of data points: 8156

Total $\chi^2/\text{dof} = 1.45$

Preliminary

Pavia 2016 perturbative

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$	$A_2(\mathcal{O}(\alpha_S^2))$	$A_3(\mathcal{O}(\alpha_S^3))$	\dots
$B_1(\mathcal{O}(\alpha_S^1))$	$B_2(\mathcal{O}(\alpha_S^2))$	\dots	
$C_0(\mathcal{O}(\alpha_S^0))$	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	\dots
$H_0(\mathcal{O}(\alpha_S^0))$	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	\dots
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	\dots